

# Pattern Recognition Assignment#1

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March 28, 2023

## Question1

The likelihood function of Gaussain variables( $N(\mu, \sigma^2)$ )

$$L(\mu, \sigma^2) = \prod_{i=1}^n \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{(x_i - \mu)^2}{\sigma^2}\right)$$

The corresponding log-likelihood function

$$\begin{aligned} \ell(\mu, \sigma^2) &= \ln L(\mu, \sigma^2) = \sum_{i=1}^n \ln \left[ \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{(x_i - \mu)^2}{\sigma^2}\right) \right] \\ &= \sum_{i=1}^n \left[ -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln \sigma^2 - \frac{1}{2} \frac{(x_i - \mu)^2}{\sigma^2} \right] \end{aligned}$$

Maximum Estimate of parameters

$$\frac{\partial \ell}{\partial \mu} = \sum_{i=1}^n \frac{x_i - \mu}{\sigma^2} = 0$$

$$\frac{\partial \ell}{\partial \sigma} = \sum_{i=1}^n -\frac{1}{\sigma} + \frac{(x_i - \mu)^2}{\sigma^3} = 0$$

$$\hat{\mu} = \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

While the expectation of estimated variance  $\hat{\sigma}^2$

$$\mathcal{E} \hat{\sigma}^2 = \frac{1}{n} \mathcal{E} \sum_{i=1}^n (x_i - \bar{x})^2 = \frac{n-1}{n} \sigma^2 \neq \sigma^2$$

Therefore, the maximum likelihood estimator of the variance of a Gaussian variable is biased.

## Question2

Use the forward algorithm of HMM

$$\alpha_1 = \pi \mathbf{A} \odot (B_{12}, B_{22}, B_{32}) = (0.095, 0.148, 0.044)$$

$$\alpha_2 = \alpha_1 \mathbf{A} \odot (B_{12}, B_{22}, B_{32}) = (0.02605, 0.04008, 0.01347)$$

$$\alpha_3 = \alpha_2 \mathbf{A} \odot (B_{11}, B_{21}, B_{31}) = (0.0058068, 0.0055954, 0.0111318)$$

$$\alpha_4 = \alpha_3 \mathbf{A} \odot (B_{13}, B_{23}, B_{33}) = (0.00045279, 0.00358617, 0.00542438)$$

Therefore, the possibility that the specific activity sequence  $O$  is observed is 0.009463346

## Question3

(a) Use the Bayesian formula

$$p(\text{wrong}) = p(\text{wrong} \mid \omega_1)p(\omega_1) + p(\text{wrong} \mid \omega_2)p(\omega_2) = 0.095$$

$$p(\omega_1 \mid \text{wrong}) = \frac{p(\text{wrong} \mid \omega_1)p(\omega_1)}{p(\text{wrong})} = 0.6316$$

$$p(\omega_2 \mid \text{wrong}) = \frac{p(\text{wrong} \mid \omega_2)p(\omega_2)}{p(\text{wrong})} = 0.3684$$

$$p(\omega_1 \mid \text{wrong}) > p(\omega_2 \mid \text{wrong})$$

Therefore, the book tends to be purchased by online shopping

(b) Calculate the risk of taking each action

$$R(\alpha_1 \mid \text{wrong}) = \lambda_{11}p(\omega_1 \mid \text{wrong}) + \lambda_{12}p(\omega_2 \mid \text{wrong}) = 2.4716$$

$$R(\alpha_2 \mid \text{wrong}) = \lambda_{21}p(\omega_1 \mid \text{wrong}) + \lambda_{22}p(\omega_2 \mid \text{wrong}) = 2.2632$$

$$R(\alpha_1 \mid \text{wrong}) > R(\alpha_2 \mid \text{wrong})$$

Therefore, the book tends to be purchased by physical stores

## Question4

(a) ii and iii are asserted by the network while i isn't, prove:

$$P(B, I, L) = P(B)P(L)P(I | B, L) \neq P(B)P(I)P(L)$$

$$\begin{aligned} P(J | G, I) &= \frac{P(J, G, I)}{P(G, I)} \\ &= \frac{\sum_{B, L} P(B)P(L)P(I | B, L)P(G | B, L, I)P(J | G)}{\sum_{B, L} P(B)P(L)P(I | B, L)P(G | B, L, I)} \\ &= P(J | G) \end{aligned}$$

$$\begin{aligned} P(L | G, B, I) &= \frac{P(L, G, B, I)}{P(G, B, I)} \\ &= \frac{P(B)P(L)P(I | B, L)P(G | B, L, I)}{\sum_L P(B)P(L)P(I | B, L)P(G | B, L, I)} \\ &= \frac{P(B)P(L)P(I | B, L)P(G | B, L, I)P(J | G)}{\sum_L P(B)P(L)P(I | B, L)P(G | B, L, I)P(J | G)} \\ &= \frac{P(L, G, B, I, J)}{P(G, B, I, J)} \\ &= P(L | G, B, I, J) \end{aligned}$$

(b)

$$P(B, I, \neg L, G, J) = P(B)P(\neg L)P(I | B, \neg L)P(G | B, \neg L, I)P(J | G) = 0.2916$$

(c) The original proposition could be expressed as

$$\begin{aligned} P(B | L, I, J) &= \frac{P(B, L, I, J)}{P(L, I, J)} \\ &= \frac{\sum_G P(B)P(L)P(I | B, L)P(G | B, L, I)P(J | G)}{P(B)P(L)P(I | B, L)} \\ &= 0.81 \end{aligned}$$

(d) The new Bayesian belief network is shown as the following Figure

