1st Iteration

Step 1:

$$m = -1, \quad b = 1 : \quad \hat{y}_1 = -1(1) + 1 = 0$$

$$\hat{y}_2 = -1(3) + 1 = -2$$

step 2:

errors:

$$e_1 = y_1 - \hat{y}_1 = 3 - 0 = 3$$

$$e_2 = y_2 - \hat{y}_2 = 6 - (-2) = 8$$

step 3:

gradients (n=2)

$$\frac{\partial I}{\partial m} = -\frac{2}{2}\left[(3)(1) + (8)(3)\right] = -(3 + 24)$$
$$= -27$$

$$\frac{\partial I}{\partial b} = -\frac{2}{2}(3 + 8) = -11$$

Step 4:

updates: $m = -1 - 0.1(-27) = -1 + 2.7$
$$= 1.7$$

$$b = 1 - 0.1(-11) = 1 + 1.1$$
$$= 2.1$$

2nd iteration

Step 1: prediction: $m = 1.7$, $b = 2.1$

$$y_1 = 1.7(1) + 2.1 = 3.8$$

$$y_2 = 1.7(3) + 2.1 = 7.2$$

Step 2: errors:

$$e_1 = 3 - 3.8 = -0.8$$

$$e_2 = 6 - 7.2 = -1.2$$

Step 3: gradients

$$\frac{\partial J}{\partial m} = -1 \left[(-0.8)(1) + (-1.2)(3) \right] = 4.4$$

$$\frac{\partial J}{\partial b} = -1 \cdot (-0.8 - 1.2) = 2.0$$

Step 4: update: $m = 1.7 - 0.1(4.4) = 1.7 - 0.44 = 1.26$

$$b = 2.1 - 0.1(2.0) = 2.1 - 0.2 = 1.9$$

3rd iteration

Step1: Predictions, $m = 1.26, b = 1.9$

$$\hat{y}_1 = 1.26(1) + 1.9 = 3.16$$

$$\hat{y}_2 = 1.26(3) + 1.9 = 5.58$$

Step2: errors; $e_1 = 3 - 3.16 = -0.16$

$$e_2 = 6 - 5.58 = 0.42$$

Step3: gradients

$$\frac{\partial J}{\partial m} = -1 \cdot [(-0.16)(1) + (0.42)(3)] = -(-0.16 + 1.26) = -1.10$$

$$\frac{\partial J}{\partial b} = -1 \cdot (-0.16 + 0.42) = 0.26$$

Step4: updates

$$m = 1.26 - 0.1(-1.10) = 1.26 + 0.11 = 1.37$$

$$b = 1.9 - 0.1(0.26) = 1.9 + 0.026 = 1.926$$

Step 1: Predictions : $m = 1.37$, $b = 1.926$

$$\hat{y}_1 = 1.37(1) + 1.926 = \mathbf{3.286}$$
$$\hat{y}_2 = 1.37(3) + 1.926 = 6.036$$

Step 2: Errors ! $e_1 = 3 - 3.296 = -0.296 = -0.296$
$$e_2 = 6 - 6.036 = -0.036$$

Step 3: Gradients $(n=2)$

$$\frac{\partial J}{\partial m} = -1 \times \left[(-0.296)(1) + (-0.036)(3) \right]$$

$$= 0.404$$

$$\frac{\partial J}{\partial m} = -1 \times (-0.296 + (-0.036)) = 0.332$$

Step 4: Updates : $m = 1.37 - 0.1(0.404) = \underline{1.3296}$
$$b = 1.926 - 0.1(0.332) = \underline{1.8928}$$