Krang Arm CoM

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1 Forward Kinematics

Check the diagram:

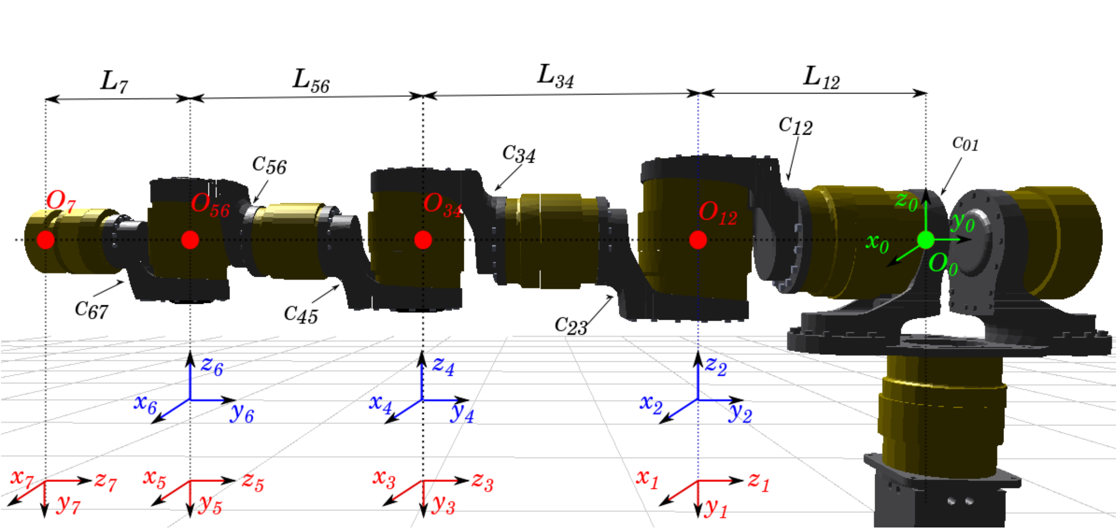

Figure 1: Local frames in Krang Arm

In the following table we have the parameters of the Denavit-Hartenberg matrices used for the arm (Proximal).
PS.- Unlike Chesster, I have defined the axis z for frame *1*, *3*, *5* and *7* in the "negative" orientation, so when the joint angles are entered, we don't have to change the sign. as we did with the previous model. As you may note, there is not difference (as it should be, given that this is just a convention). I just mention it in the case you wonder why I chose those directions (whereas in Chesster model all the z axis point towards the end effector)

1.1 Transformation Matrices

Using the D-H formula:

Link	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	$-\pi/2$	$-L_{12}$	θ_1
2	0	$\pi/2$	0	θ_2
3	0	$-\pi/2$	$-L_{34}$	θ_3
4	0	$\pi/2$	0	θ_4
5	0	$-\pi/2$	$-L_{56}$	θ_5
6	0	$\pi/2$	0	θ_6
7	0	$-\pi/2$	$-L_7$	θ_7

Table 1: *Denavit-Hartenberg Parameters*

$$
{}^{i-1}_{i}T = \begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

You can calculate the Transformation matrices for each local frame, and then, combining them:

$$
{}^{0}_{1}T = \begin{pmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ 0 & 0 & 1 & -L_{12} \\ -s\theta_1 & -c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{1}_{2}T = \begin{pmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{2}_{3}T = \begin{pmatrix} c\theta_3 & -s\theta_3 & 0 & 0 \\ 0 & 0 & 1 & -L_{34} \\ -s\theta_3 & -c\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{3}_{4}T = \begin{pmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_4 & c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{4}_{5}T = \begin{pmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & 1 & -L_{56} \\ -s\theta_5 & -c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{5}_{6}T = \begin{pmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_6 & c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^{6}_{7}T = \begin{pmatrix} c\theta_7 & -s\theta_7 & 0 & 0 \\ 0 & 0 & 1 & -L_7 \\ -s\theta_7 & -c\theta_7 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

As you may know, to get the final transformation matrices relative to frame, we have just to do the following:

$$^0_1T =^0_1 T$$
$$^0_2T =^0_1 T\cdot^1_2T$$
$$^0_3T =^0_2 T\cdot^2_3T$$
$$^0_4T =^0_3 T\cdot^3_4T$$
$$^0_5T =^0_4 T\cdot^4_5T$$
$$^0_6T =^0_5 T\cdot^5_6T$$
$$^0_7T =^0_6 T\cdot^6_7T$$

1.2 Physical and Geometric Data of the System

We use the information of the technical data for the LWA3 motors, as well as the Solidworks models for the brackets (available in the Wiki, if not, check the figures).
ACHTUNG! Some quantities are approximate, check carefully

1.2.1 Frame Data

To establish the local frames used in the calculus, refer to the figures below:

Designation	Main Lengths (m)
L_{12}	0.250
L_{34}	0.328
L_{56}	0.2765
L_7	0.171

Table 2: *Main distances for relative location of the local frames*

1.2.2 Motor Data

Using the information for the PRL modules, we have the following:

Designation	Motor Weight (kg)
M_{Motor1}	3.6
M_{Motor2}	3.6
M_{Motor3}	2.0
M_{Motor4}	2.0
M_{Motor5}	1.2
M_{Motor6}	1.2
M_{Motor7}	1.0

Table 3: *Motor Weight Parameters*

CM	x(m)	y(m)	z(m)	Frame
${}^{1}CM_{Motor1}$	0	0	(0.110 + 0.0821 - 0.008)	Frame 1
${}^{2}CM_{Motor2}$	0	0	0	Frame 2
${}^{3}CM_{Motor3}$	0	0	(0.100 + 0.059 - 0.008)	Frame 3
${}^{4}CM_{Motor4}$	0	0	0	Frame 4
${}^{5}CM_{Motor5}$	0	0	(0.080 + 0.0535 - 0.008)	Frame 5
${}^{6}CM_{Motor6}$	0	0	0	Frame 6
${}^{7}CM_{Motor7}$	0	0	(0.050 - 0.0068)	Frame 7

Table 4: *CM of the Motors relative to their local frames*

1.2.3 Connectors Data

For the calculus of the Center of Mass, we also consider the effect of the metallic connectors between the motors (which together add around 3.312 Kg to the arm weight)

Designation	Connector Weight (kg)
$M_{Connector01}$	0.635
$M_{Connector12}$	0.635
$M_{Connector23}$	0.576
$M_{Connector34}$	0.471
$M_{Connector45}$	0.420
$M_{Connector56}$	0.302
$M_{Connector67}$	0.273

Table 5: *Connectors Weight Parameters*

CM	x(m)	y(m)	z(m)	Frame
${}^{0}CM_{Connector01}$	0	$-(0.09375 - 0.05022)$	$-(0.110 - 0.05272)$	Frame 0
${}^{1}CM_{Connector12}$	0	-0.05022	$(0.110 - 0.05272)$	Frame 1
${}^{2}CM_{Connector23}$	0	$-(0.110 - 0.05655)$	-0.05313	Frame 2
${}^{3}CM_{Connector34}$	0	-0.04463	$(0.100 - 0.04963)$	Frame 3
${}^{4}CM_{Connector45}$	0	$-(0.100 - 0.05329)$	-0.04740	Frame 4
${}^{5}CM_{Connector56}$	0	-0.03815	$(0.080 - 0.03957)$	Frame 5
${}^{6}CM_{Connector67}$	0	$-(0.080 - 0.04152)$	-0.03913	Frame 6

Table 6: *CM of the Connectors relative to their local frames*

2 Calculus of the Center of Mass of the whole arm

We use the information below to calculate the center of mass of the arm.

2.1 Equations

To apply the formula:

$$CM = \frac{\sum CM_i.M_i}{\sum M_i}$$

We need the center of mass of each component relative to frame 0. So, what should be done is:

For the Motors: Transform its local coordinates to zero frame, that is:

$$^0CM_{Motor1} =^0_1 T.^1CM_{Motor1}$$
$$^0CM_{Motor2} =^0_2 T.^2CM_{Motor2}$$
$$^0CM_{Motor3} =^0_3 T.^3CM_{Motor3}$$
$$^0CM_{Motor4} =^0_4 T.^4CM_{Motor4}$$
$$^0CM_{Motor5} =^0_5 T.^5CM_{Motor5}$$
$$^0CM_{Motor6} =^0_6 T.^6CM_{Motor6}$$
$$^0CM_{Motor7} =^0_7 T.^7CM_{Motor7}$$

For the Connectors (or Brackets): The same than for the motors:

$$^0CM_{Connector01} =^0 CM_{Connector01}$$
$$^0CM_{Connector12} =^0_1 T.^1CM_{Connector12}$$
$$^0CM_{Connector23} =^0_2 T.^2CM_{Connector23}$$
$$^0CM_{Connector34} =^0_3 T.^3CM_{Connector34}$$
$$^0CM_{Connector45} =^0_4 T.^4CM_{Connector45}$$
$$^0CM_{Connector56} =^0_5 T.^5CM_{Connector56}$$
$$^0CM_{Connector67} =^0_6 T.^6CM_{Connector67}$$

With the coordinates of each center of mass relative to the zero frame (or global frame, what the heck), we find the center of mass for Krang's right arm... (left should be the same)

For the motors:

$\sum CM_M.M_M = CM_{M1}M_{M1}+CM_{M2}M_{M2}+CM_{M3}M_{M3}+CM_{M4}M_{M4}+CM_{M5}M_{M5}+ CM_{M6}M_{M6} + CM_{M7}M_{M7}$

$\sum M_M = M_{M1} + M_{M2} + M_{M3} + M_{M4} + M_{M5} + M_{M6} + M_{M7}$

Where:

- CM_Mi : Center of mass of the ith motor, relative to zero frame
- M_Mi : Mass of the ith motor

For the Connectors:

$\sum CM_C.M_C = CM_{C01}M_{C01} + CM_{C12}M_{C12} + CM_{C23}M_{C23} + CM_{C34}M_{C34} + CM_{C45}M_{C45} + CM_{C56}M_{C56} + CM_{C67}M_{C67}$

$\sum M_C = M_{C01} + M_{C12} + M_{C23} + M_{C34} + M_{C45} + M_{C56} + M_{C67}$

Where:

- CM_Cij : Center of mass of the connector between motors i and j, relative to zero frame
- M_Cij : Mass of the ith connector between motors i and j

So (finally!) we calculate the center of mass of this arm!

$$CM_C = \frac{\sum CM_M.M_M + \sum CM_C.M_C}{\sum M_M + \sum M_C}$$

If you want to see the equations, check in SVN the file KrangKinematics.mac

3 Annex

Well, this is what I draw, in case the Peekabot diagram is not clear (but I think it is)

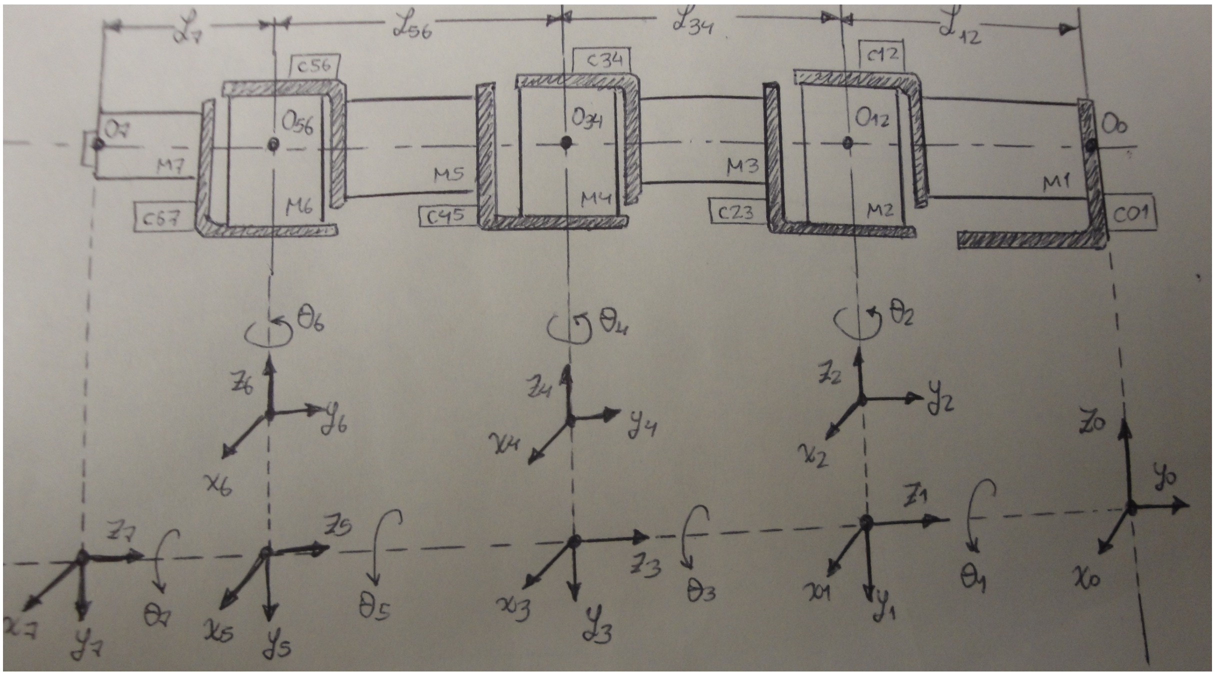

Figure 2: Local frames in Krang Arm

4 Notes

- As it was mentioned at the beginning of this pdf, there are some approximations in the calculations, for example, I am assuming that all the frame origins are aligned in an imaginary axis that goes from O_0 until the origin of the end effector frame. However that does not hold true, given that for the motors 2,4 and 6 there are deviations that we are going to check as soon as we have the Solidworks models handy.

- Another approximation is related with the center of mass of each motor. We are not considering the small deviations of the center of mass of each motor in their local frames in axis y and z. This deviations are small (the biggest 5.8 mm for the module PRL 120 and the smallest 0.2 mm for the module PRL 60, both in their y axis). Note that to take them into exact account, we should set the zero position of each module to the correspondent initial position of this offsets, but so far, we have not found a way to do this (how can we know what is the ”zero position” of the motor? at least for the PRL 120 and PRL 100 it is hard to say).

- This model has been tested with the arm alone, and -together with the controller- has given good results (balancing). Testing it with an additional weight in the end effector (2.5 kg) has given unsatisfactory results. This can be due to the mentioned approximations in the calculations, but another important source of error can be the bending of the arm when carrying the mentioned weight in the End Effector (Kasemsit mentioned that it can be at least ten degrees). Also, the zero position is not exactly aligned.

- I have checked this model and works properly! The only improvement I can do is to try to use the most exact dimensions I can find (so again I need the Solidworks model and somebody who lend me a laptop or his/her PC to use SW. If you find any mathematical error in this, please do let me know (I seriously doubt it :)