a path clears a set of obstacles ithe intersection between its line segments and the union of all these obstacles is empty. Note that the case of a robot with a cylindrical geometry of diameter d (i.e., a robot at a position P occupies a region of the plane that corresponds to a disk of diameter d centered in P, where d is a parameter of the robot) can straightforwardly be handled by growing obstacles by d/2 (i.e., by computing the Minkowski sum of the obstacles with a disk of diameter d).

The purpose of the clearance parameter ci is to provide additional information about the location of the obstacles that are avoided when moving along the segments [pi −1,pi] and [pi ,pi+1]. This is achieved by considering a disk Di that is tangent to both segments (which implies that its center belongs to the inner bisector of the angle formed by these segments), and that fully covers the obstacles cleared by the pair of segments..

Without loss of generality, we assume βi , 0 different from 0

In the case of adjacent segments forming an acute angle, it then becomes necessary to interleave an intermediate segment between them.

The speed that can be reached by the robot at some point of a path is then, among others, bounded by a function of the absolute curvature |κ| at this point, as well as the rate of variation dκ ds of this curvature with respect to the linear travelled distance. \_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_SOLUTION\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

In order to bound the absolute curvature throughout the path, we build a curve composed of straight line segments (with zero curvature) and circle arcs (with constant curvature), connected in such a way that continuity of the tangent vector is ensured everywhere. On such a curve, the curvature can be expressed as a piecewise constant function with respect to travelled distance.

This solution also applies to pairs of adjacent segments that turn in opposite directions; in such a case, small values of βi (which represent small changes of direction) lead to small circle arcs, and large values of βi to large arcs, which is geometrically sound.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_STEP 2

As already discussed, we construct such curves out of clothoids, which, as already said, correspond to the time-optimal trajectories of dierential-drive robots (i.e., the paths followed by dierential-drive robots when their wheels are driven at respectively their minimum and maximum acceleration).

Clothoids are formally dened as curves with a curvature that varies linearly with travelled distance.

The curvature κ of the clothoid is given by κ(s) = κ0 + cs, (5.4) where κ0 is the initial curvature,c is the clothoid sharpness (i.e., rate of change of curvature), and s denotes the travelled distance.

Recall from Chapter 3 that the curvature of a path is dened by κ(t) = dθ ds , where θ (s) denotes the tangential angle of the path, we have θ (s) = θ0 + κ0s + 1 2 cs2 , (5.5) where θ0 is the initial tangential angle. The general4 parametric expression of a clothoid is then     x (s) = x0 + s 0 cos π 2 + θ0 + κ0u + 1 2 cu2 du y(s) = y0 + s 0 sin π 2 + θ0 + κ0u + 1 2 cu2 du. (5.6) Figure 5.5 shows the unit clothoid (x0 = y0 = κ0 = 0, θ0 = − π 2 , c = 1). The coordinates of the points visited by a clothoid are thus expressed in terms of the integrals S (x) and C(x) S (x) = x 0 sint 2 dt, C(x) = x 0 cost 2 dt, which are known as the Fresnel integrals. These integrals cannot be evaluated analytically, but, as already mentioned, can be very eciently approximated [Mie00]. Note that, from Equations 2.5, 2.6, 2.18, 2.19 and 2.20, one can easily establish that a dierential robot with its wheels driven at respectively their minimum and maximum acceleration actually follows a clothoid curve

**Computing a Pair of Clothoids** We now show how we use clothoids in order to construct a smooth path that clears obstacles. Consider a circle arc with curvatureκC, interpolating two successive line segments [pi −1,pi] and [pi ,pi+1] within their safe zone. Let ti1 and ti2 denote the points of tangency between

this circular arc and receptively the lines segment [pi −1,pi] and [pi ,pi+1]. Let also θ1 and θ2 denote the tangential angles at these two points. Assuming w.l.o.g. κC > 0 (the case κC < 0 is handled symmetrically), we have established the following result. Theorem 5.4.1. For every rotation angle βi ∈ ]0, π 2 ] and for every pair of curvatures κ1,κ2 such that 0 ≤ κ1 < κC and 0 ≤ κ2 < κC, there exist two clothoids arcs moving respectively from the curvatures κ1 to κM and from κM to κ2, with κM > κC, the concatenation of which interpolates the path from ti1 to ti2 within the safe zone, with initial and nal tangential angles of respectively θ1 and θ2 = θ1 + βi and with continuity of the tangent vector at the junction point between the two curves. The parameters of these two clothoids arcs are uniquely determined by κ1, κ2, κC, and the rotation angle βi . Our method for characterizing the two clothoids arcs consists in reasoning on a diagram expressing the curvature of the interpolated path as a function of travelled distance. The problem is illustrated in Figure 5.6 (exaggerating the curvatures in order to make the interpolated path stand out from the circle arc). A proof of this theorem is provided in Appendix B.