Biostat 250C HW1

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Q1: $X \perp\!\!\!\perp (Y, W) \mid Z$, prove $X \perp\!\!\!\perp W \mid Z$.

Pf: $X \perp\!\!\!\perp (Y, W) \mid Z$

$\Rightarrow [X, Y, W \mid Z] = [X \mid Z][Y, W \mid Z]$

Marginalize Y on both sides:

$$[X, W \mid Z] = [X \mid Z][W \mid Z].$$

Done!

Q2: $X \perp\!\!\!\perp (Y, W) \mid Z$. Prove $X \perp\!\!\!\perp Y \mid W, Z$

Pf: By Q1 & marginalze W, we have

$$X \perp\!\!\!\perp (Y, W) \mid Z \Rightarrow \begin{cases} X \perp\!\!\!\perp Y \mid Z \\ X \perp\!\!\!\perp W \mid Z \end{cases}.$$

Thus,

$$[X, Y \mid W, Z]$$
$$= [X \mid Y, W, Z][Y \mid W, Z]$$
Def.
$$= [X \mid Z][Y \mid W, Z]$$
$X \perp\!\!\!\perp W \mid Z$
$$= [X \mid W, Z][Y \mid W, Z]$$

$\Rightarrow X \perp\!\!\!\perp Y \mid W, Z$. □

Q3: Contraction

$(X \perp\!\!\!\perp W \mid Z, Y) \wedge (X \perp\!\!\!\perp Y \mid Z)$

$\Rightarrow X \perp\!\!\!\perp (Y, W) \mid Z$.

Pf: $X \perp\!\!\!\perp Y \mid Z \Rightarrow [X \mid Z, Y] = [X \mid Z]$ (*)

$$[X, W \mid Z, Y] \overset{\text{def}}{=} [X \mid Z, Y][W \mid Z, Y]$$
$$\overset{(*)}{=} [X \mid Z][W \mid Z, Y] \quad (1)$$
$$[X, Y, W \mid Z] = [Y \mid Z][X W \mid Y, Z]$$
$$\overset{(1)}{=} [Y \mid Z][X \mid Z][W \mid Z, Y]$$
$$= [X \mid Z][Y, W \mid Z]$$

$\Rightarrow X \perp\!\!\!\perp (Y, W) \mid Z$ □.