Variable aleatoria: $X \sim Bernoulli(\theta)$

Función de probabilidad: $p(x) = \theta^x (1-\theta)^{1-x}$

Función de verosimilitud:

$$L(\theta) = p(data|\theta) = \prod_{i=1}^{n} p(x_i|\theta) = \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i}$$

Logaritmo de la verosimilitud (log-verosimilitud):

$$\begin{aligned} l(\theta) &= \log L(\theta) \\ &= log\left(\prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i}\right) = \sum_{i=1}^{n}(x_i\, log(\theta) + (1-x_i)\, log(1-\theta)) \end{aligned}$$

Derivar e igualar a cero, y despejar θ:

$$0 = \frac{dl(\theta)}{d\theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n}(1-x_i)$$

$$0 = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n}(1-x_i)$$

$$\frac{1}{\theta}\sum_{i=1}^{n} x_i = \frac{1}{1-\theta}\sum_{i=1}^{n}(1-x_i)$$

$$(1-\theta)\sum_{i=1}^{n} x_i = \theta\sum_{i=1}^{n}(1-x_i)$$

$$\sum_{i=1}^{n} x_i - \theta\sum_{i=1}^{n} x_i - \theta n + \theta\sum_{i=1}^{n} x_i = 0$$

$$\sum_{i=1}^{n} x_i - \theta n = 0$$

$$\theta n = \sum_{i=1}^{n} x_i$$

$$\theta = \frac{1}{n}\sum_{i=1}^{n} x_i$$