Pattern Recognition Assignment#2

April 13, 2023

Q1. Given a test sample $\mathbf{x} = (2, 5)^\top$ and the following eight training samples of three categories, please answer the following questions.

(a) Please determine the category of $\mathbf{x}$ using the single-nearest neighbor algorithm.

(b) Please determine the category of $\mathbf{x}$ using the k-nearest neighbor algorithm, where k = 3.

Training examples	Category	Training examples	Category
$\mathbf{a} = (3, 9)^\top$	ω_1	$\mathbf{e} = (5, 1)^\top$	ω_3
$\mathbf{b} = (1, 3)^\top$	ω_3	$\mathbf{f} = (4, 5)^\top$	ω_2
$\mathbf{c} = (1, 2)^\top$	ω_3	$\mathbf{g} = (6, 9)^\top$	ω_1
$\mathbf{d} = (3, 8)^\top$	ω_1	$\mathbf{h} = (7, 3)^\top$	ω_3

Q2. Given a set of one-dimensional training samples $D = \{x_1, x_2, ..., x_{10}\}$ as follows:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
-0.17	0.24	-0.79	0.32	0.44	0.08	0.26	0.06	-0.71	0.65

(a) Suppose these data points are independently and identically sampled from $N(\mu, 1)$. Please estimate μ by maximum-likelihood estimation and further calculate the mean squared error (MSE) between the estimated distribution $\hat{p}(x) \sim N(\hat{\mu}, 1)$ and the ground-truth distribution $p(x) \sim N(0, 1)$ over the given training samples, i.e., $\frac{1}{10}\sum_{i=1}^{10}(p(x_i) - \hat{p}(x_i))^2$.

(b) Given the window function $\varphi(u) \sim N(0, 1)$ and $h_n = \frac{1}{\sqrt{n}}$, please estimate the probability density function (pdf) by the Parzen window method. Furthermore, please calculate the MSE between the estimated Parzen pdf and the ground-truth distribution over D.

(Tips: It is recommended to write computer programs to calculate the MSEs.)