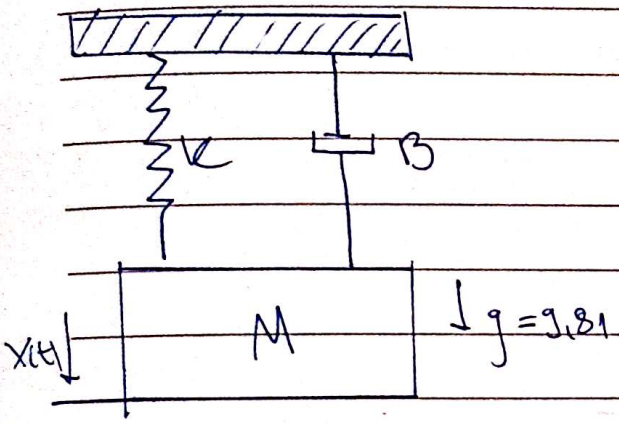
Str. 22

7.



$M = 10\ [kg]$

$k = 200\ [N/m]$

$B = 20\ [Ns/m]$

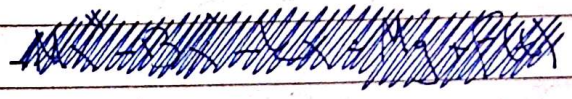
a) $10\ [N]$

b) $20\sin(t)\ [N]$

c) $20\cos(t)\ [N]$

d) $\cos t\ [N]$

a)



$$-Mx'' - Bx' - kx - f(t) + Mg = 0$$

$$-Mx'' - Bx' - kx - f(t) = -Mg$$

$$Mx'' + Bx' + kx + f(t) = Mg$$

$$Mx'' + Bx' + kx = Mg - f(t) \quad / : M$$

$$x'' + \frac{B}{M}x' + \frac{k}{M}x = g - \frac{f(t)}{M}$$

$$x'' + 2x' + 20x = 9,81 - 1$$

$$x'' + 2x' + 20x = 8,81$$

Datum/Date

Bilješke/Notes

$$s^2X(s) + 2sX(s) + 20X(s) = \frac{8,81}{s}$$

$$X(s)\left(s^2 + 2s + 20\right) = \frac{8,81}{s}$$

$$X(s) = \frac{8,81}{s(s^2+2s+20)}$$

$$\frac{8,81}{s(s^2+2s+20)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+20}$$

$$8,81 = A(s^2+2s+20) + s(Bs+C)$$

$$8,81 = As^2 + 2As + 20A + Bs^2 + Cs$$

$$8,81 = s^2(A+B) + s(2A+C) + 20A$$

$A+B=0$ $\quad 2A+C=0$ $\quad 20A=8,81$

$B=-0,44$ $\quad C=-0,88$ $\quad A=0,44$

$$\frac{8,81}{s(s^2+2s+20)} = \frac{0,44}{s} + \frac{-0,44s-0,88}{s^2+2s+20}$$

$$= \frac{0,44}{s} + \frac{-0,44s-0,88}{(s+1)^2+(\sqrt{19})^2}$$

$$x(t) = 0,44 - 0,44e^{-t}\cos\sqrt{19}t - 0,1e^{-t}\sin\sqrt{19}t$$

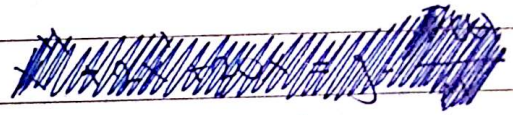
$$= \frac{-0,445 - 0,88}{(s+1)^2 + (\sqrt{19})^2} = \frac{-0,44((s+1)-1) - 0,88}{(s+1)^2 + (\sqrt{19})^2}$$

$$= \frac{-0,44(s+1) - 0,44}{(s+1)^2 + (\sqrt{19})^2} = -0,44 \frac{s+1}{(s+1)^2 + (\sqrt{19})^2} - 0,44 \frac{1}{(s+1)^2 + (\sqrt{19})^2}$$

$$= -0,44\, e^{-t} \cos\sqrt{19}t$$

$$= -0,44 \frac{\sqrt{19} \cdot \frac{1}{\sqrt{19}}}{(s+1)^2 + (\sqrt{19})^2} = -0,1\, e^{-t} \sin(\sqrt{19})t$$

b)



$$x'' + \frac{B}{M}x' + \frac{k}{M}x = g - \frac{F(t)}{M}$$

$$x'' + 2x' + 20x = 9,81 - 2\sin t$$

$$s^2X(s) + 2sX(s) + 20X(s) = \frac{9,81}{s} - \frac{2}{s^2+1}$$

$$X(s)\left(s^2 + 2s + 20\right) = \frac{9,81s^2 + 9,81 - 2s}{s(s^2+1)}$$

$$X(s) = \frac{9,81s^2 - 2s + 9,81}{s(s^2+1)(s^2+2s+20)}$$

Datum/Date

Bilješke/Notes

$$\frac{9,81s^2-2s+9,81}{s(s^2+1)(s^2+2s+20)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{(s^2+2s+20)}$$

$$9,81s^2-2s+9,81 = A(s^2+1)(s^2+2s+20) + s(Bs+C)(s^2+2s+20) + s(Ds+E)(s^2+1)$$

$$9,81s^2-2s+9,81 = A(s^4+2s^3+20s^2+s^2+2s+20) + s(Bs^3+2Bs^2+20Bs+Cs^2+2Cs+20C)$$
$$+ s(Ds^3+Ds+Es^2+E)$$

$$9,81s^2-2s+9,81 = As^4+2As^3+21As^2+2As+20+Bs^4+2Bs^3+20Bs^2+Cs^3+2Cs^2+20C$$
$$+Ds^4+Ds^2+Es^3+Es$$

$$9,81s^2-2s+9,81 = s^4(A+B+D)+s^3(2A+2B+C+E)+s^2(21A+20B+2C+D)+s(2A+20C+E)+20A$$

$A+B+D=0$

$B=-0,5-D$

$B=0$

$2A+20C+E=-2$

$E=-3-20C$

$E=-1$

$2A+2B+C+E=0$

$A+1-2D+C-3-20C=0$

$-2D-19C=3$

$D=\frac{3+19C}{-2}$

$D=-0,5$

$21A+20B+2C+D=9,81$

$20A=9,81$

$A=0,5$

$10,5-10-20D+2C+D=9,81$

$-19D+2C=9,31$

$-19\left(\frac{3+19C}{-2}\right)+2C=9,31 \quad /2$

$57+361C+4C=18,6$

$C=-0,1$

$$\frac{9{,}81s^2 - 25s + 9{,}81}{s(s^2+1)(s^2+25s+120)} = \frac{0{,}5}{s} + \frac{-0{,}1}{s^2+1} + \frac{-0{,}5s - 1}{(s^2+25s+120)}$$

$$\boxed{x(t) = 0{,}5 - 0{,}1\sin t - 0{,}5e^{-t}\cos\sqrt{19}t - 0{,}1e^{-t}\sin\sqrt{19}t}$$

$$\frac{-0{,}5s - 1}{s^2+25s+120} = \frac{-0{,}5s - 1}{(s+1)^2 + (\sqrt{19})^2} = \frac{-0{,}5((s+1)-1) - 1}{(s+1)^2 + (\sqrt{19})^2}$$

$$= \frac{-0{,}5(s+1) - 0{,}5}{(s+1)^2 + (\sqrt{19})^2} = -0{,}5\frac{s+1}{(s+1)^2+(\sqrt{19})^2} - 0{,}5\frac{1}{(s+1)^2+(\sqrt{19})^2}$$

$$= -0{,}5e^{-t}\cos\sqrt{19}t - 0{,}1e^{-t}\sin\sqrt{19}t$$

Ako precizirije zaokružimo imat ćemo i cost !!!

c)

$$Mx'' + Bx' + kx = Mg - F(t)$$

$$Ms^2X(s) + BsX(s) + kX(s) = Mg - F(t) \quad |\cdot(-1)$$

$$-X(s)(Ms^2 + Bs + k) = \boxed{F(t) - Mg} \quad \leftarrow U(s)$$

$$G_1(s) = \frac{X(s)}{U(s)} = \frac{-1}{Ms^2 + Bs + k} \rightarrow \text{Pomak}$$

$$G_1(s) = \frac{X(s)}{U(s)} \cdot s = \frac{-s}{Ms^2 + Bs + k}$$

a)

$$M\ddot{x} = g - F(t) - kx - B\dot{x}$$

$$M\ddot{x} = U - kx - B\dot{x}$$

$y_1 = x$ $\quad\rightarrow\quad$ $\dot{y}_1 = \dot{x} = y_2$

$y_2 = \dot{x}$ $\quad\quad$ $\dot{y}_2 = \ddot{x} = \frac{U}{M} - \frac{k}{M} y_1 - \frac{B}{M} y_2$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{M} & -\frac{B}{M} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} U$$

$$Y = Cx + D$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}$$