Вариант 8

In[46]:= $X = \left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\right\};$

$f[x\_] = \mathrm{Sin}[x]^3 * \mathrm{Cos}[x];$

косинус

`F = f[X];`

`n = Length[X] - 1;`

длина

$\mathrm{Do}[\{x_i = X[[i+1]], f_i = F[[i+1]]\}, \{i, 0, n\}]$

оператор цикла

In[51]:= $\mathrm{Do}\left[l_k[x\_] = \left(\prod_{i=0}^{k-1} \frac{(x - x_i)}{(x_k - x_i)}\right) \left(\prod_{i=k+1}^{n} \frac{(x - x_i)}{(x_k - x_i)}\right), \{k, 0, n\}\right]$

оператор цикла

$P[x\_] = \sum_{k=0}^{n} l_k[x]\, f_k \;\; // \; \mathrm{Expand}$

раскрыть скобки

Out[52]= $-\frac{16\, x}{\pi} + \frac{135 \sqrt{3}\, x}{16\, \pi} + \frac{176\, x^2}{\pi^2} - \frac{729 \sqrt{3}\, x^2}{8\, \pi^2} - \frac{576\, x^3}{\pi^3} + \frac{621 \sqrt{3}\, x^3}{2\, \pi^3} + \frac{576\, x^4}{\pi^4} - \frac{324 \sqrt{3}\, x^4}{\pi^4}$

In[53]:= $H[x\_] = f_0 + \sum_{j=1}^{n} \left(\sum_{k=0}^{j} \frac{f_k}{\left(\prod_{i=0}^{k-1} (x_k - x_i)\right) \left(\prod_{i=k+1}^{j} (x_k - x_i)\right)}\right) \left(\prod_{m=0}^{j-1} (x - x_m)\right) \; // \; \mathrm{Expand}$

раскрыть

Out[53]= $-\frac{16\, x}{\pi} + \frac{135 \sqrt{3}\, x}{16\, \pi} + \frac{176\, x^2}{\pi^2} - \frac{729 \sqrt{3}\, x^2}{8\, \pi^2} - \frac{576\, x^3}{\pi^3} + \frac{621 \sqrt{3}\, x^3}{2\, \pi^3} + \frac{576\, x^4}{\pi^4} - \frac{324 \sqrt{3}\, x^4}{\pi^4}$

In[54]:= $\mathrm{Tb1} = \mathrm{Table}[\{x_i, f_i\}, \{i, 0, n\}];$

таблица значений

`P1[x_] = InterpolatingPolynomial[Tb1, x] // Expand`

интерполяционный многочлен   раскрыть

`P[x] == H[x] == P1[x]`

Out[55]= $-\frac{16\, x}{\pi} + \frac{135 \sqrt{3}\, x}{16\, \pi} + \frac{176\, x^2}{\pi^2} - \frac{729 \sqrt{3}\, x^2}{8\, \pi^2} - \frac{576\, x^3}{\pi^3} + \frac{621 \sqrt{3}\, x^3}{2\, \pi^3} + \frac{576\, x^4}{\pi^4} - \frac{324 \sqrt{3}\, x^4}{\pi^4}$

Out[56]= True

In[57]:= `Unprotect[Power];`

снять защ⋯ степень

$0^0 := 1;$

In[59]:= $\mathrm{Table}\left[x^j === \left(\sum_{k=0}^{n} l_k[x]\, x_k{}^j \; // \; \mathrm{Expand}\right), \{j, 0, n\}\right]$

таблица значений   раскрыть скобки

$x^{n+1} === \left(\sum_{k=0}^{n} l_k[x] * x_k{}^{n+1} \; // \; \mathrm{Expand}\right)$

раскрыть скобки

Out[59]= {True, True, True, True, True}

Out[60]= False

In[61]:= $x_{n+1} = \frac{\pi}{5};$
$H[x_{n+1}]$
$f_{n+1} = f[x_{n+1}]$
$R1 = f[x_{n+1}] - H[x_{n+1}]$ // Expand
раскрыть скобки

$R2 = \sum_{k=0}^{n+1} \left(\frac{f_k}{\left(\prod_{i=0}^{k-1} (x_k - x_i)\right) \left(\prod_{i=k+1}^{n+1} (x_k - x_i)\right)} \right) \prod_{j=0}^{n} (x_{n+1} - x_j)$ // Expand
раскрыть

R1 == R2

Out[62]= $\frac{96}{625} + \frac{81\sqrt{3}}{10\,000}$

Out[63]= $\frac{1}{4}\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)^{3/2}\left(1 + \sqrt{5}\right)$

Out[64]= $-\frac{96}{625} - \frac{81\sqrt{3}}{10\,000} + \frac{1}{4}\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)^{3/2} + \frac{1}{4}\sqrt{5}\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)^{3/2}$

Out[65]= $-\frac{96}{625} - \frac{81\sqrt{3}}{10\,000} + \frac{1}{4}\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)^{3/2} + \frac{1}{4}\sqrt{5}\left(\frac{5}{8} - \frac{\sqrt{5}}{8}\right)^{3/2}$

Out[66]= True

Задание 2

In[67]:= $f[x\_] = \sum_{k=1}^{x} (2k)^5$ // Expand
раскрыть

Out[67]= $-\frac{8x^2}{3} + \frac{40x^4}{3} + 16x^5 + \frac{16x^6}{3}$

In[68]:= $x_0 = 1;\ x_1 = 2;\ f_0 = f[x_0];\ f_1 = f[x_1];$
$k = 1;$
While$\left[\left(r_k = \sum_{j=0}^{k} \frac{f_j}{\left(\prod_{i=0}^{j-1} (x_j - x_i)\right)\left(\prod_{i=j+1}^{k} (x_j - x_i)\right)}\right) \neq 0,\right.$
цикл–пока

$\left.\{k = k + 1,\ x_k = k + 1,\ f_k = f[x_k]\}\right]$
k

Out[71]= 7

In[72]:= $n = k - 1;$
$P[x\_] = f_0 + \sum_{k=1}^{n} \left(r_k \prod_{i=0}^{k-1} (x - x_i)\right)$ // Expand
раскрыть

Out[73]= $-\frac{8x^2}{3} + \frac{40x^4}{3} + 16x^5 + \frac{16x^6}{3}$

In[74]:= P[x] == f[x]

Out[74]= True

In[75]:=
```
True
```
истина

Out[75]=
```
True
```

In[76]:=
```
P[8]
```

Out[76]=
```
1 976 832
```