Biostat 279 HW1

Q5 part (ii)

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April 25, 2021

1 Verify the information matrix of the exponential distribution in the Technometrics paper

Let $S \sim \mathcal{E}(\lambda)$ and $\lambda^{-1} = \frac{A}{(\phi_c-\phi)^\gamma}$ so that λ is a function of $\theta = (A, \phi, \gamma)$. Define

$$x = \phi_c - \phi$$

$$\eta = -\log\lambda = \log A - \gamma \log x$$

Then according to Atkinson's paper, I have

$$\nu(\eta) = Var(\frac{\partial}{\partial\eta}\log p(s|\eta)) = 1 \tag{1}$$

$$F(S,\lambda) = \frac{\partial\eta}{\partial\theta} = (\frac{1}{A}, \frac{\gamma}{x}, -\log x)^T \tag{2}$$

To prove (1), note that by Chain rule,

$$\frac{\partial}{\partial\eta}\log p(s|\eta) = \frac{\partial\lambda}{\partial\eta}\frac{\partial\log p(s|\lambda)}{\partial\lambda}$$

So that

$$\nu(\eta) = \frac{\partial^2\lambda}{\partial\eta^2}Var(\frac{\partial\log p(s|\lambda)}{\partial\lambda})$$

By Wikipedia, the information matrix for λ in the exponential distribution is $\frac{1}{\lambda^2}$ and $\frac{\partial^2\lambda}{\partial\eta^2} = e^{-2\eta} = \lambda^2$. Thus,

$$\nu(\eta) = \lambda^2 \times \frac{1}{\lambda^2} = 1$$

To prove (2), take derivative w.r.t. to A, ϕ and γ in the link function $\eta = \log A - \gamma\log x$.

Finally, by lemma 1 in the Atkinson's paper, I have

$$\begin{aligned}\mathcal{I}(\theta) &= F(S,\lambda)\nu(\eta)F(S,\lambda)^T \\ &= \begin{pmatrix} \frac{1}{A^2} & \frac{\gamma}{Ax} & -\frac{\log x}{A} \\ \frac{\gamma}{Ax} & \frac{\gamma^2}{x^2} & -\frac{\gamma\log x}{x} \\ -\frac{\log x}{A} & -\frac{\gamma\log x}{x} & \log^2 x \end{pmatrix}\end{aligned} \tag{3}$$