Time Series
Fall 2018 November 11, 2018

Homework #4
(Due November 30th)

#1. Consider the ARMA(1,1) model

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}, \varepsilon \sim N(0, \sigma^2).$$

Compute its optimal forecast $y_{T+h,h}$ and forecast error variance for all h.

#2. **Numerical Experiment**

The file VIX.csv contains the data of the VIX for 300 trading days, from 1/2/2004 to 3/10/2005. Our goal is to find a time series model that fits this data. We shall use the data from 1/2/2004 to 12/31/2004 as the in-sample data and that from 1/3/2005 to 3/10/2005 as the out-of-sample data. Do the following time series analysis in R. You need to submit your R code and a detailed report on your numerical results.

(a) Detrend the VIX time series.

(b) Fit the residuals (after removing the trend) to ARMA(p,q) for $0 \leq p \leq 1$, $0 \leq q \leq 1$.

(c) Among these ARMA models, which model will you choose for the liquor sale forecasting and explain why? (You can make your choice based on the AIC, Box test or any other reasonable criteria.) Plot the (true value, fitted value, residual) triplet of your best model.

(d) Use the model you have chosen to forecast VIX from 1/3/2005 to 3/10/2015. (Remember to include the trend.) Compare it with the true data.

#3. Consider the following VAR(1) model:

$$y_{1,t} = 0.5y_{1,t-1} + 0.2y_{2,t-1} + \varepsilon_{1,t},$$
$$y_{2,t} = -0.1y_{1,t-1} + 0.6y_{2,t-1} + \varepsilon_{2,t},$$

and $\varepsilon_1 \sim WN(0,1)$, $\varepsilon_2 \sim WN(0,1)$ and $\text{corr}(\varepsilon_{1,t}, \varepsilon_{2,t}) = 0.6$.

(a) Check that this VAR(1) model satisfies the stationarity condition:

$$\det\left(I - \sum_{i=1}^{p} A_i z^i\right) \neq 0 \text{ for all} |z| \leq 1.$$

(b) Compute the autocovariance *matrix* $\Gamma(h)$ for $h = 0$ and 1.

(c) Choose the right Cholesky factor and compute the impulse response function $b_{ij}(h)$ of y_1 and y_2 to ε_1 respectively, for $h = 1, 2$ respectively.