Lec 15 Satterwaite Approx

μ_1 $\quad$ μ_2
$\bar{x}_1$ $\quad$ $\bar{x}_2$
n_1 $\quad$ n_2
σ_1^2 $\quad$ σ_2^2
$\sigma_1 \neq \sigma_2$

Test $\mu_1 = \mu_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \overset{H_0}{\sim} t_b$$

$$b = \left(S_1^2/n_1 + S_2^2/n_2\right)^2 \Big/ \left[\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}\right]$$

Frequently, $\frac{r_i MSE_i}{\sigma^2} \sim \chi^2$,

Let $u_1, \dots, u_k$ be independent χ^2 rvs with $r_1, \dots, r_k$ df respectively.

Let $u = \sum_{i=1}^{k} a_i u_i$. Find constants v such that $vu/\mathbb{E}u \sim \chi_b^2$

$$\mathbb{E}\frac{vu}{\mathbb{E}u} = b \Rightarrow v = b$$

Take variance on both sides.

$$2b = Var\,\chi_b^2 = Var\frac{vu}{\mathbb{E}u} = \frac{v^2}{(\mathbb{E}u)^2}\sum_i a_i^2 Var\,u_i$$

$$= \frac{v^2}{(\mathbb{E}u)^2}\sum_i a_i^2 (2r_i)$$

$$\Rightarrow b = \frac{v^2}{(\mathbb{E}u)^2}\sum_{i=1}^{k}(a_i \mathbb{E}u_i)^2 / r_i$$

$$\Rightarrow \hat{b} = \frac{(\sum_i a_i u_i)^2}{(\sum_{i=1}^{k} a_i^2 u_i^2)/r_i}$$

METHOD OF MOMENTS