Problem Solving with AI Techniques
Reasoning under Uncertainty

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UM-SJTU Joint Institute

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1. Reasoning under Uncertainty
2. Example
3. Graphical Models
4. Applications

Why Reasoning under Uncertainty?

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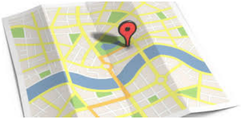

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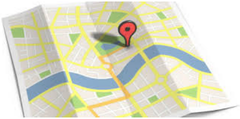

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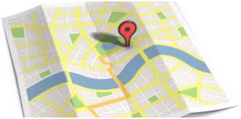

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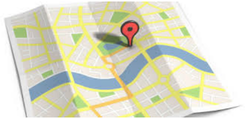

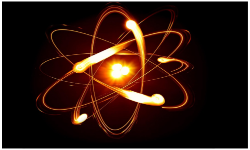

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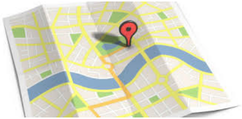

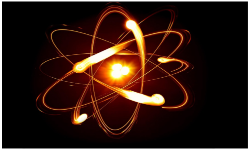

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- Previous search algos are therefore not directly applicable anymore

What is Reasoning under Uncertainty?

- A state is an assignment of $X_{1:n} = (X_1, \ldots, X_n)$
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- Issue: memory requirement to store $\mathbb{P}$

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Medical Diagnosis of Lung Cancer

Variables	Type	Values
Pollution (P)	Binary	{Low, High}
Smoker (S)	Boolean	{True, False}
Cancer (C)	Boolean	{True, False}
Dyspnoea[1] (D)	Boolean	{True, False}
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- We can then easily compute probabilities for any event: e.g., $\mathbb{P}(C = True), \mathbb{P}(D = True), \mathbb{P}(C = True, S = F)$...
- Even better, we can compute conditional probabilities: e.g., $\mathbb{P}(C = True \,|\, D = True)$, $\mathbb{P}(C = False \,|\, X = True)$, $\mathbb{P}(C = True \,|\, S = True, X = False)$...

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$$\mathbb{P}(P, S, C, D, X) = \mathbb{P}(P) \times \mathbb{P}(S) \times \mathbb{P}(C \mid P, S) \times \mathbb{P}(D \mid C) \times \mathbb{P}(X \mid C)$$

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  what is the space requirement now?
- Huge saving in large domains with many (conditional) independences

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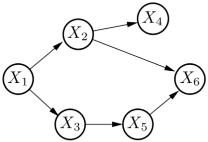
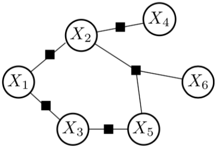
Graphical Models

- What is a graphical model?
  - Graphical notation to express all the (random) variables and their (conditional) (in)dependences
  - (Compact) representation of a joint distributions over the variables
- Two basic variants:
  - Directed model: Bayesian network or belief network
  - Undirected model: factor graph or Markov random field



$P(x_{1:6}) = P(x_1)\ P(x_2|x_1)\ P(x_3|x_1)\ P(x_4|x_2)\ P(x_5|x_3)\ P(x_6|x_2, x_5)$



$P(x_{1:6}) = f_1(x_1, x_2)\ f_2(x_3, x_1)\ f_3(x_2, x_4)\ f_4(x_3, x_5)\ f_5(x_2, x_5, x_6)$

From Marc Toussaint

Why are Graphical Models Useful?

- Why do we need graphical models?
  - Real problems have many variables
  - Real problems have some structure (i.e., conditional probabilistic independences)

- What can we do with graphical models?
  - Are two variables independent given a third one?
  - What is the probability of some event?
  - What is the probability of some event given some observation?

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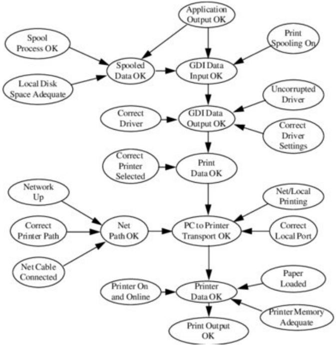
Diagnostics

- Medical diagnosis
- Printer troubleshooting

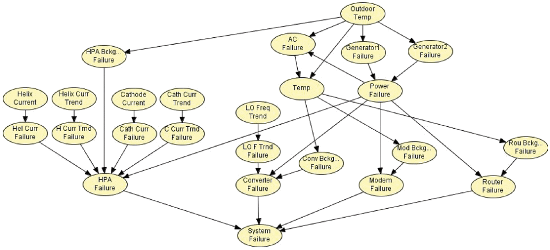


Heckerman and Breese, 1996

- Equipment failure
- Intelligent tutoring

Prediction and decision-making

- Failure prediction in satellite



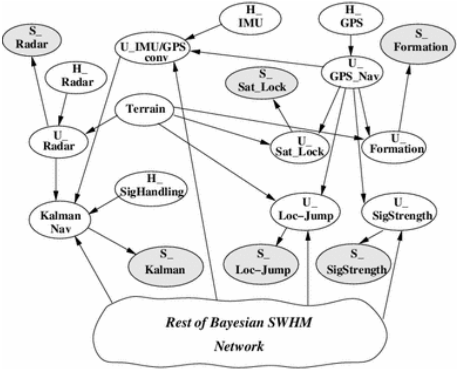
Bottone et al., 2008

- Disease prediction
- Price evolution in stockmarket

- With utilities, optimal decisions with influence diagrams
- More about this, in the last part of this course

Anomaly Detection

- Software health management in UAVs

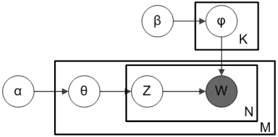


Schumann et al., 2013

- Breakdown detection
- Fraud detection in banking and finance
- Spam detection

Machine Learning Models

- Any probabilistic ML model could exploit Bayes net
- Many applications in computer vision, speech recognition, bioinformatics...
- e.g., Latent Dirichlet Allocation in NLP



α parameter of Dirichlet prior of per-document topic distribution
β parameter of Dirichlet prior of per-topic word distribution
θ_m topic distribution for document m
ϕ_k word distribution for topic k
z_{mn} topic for for n-th word for document m
w_{mn} n-th word for document m