In[1]:=
```
n = 3; m = 3; μ = (n + m) ! / (n ! m !)
SS = Table[S_i = i, {i, 0, m}];
t = {}; i = 1;
Do[If[j + k + q < m + 1, {t = Join[t, {{S_j, S_k, S_q}}], φ_i[x_, y_, z_] = x^S_j y^S_k z^S_q, i = i + 1}],
 {j, 0, m}, {k, 0, m}, {q, 0, m}]
t
Table[φ_i[x, y, z], {i, 1, μ}]
```

Out[1]=
```
20
```

Out[5]=
```
{{0, 0, 0}, {0, 0, 1}, {0, 0, 2}, {0, 0, 3}, {0, 1, 0}, {0, 1, 1},
 {0, 1, 2}, {0, 2, 0}, {0, 2, 1}, {0, 3, 0}, {1, 0, 0}, {1, 0, 1}, {1, 0, 2},
 {1, 1, 0}, {1, 1, 1}, {1, 2, 0}, {2, 0, 0}, {2, 0, 1}, {2, 1, 0}, {3, 0, 0}}
```

Out[6]=
```
{1, z, z^2, z^3, y, y z, y z^2, y^2, y^2 z, y^3, x, x z, x z^2, x y, x y z, x y^2, x^2, x^2 z, x^2 y, x^3}
```

In[7]:=
```
V = Table[φ_j[t[[i, 1]], t[[i, 2]], t[[i, 3]]], {i, 1, μ}, {j, 1, μ}];
Det[V] ≠ 0
```

Out[8]=
```
True
```

In[9]:=
```
eqv := Table[Sum_{i=1}^{μ} (b_i * φ_i[t[[k, 1]], t[[k, 2]], t[[k, 3]]]) ==
   f[t[[k, 1]], t[[k, 2]], t[[k, 3]]], {k, 1, μ}]
koef := Solve[eqv, {}] // Flatten
P[x_, y_, z_] := Sum_{i=1}^{μ} (b_i * φ_i[x, y, z]) /. koef
```

In[12]:=
```
f[x_, y_, z_] = 3^(x*y) * Sin[y + z];
P1[x_, y_, z_] = P[x, y, z] // N
```

Out[13]=
```
1.20751 y - 1.95425 x y + 1.68294 x^2 y - 0.355643 y^2 + 1.95425 x y^2 - 0.0103932 y^3 + 1.20751 z -
 0.711285 y z + 0.135653 x y z - 0.0311797 y^2 z - 0.355643 z^2 - 0.0311797 y z^2 - 0.0103932 z^3
```

In[14]:=
```
Table[P1[t[[i, 1]], t[[i, 2]], t[[i, 3]]] - f[t[[i, 1]], t[[i, 2]], t[[i, 3]]],
   {i, 1, μ}] // Simplify // Chop
```

Out[14]=
```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

In[15]:=
```
a;
```

In[16]:=
```
arg1 = {x → t[[m - 1, 1]], y → t[[m - 1, 2]], z → t[[m - 1, 3]]};
D1 = D[P1[x, y, z], {x, 1}] //. arg1 // N
```

Out[17]=
```
0.
```

In[18]:=
```
б;
```

In[19]:=
```
arg2 = {x → t[[m, 1]], y → t[[m, 2]], z → t[[m, 3]]};
D2 = D[D[D[P1[x, y, z], {x, m - 2}], {y, 1}], {z, 1}] //. arg2
```
дифференциировать

Out[20]=
```
0.135653
```

In[21]:=
```
в;
```

In[22]:=
```
arg3 = {x → t[[μ, 1]], y → t[[μ, 2]], z → t[[μ, 3]]};
D3 = D[D[P1[x, y, z], {x, μ - 1}], {y, 1}] //. arg3
```
дифференциировать

Out[23]=
```
0
```

In[24]:=
```
а;
```

In[25]:=
```
DD1 = ∑_{k=1}^{μ} A_k * f[t[[k, 1]], t[[k, 2]], t[[k, 3]]];
eqv1 = Table[(D[φ_i[x, y, z], {x, 1}] //. arg1) ==
```
таблиц дифференциировать
```
    ∑_{k=1}^{μ} A_k * φ_i[t[[k, 1]], t[[k, 2]], t[[k, 3]]], {i, 1, μ}];
koef = Solve[eqv1, {}] // Flatten;
```
решить уравнения уплостить
```
DD1 = DD1 /. koef // N
```
численное приближение
```
D1 == DD1
```

Out[28]=
```
0.
```

Out[29]=
```
True
```

In[30]:=
```
б;
```

In[31]:=
```
DD2 = ∑_{k=1}^{μ} A_k * f[t[[k, 1]], t[[k, 2]], t[[k, 3]]];
eqv1 = Table[(D[D[D[φ_i[x, y, z], {x, m - 2}], {y, 1}], {z, 1}] //. arg2) ==
```
таблиц дифференциировать
```
    ∑_{k=1}^{μ} A_k * φ_i[t[[k, 1]], t[[k, 2]], t[[k, 3]]], {i, 1, μ}];
koef = Solve[eqv1, {}] // Flatten;
```
решить уравнения уплостить
```
DD2 = DD2 /. koef // N
```
численное приближение
```
D2 == DD2
```

Out[34]=
```
0.135653
```

Out[35]=
```
True
```

In[36]:=
```
в;
```

In[37]:=
```
DD3 = Sum[A_k * f[t[[k, 1]], t[[k, 2]], t[[k, 3]]], {k, 1, μ}];
eqv1 = Table[(D[D[φ_i[x, y, z], {x, μ - 1}], {y, 1}] //. arg3) ==
    Sum[A_k * φ_i[t[[k, 1]], t[[k, 2]], t[[k, 3]]], {k, 1, μ}], {i, 1, μ}];
koef = Solve[eqv1, {}] // Flatten;
DD3 = DD3 /. koef // N
D3 == DD3
```

Out[40]=
```
0.
```

Out[41]=
```
True
```

In[42]:=

In[43]:=
```
a;
```

In[44]:=
```
Toch1 = D[f[x, y, z], {x, 1}] //. arg1 // N
Abs[D1 - Toch1]
```

Out[44]=
```
0.
```

Out[45]=
```
0.
```

In[46]:=
```
б;
```

In[47]:=
```
Toch2 = D[D[D[f[x, y, z], {x, m - 2}], {y, 1}], {z, 1}] //. arg2
Abs[D2 - Toch2]
```

Out[47]=
```
Cos[2] Log[3]
```

Out[48]=
```
0.592837
```

In[49]:=
```
в;
```

In[50]:=
```
Toch3 = D[D[f[x, y, z], {x, μ - 1}], {y, 1}] //. arg3
Abs[D3 - Toch3]
```

Out[50]=
```
0
```

Out[51]=
```
0
```