Pattern Recognition Assignment#1

61821313 Zihao Wang

March 27, 2023

Question1

The likelihood function of Gaussain variables($N(\mu,\ \sigma^2)$)

$$L(\mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2})$$

The corresponding log-likelihood function

$$\ell(\mu, \sigma^2) = \ln L(\mu, \sigma^2) = \sum_{i=1}^{n} \ln\left[\frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2})\right]$$

$$= \sum_{i=1}^{n}\left[-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\sigma^2 - \frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}\right]$$

Maximum Estimate of parameters

$$\frac{\partial \ell}{\partial \mu} = \sum_{i=1}^{n} \frac{x_i-\mu}{\sigma^2} = 0$$

$$\frac{\partial \ell}{\partial \sigma} = \sum_{i=1}^{n} -\frac{1}{\sigma} + \frac{(x_i-\mu)^2}{\sigma^3} = 0$$

$$\hat{\mu} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\hat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

While the expectation of estimated variance $\hat{\sigma}^2$

$$\mathcal{E}\hat{\sigma}^2 = \frac{1}{n}\mathcal{E}\sum_{i=1}^{n}(x_i-\bar{x})^2 = \frac{n-1}{n}\sigma^2 \neq \sigma^2$$

Therefore, the maximum likelihood estimator of the variance of a Gaussian variable is biased.

Question2

Use the forward algorithm of HMM

$$\alpha_1 = \pi\mathbf{A} \odot (B_{12},\ B_{22},\ B_{32}) = (0.095,\ 0.148,\ 0.044)$$

$$\alpha_2 = \alpha_1\mathbf{A} \odot (B_{12},\ B_{22},\ B_{32}) = (0.02605,\ 0.04008,\ 0.01347)$$

$$\alpha_3 = \alpha_2\mathbf{A} \odot (B_{11},\ B_{21},\ B_{31}) = (0.0058068,\ 0.0055954,\ 0.0111318)$$

$$\alpha_4 = \alpha_3\mathbf{A} \odot (B_{13},\ B_{23},\ B_{33}) = (0.00045279,\ 0.00358617,\ 0.00542438)$$

Therefore, the possibility that the specific activity sequence O is observed is 0.009463346

Question3

(a) Use the Bayesian formula

$$p(\text{wrong}) = p(\text{wrong} \mid \omega_1)p(\omega_1) + p(\text{wrong} \mid \omega_2)p(\omega_2) = 0.095$$

$$p(\omega_1 \mid \text{wrong}) = \frac{p(\text{wrong} \mid \omega_1)p(\omega_1)}{p(\text{wrong})} = 0.6316$$

$$p(\omega_2 \mid \text{wrong}) = \frac{p(\text{wrong} \mid \omega_2)p(\omega_2)}{p(\text{wrong})} = 0.3684$$

$$p(\omega_1 \mid \text{wrong}) > p(\omega_2 \mid \text{wrong})$$

Therefore, the book tends to be purchased by online shopping

(b)Calculate the risk of taking each action

$$R(\alpha_1 \mid \text{wrong}) = \lambda_{11}p(\omega_1 \mid \text{wrong}) + \lambda_{12}p(\omega_2 \mid \text{wrong}) = 2.4716$$

$$R(\alpha_2 \mid \text{wrong}) = \lambda_{21}p(\omega_1 \mid \text{wrong}) + \lambda_{22}p(\omega_2 \mid \text{wrong}) = 2.2632$$

$$R(\alpha_1 \mid \text{wrong}) > R(\alpha_2 \mid \text{wrong})$$

Therefore, the book tends to be purchased by physical stores

Question4

(a) ii and iii are asserted by the network while i isn't, prove:

$$P(B, I, L) = P(B)P(L)P(I \mid B, L) \neq P(B)P(I)P(L)$$

$$\begin{aligned}
P(J \mid G, I) &= \frac{P(J, G, I)}{P(G, I)} \\
&= \frac{\sum_{B,L} P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)P(J \mid G)}{\sum_{B,L} P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)} \\
&= P(J \mid G)
\end{aligned}$$

$$\begin{aligned}
P(L \mid G, B, I) &= \frac{P(L, G, B, I)}{P(G, B, I)} \\
&= \frac{P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)}{\sum_L P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)} \\
&= \frac{P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)P(J \mid G)}{\sum_L P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)P(J \mid G)} \\
&= \frac{P(L, G, B, I, J)}{P(G, B, I, J)} \\
&= P(L \mid G, B, I, J)
\end{aligned}$$

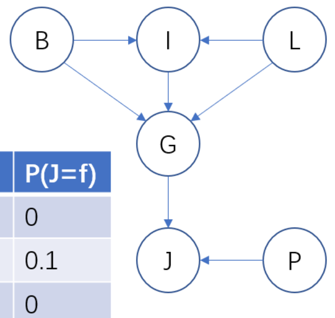
(b)

$$P(B, I, \neg L, G, J) = P(B)P(\neg L)P(I \mid B, \neg L)P(G \mid B, \neg L, I)P(J \mid G) = 0.2916$$

(c) The original proposition could be expressed as

$$P(B, L, I, J) = \sum_G P(B)P(L)P(I \mid B, L)P(G \mid B, L, I)P(J \mid G) = 0.06561$$

(d) The new Bayesian belief network is shown as the following Figure



G	P	P(J=t)	P(J=f)
t	t	1	0
t	f	0.9	0.1
f	t	1	0
f	f	0	1

P(P=t)	P(P=f)
0.01	0.99