# Pattern Recognition Assignment#1

March 22, 2023

The format of your report is up to you. In general, your report should clearly show how you have obtained the results and a detailed analysis of your solutions. If you feel a bit inexperienced with writing scientific reports, have a look at the line[1]. I recommend chapter 4 of this document if (like me) English is not your mother language.

Q1. The maximum likelihood estimate is not always an unbiased estimator. Prove that the maximum likelihood estimator of the variance of a Gaussian variable is biased.

Q2. People often decide their outdoor activities according to the weather conditions. Suppose you have a friend in London, where the weather conditions, denoted by $W = (\omega_1, \omega_2, \omega_3)$, is unknown. His activities option, denoted by $V = (v_1, v_2, v_3)$, is decided by the weather conditions. The initial state of the weather is $\pi = [0.3, 0.4, 0.3]$. Given the Hidden Markov model $\theta = (A, B, \pi)$, calculate the probability that you observe a specific activity sequence $O = [v_2, v_2, v_1, v_3]$ of your friend over the past four days, where $A_{i,j}$ is the transition probability from $\omega_i$ to $\omega_j$, $B_{i,j}$ is the probability of observing the activity $v_j$ under the state $\omega_i$.

$$A = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.1 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.2 & 0.4 & 0.4 \\ 0.3 & 0.1 & 0.6 \end{bmatrix}.$$

Q3. Suppose your friend buys a book and the book is either from online shopping (i.e., $\omega_1$) or physical store shopping (i.e., $\omega_2$). His satisfaction with the two methods is 0.3 and 0.7, respectively. It's known that online shopping has a 20% probability of delivering the wrong goods, while physical stores only have a 5% probability of doing so. Now he receives a book and finds it is wrong.

(a) Which method do you suspect the book is purchased by?

(b) The loss function is defined as: $\lambda_{11}$=1, $\lambda_{12}$=5, $\lambda_{21}$=3, $\lambda_{22}$=1. Considering the minimum risk Bayesian decision, which method do you suspect the book is purchased by?

Q4. Consider the Bayesian belief network as shown in Figure 1.

(a) Which of the following are asserted by the network structure and why?

i. $P(B, I, L) = P(B)P(I)P(L)$.

ii. $P(J|G) = P(J|G, I)$.

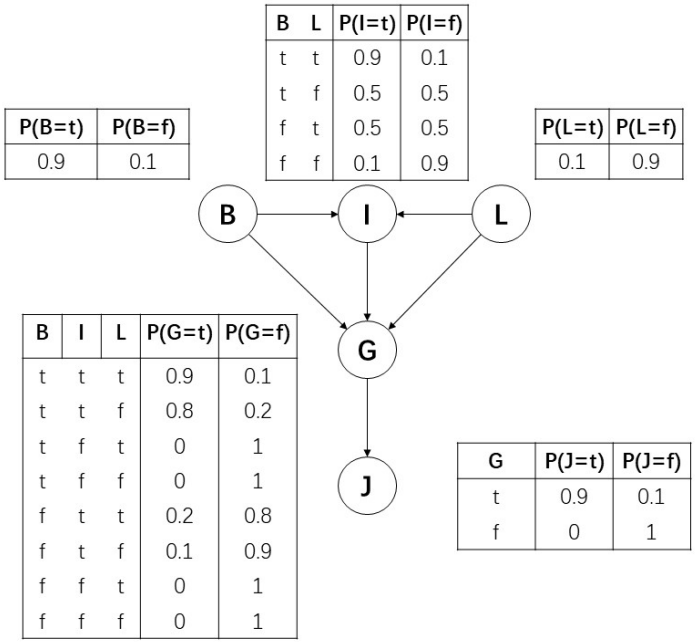
[1] http://www.cs.joensuu.fi/pages/whamalai/sciwri/sciwri.pdf

iii. $P(L|G,B,I) = P(L|G,B,I,J)$.

(b) Calculate the value of $P(B = t, I = t, L = f, G = t, J = t)$.

(c) Calculate the probability that someone will go to jail given that he/she has broken the law, has been indicted, and his/her lawyer is inexperienced.

(d) Suppose we want to add the variable $P$ = Presidential Pardon to the network, draw the new network and briefly explain any links you add.




| B | L | P(I=t) | P(I=f) |
|---|---|---|---|
| t | t | 0.9 | 0.1 |
| t | f | 0.5 | 0.5 |
| f | t | 0.5 | 0.5 |
| f | f | 0.1 | 0.9 |

| P(B=t) | P(B=f) |
|---|---|
| 0.9 | 0.1 |

| P(L=t) | P(L=f) |
|---|---|
| 0.1 | 0.9 |

| B | I | L | P(G=t) | P(G=f) |
|---|---|---|---|---|
| t | t | t | 0.9 | 0.1 |
| t | t | f | 0.8 | 0.2 |
| t | f | t | 0 | 1 |
| t | f | f | 0 | 1 |
| f | t | t | 0.2 | 0.8 |
| f | t | f | 0.1 | 0.9 |
| f | f | t | 0 | 1 |
| f | f | f | 0 | 1 |

| G | P(J=t) | P(J=f) |
|---|---|---|
| t | 0.9 | 0.1 |
| f | 0 | 1 |

Figure 1: A Bayesian belief network with Bernoulli-distributed variables $B$ = Broke Law, $I$ = Indicted, $L$ = Inexperienced Lawyer, $G$ = Found Guilty, $J$ = Jailed.