Control of Inverted Pendulum System Using LabVIEW

Research · February 2018

DOI: 10.13140/RG.2.2.31142.22083

3 authors:

Devendra Rawat
University of Petroleum & Energy Studies

Deepak Kumar
University of Petroleum & Energy Studies

Deepali Yadav
University of Petroleum & Energy Studies

ISSN 2229-5518

Control of Inverted Pendulum System Using LabVIEW

Devendra Rawat[a], Deepak Kumar[a*], Deepali Yadav[a]
[a] Department of Electrical engineering, School of Engineering
University of Petroleum & Energy Studies, Dehradun
Uttrakhand, India-248007

d.rawat@ddn.upes.ac.in deepak.kumar@ddn.upes.ac.in dyadav@ddn.upes.ac.in

Abstract— Advantageous use of PC in last two decades has resulted a revolution in instrumentation for measurement and automation. Virtual instrumentation (VI) is the result of this revolution. VI is the use of measurement and control hardware and an industry standard computer with powerful application software to create applications as per the need of user. LabVIEW is application software used in VI system. LabVIEW is an essential part of a VI because it offers data acquisition and better interfacing with hardware. Owing to its non-linear behavior, Inverted Pendulum (IP) is highly unstable and observed to be the potential candidate for control problem. This paper describes the modeling of IP system followed by the pole placement based state feedback controller and Linear Quadratic Regulator (LQR) is simulated in LabVIEW. System states are estimated and observer based controller is designed for the system. The performance of controllers is observed with the help of virtual instrument.

Keywords-- *Inverted Pendulum, LQR, Lab-VIEW, Lagrangian, Pole placement, State Feedback, State observer, Virtual Instrumentation*

I. Introduction

IP is one of the most haunted problems of classical control engineering. IP on a cart is a nonlinear and unstable control problem. The complexity of the problem depends also on the flexibility of pendulum rod .The problem is to balance a pole on a mobile platform that can move in only two directions, to the left or to the right. The basic principle in rocket or missile propulsion uses the concept of this inverted pendulum system. Virtual instrumentation is the measurement technique with which we can perform real time measurements by creating user defined applications in LabVIEW software. LabVIEW is a graphical programming environment suited for high level or system level design. Software component of a virtual instrument is the main difference between natural and virtual instrumentation. With the help of software complex and expensive equipment are being replaced by simpler and less expansive hardware. In [3, 5, 9, and 10] the inverted pendulum on a moving cart system was stabilized by LQR controller with linearized state space model. In [12] Pole Placement based State feedback control system design with and without estimation has been introduced. Robustness is essential feature for inverted pendulum system due to its intrinsic non linearity, so in [14], a robust compensation has been developed by seeming dry friction and endeavored to implement it in the IP system. A robust periodic controller with zero placement capability for IP system has been designed by authors in [15]. Various control schemes have been developed by authors like Fuzzy, Sliding Mode, Expert systems and Neural networks [13] and their comparative study has been attempted in [16].

In this paper, the inverted pendulum system modeling i.e. the mathematical model of the pendulum system is described in the section II. The mathematical modeling is based on the Lagrangian which is the conservation of energy. In III and IV section the pole placement based state feedback controller and LQR controllers are described. Section V describes the state observer and observer based state feedback controller. The simulation results for the pole placement, LQR and observer based controllers are given in the section VI and conclusion is given in section VII. Finally the references are given in section VIII.

II. ARCHITECTURE OF INVERTED PENDULUM

An inverted pendulum (IP) is a basic as well as important problem of control system shown in Fig.1. This is a non linear and highly unstable process. It has one input and two output signals. Here our objective is to balance a pendulum vertically on a motor driven wagon. When the wagon moves along the x direction the pendulum should

ISSN 2229-5518

not fall. A DC motor is used to drive the wagon; a controller is controlling the motor.

The objective here is to balance the pendulum at a vertical position on a motor driven cart, with specific desired settling time for angular position of pendulum and displacement of cart. The angular displacement of the inverted pendulum from vertical should be zero. The linearized model is derived using the Lagrangian equations of motion. The Lagrangian can be defined as

$$\lambda = K - U \qquad (1)$$

Where, K is kinetic energy of system and U is the potential energy. The Lagrangian equations can be written as:

$$\frac{\partial}{\partial t}\left(\frac{\partial \lambda}{\partial \dot{\theta}}\right) - \frac{\partial \lambda}{\partial \theta} = 0 \qquad \frac{\partial}{\partial t}\left(\frac{\partial \lambda}{\partial \dot{x}}\right) - \frac{\partial \lambda}{\partial x} = u \qquad (2)$$

These lagrangian equations are just an equivalent representation of newtons equations of motion. The system parameters for inverted pendulum on a moving cart are given following.

X: position of the cart or displacement of the cart

θ : Pendulum angle from vertical

F: force applied on the cart

M: mass of the cart

m: mass of the pendulum

l: length to pendulum centre of mass

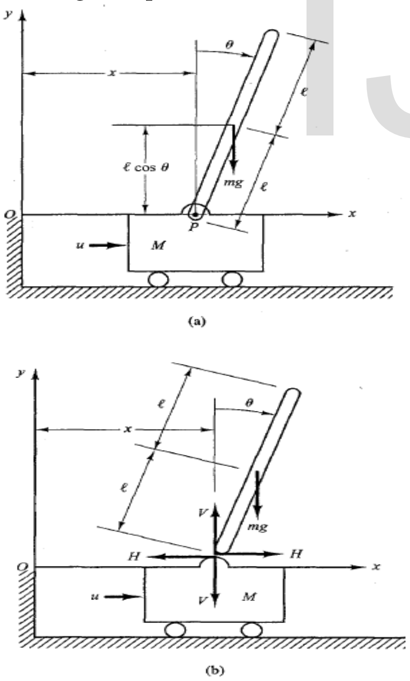

Fig.1. (a) Inverted Pendulum System (b) Free Body Diagram

The system equations in state space form can be written as following

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \left(\frac{mg}{Ml} + \frac{g}{l}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{mg}{M}\right) & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + [0]u \qquad (3)$$

III. Pole placement based controller for inverted pendulum

The pole placement design [8] is a method to place all the closed loop poles at the desired locations. The sufficient and necessary condition for the arbitrary placement of closed loop poles (in the complex plane) is such that:

$$\dot{X}(t) = AX(t) + Bu(t) \qquad (4)$$
$$Y(t) = CX(t)$$

is controllable, If all n state variables x_1, x_2,.., x_n can be accurately measured for any instant of time, according to linear control law of the form

$$u(t) = -k_1x_1(t) - k_2x_2(t) - \ldots\ldots - k_nx_n(t) = -kX(t) \qquad (5)$$

$$K = [k_1, k_2, \ldots k_n] \qquad (6)$$

Where, K is a constant state feedback gain matrix. This K can be obtained with Ackerman's formula [8].The closed loop system is described by the state differential equation:

$$\dot{X}(t) = (A - BK)X(t) \qquad (7)$$

The feedback gain matrix [K] should be insensitive to any changes in system. It should be robust in nature. The closed loop should not be affected by any disturbance.

IV. LQR controller for inverted pendulum

LQR is a state feedback controller which minimizes the performance index J of a controllable and linear system [8][3][5]. Consider the linear system the quadratic objective function J (or cost function) as

$$J = \frac{1}{2}\int_0^T x^T Q x + u^T R u)dt \qquad (8)$$

LQR minimize J with respect to the control input u (t). Now we see that J represents the weighted sum of energy of the state and control. In general case, Q and R represent respective weights on different states and control channels. The main design parameters are Q and R, such that Q be symmetric positive semi definite and R symmetric positive

definite for a meaningful optimization problem. The controller of the feedback form is given as
$U(t) = -k(t)\,X(t)$ where $K(t) = R^{-1}B^{T}P(t)$ (9)

Here also k state feedback gain matrix is of the form

$$K = [k_1, k_2, \ldots k_n] \quad (10)$$

LQR approach gives us optimal value of K for which system behaves could be stable under applied constraints.

V. obserevr based controller for inverted pendulum

While designing pole placement based state feedback controller author(s) have assumed the availability of all the state variables for feedback but, in practice, this is not the case (a few seems to be unavailable). So author(s) need to estimate these unavailable state variables. The state observer estimates the state variables based on the measurement of outputs and control variables of the system [8].
Mathematical model for observer can be written as following by considering the plant as equation no. 4.

$$\tilde{X} = (A - LC)\tilde{X} + Bu + LY \quad (11)$$

where $\tilde{X}$ estimated state and $C\tilde{X}$ is the estimated output. L is observer gain matrix.
The Eigen values of *(A-LC)* are chosen in such a way that the dynamic behavior of error vector is asymptotically stable.
Consider completely state controllable and completely observable system.

$$\dot{X}(t) = AX(t) + Bu(t)$$
$$Y(t) = CX(t) \quad (12)$$

The state feedback control based on the observed-state is stated as:

$$u = -K\tilde{X} \quad (13)$$

Assuming this state of control, the state equation becomes:

$$\dot{X} = (A - BK)X + BK(X - \tilde{X}) \quad (14)$$

The difference between true state and observer state can be written as $e(t) = X(t) - \tilde{X}(t)$ (15)
By putting this error in observer state equation it becomes:

$$\dot{X} = (A - BK)X + BKe \quad (16)$$

The equation describing observer-error is:

$$\dot{e} = (A - LC)e \quad (17)$$

So the dynamics of observed state feedback control system can be written as

$$\begin{bmatrix} \dot{X} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} X \\ e \end{bmatrix} \quad (18)$$

VI. simulation results for pole placement,lqr and obserever based controller

System parameters for inverted pendulum system are taken as given in table I.

TABLE I
VALUES OF THE SYSTEM PARAMETERS

Symbol	Parameter	Value
M	Mass of cart	10 kg
m	Mass of pendulum rod	1 kg
l	Length of pendulum	0.5 m
g	Gravitational constant	9.8 m/s2

The state space representation for the system is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 21.56 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.98 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ -0.20 \\ 0 \\ .10 \end{bmatrix} \quad (19)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} D = [0] \quad (20)$$

These are the parameter coefficient matrices which describe the system with chosen parameter.

A. controllabilty and observabilty test

The necessary and sufficient condition of the design of state feedback based controllers is that the system must be completely state controllable[8] and for observer based controller is that the system must be completely state controllable and observable. A Linear time invariant system is controllable if rank of the controllability matrix is n.

$$rank\left[B : AB : \ldots\ldots A^{n-1}B\right] = n \quad (21)$$

A Linear time invariant system is observable if rank of the observability matrix is n.

$$rank\left[C^T : A^T C^T : \ldots\ldots (A^T)^{n-1} C^T\right] = n \quad (22)$$

So the controllability and observability for the system are checked with LabVIEW. As a result system is found to be controllable, observable, detectable and stablizable.

International Journal of Scientific & Engineering Research Volume 9, Issue 1, January-2018
ISSN 2229-5518

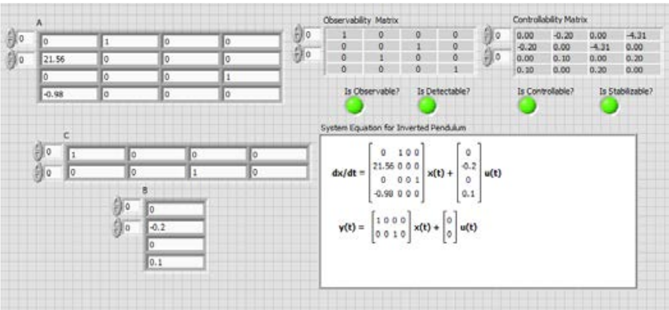

Fig.2. Screen shot of LabVIEW Front panel showing that the Inverted pendulum system is controllable and observable.

B. Pole placement controller

The pole placement based method successfully gives the stable responses for pendulum angle and cart position for inverted pendulum system. In designing this pole placement controller we have placed the system poles at a location given by matrix *P*.

P= [-4+4i, -4-4i, -7, -8]

The desired pole locations have been calculated by considering the transient response characteristics and comparing it to standard characteristic equation [8]. We obtained the state feedback gain matrix using Ackerman's formulae as

$$K = \begin{bmatrix} -1604.9 & -357.73 & -914.29 & -473.47 \end{bmatrix} \tag{23}$$

The Block diagram for pole placement controller has been shown in Figure. 3 and the closed loop responses for pendulum angle and cart position have been shown in Fig. 4. Our objective is to get the settling time near to 5 seconds. The desired settling time has been obtained for both output variables but cart position has steady state error. While designing this pole placement controller we have assumed that all the states are available for feedback.

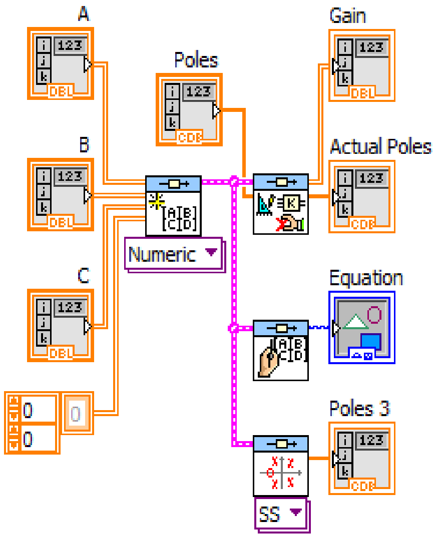

Fig.3. Screen shot of LabVIEW Block diagram for pole placement of Inverted pendulum system

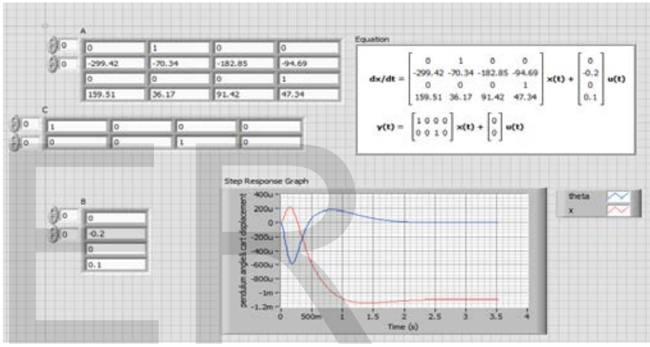

Fig.4. Screen shot of LabVIEW Front panel for pole placement response of Inverted pendulum system

C. LQR Controller

For LQR controller we need to decide the state weighted matrix Q as following.

$$Q = C^T * C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{24}$$

and *R*=1(assumption). The state feedback gain matrix is procured to be:

$$K_1 = \begin{bmatrix} -236.36 & -51.13 & -1.0 & -5.12 \end{bmatrix} \tag{25}$$

The block diagram for LQR controller has been shown in Fig. 5. The closed loop step response of for both the system's output pendulum angle and cart position is shown in Fig. 6.

The settling time for both the pendulum angle and position of cart is about 20 seconds but we want it to be less than 5 seconds. So the *Q* matrix will be tuned for improving the settling time and other specification. Now we select *Q* as Q_2

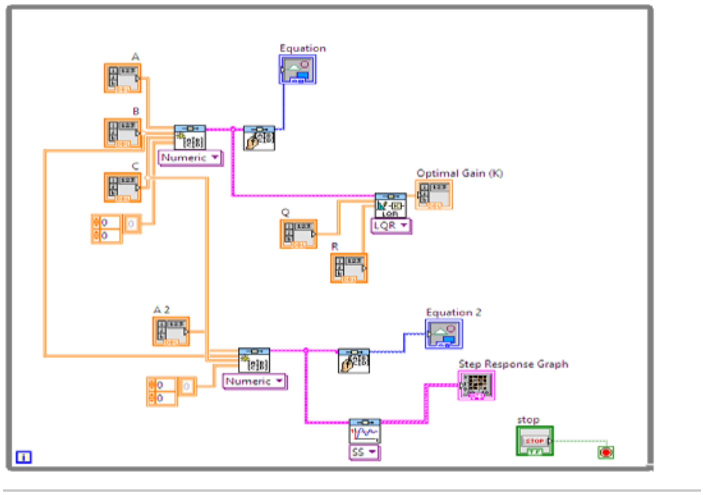

Fig.5. Screen shot of LabVIEW Block diagram for pole placement of Inverted pendulum system

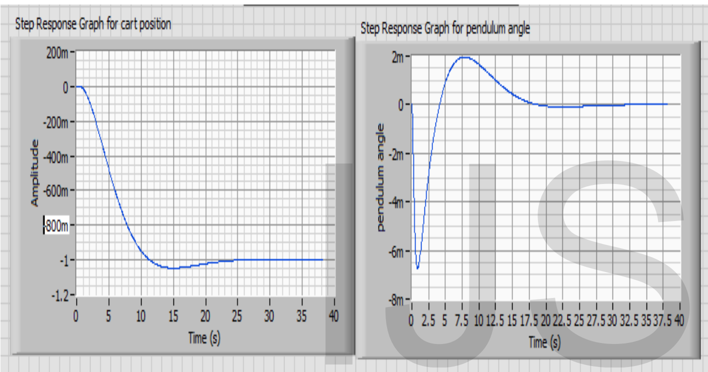

Fig.6. Screen shot of LabVIEW Front panel for step response of LQR for Inverted pendulum system with Q_1.

$$Q_2 = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (26)$$

We can do this by increasing the non-zero diagonal values in the matrix Q [6].

Q the state weighted matrix must be symmetric and positive semidefinite as to keep the error squared positive.

q_{11}- weights to the position of cart;

q_{33}- weights to the pendulum angle;

Since there is a constraint on position of cart it has to be in a range so this factor is very important in tuning of *Q* so we will weight to it more $q11 >>> q_{33}$.

State feedback gain matrix in this case can be obtained as:

$$K_2 = \begin{bmatrix} -307.39 & -66.80 & -14.14 & -24 \end{bmatrix} \qquad (27)$$

In Fig. 6 the closed loop response for both the outputs i.e pendulum angle and cart position is shown and we obtain the settling time for pendulum angular position is 6 seconds and for cart position is 7 seconds which is not as per our requirement so we need to further tune the matrix *Q*.

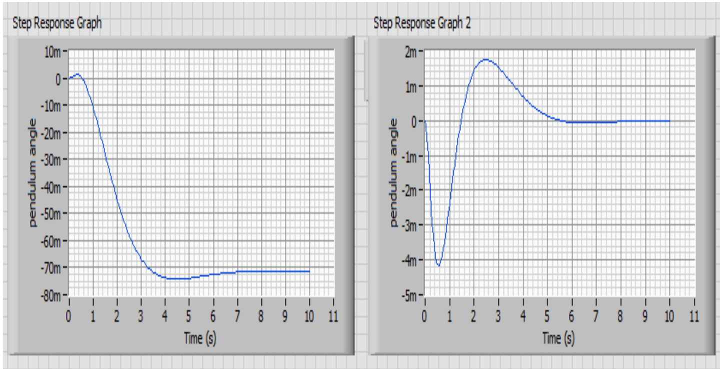

Fig.7. Screen shot of LabVIEW Front panel for step response of LQR for Inverted pendulum system with Q_2.

If we further tuned the Q the settling time can be reduced. Now we take Q as Q_3.

$$Q_3 = \begin{bmatrix} 5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (28)$$

We get the state feedback gain matrix as

$$K_3 = \begin{bmatrix} -321.1209 & -68.4466 & -14.1421 & -24.8128 \end{bmatrix} \qquad (29)$$

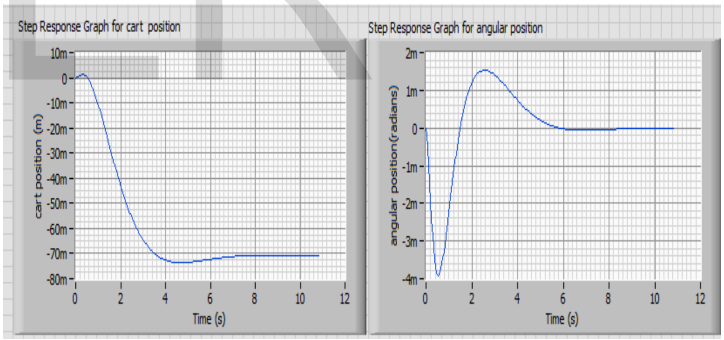

Fig 8.Screen shot of LabVIEW Front panel for step response of LQR for Inverted pendulum system with Q_3.

Fig 7 shows LQR response for both the pendulum angle and cart position with increased Q_3. The settling time as seen from above figure is near to 5 seconds. The steady state error for angular position is zero but a finite steady state error exists for cart position. Now we have eliminated this steady state error by adding a pre-compensation in reference.

D. Ellimination of steady state error by adding precompensation

The pole placement and LQR based state feedback controller gives desired transient response specifications but the steady state error for cart position is an issue in designing the controllers. To eliminate this steady state error we add a pre-compensation in reference input [18]. A constant gain N is added after the reference. We can find this *N* with the help of MATLAB user defined function rscale.m. We obtained the value of *N= -14.1421*

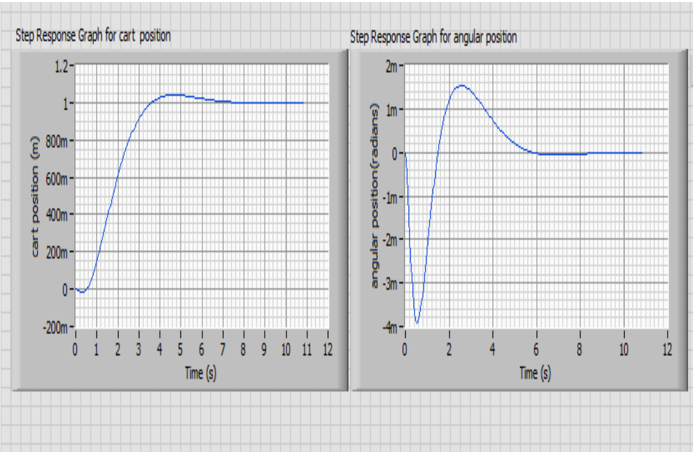

Fig.9. Screen shot of LabVIEW Front panel for step response of LQR for Inverted pendulum system with Pre-compensation.

Thus, the pre-componsator N has eliminated the steady state error and settling and rise time is as per our requirement. The pendulum overshoot is found to be in the design range.

E. Observer based controller

In the design of observer based controller the observer gain matrix L is obtained as shown in equation below.

$$L = \begin{bmatrix} 17.6013 & 1.0249 \\ 98.2967 & 9.1610 \\ 0.9355 & 17.9987 \\ 7.3272 & 80.1815 \end{bmatrix} \qquad (30)$$

The observer gain matrix *L* is a weighting matrix involving the difference between the estimated output and the measured output. This term consistently rectifies the model output and improvises the performance of the observer. Author(s) have taken state feedback gain matrix *K as* K_3. The observer-poles are taken with the help of Eigen values of closed loop equation [18] i.e. the closed loop poles of *A-BK*. Observer-poles are taken as

$$P=[-7.4,-8.4,-9.4,-10.4] \qquad (31)$$

So the parameter coefficient matrices describe the complete dynamics of the observer based state feedback control system using equation no. 18 can be obtained as follows.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -42.66 & -13.69 & -2.8 & -4.9 & 64.22 & 13.69 & 2.8 & 4.7 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 31.13 & 6.84 & 1.41 & 2.48 & -32.11 & -6.84 & -1.41 & -2.48 \\ 0 & 0 & 0 & 0 & -17.6 & 1 & -1.025 & 0 \\ 0 & 0 & 0 & 0 & -76.74 & 0 & -9.16 & 0 \\ 0 & 0 & 0 & 0 & -0.93 & 0 & -18 & 1 \\ 0 & 0 & 0 & 0 & -8.30 & 0 & -80.18 & 0 \end{bmatrix} \qquad (32)$$

$$B = \begin{bmatrix} 0 \\ 2.82 \\ 0 \\ -1.41 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (33)$$

The above matrices are the complete dynamics of observer based state feedback controller.

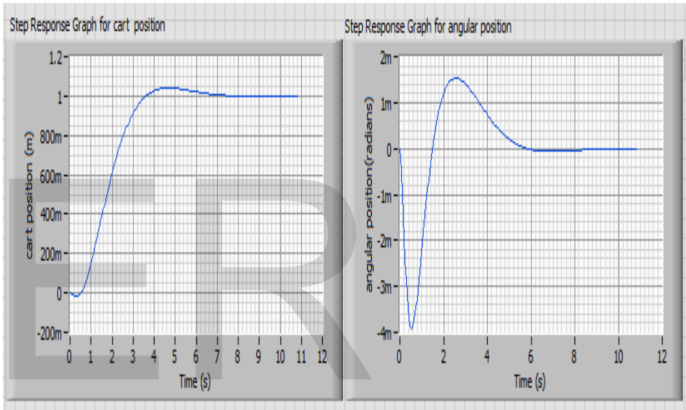

Fig 10.Screen shot of LabVIEW Front panel for step response of observer based controller Inverted pendulum system.

Fig 10 shows that both the output variables i.e. pendulum angle and cart position are stabilized and settling time is about 5 seconds. The systems states are estimated successfully with no error. This state estimation based observer controller has same response as LQR response with state feedback gain matrix K_3. The settling time obtained for various controllers are summarized in Table II.

TABLE III

SETTLING TIME ANALYSIS FOR CONTROLLERS

Controllers	Settling Time (seconds)	
	Pendulum angle	Cart position
Pole placement	1.37	1.4
LQR with Q_1	18.1	20.7
LQR with Q_2	5.41	6.19
LQR with Q_3	5.44	6.16
Observer Controller	5.5	3.64

VII. conclusion

Inverted Pendulum is very difficult system to control due its intrinsic non linearity and instability. Two state feedback based control techniques namely pole placement and linear quadratic regulator is simulated with the help of LabVIEW. The sate feedback gain matrices are obtained and then closed loop responses for both cart position and angle of IP are found to be satisfactory. Then the system states are estimated and observer based controller is designed. The observer based controller response is found to be same as state feedback closed loop response. Thus the system states are estimated successfully.

References

[1] National Instruments, "*LabVIEW Course LV1 & LV2*", 2011.

[2] www.ni.com /white-paper/4752/en

[3] Hongliang Wang, Haobin Dong, Lianghua He, Yongle Shi, Yuan Zhang, "Design and Simulation of LQR Controller with the Linear Inverted Pendulum," *International Conference on Electrical and Control Engineering (ICECE)*, pp. 699-702, June 2010.

[4] Stanislaw H. Zak, "Systems and Control" Oxford University Press, New York, 2003.

[5] Chandan Kumar, Santosh Lal, Nilanjan Patra, Kaushik Halder, Motahar Reza "Optimal Controller Design for Inverted Pendulum System based on LQR Method",*International Conference on Advanced Communication Control and Computing Technologies(ICACCCT)*,DOI:10.1109/ICACCCT.2012.6320782,Ramnathapuram,Tamilnadu, August 2012.

[6] Changkai Xu; Ming Li ; Fangyu Pan " The System Design and LQR control of a Two- wheels Self-balancing Mobile Robot", Electrical and Control Engineering (ICECE),2011 International Conference on, page(s): 2786 – 2789

[7] Zhang Lin, Song Yin, "Design of PID Temperature Controlling System Based on Virtual Instrument Technique", The Eighth International Conference on Electronic Measurement and Instruments, 2005.

[8] K. Ogata, System Dynamics, 4th ed, Pearson Education (Singapore) Pvt. Ltd., New Delhi, 2004

[9] D. Chatterjee, A. Patra, H. K. Joglekar, "Swing-up and Stabilization of a cart-pendulum system under restricted cart length", *Systems and Control Letters*, vol. 47, pp. 355-364, July 2002.

[10] F. L. Lewis, "Linear Quadratic Regulator (LQR) State Feedback Design", Lecture notes in Dept. Elect. Engineering, University of Texas, Arlington, Oct 2008.

[11] M.G. Henders, A.C. Soudack, "Dynamics and stability state-space of a controlled inverted pendulum", *International Journal of Non-Linear Mechanics*, vol.31, no.2, pp. 215-227, March 1996.

[12] M. W. Dunnigan , "Enhancing state-space control teaching with a computer-based assignment," *IEEE Transactions on Education*, vol.44, no.2, pp.129-136, May 2001

[13] Chen Wei Ji., Fang Lei, and Lei Kam Kin, "Fuzzy Logic Controller for An Inverted Pendulum System", IEEE International Conference on Intelligent Processing Systems, pp.185-189, 1997.

[14] G. W. Van der Linden and P.F Lambrechts, "H/sub ∞/ control of an experimental inverted pendulum with dry friction," *Control Systems IEEE* , vol.13, no.4, pp.44-50, Aug. 1993

[15] S. K. Das, K. K. Paul, "Robust compensation of a Cart–Inverted Pendulum system using a periodic controller: Experimental results", *Automatica*, vol. 47, no. 11, pp. 2543-2547, November 2011.

[16] C. C. Hung, B. Fernandez, "Comparative Analysis of Control Design Techniques for a Cart-Inverted-Pendulum in Real-Time Implementation," *American Control Conference*,pp.1870-1874, 2-4 June 1993.

[17] K.J. Astrom, K. Furuta, "Swinging up a pendulum by energy control" , *Automatica*, vol. 36, no. 2, pp. 287-295, February 2000

[18] CTM Example: Inverted Pendulum Animation
www.engin.umich.edu/group/ctm/gui/pend/invGUI.html