VE281

Data Structures and Algorithms

Bloom Filter

Learning Objectives:

- Know what Bloom filter is and how it works
- Know the advantages and disadvantages of Bloom filter

Bloom Filter

- Invented by Burton Bloom in 1970
- Supports **fast insert** and **find**
- Comparison to hash tables:
  - Pros: more space efficient
  - Cons:

1. Can't store an associated object
2. No deletion (There are variations support deletion, but this operation is complicated)
3. Small **false positive** probability: may say x has been inserted even if it hasn't been
   - But no false negative (x is inserted, but says not inserted)

Bloom Filter Applications

- When to use bloom filter?
  - If the false positive is not a concern, no deletion, and you look for space efficiency
- Original application: spell checker
  - 40 years ago, space is a big concern, it's OK to tolerate some error
- Canonical application: list of forbidden passwords
  - Don't care about the false positive issue
- Modern applications: network routers
  - Limited memory, need to be fast
  - Applications include keeping track of blocked IP address, keeping track of contents of caches, etc.

Bloom Filter Implementation: Components

- An array of n **bits**. Each bit 0 or 1
  - $n = b|S|$, where b is small real number. For example, $b \approx 8$ for 32-bit IP address (That's why it is space efficient)
- k hash functions $h_1, \ldots, h_k$, each mapping inside $\{0,1, \ldots, n-1\}$.
  - k usually small.

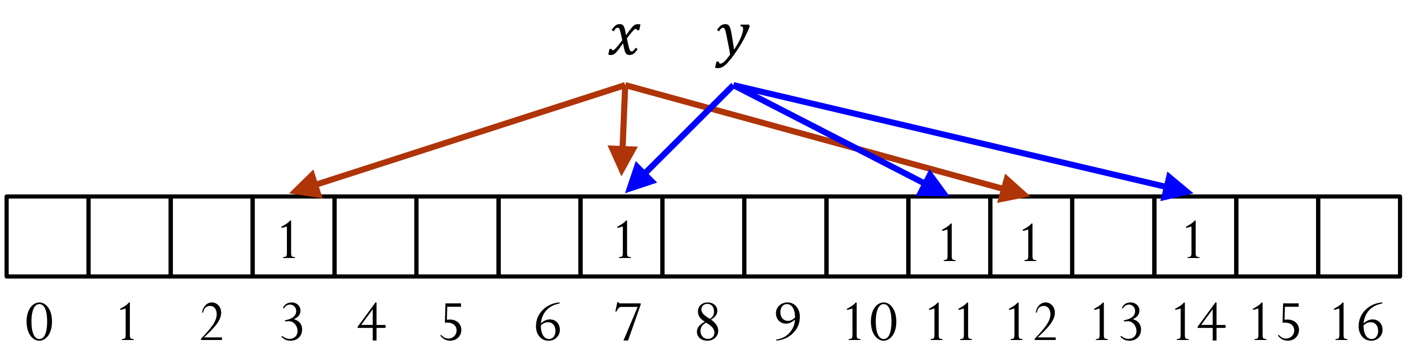
Bloom Filter Insert

- Initially, the array is all-zero.
- Insert x: For $i = 1,2, \ldots, k$, set $A[h_i(x)] = 1$
  - No matter whether the bit is 0 or 1 before

<u>Example</u>: $n = 17$, 3 hash functions

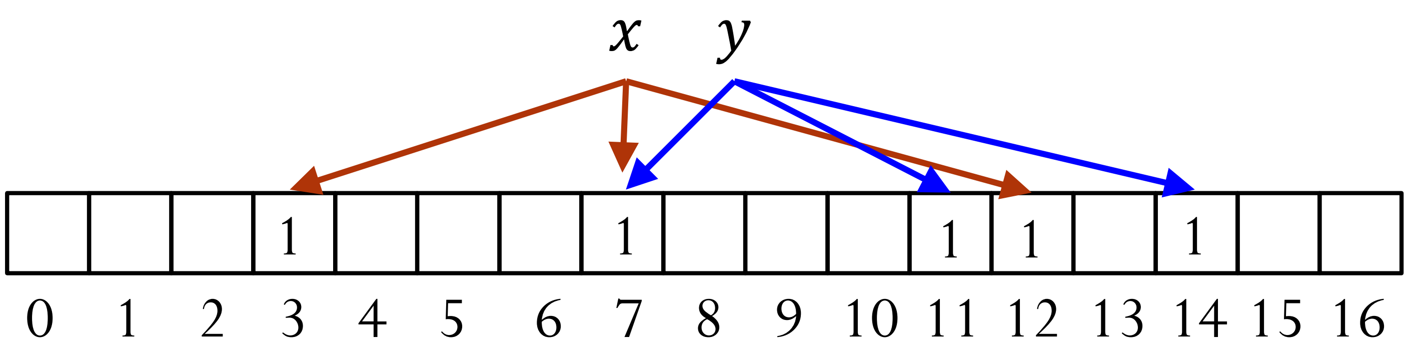
$h_1(x) = 7, h_2(x) = 3, h_3(x) = 12$

$h_1(y) = 11, h_2(y) = 14, h_3(y) = 7$



Bloom Filter Find

- Find x: return true if and only if $A[h_i(x)] = 1, \forall i = 1, \dots, k$



Suppose $h_1(x) = 7, h_2(x) = 3, h_3(x) = 12$. Find x? Yes!

Suppose $h_1(z) = 3, h_2(z) = 11, h_3(z) = 5$. Find z? No!

- No false negative: if x was inserted, find(x) guaranteed to return true
- False positive possible: consider $h_1(w) = 11, h_2(w) = 12, h_3(w) = 7$ in the above example

Heuristic Analysis of Error Probability

- Intuition: should be a trade-off between space (array size) and false positive probability
  - Array size decreases, more reuse of bits, false positive probability increases
- Goal: analyze the false positive probability
- Setup: Insert data set S into the Bloom filter, use k hash functions, array has n bits
- Assumption: All k hash functions map keys uniformly random and these hash functions are independent

Probability of a Slot Being 1

- For an arbitrary slot j in the array, what's the probability that the slot is 1?

- Consider when slot j is 0
  - Happens when $h_i(x) \neq j$ for all $i = 1, \dots, k$ and $x \in S$
  - $\Pr(h_i(x) \neq j) = 1 - \frac{1}{n}$
  - $\Pr(A[j] = 0) = \left(1 - \frac{1}{n}\right)^{k|S|} \approx e^{-\frac{k|S|}{n}} = e^{-\frac{k}{b}}$
    - $b = \frac{n}{|S|}$ denotes # of bits per object
- $\Pr(A[j] = 1) \approx 1 - e^{-\frac{k}{b}}$

False Positive Probability

- For x not in S, the false positive probability happens when all $A[h_i(x)] = 1$ for all $i = 1, \ldots, k$
  - The probability is $\epsilon \approx \left(1 - e^{-\frac{k}{b}}\right)^k$

- For a fixed b, ϵ is minimized when $k = (\ln 2) \cdot b$
- The minimal error probability is $\epsilon \approx \left(\frac{1}{2}\right)^{\ln 2 \cdot b} \approx 0.6185^b$
  - Error probability decreases exponentially with b
- Example: $b = 8$, could choose k as 5 or 6. Min error probability $\approx$ 2%