Problem Solving with AI Techniques
Uninformed Search

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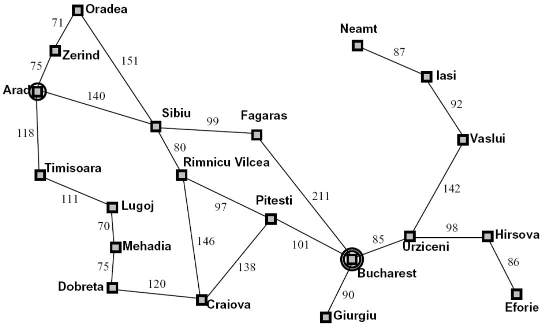
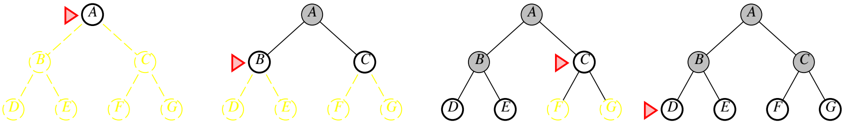
Traveling in Romania

- State space
  - Cities
- Successor function
  - Roads: Go to adjacent city with cost = distance
- Start state
  - Arad
- Goal test:
  - is state = Bucharest?
- Solution?

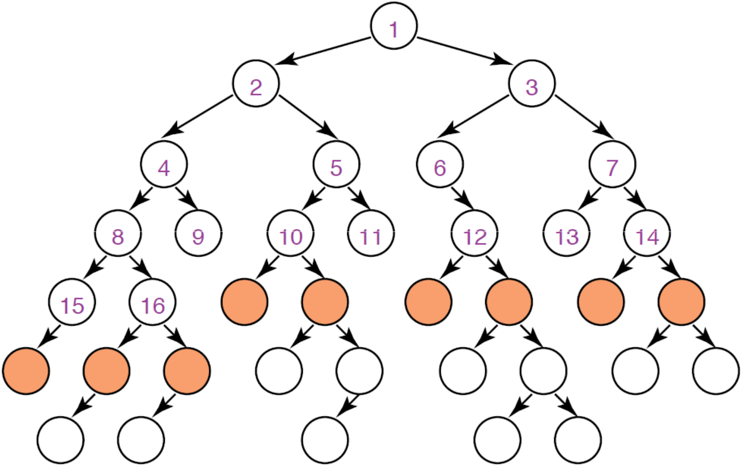


Principle of Breadth-First Search (BFS)

- Strategy: expand shallowest unexpanded node
- Implementation: fringe = queue (FIFO); put successors at end



Other Illustrative Example of Breadth-First Search



[Poole Mackworth 2010]

Properties of Breadth-First Search

- Complete? Yes (if branching factor b is finite)
- Time? $\mathcal{O}(1 + b + b^2 + \ldots + b^d + b(b^d - 1)) = \mathcal{O}(b^{d+1})$
  exponential in d
- Space? $\mathcal{O}(b^{d+1})$, keeps every node in memory
- Optimal? Yes, with all equal action costs (e.g., $= 1$)

- *Space* is the bigger problem (more than time)

Principle of Uniform-Cost Search (UCS)

- Breadth-first search optimal only if path cost is non-decreasing function of depth
  for all nodes n_{d-1} at depth $d-1$ and n_d at depth d
  $cost(n_0, \ldots, n_d) \geq cost(n_0, \ldots, n_{d-1})$
- Can we guarantee optimality for any positive action cost?

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  $cost(n_0, \ldots, n_d) \geq cost(n_0, \ldots, n_{d-1})$
- Can we guarantee optimality for any positive action cost?

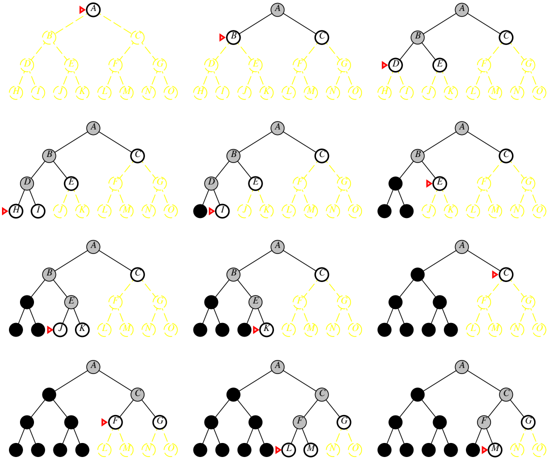
- Strategy: expand least-cost unexpanded node
- Implementation: fringe = priority queue ordered by path cost, lowest first
- Equivalent to breadth-first search if action costs all equal
- Cost-aware breadth-first search

Properties of Uniform-Cost Search

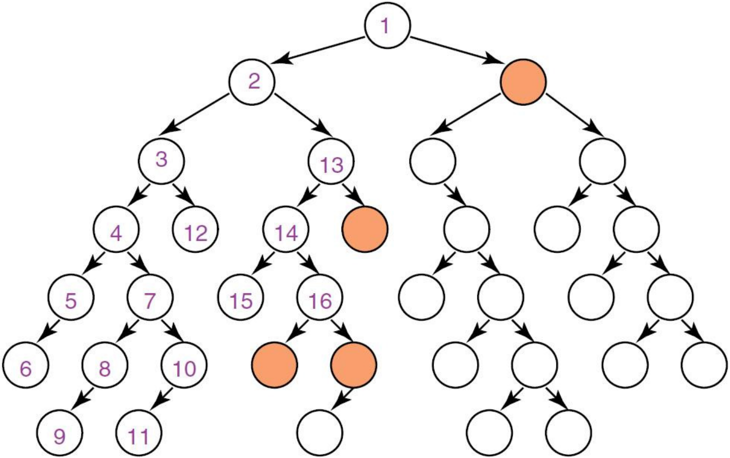
- Complete? Yes, if action cost $\geq \varepsilon$ (i.e., strictly positive)
- Time? $\#$ of nodes with $g \geq$ cost of optimal solution c^*, $\mathcal{O}(b^{\lceil \frac{c^*}{\varepsilon} \rceil})$
- Space? $\#$ of nodes with $g \geq$ cost of optimal solution c^*, $\mathcal{O}(b^{\lceil \frac{c^*}{\varepsilon} \rceil})$
- Optimal? Yes, nodes expanded in increasing order of $g(n)$ where $g(n)$ cost of subpath from root to n

Principle of Depth-First Search (DFS)

- Strategy: expand deepest unexpanded node
- Implementation: fringe = stack (LIFO); put successors at front



Other Illustrative Example of Depth-First Search



[Poole Mackworth 2010]

Properties of Depth-First Search

- Complete? No: fails in infinite-depth state-spaces and state-spaces with loops

  Can be modified to avoid repeated states along path $\Rightarrow$ complete in finite state-space graphs
- Time? $\mathcal{O}(b^m)$ where b is maximum branching factor of search tree

  bad if $m >> d$, but may be much faster than breadth-first search if solutions are dense
- Space? $\mathcal{O}(bm)$, i.e., linear space!
- Optimal? No

Principle of Depth-Limited Search (DLS)

- Depth-first search complete only for finite state-space graphs
- Can we modify it to make it complete?
- Idea: depth-limited search with depth limit l, i.e., nodes at depth l are treated as having no successors
- Depth-limited search is complete if we know l such that a solution exists at depth lower or equal than l

Algorithm

```
function DEPTH-LIMITED-SEARCH(problem, limit) returns soln/fail/cutoff
    RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
    cutoff-occurred? ← false
    if GOAL-TEST(problem, STATE[node]) then return node
    else if DEPTH[node] = limit then return cutoff
    else for each successor in EXPAND(node, problem) do
        result ← RECURSIVE-DLS(successor, problem, limit)
        if result = cutoff then cutoff-occurred? ← true
        else if result ≠ failure then return result
    if cutoff-occurred? then return cutoff else return failure
```

Principle of Iterative Deepening Search (IDS)

- What if we don't know such l?
- Idea: Try them all: 0, 1, 2, 3, ...

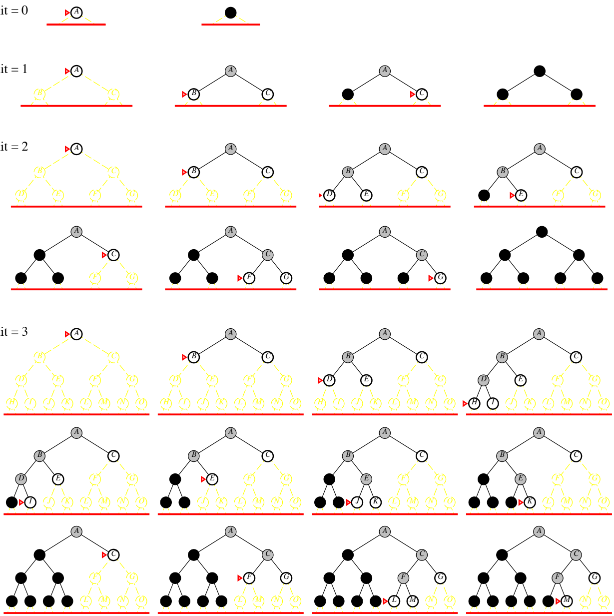
```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH(problem, depth)
        if result ≠ cutoff then return result
    end
```

Limit = 0

Limit = 1

Limit = 2

Limit = 3



Properties of Iterative Deepening Search

- Complete? Yes, if a solution exists at a finite depth
- Time? $\mathcal{O}((d+1)b^0 + db^1 + (d-1)d^2 + \ldots + b^d) = \mathcal{O}(b^d)$
- Space? $\mathcal{O}(bd)$
- Optimal? Yes, if all equal action costs

Comparison IDS/DLS

- Number of nodes generated in a depth-limited search to depth d with branching factor b:

$$N_{DLS} = b^0 + b^1 + b^2 + \ldots + b^{d-2} + b^{d-1} + b^d$$

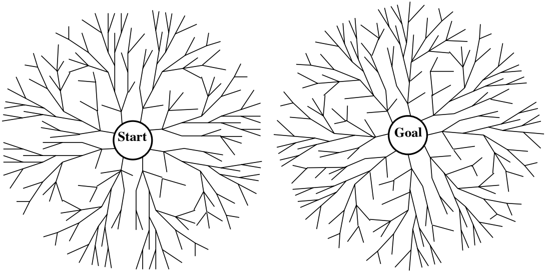
- Number of nodes generated in an iterative deepening search to depth d with branching factor b:

$$N_{IDS} = (d+1)b^0 + db^1 + (d-1)b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Example: for $b = 10, d = 5$:
  - $N_{DLS} = 1 + 10 + 100 + 1,000 + 10,000 + 100,000 = 111,111$
  - $N_{IDS} = 6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456$
- Overhead $= (123,456 - 111,111)/111,111 = 11\%$

Principle of Bidirectional Search

- Strategy: run two simultaneous searches, one forward from initial state and the other backward from goal state, stop when they meet
- Implementation: one fringe stored as hash table



Properties of Bidirectional Search

- Complete? Yes, with two breadth-first searches
- Time? $\mathcal{O}(b^{\frac{d}{2}})$
- Space? $\mathcal{O}(b^{\frac{d}{2}})$
- Optimal? Yes, with two breadth-first searches if all action costs equal

- Space requirement still exponential
- For $d = 6$, $b = 10$, $22,200$ generated nodes vs $11,111,100$ for BFS
- Issue: How to search backward?

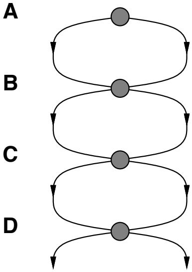
Summary

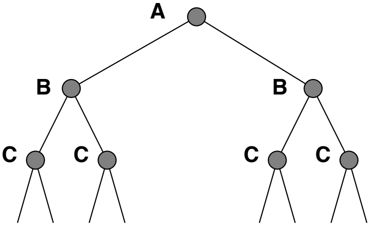
Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional
Complete	Yes[1]	Yes[1,2]	No	Yes[3]	Yes[1]	Yes[1,4]
Time	b^{d+1}	$b^{\lceil \frac{c^*}{\varepsilon} \rceil}$	b^m	b^l	b^d	$b^{\frac{1}{2}}$
Space	b^{d+1}	$b^{\lceil \frac{c^*}{\varepsilon} \rceil}$	bm	bl	bd	$b^{\frac{1}{2}}$
Optimal?	Yes[5]	Yes	No	No	Yes[5]	Yes[4,5]

[1] If b finite
[2] If costs > 0
[3] If $l \geq d$
[4] If both directions use breadth-first search
[5] If costs identical

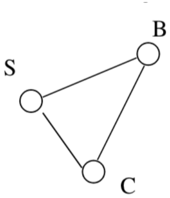
Repeated States

- Failure to detect repeated states can turn a linear problem into an exponential one!

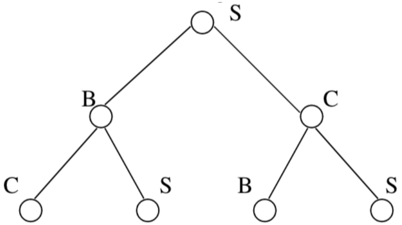




Repeated States: Cycles



State Space



Example of a Search Tree

- During search, never regenerate a visited state
  - must keep track of all visited states (needs a lot of memory)
  - approximation for DFS/DLS: only avoid states in a fixed limited memory
  - optimal for BFS and UCS, not for DFS

Graph Search

```
function GRAPH-SEARCH(problem, fringe) returns a solution, or failure

    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            fringe ← INSERTALL(EXPAND(node, problem), fringe)
    end
```