e) $x(t) = \sin^2 t$

$$x(t) = \frac{1}{2} - \frac{1}{2}\cos 2t$$

sinjal periodik me periodë të çfarëdoshme.

$$x(t) = \sum_{n=-\infty}^{\infty} e^{jn2t}$$

$$x(t) = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{2}e^{j2t} + \frac{1}{2}e^{-j2t}\right)$$

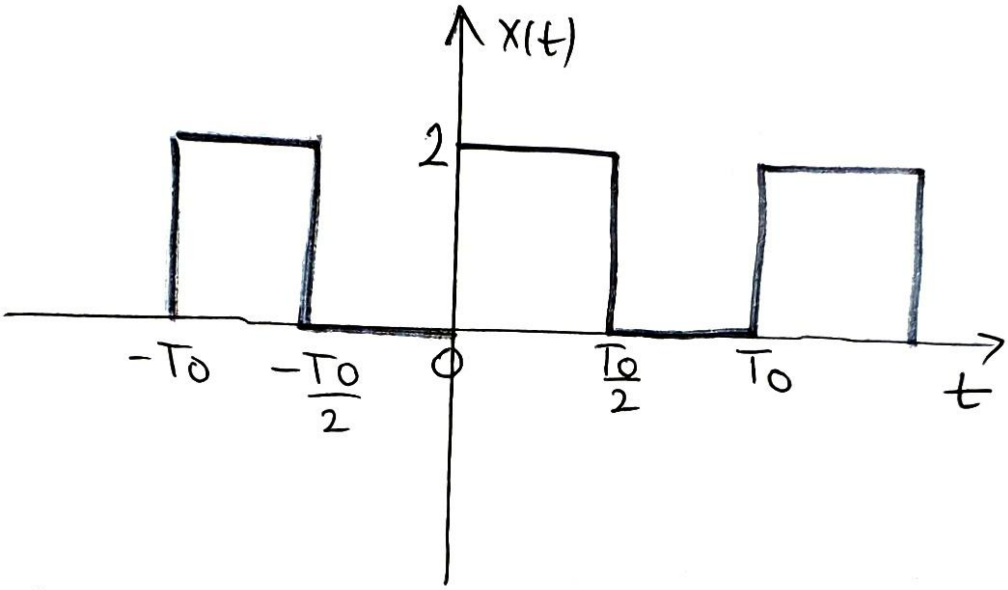
$$= \underbrace{\frac{1}{2}}_{n=0} - \underbrace{\frac{1}{4}e^{j2t}}_{n=1} - \underbrace{\frac{1}{4}e^{-j2t}}_{n=-1}$$

$C_0 = \frac{1}{2}$, $C_1 = -\frac{1}{4}$, $C_{-1} = -\frac{1}{4}$

5.5.

a

Të shkruhet forma eksponenciale për sinjalin $x(t)$ të dhënë në figurë.



Zgjidhje

Perioda e sinjalit është $T = T_0$; $\omega = \frac{2\pi}{T} = \frac{2\pi}{T_0}$

$$C_n = \frac{1}{T}\int_T x(t) e^{-jn\omega_0 t} dt$$

Në rastin tonë kemi:

$$C_n = \frac{1}{T_0}\int_0^{T_0/2} 2e^{-jn\omega_0 t} dt = \frac{2}{T_0}\int_0^{T_0/2} e^{-jn\omega_0 t} dt$$

$$= \frac{2}{T_0}\left(-\frac{1}{jn\omega_0} e^{-jn\omega_0 t}\Big|_0^{T_0/2}\right)$$

$$= \frac{2}{T_0}\left[-\frac{1}{jn\omega_0}\left(e^{-jn\omega_0\frac{T_0}{2}} - e^{0}\right)\right]$$

$$= \frac{2}{T_0}\left(\frac{1 - e^{-jn\frac{\omega_0 T_0}{2}}}{jn\omega_0}\right) \xrightarrow{\text{zëvendesojmë}} e^{-jn\pi}$$

zëvendesojm me $\frac{2\pi}{\omega}$

$$= \frac{2}{\frac{2\pi}{\omega_0}}\left(\frac{1 - e^{-jn\pi}}{jn\omega_0}\right) \xrightarrow{\text{zëvendesojm}} (-1)^n$$

$$= \frac{1}{j\pi n}\left(1 - (-1)^n\right)$$

$$\boxed{= \frac{(1-(-1)^n)}{j\pi n}}$$

Për

$n = 2k \rightarrow$ Çift.

$$\boxed{C_{2k} = 0}$$

Për

$n = 2k+1$

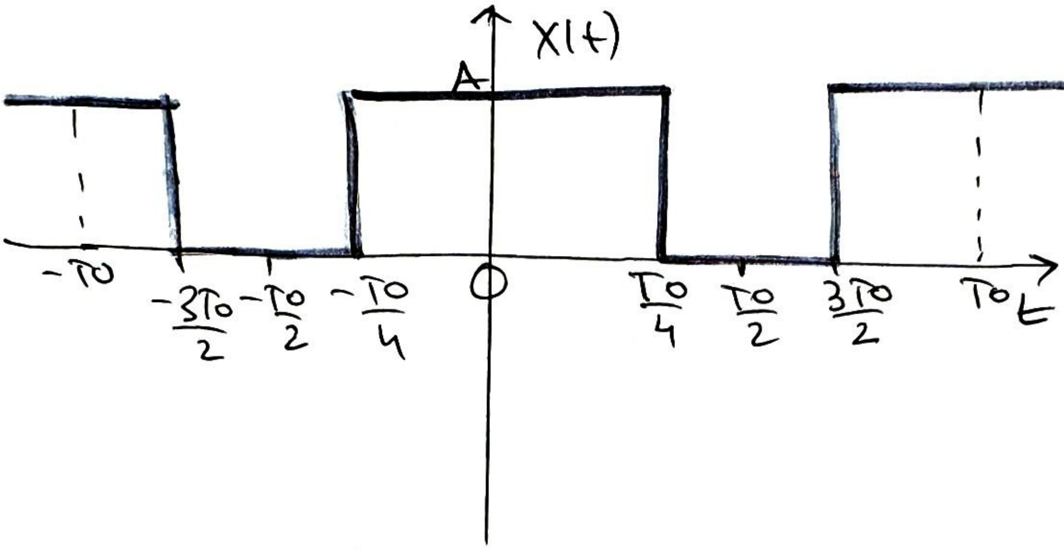
$$C_{2k+1} = \frac{2}{j\pi(2k+1)}$$

$$C_0 = \frac{1}{T_0}\int_0^{T_0/2} 2\,dt = \frac{2}{T_0}\cdot\frac{T_0}{2} = 1$$

$$\boxed{X(t) = 1 + \sum_{k=-\infty}^{\infty} \frac{2}{j\pi(2k+1)} e^{j(2k+1)\omega_0 t}}$$

5.6

Të shkruhet forma eksponenciale për sinjalin $x(t)$ të dhënë në figurë:



Zgjidhje

Perioda e sinjalit është $T = T_0$; $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{T_0}$

Forma eksp: $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$C_n = \frac{1}{T} \int_T x(t) e^{-jn\omega_0 t} dt$$

Në rastin tonë:

$$C_n = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} A e^{-jn\omega_0 t} dt = \frac{A}{T_0}\left(\frac{-1}{jn\omega_0} e^{-jn\omega_0 t} \Big|_{-\frac{T_0}{4}}^{\frac{T_0}{4}} \right)$$

$$= \frac{A}{\frac{2\pi}{\cancel{\omega_0}}} \left[-\frac{1}{jn\cancel{\omega_0}} \left(e^{-jn\omega_0 \frac{T_0}{4}} - e^{jn\omega_0 \frac{T_0}{4}} \right) \right]$$

$\frac{\pi}{2}$ $\frac{\pi}{2}$

$$= \frac{A}{2\pi j n}\left(e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}\right) = \frac{A}{\pi n}\left(\frac{e^{jn\frac{\pi}{2}} - e^{-jn\frac{\pi}{2}}}{2j}\right)$$

$$\boxed{= \frac{A}{\pi n} \sin n\frac{\pi}{2}}$$

Për $n = 2k \rightarrow$ çift

$C_{2k} = 0$

Për $n = 2k+1 \longrightarrow$ tek

$$C_{2k+1} = \frac{A(-1)^k}{\pi(2k+1)}$$

$$C_0 = \frac{1}{T_0}\int_{-\frac{T_0}{4}}^{T_0/4} A\,dt = \frac{A}{2}$$

$$\boxed{X(t) = \frac{A}{2} + \sum_{k=-\infty}^{\infty} \frac{A(-1)^k}{\pi(2k+1)} e^{j(2k+1)\omega_0 t}}$$

① Gjeni koeficientet e seris furie për singalet në vazhdim nëse $\omega_0 = 2\pi$

a) $x(t) = 1 + \sin(8\pi t + \frac{\pi}{3})$

$$= 1 + \frac{1}{2j} e^{j(8\pi t + \frac{\pi}{3})} - \frac{1}{2j} e^{-j(8\pi t + \frac{\pi}{3})}$$

$$= 1 + \frac{1}{2j} e^{j8\pi t} \cdot e^{j\frac{\pi}{3}} - \frac{1}{2j} e^{-j8\pi t} \cdot e^{-j\frac{\pi}{3}}$$

$n=0$ $\quad n=4$ $\quad n=-4$

$C_0 = 1, \quad C_4 = \frac{1}{2j} e^{j\frac{\pi}{3}}, \quad C_{-4} = -\frac{1}{2j} e^{-j\frac{\pi}{3}}$

b) $x(t) = [1 + \cos(2\pi t)][\sin(10\pi t + \frac{\pi}{6})]$

$$= \sin(10\pi + \frac{\pi}{6}) + \cos(2\pi t)\sin(10\pi t + \frac{\pi}{6})$$

$$= \left(\frac{e^{j\pi/6}}{2j} \cdot e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j} e^{-j2\pi t5} \right) + \left(\frac{1}{2} e^{j2\pi t} + \frac{1}{2} e^{-j2\pi t} \right)$$

$$\cdot \left(\frac{e^{j\pi/6}}{2j} e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j} e^{-j2\pi t5} \right)$$

$$= \frac{e^{j\pi/6}}{2j} e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j} e^{-2\pi t5} + \frac{e^{j\pi/6}}{4j} e^{j2\pi t6} - \frac{e^{-j\pi/6}}{4j} e^{-2\pi t4}$$

$$+ \frac{e^{j\pi/6}}{4j} e^{j2\pi t4} - \frac{e^{-j\pi/6}}{4j} e^{-2\pi t6}.$$

$$C_5 = \frac{e^{j\pi/6}}{2j} \; ; \quad C_{-5} = -\frac{e^{-j\pi/6}}{2j}$$

$$C_6 = \frac{e^{j\pi/6}}{4j} \; ; \quad C_{-4} = -\frac{e^{-j\pi/6}}{4j}$$

$$C_4 = \frac{e^{j\pi/6}}{4j} \; ; \quad C_{-6} = -\frac{e^{j\pi/6}}{4j}$$

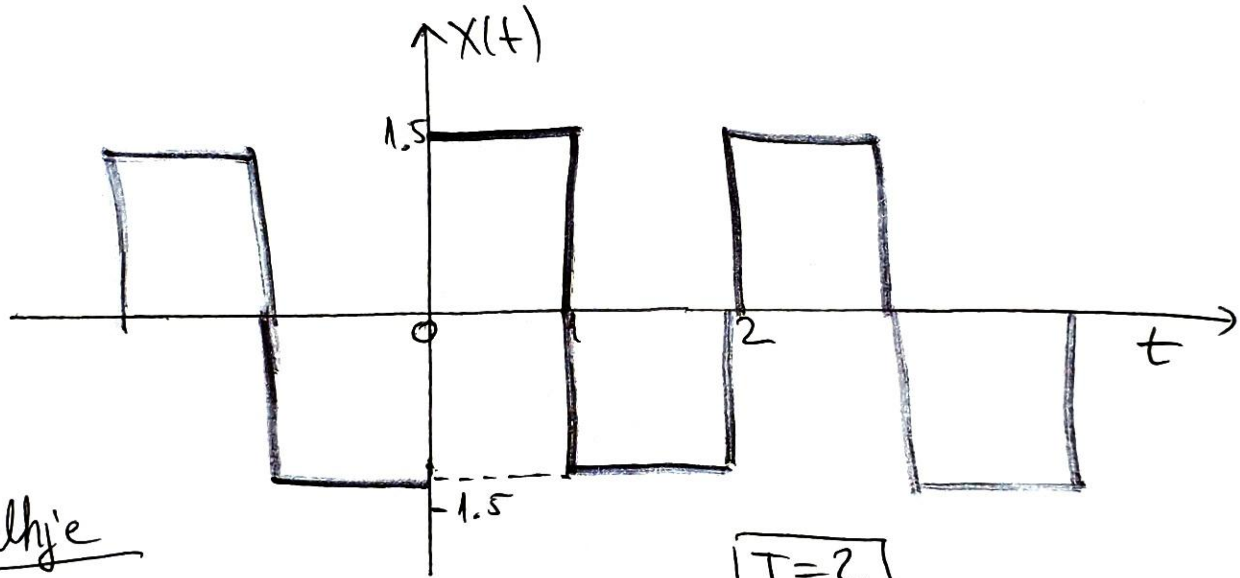
D.P

1. Për sinjalin periodik $x(t)$, vlerat e sinjalit brenda një periode janë dhënë si në vijim:

$$x(t) = \begin{cases} 1{,}5 & 0<t<1 \\ -1{,}5 & 1\le t<2 \end{cases}$$

Përcaktoni koefiçentët e Serisë eksponenciale komplekse furie C_n.

<u>Zgjidhje</u>



<u>Zgjidhje</u>

$\boxed{T=2}$

$$\omega_0 = \frac{2\pi}{T} = \frac{\cancel{2}\pi}{\cancel{2}} = \pi$$

$$C_n = \frac{1}{2}\left[\int_{-1}^{0}\left(-\frac{3}{2}\right)e^{-jn\omega_0 t}\,dt + \int_{0}^{1}\frac{3}{2}e^{-jn\omega_0 t}\,dt\right]$$

$$= -\frac{3}{4}\left[-\frac{1}{jn\omega_0}\left(1 - e^{jn\omega_0}\right)\right] + \frac{3}{4}\left[-\frac{1}{jn\omega_0}\left(e^{-jn\omega_0} - 1\right)\right]$$

$$= \frac{3}{4jn\omega_0}\left(1 - e^{jn\omega_0}\right) - \frac{3}{4jn\omega_0}\left(e^{-jn\omega_0} - 1\right)$$

$\boxed{\omega_0 = \pi}$

$$= \frac{3}{4jn\pi}\left(1 - e^{jn\pi}\right) - \frac{3}{4jn\pi}\left(e^{-jn\pi} - 1\right)$$

$$= \frac{3}{4jn\pi} - \frac{3}{4jn\pi}e^{jn\pi} - \frac{3}{4jn\pi}e^{-jn\pi} + \frac{3}{4jn\pi}$$

mbledhim

$$= \frac{3}{2jn\pi} - \frac{3}{2jn\pi}\left(\frac{e^{jn\pi} + e^{-jn\pi}}{2}\right) \rightarrow \cos n\pi \rightarrow (-1)^n$$

$$= \frac{3}{2jn\pi} - \frac{3}{2jn\pi}(-1)^n = \boxed{\frac{3}{2jn\pi}\left(1 - (-1)^n\right).}$$

$Re\{C_n\} = 0$

$$|C_n| = \frac{3}{2n\pi}\left(1 - (-1)^n\right]$$

Komplekse = Imagjinar