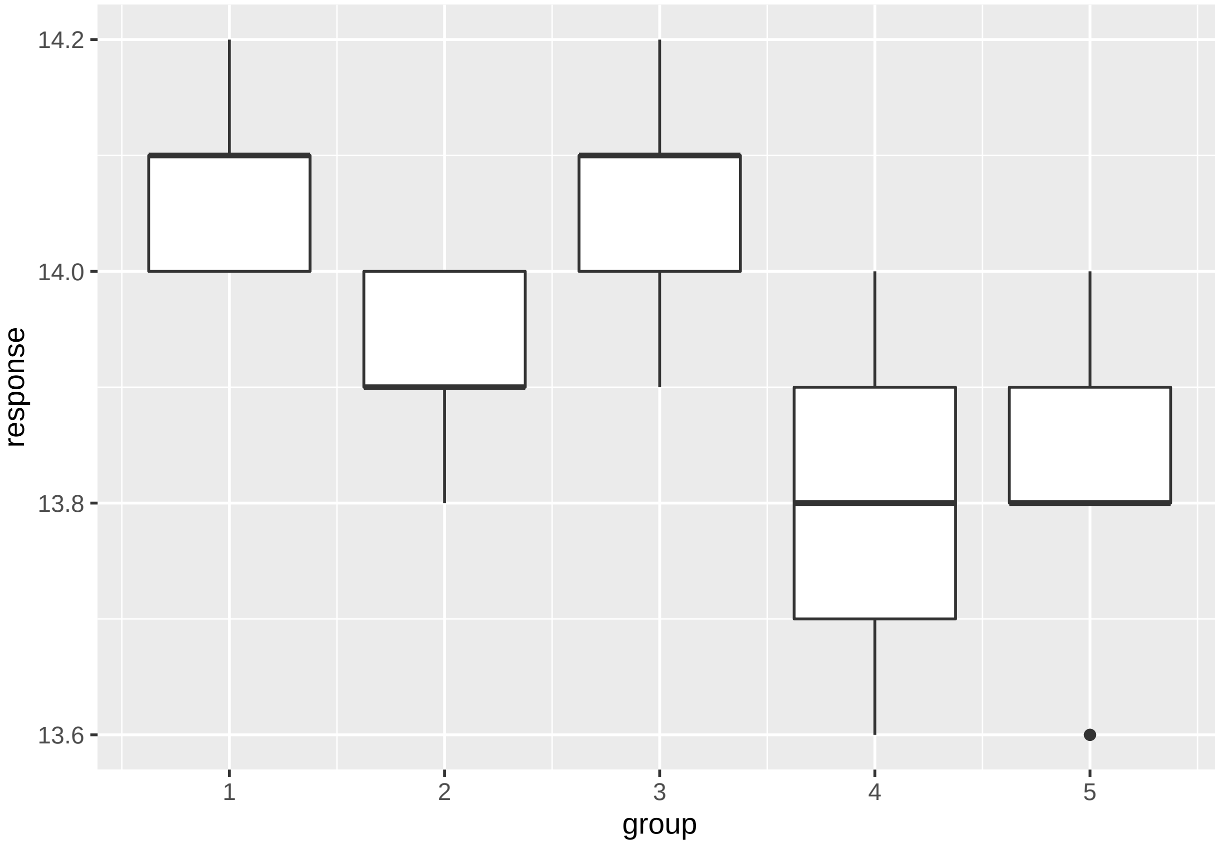
2

```r
# read in data
q2 <- read_dta("Data_for_HW4_Q2.dta")
```

a.

```r
ggplot(q2, aes(x = group, y = response, group = group)) +
  geom_boxplot()
```




```r
# Hartley's test
hartleyTest(response ~ factor(group), data = q2)
```

```
##
##  Hartley's maximum F-ratio test of homogeneity of variances
##
## data:  response by factor(group)
## F Max = 3.5714, df = 4, k = 5, p-value = 0.7576
```

```r
# Levene's test
leveneTest(response ~ factor(group), data = q2, center=mean)
```

```
## Levene's Test for Homogeneity of Variance (center = mean)
##       Df F value Pr(>F)
## group  4  0.6738 0.6179
##       20
```

```r
# Brown-Forsythe's test
leveneTest(response ~ factor(group), data = q2, center=median)
```

```
## Levene's Test for Homogeneity of Variance (center = median)
##       Df F value Pr(>F)
## group  4  0.5667 0.6897
##       20
```

```r
# Bartlett's Test
bartlett.test(response ~ factor(group), data = q2)
```

```
##
##  Bartlett test of homogeneity of variances
##
## data:  response by factor(group)
## Bartlett's K-squared = 2.5689, df = 4, p-value = 0.6323
```

None of the 4 test provides a p-value smaller than 0.05, indicating no evidence to reject the null hypothesis that the variance are the same among 5 groups, which resonates the observational results from the boxplot above.

b. O'Brien's Test for Homogeneity of Variance in 1-way ANOVA

In 1979, O'Brien proposed a test that transforms original scores so they represnet sample variance. And a standard one-way ANOVA on the transformed score can be used to test if all groups share a same variance. This procedure is named after O'Brien as the O'Brien's Test for homogeneity of variance. In the one-way ANOVA (need not be balanced) setting, the transformed scores are calculated by:

$$
\begin{aligned}
r_{ij} &= \frac{(w+n_j-2)n_j(x_{ij}-\bar{x_j})^2 - wS_j^2(n_j-1)}{(n_j-1)(n_j-2)} \\
\text{where } w &= \text{weight parameter} \\
n_j &= \text{size of the jth group} \\
S_j^2 &= \text{variance of the jth group} \\
x_{ij} &= \text{original score (response) of the ith subject in the jth group} \\
\bar{x_j} &= \text{mean of the jth group}
\end{aligned}
$$

The intuitive understanding of this tranformed score r_{ij} is that the mean of tranformed values per group equals to the variance computed from that group ($\bar{r_j} = \sum r_i j/n_j = S_j^2$). Therefore, a one-way ANOVA on the tranformed score can test for the variance difference across groups. The weight parameter represents the anticipation of departure from kurtosis = 0, where w = 0.5 is the default setting in many mainstream statistical packages, standing for a moderate departure. However in practice, tuning for the weight parameter w didn't make critical difference in testing resutls.

If the null hypothesis $\sigma_1^2 = \cdots = \sigma_k^2$ is rejected, O'Brien suggested that researcher can resort to Welch ANOVA for testing each contrast between two groups, to furtherly figure out the pair-wise difference of vairance between groups.

Reference

Fourteen Homogeneity of Variance Tests: When and How To Use Them. Zhang Shuqiang. (1998). Annual Meeting of the American Educational Research Association.

O' Brien Test for Homogeneity of Variance. Herve Abdi. Neil Salkind (Ed.) (2007). Encyclopedia of Measurement and Statistics.

3

a. Statistical Setup and Basics of Score Test

A general score test assesses constraints on statistical parameters based on the gradient of the likelihood function, a.k.a the score, evaluated at the hypothesized parameter value under the null hypothesis. The intuitive explaination is that, if the restricted estimator is near the true maximum likelihood estimation, the score shouuld not differ from zero too far, the likelihood function reaches its minimum value near the hypothesized parameter. Proved by C.R.Rao in 1948, the score statistic is asymptotic χ^2 distribution, offering an explicit rejection rule for any well defined score test.

In an explicit form, the test statistic for $H_0 : \theta = \theta_0$ can be written as following. Let L be the likelihood function only depends on parameter vector θ, and X be the data.

$$
\begin{aligned}
S(\theta_0) &= U'(\theta_0)I^{-1}(\theta_0)U(\theta_0) \sim \chi^2_k \\
\text{where } U(\theta_0) &= \frac{\partial logL(\theta_0|x)}{\partial\theta} \text{ is the score vector} \\
I(\theta_0) &= -E[\frac{\partial^2}{\partial\theta^2}logL(X;\theta)|\theta] \text{ is the fisher information.}
\end{aligned}
$$

In Cook and Weisberg test for heteroscedaticity paper, they convert the non-parametric testing for non-constanst variance in linear regression model to a parametric problem by assuming the observation variance follow certain parameric functional form. The functional for variance $w_i = w(z_i, \lambda)$ is based on an unknown parameter λ and some additional covariates z_i, which will always lead to $w_1 = w_2 = \cdots = w_n$ when $\lambda = \lambda^*$. Therefore, the testing for $H_0 : w_1 = \cdots = w_n$ is equivalent with testing $H_0 : \lambda = \lambda^*$. Since the latter one is testing for the constraint of $\lambda = \lambda^*$ on parameter λ, we can derive the score statistic $S(\lambda^*)$ from the likelihood function and calculate an asymptotic significance level by the χ^2 distribution.

b. Test Statistic for Score test

5

There are many datasets in the link (http://sssl-staffweb.tees.ac.uk/u0011128/Fingertips.data.and.analysis/R.zip) presented by the paper, and no simple instructions, so I didn't spend the extra amount of time to figure out how to represent the paper results.