Variable aleatoria: $X \sim Normal(\mu, \sigma^2)$

Función de densidad: $f_X = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Función de verosimilitud:

$$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

Logaritmo de la verosimilitud (log-verosimilitud):

$$l(\theta) = log\, L(\theta) = log\left(\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}\right) = \sum_{i=1}^{n} log\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}\right)$$

$$= \sum_{i=1}^{n} log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + log\, e^{-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

$$= \sum_{i=1}^{n} log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2$$

$$= -\frac{1}{2}\left[log(2\pi\sigma^2)\sum_{i=1}^{n} 1 - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]$$

$$= -\frac{1}{2}\left[n\ log(2\pi\sigma^2) - \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2\right]$$

Derivar e igualar a cero, y despejar μ:

$$0 = \frac{\partial l}{\partial \mu} = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)\frac{1}{\sigma}$$

$$0 = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)\frac{1}{\sigma}$$

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$Función\ de\ log - verosimilitud\ con\ respecto\ a\ v = \sigma^2$:

$$l(v) = -\frac{1}{2}\left[n\ \log(2\pi v) - \frac{1}{v}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

Derivar e igualar a cero, y despejar v:

$$\frac{\partial l}{\partial v} = -\frac{1}{2}\left[n\frac{1}{2\pi v}2\pi - \frac{1}{v^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

$$0 = -\frac{1}{2}\left[n\frac{1}{v} - \frac{1}{v^2}\sum_{i=1}^{n}(x_i-\mu)^2\right]$$

$Resultado$:

$$\hat{v} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2$$