VE281

Data Structures and Algorithms

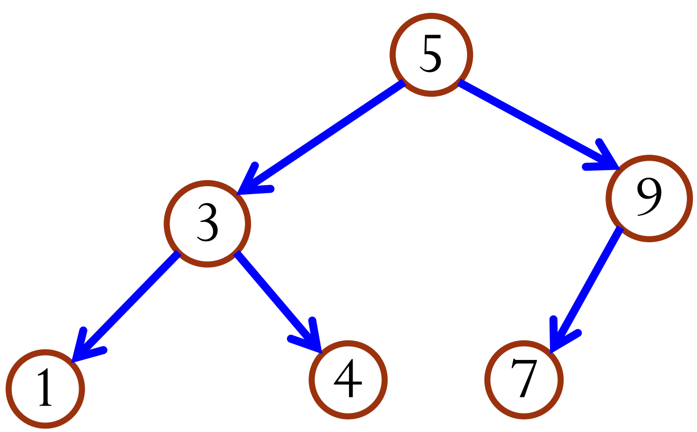
Binary Search Trees

Learning Objectives:

- Know what a binary search tree is
- Know how to do search, insertion, and removal for a binary search tree

Binary Search Tree

- A **binary search tree (BST)** is a binary tree with the following properties:
  - Each node is associated with a **key**.
    - A key is a value that can be compared.
    - **Assume**: all the keys are **distinct**.
  - The key of **any** node is greater than the keys of all nodes in its left subtree and smaller than the keys of all nodes in its right tree.



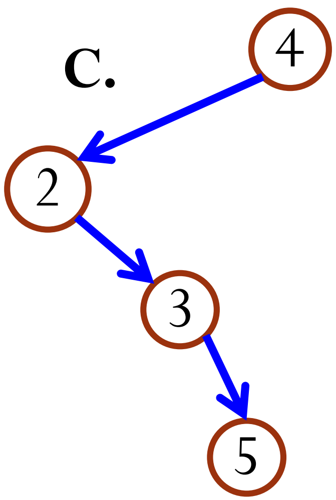
Which of the Following Trees Are BST?

- Select all the BSTs.

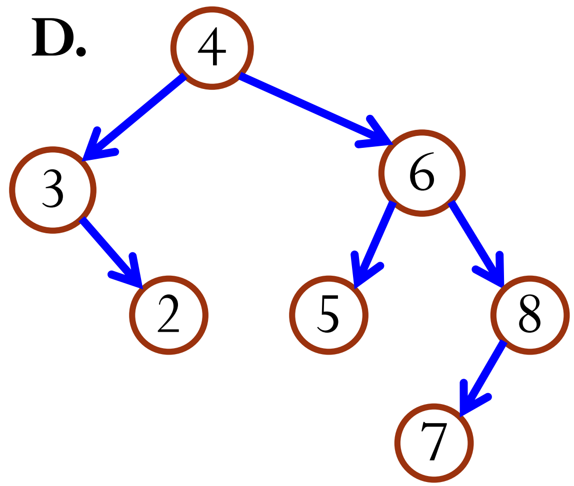
A. an empty tree

B.

C.



D.



Basic Binary Search Tree Operations

- A BST allows search, insertion, and removal by key.
  - The **average case** time complexities for these operations are $O(\log n)$.
  - **Average over all possible BSTs.**

Binary Search Tree

Search

```
node *search(node *root, Key k)
// EFFECTS: return the node whose key is k.
// If no matching node, return NULL.
```

- Procedure: Compare the search key with the key of the root
  - If they are equal, return the root.
  - If search key < root key, search the left subtree.
  - If search key > root key, search the right subtree.
  - Recursively applying the above procedure.

Binary Search Tree

Search

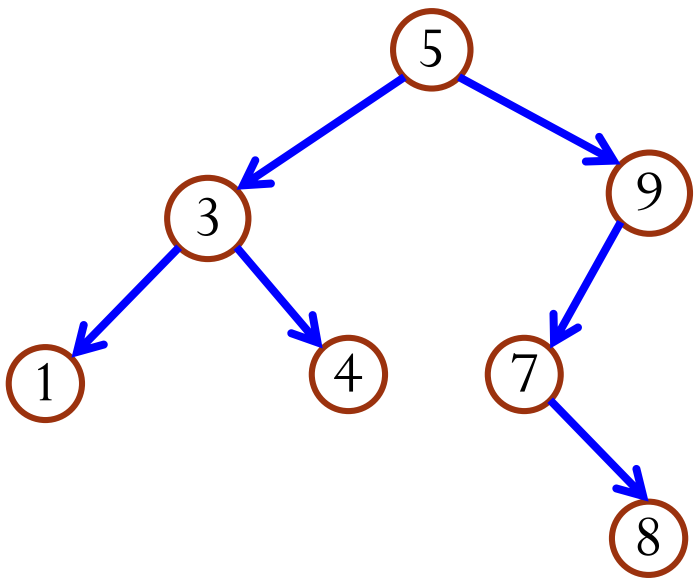
```
struct node {
  Item item;
  node *left;
  node *right;
};
```

```
struct Item {
  Key key;
  Val val;
};
```

```
node *search(node *root, Key k) {
  if(root == NULL) return NULL;
  if(k == root->item.key) return root;
  if(k < root->item.key)
    return search(root->left, k);
  else return search(root->right, k);
}
```

Binary Search Tree
Insertion

- Insertion inserts the item **as a leaf** of the BST.
- It inserts at a proper location in the BST, maintaining the BST properties.
  - **Pretend** we are searching the key.



Insert a node with key = 8

Binary Search Tree

Insertion

```
void insert(node *&root, Item item)
// EFFECTS: insert the item as a leaf,
// maintaining the BST property.
{
  if(root == NULL) {
    root = new node(item);
    return;
  }
  if(item.key < root->item.key)
    insert(root->left, item);
  else if(item.key > root->item.key)
    insert(root->right, item);
}
```

Question: why define root as the reference-to-pointer?

Question: what happens if the key is already in the BST?

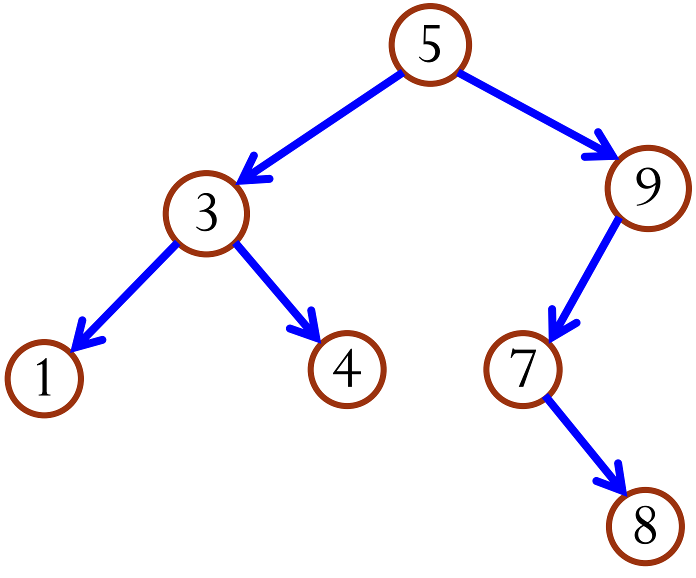
Binary Search Tree

Removal

reference to pointer

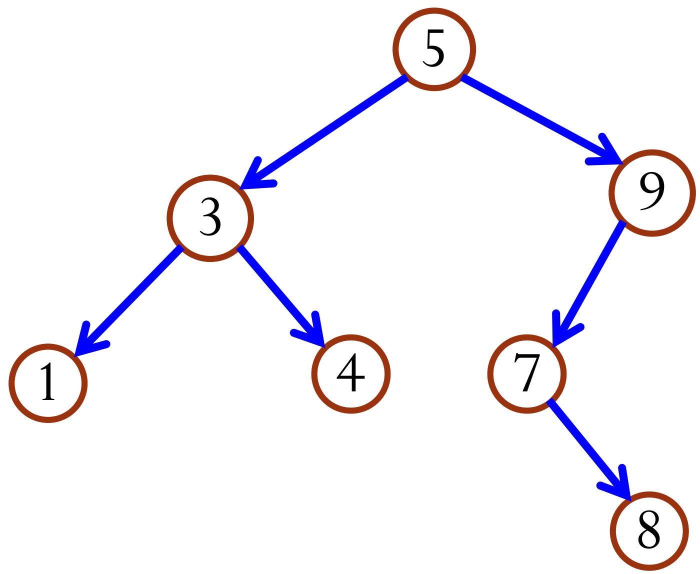
```
void remove(node *&root, Key k) {
  if(root == NULL) return;
  if(k < root->item.key) remove(root->left, k);
  else if(k > root->item.key)
    remove(root->right, k);
  else { // root->item.key == k
    // What to do when root->item.key == k?
  }
}
```

- How will you remove 8?
- How will you remove 9?
- How will you remove 5?



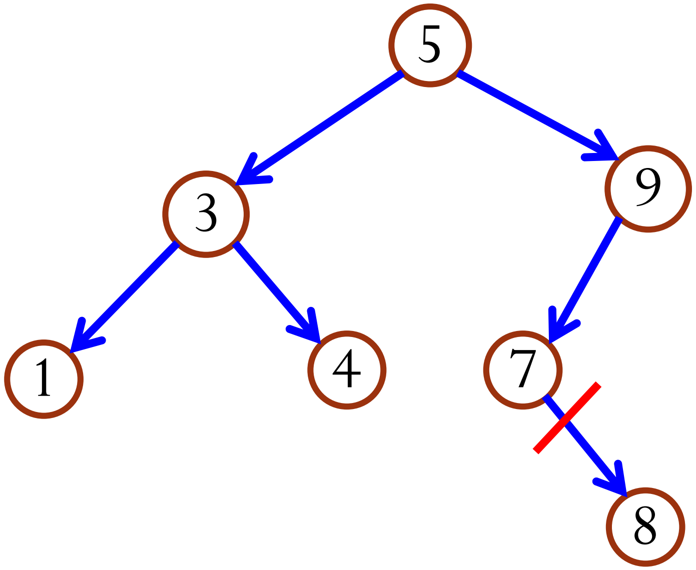
Binary Search Tree
Removal

- We distinguish three cases:
  - Node to be removed is a leaf.
  - Node to be removed is a degree-one node.
  - Node to be removed is a degree-two node.



Remove A Leaf

- Remove node 8

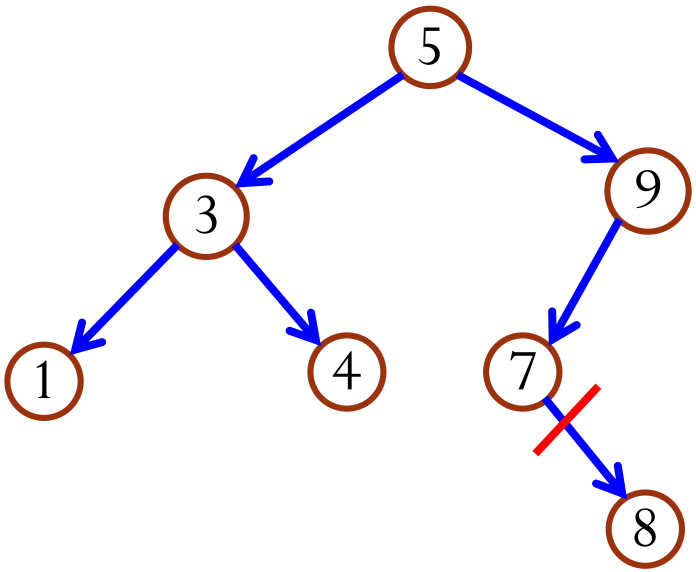


Remove A Leaf

Code

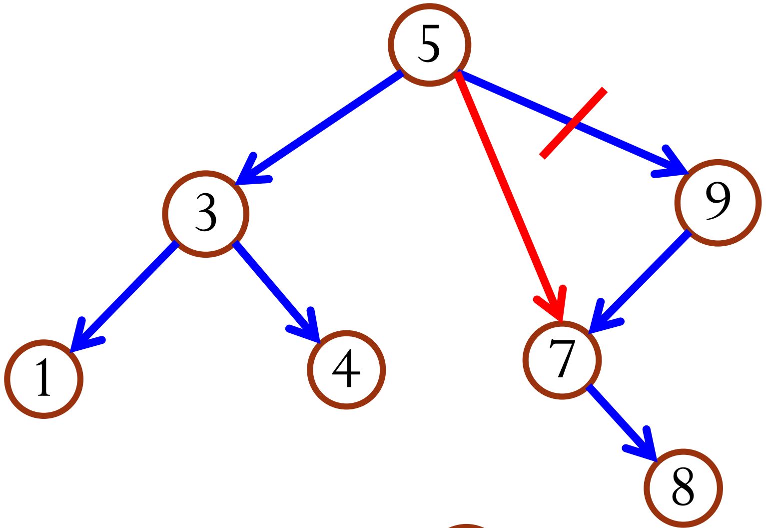
```
else { // root->item.key == k
  if(isLeaf(root)) {
    delete root;
    root = NULL;
  }
  else { // remove degree-one or two node
    ...
  }
}
```

Note: **`root`** is a **reference to a pointer**, which could be its parent's **`left`** pointer or **`right`** pointer. Our code effectively changes that pointer to NULL.

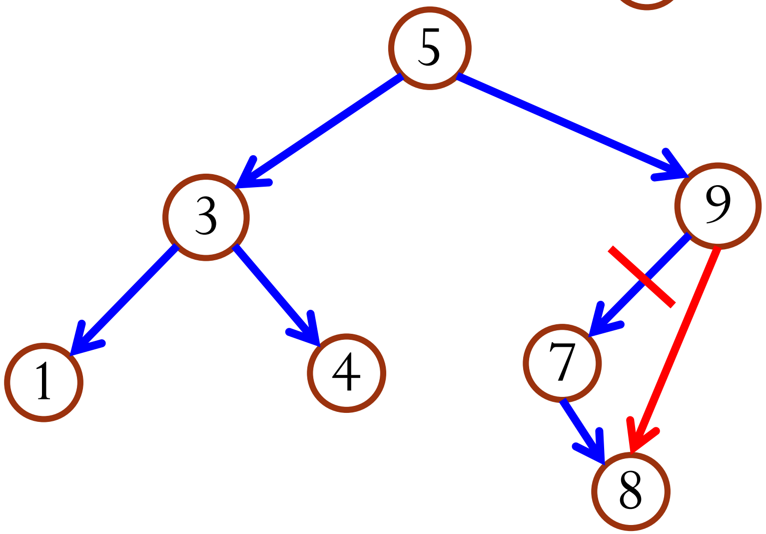


Remove A Degree-One Node

- Remove node 9



- Remove node 7



Remove A Degree-One Node

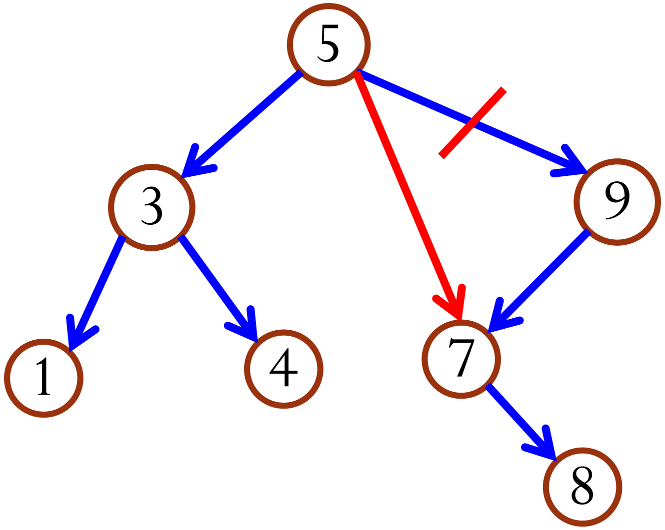
Code

```
else { // remove degree-one or two node
  if(root->right == NULL) { // no right child
    node *tmp = root;
    root = root->left;
    delete tmp;
  }




}
```

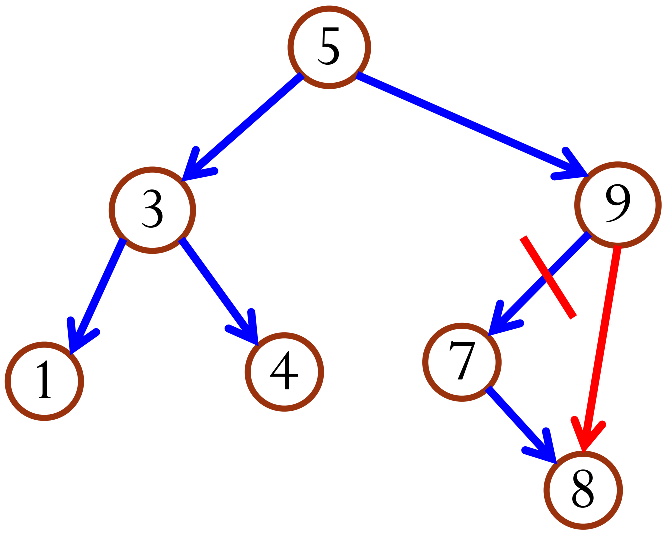
Note the order!



Remove A Degree-One Node

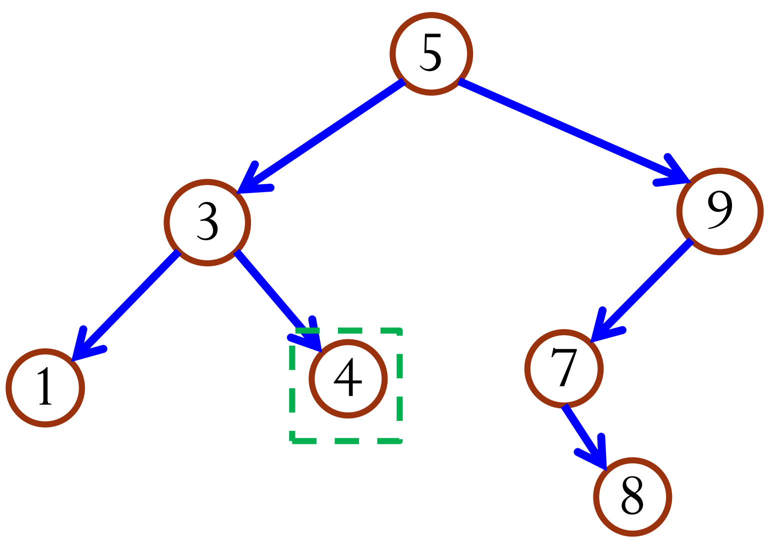
Code

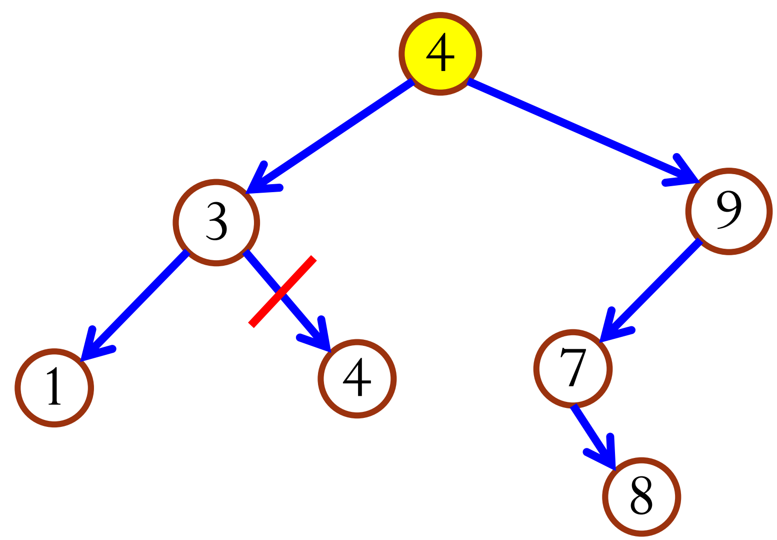
```
else { // remove degree-one or two node
  if(root->right == NULL) { // no right child
    node *tmp = root;
    root = root->left;
    delete tmp;
  }
  else if(root->left == NULL) { // no left child
    node *tmp = root;
    root = root->right;
    delete tmp;
  }
  else {
  // remove degree-two node
  }
}
```



Remove A Degree-Two Node

- Remove node 5 How shall we do this?
- Idea: Replace with the largest key in the left subtree.
  - or replace with the smallest key in the right subtree.
- Claim: The largest key must be in a leaf node or in a degree-one node. Great! We know how to remove such a node!



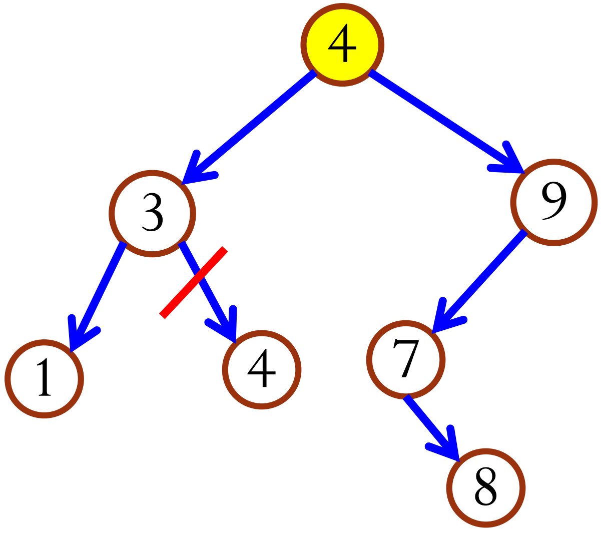


Remove A Degree-Two Node

Code

```
else { // remove degree-two node
  node *&replace = findMax(root->left);
  root->item = replace->item;
  node *tmp = replace;
  replace = replace->left;
  // both leaf and degree-one node are OK
  delete tmp;
}
```

```
node *&findMax(node *&root)
// REQUIRES: tree is non-empty.
// EFFECTS: return the reference
// to the left/right pointer of
// the parent of the node
// that has the largest key in
// the tree rooted at root
```

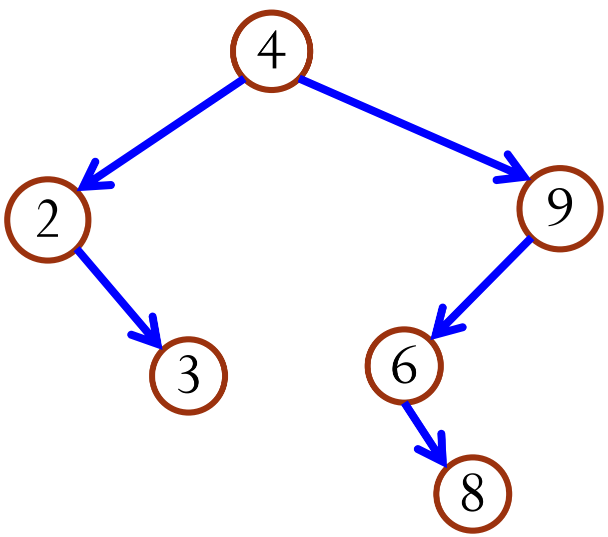


Remove A Degree-Two Node

Code

- How do you implement the function **`findMax()`**?

```
node *&findMax(node *&root) {
  if(root->right == NULL) return root;
  return findMax(root->right);
}
```



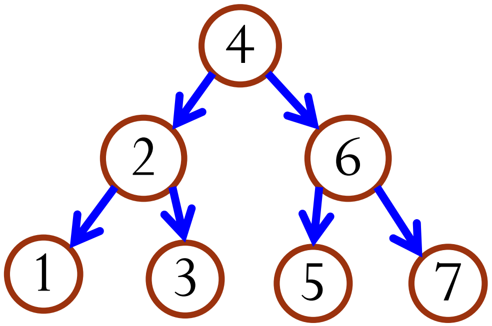
Removal of Binary Search Tree

Summary

- Node to be removed is a leaf.
  - Delete the node.
- Node to be removed is a degree-one node.
  - "Bypass" the node from its parent to its child.
- Node to be removed is a degree-two node.
  - Replace the node key with the largest key in the left subtree and remove the node with the largest key

Exercise

- Insert 4, 2, 6, 3, 7, 1, 5



- Delete 2, insert 9, delete 5, delete 1