

# FPGA-based, 4-channel, High-speed Phasemeter for Heterodyne Interferometry

by

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## Biographical Sketch

The author was born in Lucheng, Shanxi, China. He attended Harbin Institute of Technology, and graduated with a Bachelor of Engineering degree in Measurement, Control Technique and Instruments in 2011. He began master's studies in Electrical Engineering at the University of Rochester in 2011. He pursued his research in Precision Instrumentation under the direction of Jonathan D. Ellis.



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# Abstract

A phasemeter is a device for performing phase measurements by extracting the relative phase between two alternating signals. It is widely used with heterodyne interferometry to measure displacement. Its performance determines the quality of entire displacement measurement. The aim of this work is to design a high-speed, high-precision, compact, and economical, user-friendly interface phasemeter prototype, which could outperform the currently-used commercial phase measurement solution in the laboratory.

The phasemeter was designed in three crucial parts, which includes detection and analog signal processing circuitry, digital signal processing algorithm of phase measurement, and Ethernet transmission. The detection and processing circuitry employed a large active area photodiode for detecting the incident laser beam with high spatial sensitivity for differential wavefront sensing, and analog signal processing circuits, such as filters, gain, buffers to optimize the analog signal. The phase measurement algorithm was implemented in an FPGA board with high processing speed, flexible and parallel performance. The Ethernet transmission was based on user datagram protocol (UDP), whose transmitting end was implemented by an embedded system in the FPGA and receiving end was implemented by a Matlab xPC Target.

This work has achieved a phasemeter prototype with an ability to be utilized in differential wavefront sensing, high phase processing speed, and user-friendly, easily

accessible interface, which is competent to replace the currently-used commercial phase measurement solution in the laboratory.

## Contributors and Funding Sources

This work was supervised by a dissertation committee consisting of Professor Jonathan D. Ellis (advisor) of the Department of Mechanical Engineering and The Institute of Optics, Professors Qiang Lin and Tolga Soyata of the Department of Electrical and Computer Engineering, and Professor Nick Vamivakas of The Institute of Optics. All work for the dissertation was completed independently by the student. The graduate study and research were supported, in part, by the National Institute of Standards and Technology (NIST) under cooperative agreement number 70NANB12H186.



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# 1 Introduction

## 1.1 Overview

Presently, heterodyne interferometry is widely used in precision displacement measurement. The displacement of the moving target is correlated to the optical path, and then correlated to the phase of optical signal. The precision of the phase measurement determines the entire precision of the displacement measurement. Hence, the phase measurement is vital technique in displacement measurement.

A phasemeter is a device for performing the phase measurement by extracting the phase difference between reference and measurement signals, which could be two alternating currents or voltages (Figure 1.1). The primary function of a phasemeter is to provide a high precision measurement of the relative phase of the input signals in real time for the intended measurement.

### 1.1.1 Evolution

Before Very Large Scale Integration (VLSI) techniques began appearing, high-speed processors, high-speed A/D converters, and other high performance electronic devices

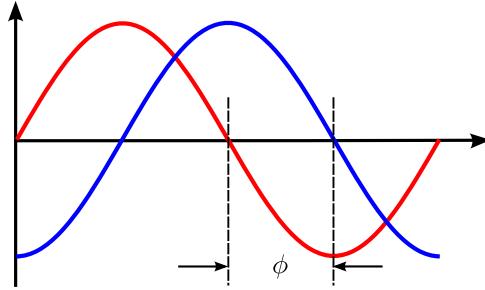
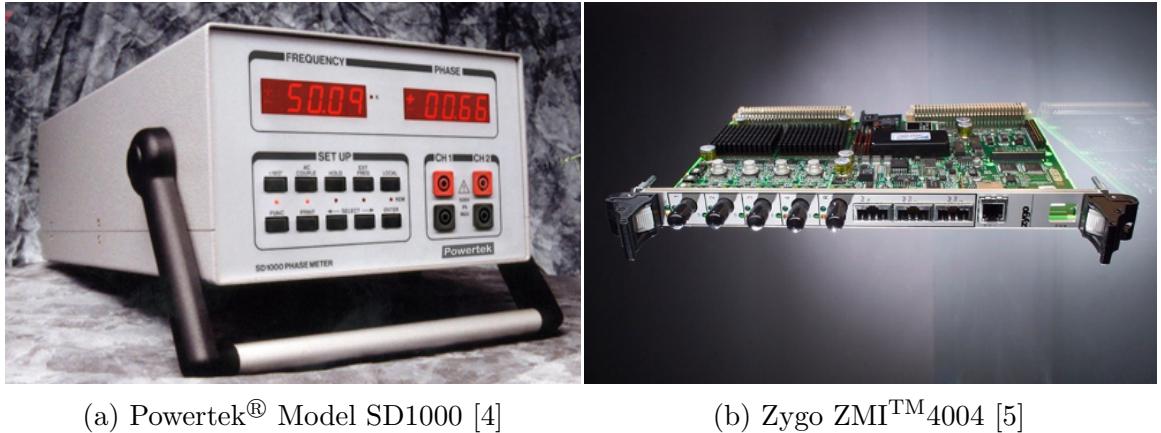


Figure 1.1: A phase difference  $\phi$  between two alternating input signals: the reference signal (red) and the measurement signal (blue).

were not available for instrumentation. Phasemeters could only be implemented based on analog circuitry. One advantage of the analog phasemeter is that there is no loss of information due to digitization; thus it keeps the continuous nature of the signals. However, in an analog circuit, some components or features cause artificial phase shifts, such as oscillators, filters, etc. Devices whose responses are not linear on all measuring intervals create a more significant problem [1]. Because of these issues, analog phasemeters can only typically achieve a resolution of approximately  $0.5^\circ$  [2; 3].

After the development of high-performance digital devices, it is possible to implement a digital phasemeter with high resolution, high precision, and high bandwidth. Currently, there are two main phasemeters widely used in commercial applications, one is a classical stand-alone instrument (Figure 1.2(a)) and the another is a virtual instrument (Figure 1.2(b)).

The classical stand-alone instruments include common oscilloscopes, function generators, etc., whose entire functional circuitries are held in cuboid cases. Panel connectors to detect or transmit signals are typically mounted on the front panel with a measurement displayed as shown in Figure 1.2(a). This kind of phasemeter is easy and simple to operate, while its functions are fixed after manufacturing, with little



(a) Powertek® Model SD1000 [4]

(b) Zygo ZMI™4004 [5]

Figure 1.2: The classical stand-alone instrument phasemeter (a) and virtual instrument phasemeter (b).

modifications or upgrades.

Virtual instruments represent a new method to develop instruments which are based on the modular measurement hardware. These are usually plug-in boards (Figure 1.2(b)) with a host PC or specific chassis (for instance, a NI VME Chassis) with the software running on a host PC to execute their measurements. These results are then either displayed a monitor, directly transmitted to a controller, or recorded for post-processing. One of its advantages is that the functions of the instruments are user-customized to some degree. By modifying and reconfiguring the algorithm with software, different measurement and data processing can be implemented on the same hardware. The virtual instruments decrease the volume of the measurement system, but increase the dependence on computers.

### 1.1.2 Application

One application area for a phasemeter is with displacement interferometers to measure displacements with high dynamic range and/or to calibrate other measurement tools. Two example applications are the Laser Interferometer Space Antenna (LISA) [6] and

stage metrology for photolithography [7].

LISA is designed for detecting and studying gravitational waves in a frequency range between  $10^{-4}$  and  $10^{-1}$  Hz [6], which are from sources throughout the universe such as black holes. When a gravitational wave passes through the plane of the LISA antenna, it changes the distances between the spacecrafts with a nominal arm length of  $5 \times 10^6$  km [8], thus, changes the phase of the interferometric fringe formed at the LISA interferometers. LISA will measure the phase as a time series signal. From this time series, the phasemeter can be extrapolated to determine gravitational wave information. LISA is expected to have a strain sensitivity in the order of  $10^{-20}/\sqrt{\text{Hz}}$ , therefore, it should have capability to measure the displacement variation with a sensitivity about  $12 \text{ pm}/\sqrt{\text{Hz}}$  [9].

A photolithography stepper machine is used for the fabrication of semiconductor chips. The stepper is usually equipped with a number of heterodyne laser interferometers to measure and control the motions of the wafer stage, reticle stage, and other components [7]. The interferometers provide the motion feedback using frequency modulated interferometry. This is used to support the projection of fine patterns of integrated circuits onto silicon wafers (Figure 1.3). The semiconductor device fabrication technology has reached 22 nm in 2012, and it is approaching to next node at 14 nm [10]. The interferometer and the phasemeter must have improved resolution, precision, and synchronization to aid in achieving that.

One displacement measuring interferometer (DMI) used in the Precision Instrumentation Group at the University of Rochester is a custom configuration used to measure displacement and rotation angle changes with a compact architecture. Figure 1.4 shows the configuration of the interferometer [12]. In this configuration, the two photodiodes  $\text{PD}_r$  and  $\text{PD}_m$  measure the reference and measurement interference

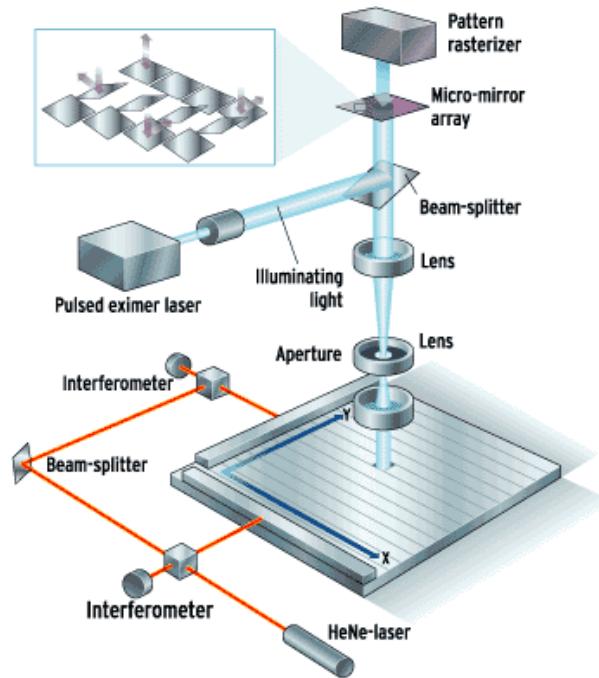


Figure 1.3: Photolithography stepper machine with interferometers. The interferometers and their phasemeters serve for orientation and control [11].

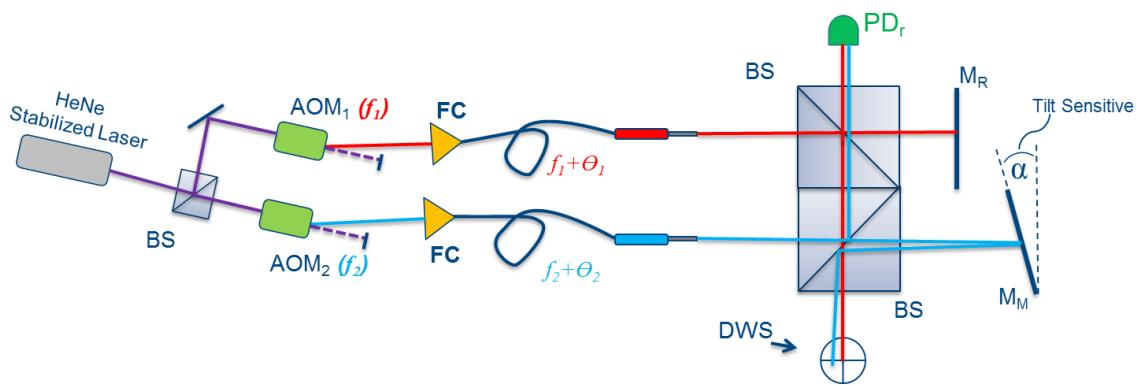


Figure 1.4: The configuration of the displacement measurement interferometer used in the lab [12].

signals, which are

$$u_r(t) = U_r \sin(2\pi f_s t) \text{ and} \quad (1.1)$$

$$u_m(t) = U_m \sin(2\pi f_s t + \phi), \quad (1.2)$$

where  $U_r$  and  $U_m$  are the amplitudes of the reference and measurement signals,  $f_s$  is the nominal split frequency of the laser source, and  $\phi$  is the phase shift, which contains the displacement of the moving target. The relationship between the phase difference and the displacement of target is

$$\phi = \frac{2\pi N \Delta x \eta f}{c}. \quad (1.3)$$

where  $\phi$  is the phase difference between the reference and measurement signals,  $N$  is the interferometer constant (two for this interferometer),  $\eta$  is the refractive index along the optical path difference,  $f$  is the nominal frequency of the laser source,  $c$  is the speed of light, and  $\Delta x$  is the displacement of the target.

The phasemeter is the measurement instrument used to extract this phase difference between the reference and measurement signals based on signal processing algorithms. There are three mainstream algorithms used to extract the phase information which are discussed in the following sections.

## 1.2 Zero-crossing Algorithm

A zero-crossing algorithm focuses on specific points on the reference and measurement signals within the waveform and measures the delay between these points. The chosen point is commonly the zero-crossing point within the waveform. The schematic

diagram of this algorithm is shown in Figure 1.5 [13].

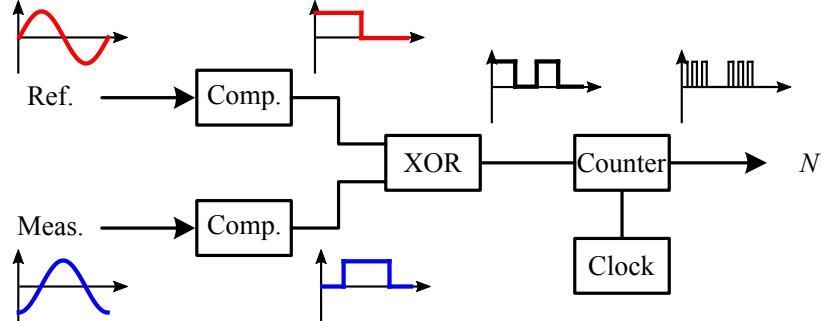


Figure 1.5: The schematic diagram of the zero-crossing algorithm.

First, the digitalized reference and measurement signals are fed into comparators. These two comparators compare the input signals with the zero level. If the signal is positive, it is converted to a high level (logic “1”) signal, or to a low level signal (logic “0”) when it is negative. This step realizes zero point detection [13]. Comparators typically have artificial hysteresis built into the circuit to prevent jitter from creating false zero crossings [14].

Next, one important step of this method is the “Exclusive OR” (XOR) logic operation between reference and measurement logic signals [1]. XOR operation produces a value of “1” only when the truth values of two operands are different. This step realizes the phase difference extraction, because the pulse duration of XOR product is equal to the phase difference between the input signals. Figure 1.6 demonstrates this logic operation.

The result of the XOR operation is a series of pulses. Then a counter records the amount of clock cycles during one pulse [15]. This step realizes the phase difference measurement. Assuming the amount of clock cycles during one pulse is  $N$  and the amount of clock cycles during one reference signal period is  $M$ , the phase difference

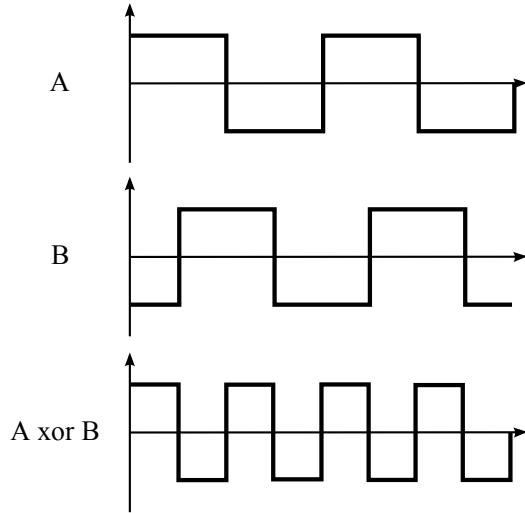


Figure 1.6: The Exclusive OR operation of two logic signals.

$P$  is

$$P = \frac{N}{M} \times 2\pi. \quad (1.4)$$

Therefore, the phase difference can be calculated through the zero-crossing algorithm.

Interpolation can be used to determine fractions of a clock cycle [16].

The limitation of this algorithm essentially comes from the counter clock (sampling rate). This method needs an ultra-high frequency counter clock when both high input signal frequency and high phase resolution are required by the phasemeter simultaneously. The product of the reciprocal of the phase resolution and the input signal frequency equals the required frequency of the counter clock. For instance, a digital zero-crossing phasemeter at 1 MHz input and 1/1000 resolution would require a 1 GHz clock. However, a digital circuit working at those high speeds in practice must have high-speed signal integrity analysis and a specifically designed PCB. Even then, it may still have some unpredictable problems. Even if the speed of clock is not a problem, the precision of its signal edge still may limit the measurement precision [1].

### 1.3 Phase-locked Loop based Lock-in Detection Algorithm

Lock-in detection algorithms apply a phase-locked loop (PLL) as the crucial part of the entire algorithm. Though it relies on a series of complex signal processing steps, this algorithm can capture the phase difference between two input signals. The schematic diagram of the algorithm is shown in Figure 1.7 [17].

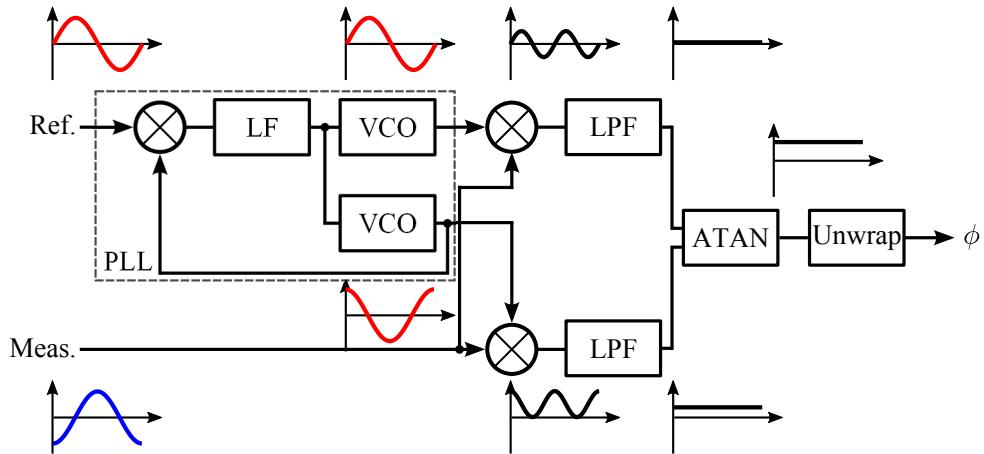


Figure 1.7: The schematic diagram of the phase-locked loop algorithm.

The input reference signal  $u_r(t)$  and measurement signal  $u_m(t)$  are the same as those shown in Equations 1.1 and 1.2.

First, the reference signal is fed into the PLL. The PLL is a module which is able to lock the incoming signal's frequency  $f_s$  and phase. Two voltage controlled oscillators (VCO) generate in-phase ( $I$ ) and quadrature ( $Q$ ) signals at frequency  $f_s$ . One VCO signal is used to feedback into the PLL. More details about principle of the PLL will be introduced in following section. As Figure 1.7 shows, the PLL generates

the in-phase  $I_r(t)$  and quadrature  $Q_r(t)$  signals,

$$I_r(t) = \sin(2\pi f_s t) \text{ and} \quad (1.5)$$

$$Q_r(t) = \cos(2\pi f_s t), \quad (1.6)$$

where the amplitude of the reference signal  $U_r$  and measurement signal  $U_m$  have been scaled by amplifiers or other digital methods.

Then the measurement signal is separately multiplied by each of two PLL output signals. Based on the trigonometric product-to-sum identity, the products equal to a sum of one high-frequency ( $4\pi f_s$ ) component and one low-frequency or DC ( $\phi$ ) component. The products are

$$\begin{aligned} I(t) &= u_m(t) \cdot Q_r(t) \\ &= \sin(2\pi f_s t + \phi) \cdot \cos(2\pi f_s t) \\ &= \frac{1}{2} \sin(4\pi f_s t + \phi) + \frac{1}{2} \sin \phi, \text{ and} \end{aligned} \quad (1.7)$$

$$\begin{aligned} Q(t) &= u_m(t) \cdot I_r(t) \\ &= \sin(2\pi f_s t + \phi) \cdot \sin(2\pi f_s t) \\ &= -\frac{1}{2} \cos(4\pi f_s t + \phi) + \frac{1}{2} \cos \phi. \end{aligned} \quad (1.8)$$

Next, the following low-pass filters block these high-frequency components at  $4\pi f_s$ . The phase difference information from only the DC components is needed, thus the remaining in-phase  $I$  and quadrature  $Q$  signals contains phase difference information.

$$I = \frac{1}{2} \sin \phi, \text{ and} \quad (1.9)$$

$$Q = \frac{1}{2} \cos \phi. \quad (1.10)$$

In the end, an inverse tangent operation of  $Q/I$  will extract the phase difference  $\phi$  by

$$\phi = \arctan\left(\frac{Q}{I}\right) = \arctan\left(\frac{\cos\phi}{\sin\phi}\right). \quad (1.11)$$

Because of the inverse tangent operation with a principal value in the range  $(-\pi, \pi]$ , the output signal is wrapped to the interval  $(-\pi, \pi]$ . Thus, an unwrap function must follow the inverse tangent operation to make the instantaneous phase difference continuous.

The following section details the principles of a PLL.

### 1.3.1 PLL

A phase-locked loop is a crucial part in this algorithm, which is a circuit synchronizing an output signal with input signal in phase. Frequency is the time derivative of phase. Locking the input and output phases implies locking frequencies, otherwise, it is meaningless to compare the phases of two signals with different frequencies. In “locked” state, the frequency error between the output signal and input signal is zero, and the phase error remains constant. Hence, it is used to generate the input signal’s in-phase and quadrature signals in this algorithm.

A PLL consists of three basic functional blocks. Regardless of the type of PLL, analog or digital, hardware or software, the mechanisms are all the similar. The structure of PLL is shown in Figure 1.8, which includes a voltage-controlled oscillator (VCO) or numerically controlled oscillator (NCO), a phase detector (PD), and loop filter (LF).

To illustrate the principle of the PLL, the following signals must be considered

- The input (reference) signal  $u_i(t)$ ,

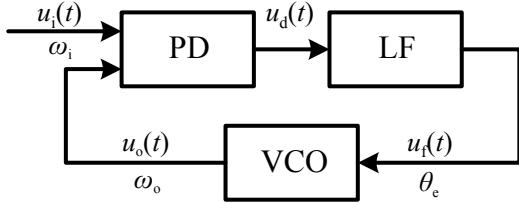


Figure 1.8: The structure of the PLL.

- The angular frequency  $\omega_i$  of the input (reference) signal,
- The output signal  $u_o(t)$  of the VCO,
- The angular frequency  $\omega_o$  of the output signal,
- The output signal  $u_d(t)$  of the phase detector,
- The phase error  $\theta_e$ , defined as the phase difference between signals  $u_i(t)$  and  $u_o(t)$ .

The following section mainly focuses on the expression and derivation of these signals to describe the functionality of the PLL [18].

## Phase Detector

The Phase Detector (PD) compares the phase of the output signal with the phase of the input signal and generates an output signal  $u_d(t)$ , which consists of a low-frequency or DC component and a high-frequency AC component. The AC component is undesired, hence it is removed by the low-pass loop filter.

The forms of the PD circuit are diverse, including analog and digital variants. For a basic analysis, the PD with a sinusoidal response characteristic is commonly used, because of its mature theory and straightforward analysis. Theoretically, an

ideal multiplier can be regarded as a PD with a sinusoidal response characteristic. Therefore, the multiplier is chosen as an example to explain how PD works in a PLL.

In a PLL, the input signal  $u_i(t)$  is mostly a sine wave and is given by

$$u_i(t) = U_i \sin(\omega_i t + \theta_i(t)), \quad (1.12)$$

where  $U_i$  is the amplitude of input signal,  $\omega_i$  is angular frequency and  $\theta_i(t)$  is its phase.

The second input signal  $u_o(t)$  is the VCO output signal and is given by

$$u_o(t) = U_o \cos(\omega_o t + \theta_o(t)), \quad (1.13)$$

where  $U_o$  is the amplitude,  $\omega_o$  is the quiescent frequency of the VCO, and  $\theta_o(t)$  is its phase.

The product of these two signals, which is the output of multiplier (PD), is given by

$$\begin{aligned} u_d(t) &= K_m u_i(t) u_o(t) \\ &= K_m U_i \sin(\omega_i t + \theta_i(t)) U_o \cos(\omega_o t + \theta_o(t)) \\ &= \frac{1}{2} K_m U_i U_o \sin((\omega_i + \omega_o)t + \theta_i(t) + \theta_o(t)) \\ &\quad + \frac{1}{2} K_m U_i U_o \sin((\omega_i - \omega_o)t + \theta_i(t) - \theta_o(t)), \end{aligned} \quad (1.14)$$

where  $K_m$  represents the gain of the multiplier, whose physical unit is the reciprocal of voltage,  $u_d$  consists two terms, one with high-frequency  $\omega_i + \omega_o$  and one with low-frequency  $\omega_i - \omega_o$  or DC level.

Comparing this procedure to phase demodulation, if  $\omega_i = \omega_o$ , PD extracts the phase information (phase difference) from the carrier waves (two input signals).

## Loop Filter

As discussed above, the output signal of the PD consists of a low-frequency or DC component and a high-frequency AC component. The DC component is approximately proportional to phase difference. The AC component has high frequency  $\omega_i + \omega_o$ , which is an unwanted signal. It must be filtered out by a loop filter, which must pass the lower frequency and block the higher frequency. Hence, it must be a low-pass filter. In most PLL designs, a first-order low-pass filter is used [18].

The output signal of the PD (Equation (1.14)) is fed into a low-pass filter and the output signal of the filter<sup>1</sup> is

$$u_f(t) = \frac{1}{2} K_m U_i U_o \sin((\omega_i - \omega_o)t + \theta_i(t) - \theta_o(t)). \quad (1.15)$$

If

$$K_d = \frac{1}{2} K_m U_i U_o, \quad (1.16)$$

$$\omega_e = \omega_i - \omega_o, \text{ and} \quad (1.17)$$

$$\theta_e(t) = \theta_i(t) - \theta_o(t), \quad (1.18)$$

where  $K_d$  is the phase detection sensitivity in volt per radian,  $\omega_e$  is the frequency difference between the two PD input signals, and  $\theta_e(t)$  is the instantaneous phase difference between the two PD input signals.

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<sup>1</sup>The filter has a transfer function  $F(s)$ . In fact, the low-pass filter does not just pass the low-frequency components and block the high-frequency components, it also introduces the phase delay depended on the type of filter. In the PLL, the phase delay influences the phase of the output signal of the filter  $u_f(t)$  rather than the output  $u_o(t)$  of the entire PLL. Hence, for the simplicity, the following derivations just ignore the phase delay from  $F(s)$ , and reach the same conclusion.

Therefore,  $u_f(t)$  can be expressed as

$$u_f(t) = K_d \sin(\omega_e t + \theta_e(t)). \quad (1.19)$$

which indicates that the PD (with LP) has a sinusoidal response. Figure 1.9 shows the characteristic of response of the PD (with LP), when  $\omega_e = 0$ . As shown in the

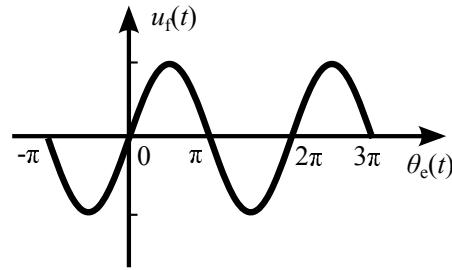


Figure 1.9: The characteristic of the response of PD with LP.

figure,  $u_f(t)$  is approximately linear in a limited interval, where  $\omega_e t + \theta_e(t)$  is very small. Thus, the sine function can be replaced by its argument around zero:

$$u_f(t) = K_d(\omega_e t + \theta_e(t)). \quad (1.20)$$

### Voltage-Controlled Oscillator

In a PLL, a VCO is used for adjusting the frequency through the input voltage. The VCO oscillates at an angular frequency  $\omega_2$ , which is determined by input voltage. The DC level output of a low-pass filter (loop filter) is applied as control signal to the VCO. The output angular frequency  $\omega_2$  of the VCO is directly proportional to input DC level  $u_f(t)$  and is given by

$$\omega_2 = \omega_o + K_o u_f(t), \quad (1.21)$$

where  $K_o$  is called the VCO gain, and its units are radian per second per volt. The unit radian is often omitted because it is a dimensionless quantity. The quiescent angular frequency of the PLL is  $\omega_o$ . Figure 1.10 shows an idealized representation of  $\omega_2$  as a function of  $u_f(t)$  of a VCO. It is assumed that the range of the control signal is symmetrical around  $u_f(t) = 0$ .

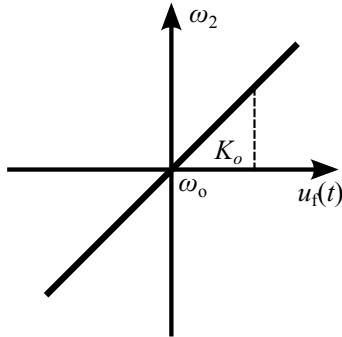


Figure 1.10: An idealized characteristic of a VCO.

In a PLL, the PD, LP, and VCO are implemented in a closed loop with negative feedback. The mathematical model of PLL is shown in Figure 1.11. In the following section, the mechanism how to track the input signal's phase and frequency will be discussed.

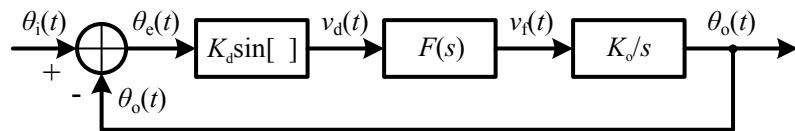


Figure 1.11: The mathematical model of PLL.

### 1.3.2 Phase locking Mechanism

First, assuming the frequency of the input (reference) signal  $\omega_i$  is equal to the quiescent frequency of the VCO  $\omega_o$ <sup>2</sup>, the frequency difference  $\omega_e$  is zero

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<sup>2</sup>The frequency of the VCO output signal  $\omega_o$  is equal to its quiescent frequency  $\omega_v$  initially.

(Equation (1.17)). If the phase difference  $\theta_e$  is zero, the output signal of the PD  $u_d(t)$  only has a high frequency component (Equation (1.14)). Consequently, the output signal of the low-pass loop filter  $u_f(t)$  will also be zero. The output signal is exact same as input signal, which means the phase has been locked.

Then, if the phase error  $\theta_e$  was not zero initially, the PD would develop a nonzero output signal  $u_d(t)$ , and the LP would also produce a finite signal  $u_f(t)$ . This would cause the VCO to change its operating frequency in such a way the phase difference finally vanishes. The phase of the VCO output signal is adjusted until it becomes equal to the phase of input signal.

Generally, the frequencies of these two input signals of PLL are different initially and it is meaningless to compare the phases under the condition of different frequencies. In order to compare these two phases, the instantaneous phase of  $u_i(t)$  must be redefined based on the VCO's quiescent angular frequency  $\omega_o$ ,

$$\begin{aligned}\omega_i t + \theta_i(t) &= \omega_o t + [(\omega_i - \omega_o)t + \theta_i(t)] \\ &= \omega_o t + \theta_1(t),\end{aligned}\tag{1.22}$$

where

$$\begin{aligned}\theta_1(t) &= (\omega_i - \omega_o)t + \theta_i(t) \\ &= \omega_e t + \theta_i(t).\end{aligned}\tag{1.23}$$

The instantaneous phase,  $\theta_1(t)$ , is based on VCO's output frequency as a reference.

Then, the VCO output signal is rewritten by replacing  $\theta_o(t)$  with  $\theta_2(t)$ , and is given by

$$\omega_o t + \theta_o(t) = \omega_o t + \theta_2(t).\tag{1.24}$$

The following is a mathematical approach to describe the whole mechanism.

Phase is the time integral of frequency, thus, integrating both sides of Equation (1.21) yields

$$\int_0^t \omega_2(t)dt = \omega_o t + K_o \int_0^t u_f(t)dt. \quad (1.25)$$

Therefore, according to Equation (1.13), the instantaneous phase  $\theta_2(t)$  of  $u_o(t)$  when the reference frequency is  $\omega_o$ , is given by

$$\theta_2(t) = K_o \int_0^t u_f(t)dt. \quad (1.26)$$

Here, for simplicity, assume that the difference between the angular frequency of the input signal and the quiescent angular frequency of the VCO  $\omega_e$  is constant as well as the phase of input signal  $\theta_i$ .

Inserting these substitutions from Equation (1.18) and Equation (1.20) into Equation (1.26), results in the phase output, given by

$$\begin{aligned} \theta_2(t) &= K_o \int_0^t u_f(t)dt \\ &= K_o \int_0^t K_d(\omega_e t + \theta_e(t))dt \\ &= K_o \int_0^t K_d(\theta_1(t) - \theta_2(t))dt. \end{aligned} \quad (1.27)$$

Assuming  $K = K_o K_d$ , and the Laplacian is taken of both sides,

$$\Theta_2(s) = \frac{K(\Theta_1(s) - \Theta_2(s))}{s}. \quad (1.28)$$

The transfer function  $H(s)$ <sup>3</sup> is given by

$$H(s) = \frac{\Theta_2(s)}{\Theta_1(s)} = \frac{K}{K + s}. \quad (1.29)$$

In the time domain, the transfer function is given by

$$h(t) = K e^{-Kt} u(t). \quad (1.30)$$

The phase of output signal is a convolution of the phase of the input signal and the transfer function. Inserting these substitutions (Equation (1.23) and Equation (1.30)), the instantaneous phase of the output signal is given by

$$\begin{aligned} \theta_o(t) &= \theta_2(t) = \int_0^t \theta_1(\tau) h(t - \tau) d\tau \\ &= \int_0^t (\omega_e \tau + \theta_i(\tau)) K e^{-K(t-\tau)} d\tau \\ &= \omega_e \left( t - \frac{1}{K} + \frac{e^{-Kt}}{K} \right) + (1 - e^{-Kt}) \theta_i(t). \end{aligned} \quad (1.31)$$

When time  $t$  is sufficiently long,

$$\lim_{t \rightarrow \infty} \theta_2(t) = \omega_e t + \theta_i(t) = \theta_1(t), \quad (1.32)$$

the instantaneous phase of the output signal approaches to that of the input signal. This duration is not necessary very long and depends on a proper factor  $K$ .

After  $\theta_2$  approaches to  $\theta_1$ , the state is so-called “phase-locked”. Because the

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<sup>3</sup>For the simplicity, it ignores the transfer function  $F(s)$  of the loop filter, but the trend of  $\theta_2(s)$  approaching to  $\theta_1(s)$  is not changed.

output signal is assumed initially as

$$u_o(t) = U_o \cos(\omega_o t + \theta_o(t)) \quad (1.33)$$

when the phase is locked, the actual output signal is the quadrature signal of the input signal. That is the crucial mechanism of PLL to generate the quadrature signal.

To generate the in-phase signal of the input signal, it can simply set the initial phase of Equation (1.33) to advance or delay 90°. The rest mechanism is identical with that of quadrature signal.

## 1.4 Single-bin DFT Algorithm

A single-bin discrete Fourier transform (SBDFT) has been used in the LISA phasemeter [19; 20], which uses the phase of certain bin in the Fourier spectrum to trace the phase change of the input reference and measurement signals.

According to Fourier theory, for a sinusoidal signal, the energy in the frequency spectrum is concentrated at the nominal signal frequency. The phase of the signal at a specific time is the phase of the Fourier transform at the point where the energy is concentrated. The phase measurement can be implemented by comparing the phase of the reference signal and phase of the measurement signal at any time [21].

The phasemeter only focuses on a point at or a limited range around nominal frequency of the signal rather than the entire frequency spectrum. Thus, it can calculate the Fourier transform only in that range. That range is constrained by a single bin. Frequency bins are even intervals spaced by frequency lines in the discrete Fourier Transform (DFT) spectrum, which can be computed by the sampling rate

$f_{\text{sample}}$  and number of samples  $N_{\text{sample}}$ ,

$$\text{bin} = \frac{f_{\text{sample}}}{N_{\text{sample}}}. \quad (1.34)$$

For example, assuming 10,000 points are sampled at 100 MHz, each frequency bin is 10 kHz. Only calculating the Fourier transform in a single bin saves time and resources.

The input measurement signal  $u_m(t)$  and reference signal  $u_r(t)$  are of the forms

$$u_m(t) = \cos(2\pi f_s t + \phi_m), \text{ and} \quad (1.35)$$

$$u_r(t) = \cos(2\pi f_s t + \phi_r), \quad (1.36)$$

where  $f_s$  is the nominal split frequency of the laser source.

For the simplicity, the following uses a continuous Fourier transform to express the algorithm. However, it reaches the same conclusion as using a DFT.

The Fourier transform of the measurement signal is

$$U_m(f) = \int_{-\infty}^{\infty} u_m(t) e^{-i2\pi f t} dt. \quad (1.37)$$

Because only the Fourier transform at the nominal frequency<sup>4</sup> is needed, specifically,

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<sup>4</sup>It can be regarded equivalent to the bin, which covers the nominal frequency in DFT spectrum.

the split frequency here. Its Fourier transform at split frequency  $f_s$  is [22]

$$\begin{aligned}
U_m(f_s) &= \int_{-\infty}^{\infty} u_m(t) e^{-i2\pi f_s t} dt \\
&= \int_{-\infty}^{\infty} \cos(2\pi f_s t + \phi_m) e^{-i2\pi f_s t} dt \\
&= \frac{1}{2} \int_{-\infty}^{\infty} (e^{i(2\pi f_s t + \phi_m)} + e^{-i(2\pi f_s t + \phi_m)}) \cdot e^{-i2\pi f_s t} dt \\
&= \frac{1}{2} e^{i\phi_m} + \frac{1}{2} \int_{-\infty}^{\infty} e^{-i(4\pi f_s t + \phi_m)} dt.
\end{aligned} \tag{1.38}$$

The Fourier transform contains a DC and a high frequency component  $4\pi f_s$ . Applying a low-pass filter to block the high frequency component in the latter term, results in only the term containing  $\phi_m$  remaining. Then applying an arctangent operation on the real and imaginary parts of  $e^{i\phi_m}$ , the result is the phase  $\phi_m$  of the measurement signal.

Likewise, the phase of the reference signal  $\phi_r$  can be obtained by this method. Then an additional subtraction operation can extract the phase difference  $\phi$  between the measurement and reference signals,

$$\phi = \phi_m - \phi_r. \tag{1.39}$$

According to Euler's formula, the real and imaginary parts of the term  $e^{-i2\pi f_s t}$  in the Fourier transform calculation actually is a quadrature signal  $Q_s$  and an in-phase  $I_s$  [23],

$$Q_s(t) = \text{Re}\{e^{-i2\pi f_s t}\} = \cos(2\pi f_s t), \text{ and} \tag{1.40}$$

$$I_s(t) = \text{Im}\{e^{-i2\pi f_s t}\} = -\sin(2\pi f_s t). \tag{1.41}$$

Hence, the single-bin DFT algorithm is 2-channel lock-in detection algorithm essentially, but without a PLL. It replaces the PLL with the in-phase  $I_s$  and quadrature  $Q_s$  signals, and treats them as in-phase and quadrature signals of the reference signal in the PLL algorithm. It treats the measurement and reference signals both as the measurement signals in PLL algorithm (Equation (1.7) to Equation (1.11)). Unlike the PLL algorithm which calculates the phase difference directly, the single-bin DFT algorithm first compares the phases of the measurement and reference signals with a signal with the split frequency to calculate the phase differences separately, then performs a subtraction operation to obtain the difference between these two phase differences, which is the relative phase difference between measurement and reference signals. Figure 1.12 shows the schematic diagram of the algorithm.

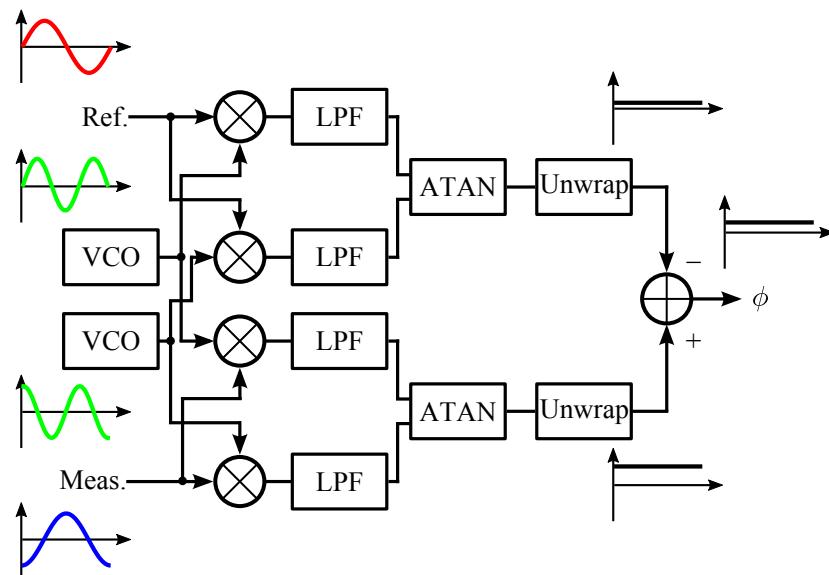


Figure 1.12: The schematic diagram of the phase-locked loop algorithm.

## 1.5 Motivation and Goals

The phase measurement solution currently used in the Precision Instrumentation Group Lab includes a commercial single element photodetector, a commercial quadrant photodetector, lock-in amplifiers, a NI PCI or USB data acquisition card, and a target PC. Some of them have limited performances or shortcomings, which will attempt to be improved and addressed in this work.

1. The commercial single element and quadrant photodetectors are used for detecting the reference and measurement signals respectively. They convert these interference signals to electrical signals for post-processing. The commercial quadrant photodetector currently used has four silicon photodiodes, each with 2.5 mm side square pixels in a  $2\times 2$  arrangement. For differential wavefront sensing, this area can only provide a limited spatial sensitivity [24]. Additionally, the performance of this system is less than adequate with op-amp oscillations and a lack of decoupling capacitors.
2. The lock-in amplifiers are used to extract the phase difference between reference and measurement signals. The lock-in amplifiers currently used are model SRS SR830 from Stanford Research Systems. It has a limited bandwidth 1 mHz to 102.4 kHz, which is sufficient for locking 70 kHz split frequency signal, but is not suited for higher split frequency 5 MHz. Also, it is an instrument with large dimensions (17" W  $\times$  5.25" H  $\times$  19.5" D), heavy weight (23 lbs), significant power consuming (40 W) [25], and high price (about \$5000). In practice, the consumed power converts into heat partly, the fan inside the chassis generates noise and vibration, which may cause problem for ultra-precision application. When

stacking four these instruments for processing differential wavefront sensing signals, there is the potential for synchronization issues.

3. The transmission interface is used to exchange the measurement data between measurement instrument and computer for post-processing, analysis, control. The transmission solution currently used is a NI PCI data acquisition card, which acquires the lock-in amplifier's analog output signal first, and then transmits the measurement data to computer through PCI interface. This solution costs an extra \$1000 to employ this extra hardware to convert the analog signal to digital signal and transmit it, which may be replaced by readily available hardware in computer, if measurement instrument outputs digital signal directly. Other virtual instrument type phasemeters usually equip with an instrument specific interface, such as the Zygo ZMI 4004 phasemeter with a VME interface, which are not equipped in common computers. This type of phasemeter must collaborate with specific chassis with these interface slots. The chassis generally costs thousands of dollars. It usually has several slots, while only one is needed for this particular application, others are spare. Hence, these data transmission solutions are not economical, user-friendly, or/and easily accessible.
4. The entire phase measurement solution currently used now is large-volume, dispersed, complicated-operation, and expensive as a system.

The goal of this work is to improve those shortcomings to some degree, and design a high-speed, high-precision, compact, economical and user-friendly interface phasemeter prototype to measure, process, and transmit data.

1. To achieve high spatial sensitivity for differential wavefront sensing, a large

active area quadrant photodiode will be used for building the measurement photodetector. To achieve a low noise level output signal, a specific analog signal processing board will be designed for the photodetector. In the initial prototype, the photodiode will work in photovoltaic mode for sensing 70 kHz signal. The processing board will be able to adjust the output signal voltage for the incident optical power 1  $\mu\text{W}$  to 50  $\mu\text{W}$ , maintain the output voltage as 1 to 2 V<sub>p-p</sub>. Chapter 2 will discuss the photodetector and analog signal processing board design, device selection, printed circuit board design, and the simulation and verification below in detail.

2. A FPGA board will implement the PLL and SBDFT algorithm to extract the phase difference. As the digital signal processing module of the phasemeter system, it will be high-precision (sub-nanometer), high-speed (50 MSPS), wide-bandwidth (5 MHz), small-volume, silent, user-customized, and economical. It is sufficient to replace the lock-in amplifier. In the initial prototype, due to the limited resource on the FPGA chip used now, only one channel measurement signal will be processed. Chapter 3 will introduce the hardware and software used to implement the algorithm, discuss the fixed-point models design, and simulation and verification below in detail.
3. This phasemeter system will use an Ethernet interface to transmit measurement data between FPGA board and computer through UDP packets. The transmitter will be implemented by a soft-core processor in the FPGA, the receiving end will be implemented in a target PC. An Ethernet cable connects the Ethernet port on FPGA board with the Ethernet adapter card in the target PC. There will be no extra hardware used in this system. It is economical, user-friendly, and easy-access. Chapter 4 will introduce the mechanism of UDP,

discuss the transmitter and receiving end implementations, and the test results.



## 2 Detection and Processing

### Board Design

Before calculating the phase difference between reference and measurement signals, the optical signals must be converted into electrical signals for further electrical circuitry to process initially. Additionally, some filters and amplifiers circuits are required to keep the converted electrical signals in good quality, because stray light in environment mixed in the laser beams may also be converted, and the optical devices in previous configuration and the electronic components on-board introduce noise inherently. Thus, a detection and analog signal processing circuit must be built in this design firstly. Figure 2.1 shows the diagram of this circuitry.

This work only designed the circuitry for the measurement beam. Following sections will discuss the principle of this circuitry, devices selection, and printed circuit board design in detail.

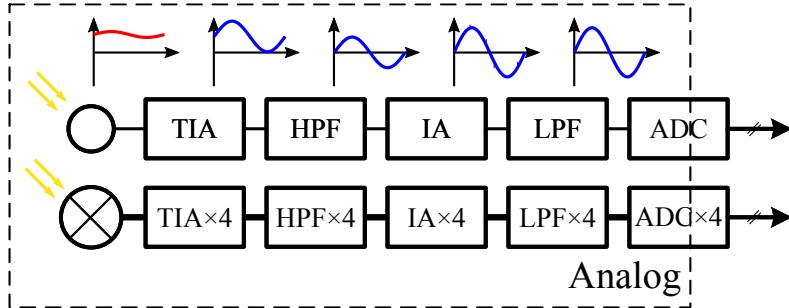


Figure 2.1: The diagram of the detection and processing circuitry. The reference and measurement beams incident one signal-element and one quadrant photodiode respectively, and then are converted to weak electrical current signals. And the following processing circuits are based on operation amplifiers, which are transimpedance amplifiers (TIA), high-pass filters (HPF), inverting amplifiers (IA), and low-pass filters (LPF). Finally, the signals will be fed into ADCs to convert to digital signals for further computation (the ADCs are on a separate board in this design). This work mainly designs the circuitry for measurement signals channels.

## 2.1 Detection and Processing Principles

### 2.1.1 Photodiodes

Silicon photodiodes are semiconductor devices with p-n junctions or PIN structures. They operate by absorbing photons or charged particles and generating photocurrent in an external circuit, which is proportional to the incident optical power. This mechanism is also known as the inner photoelectric effect [26].

A silicon photodiode can be represented by an ideal diode in parallel with a current source and some resistors and a capacitor. The current source generates the photocurrent corresponding to the incident optical power and the diode represents the p-n junction or PIN structure. In addition, a junction capacitor and a shunt resistor are parallel to the current source and ideal diode. A series resistor and all other components in this model are in a series connection. An equivalent circuit of a photodiode is shown in Figure 2.2 [27; 28].

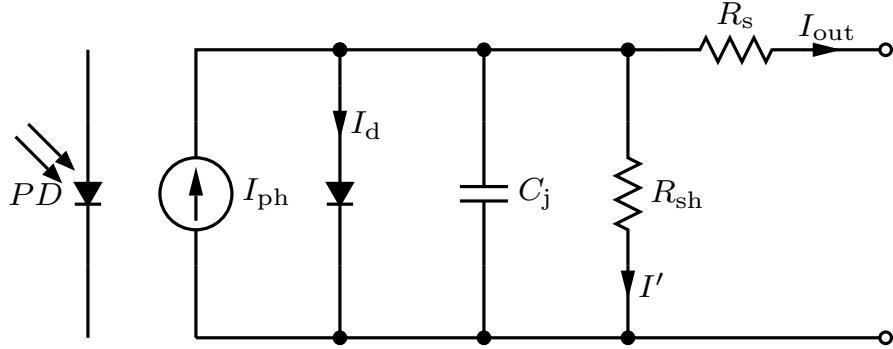


Figure 2.2: Photodiode equivalent circuit.  $I_{ph}$  is the generated photocurrent;  $I_d$  is the dark current;  $I'$  is the shunt resistor current;  $I_{out}$  is the output current;  $C_j$  is the junction capacitor, where the value depends on the applied reverse bias voltage and determines the response bandwidth of the photodiode;  $R_{sh}$  is the shunt resistor, its actual value ranges from 10's to 1000's of megaohms;  $R_s$  is the series resistor, its typical value ranges from 10 to 1000's ohms.

Using the equivalent circuit shown in Figure 2.2, the output current  $I_{out}$  is

$$I_{out} = I_{ph} - I_d - I' = R_\lambda P - I_s \left( e^{\frac{eV_d}{kT}} - 1 \right) - I', \quad (2.1)$$

where  $R_\lambda$  is the responsivity or photosensitivity of the photodiode, measuring the effectiveness of the conversion of optical power into photocurrent, expressed in A/W. Its value depends on the wavelength of the incident light, applied reverse bias voltage, and temperature. Also,  $P$  is the incident optical power,  $V_d$  is the applied reverse bias voltage across the diode,  $I_s$  is the photodiode reverse saturation current,  $e$  is the electron charge,  $k$  is Boltzmann's constant, and  $T$  is the absolute temperature.

A photodiode can be operated in two modes: photoconductive (reverse bias) or photovoltaic (zero bias). The mode selection depends on the application's response speed and sensitivity requirements. Photoconductive mode achieves the fastest response and greatest bandwidth, while introducing dark and noise current that harm the photodiode sensitivity. Photovoltaic mode achieves the highest sensitivity but has a slower response [29].

In this design, photovoltaic mode was selected because the photodiode is used in a low frequency regime (up to 200 kHz) and a precision application. The photocurrents in this mode have less variation in responsivity with temperature, no dark current is generated by this mode, and the shunt resistor current is negligible. Thus, Equation (2.1) simplifies to

$$I_{\text{out}} = R_{\lambda}P. \quad (2.2)$$

The photodiode can operate in zero bias to eliminate any additional noise current to achieve a high sensitivity.

Photodiodes can be used for more than sensing the presence or absence of light at certain wavelengths; they also can be used to quantify light intensities below 1 pW/cm<sup>2</sup> to intensities above 100 mW/cm<sup>2</sup> for extremely accurate measurements. In this design, a quadrant photodiode is used to perform differential wavefront sensing, measuring the target mirror displacement, pitch, and yawthrough measuring four spatially separated interference signals on the four elements of the quadrant photodiode [24; 30; 31]. Silicon photodiodes can also be utilized in other diverse applications such as spectroscopy, photography, analytical instrumentation, optical position sensors, beam alignment, surface characterization, laser range finders, optical communications, and medical imaging instruments [28].

### 2.1.2 Transimpedance Amplifier

A transimpedance amplifier is an amplifier circuit that generates an output voltage proportional to the input current. The proportionality of this conversion is called transimpedance or transresistance, expressed in ohms. Figure 2.3 shows a configuration for a transimpedance amplifier [32].

The photodiode is operated in photovoltaic mode (zero bias). This amplifier circuit

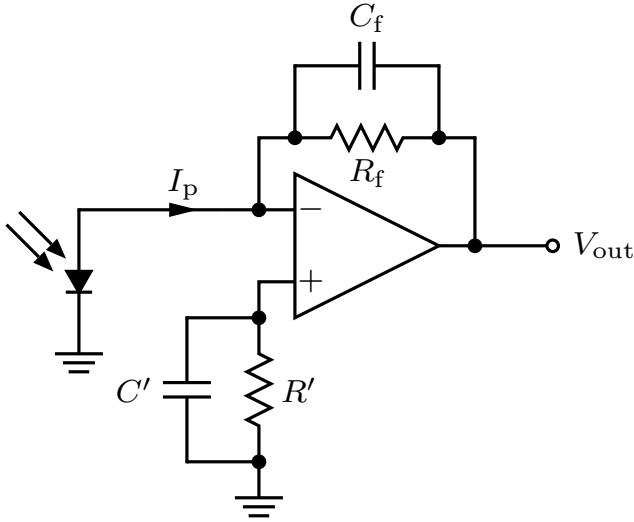


Figure 2.3: Configuration for a photovoltaic transimpedance amplifier.

provides approximate zero input impedance  $R_f/A$ , because of the operation amplifier (op-amp) properties: virtual ground and very high open loop gain  $A$ . Compared with the output resistance of photodiodes, the input resistance of the amplifier circuit is negligible, despite  $R_f$  is generally very large. This results in no voltage drawn down across the diode and then no diode leakage current basically. The temperature coefficient of the amplifier input leads to a thermal DC voltage drift, an equal resistance  $R'$  connected in series with op-amp noninverting input could compensate it, and a bypass capacitor  $C'$  could remove most of its noise. However, this may create a voltage drop across the diode and results in diode leakage current [33]. Additionally, in order to suppresses potential oscillation or gain peaking, a small capacitor  $C_f$  is placed across  $R_f$  to act as a low-pass filter cooperating with  $R_f$ . This can affect the bandwidth of the system [29].

The relationship between input current and output voltage is given by

$$V_{\text{out}} = -R_f I_{\text{in}}. \quad (2.3)$$

In this design, the input current is microamps and the output voltage is several volts. The values for  $R_f$  and  $C_f$  must be carefully selected to achieve enough gain and bandwidth.

Transimpedance amplifiers are usually used in optical communications receivers or after photodetectors to convert the photocurrent into a voltage signal for further manipulation. The motivation to implement transimpedance amplifiers is that a voltage signal is generally easier to process than microamps of photocurrent signal for following stages.

### 2.1.3 Buffer Amplifier

A buffer amplifier (sometimes simply called a buffer) is a circuit that provides electrical impedance isolation or matching between previous and following stage circuits. Two main types of buffers exist: the voltage buffer and the current buffer. This design employs voltage buffers.

The circuit schematic of a buffer amplifier is shown in Figure 2.4. In this

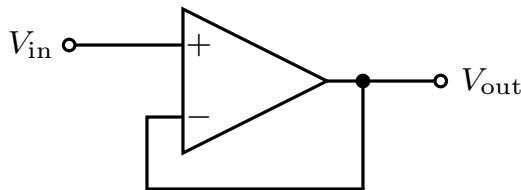


Figure 2.4: Schematic of a buffer amplifier. Its output connects to its inverting input, and the output of previous stage connects to its non-inverting input. This constructs a full series negative feedback to the op-amp, implementing a unity gain buffer amplifier.

configuration, the output voltage is connected in series with the input voltage. According to Kirchhoff's voltage law (KVL) and properties of an op-amp, the difference of the two voltages,  $V_{in}$  ( $V_+$ ) and  $V_{out}$  ( $V_-$ ) is proportional to the op-amp

differential input based on its open loop gain  $A$ . Amplifying  $A$  times to  $V_{\text{out}}$ , the relationship between  $V_{\text{out}}$  and  $V_{\text{in}}$  is [34]

$$V_{\text{out}} = \frac{A}{1+A} V_{\text{in}}, \quad (2.4)$$

where  $A$  is the open-loop gain of the op-amp. Because  $A$  is very large,  $V_{\text{out}}$  is approximately  $V_{\text{in}}$ . Thus, the closed-loop gain is unity (0 dB). Although the voltage gain of a voltage buffer amplifier is approximately unity, it usually provides considerable current gain and thus power gain.

In the Figure 2.4, it is the operation amplifier, an active device, whose properties determine the buffer function. According to the voltage divider rule, the input impedance of the op-amp is very high ( $1 \text{ M}\Omega$  to  $10 \text{ T}\Omega$ ), which means that the input of the op-amp draws only minimal current from voltage source, thus it does not load the voltage source (does not influence output voltage of source). The output impedance of the op-amp is very low, which means it drives the load as if it were a perfect voltage source (any load does not influence its output voltage). Therefore, the output impedance of the previous stage and the input impedance of following stage do not affect each other due to the buffer. This phenomenon is so-called impedance isolation or impedance matching.

The purpose of placing a buffer at the end of circuit is to avoid the influence from unknown following circuits. In other words, performing a measurement or processing a voltage does not disturb the circuit producing the voltage to be measured or processed. The output signal may propagate through a cable to other analog circuits or instruments, which are variable, the input impedance of those following stages are variable as well. A buffer helps to maintain or even promote the performance of the detection and processing circuitry, especially the drive capability, regardless of the

following stage.

### 2.1.4 High-pass Filter

A high-pass filter (HPF) is an electronic frequency selective circuit that passes signals with frequencies higher than the cutoff frequency but attenuates signals with frequencies lower than the cutoff frequency. The actual amount of attenuation for each frequency varies depending on the configuration of the filter. High-pass filters are widely used in signal processing, such as blocking DC level signals from non-zero average voltages sensitive circuitry.

In this design, first-order, noninverting high-pass filters with unity gain were applied. Compared with other inverting configurations, the noninverting configuration has a simpler structure with fewer components to achieve the unity gain in the passband. Figure 2.5 shows a first-order, noninverting high-pass filter configuration [35].

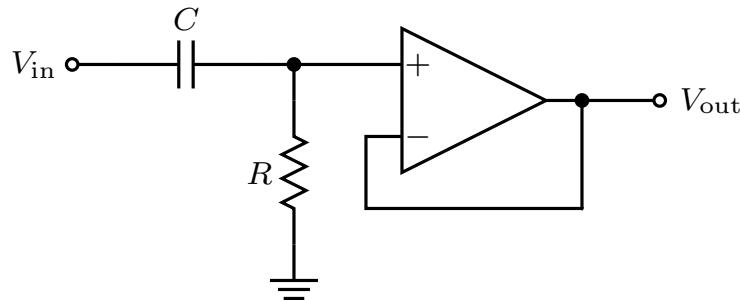


Figure 2.5: First-order noninverting high-pass filter with unity gain. With an op-amp, this is an active first-order high-pass filter. It consists of a highpass RC network and a voltage buffer. The buffer serves to provide impedance isolation so the RC network is not loaded down by the following stages and the output voltage of the RC network is transferred to the buffer's output without attenuation. Without the buffer, the frequency response of a simple RC network on its own would be varied depending on the load resistance, which is in parallel with the shunt resistor  $R$ .

The circuit transfer function of this high-pass filter is

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{s}{s + \frac{1}{RC}} = \frac{s}{s + 2\pi f_c}, \quad (2.5)$$

where  $R$  is the resistance in ohms,  $C$  is the capacitance in Farads and  $f_c$  is the cut-off frequency in Hertz.

The purpose of employing a high-pass filter in this design is to remove the DC component. Since input optical power varies and the output of the whole analog signal processing circuit must be fed into an ADC with a fixed range of 1-2 V<sub>p-p</sub> [36], removing the DC component is more straightforward for adjusting the amplitude of the signal in the following stage. Both high-pass filters and low-pass filters have the capability to adjust the signal gain. However, the high-pass filter has the limitation that the gain cannot be lower than unity (specifically in the noninverting configuration) and changing the gain in a wide range influences the cutoff frequency (specifically in the inverting configuration). So a unity gain high-pass filter and an independent inverting amplifier are used in the current and following stages. The cutoff frequency of the high-pass filter is 1 kHz, which removes the DC component effectively and passes the desired frequency of nominally 70 kHz.

### 2.1.5 Inverting Amplifier

An inverting amplifier scales and inverts the input signal. If the op-amp open-loop gain is very large, the closed-loop gain of this amplifier circuit is determined by two stable external resistors (the feedback resistor  $R_f$  and the input resistor  $R_{\text{in}}$ ) and is largely independent from op-amp parameters which are highly temperature sensitive. Figure 2.6 shows the schematic of the inverting amplifier.

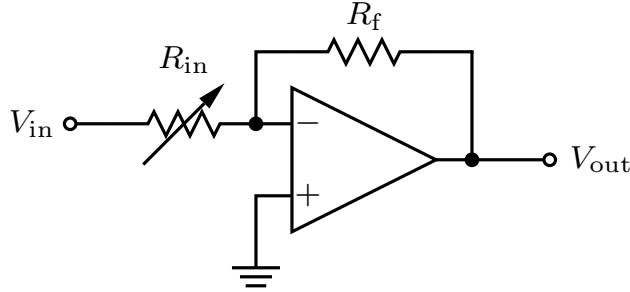


Figure 2.6: Schematic of inverting amplifier. The value for  $R_{in}$  in this design is given by a potentiometer, makes the voltage gain of circuit adjustable.

The noninverting input of the inverting amplifier circuit is grounded. According to the two assumptions of op-amp properties, virtual short and virtual open, the feedback keeps the inverting input of the op-amp at a virtual ground (noninverting input and inverting input are virtual short), and no current flows in the input leads (noninverting input and inverting input are virtual open). Hence the current flowing through  $R_{in}$  is assumed to equal the current flowing through  $R_f$ . Based on Kirchhoff's law, the voltage gain is

$$G = -\frac{R_f}{R_{in}} \quad (2.6)$$

and the minus sign here is inserted because this configuration opposes the polarity of the input signal [37].

The purpose of using an inverting amplifier is scaling the amplitude of the output signal. This reason was introduced in high-pass filter section. In order to adjust the gain of the circuit, the value of  $R_f$  or  $R_{in}$  must be adjustable as well. If a potentiometer (used as a variable resistor) is used to drive  $R_f$ , the gain linearly correlates the resistance of potentiometer, which could not be very high. That means the range of the gain could not be very wide. Using a potentiometer at  $R_{in}$ , the upper limit of gain is determined by the reciprocal of the minimum potentiometer resistance. Therefore, the range of gain can be very wide. Meanwhile, the adjustment process is

more efficient, because of the inversely proportional relationship. Since the input to this stage is buffered and the output is processed in an active filter, the issues with impedance change should be minimal.

### 2.1.6 Low-pass Filter

A low-pass filter is an electronic frequency selective circuit that passes signals with frequencies lower than the cutoff frequency but attenuates signals with frequencies higher than the cutoff frequency. The actual amount of attenuation for each frequency varies depending on the filter configuration.

In this design, a low-pass filter with a Sallen-Key topology was applied, which is a second-order filter. A second-order filter has narrower transition band and a steeper frequency response than a first-order filter. There are two typical topologies for a second-order low-pass filter: the Sallen-Key and the multiple feedback (MFB) [37]. The Sallen-Key topology, also known as a voltage control, voltage source (VCVS), is shown in Figure 2.7. The reason of choosing this topology is that its performance is relatively independent from performance of the op-amp, specifically, which has relatively loose gain-bandwidth requirements of the op-amp. Another advantage of this topology is that component spread is low (the ratio between the two resistor and capacitor values), which is beneficial for manufacturability [38].

The transfer function and cutoff frequency of the Sallen-Key low pass filter are

$$H(s) = \frac{1}{1 + C_2(R_1 + R_2)s + R_1R_2C_1C_2s^2}; \text{ and} \quad (2.7)$$

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}, \quad (2.8)$$

where  $R_1$  and  $R_2$  are the resistance in ohms,  $C_1$  and  $C_2$  are the capacitance in Farads

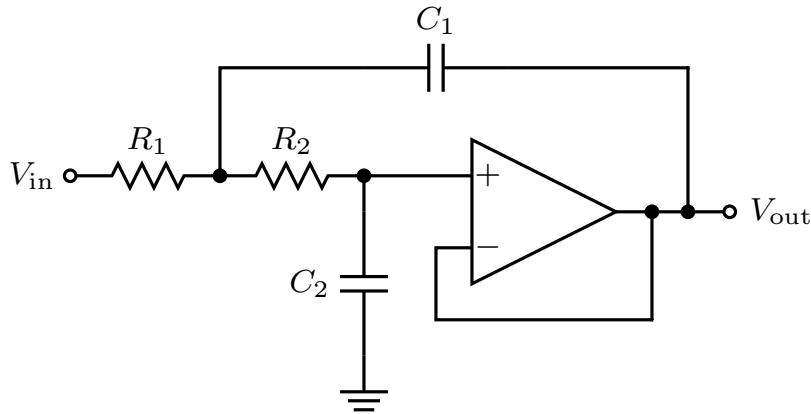


Figure 2.7: Sallen-Key low-pass filter with unity gain. This topology can be treated as containing two RC networks stages, which have 2 poles, and an op-amp configured as a voltage buffer.

and  $f_c$  is the cutoff frequency in Hertz.

The purpose of applying a low-pass filter in this circuit is to remove the high-frequency noise efficiently, whether it is introduced from the photodiode or produced by the printed circuit board (PCB) and previous stages. The whole circuit processes a signal with 70 kHz spilt frequency plus varied Doppler frequency. Thus, the cutoff frequency has been set to 200 kHz to reduce phase delay and gain roll-off.

## 2.2 Device Selection

### 2.2.1 Photodiode Selection

As discussed previously, a quadrant photodiode is employed to perform differential wavefront sensing to measure target mirror displacement, and changes in pitch and yaw. To achieve a high spatial sensitivity to measure pitch and yaw, a large active area, specifically, a large center-to-center distance between each element in the quadrant photodiode is needed. However, a large active area leads to large inherent

capacitance, which narrows the response bandwidth of the photodiode. Thus, these are tradeoffs that must be balanced when selecting the photodiode.

In this work, the Hamamatsu S5981 was selected for this design. It is a Si PIN multi-element photodiode for surface mounting. It has larger active area than other similar photodiodes, which is a  $100\text{ mm}^2$  square including four elements (quadrants), and has a 20 MHz bandwidth when operated with a 10 V reverse bias. Its photosensitivity is 0.43 A/W at the wavelength of a red HeNe laser at 633 nm [39].

The photodiode in this design senses a 70 kHz signal with a varied Doppler frequency at a wavelength of 633 nm, and the optical power of it varies from 1 to 50  $\mu\text{W}$ . The photodiode is configured in photovoltaic mode to achieve high sensitivity but a narrow bandwidth. From the datasheet, it has 20 MHz bandwidth but only with a 10 V reverse bias. It still must be tested whether it has at least a 200 kHz bandwidth with a zero bias. Theoretically, it generates 0.43 to 21.5  $\mu\text{A}$  photocurrent, depending on the incident optical power.

### 2.2.2 Op-amp Selection

It is important to choose op-amps that can provide the necessary DC precision, gain, speed, distortion, and noise. The principles introduced in the previous section assume ideal op-amps are used, which have following properties:

- Infinite open-loop gain
- Infinite voltage range available at the output
- Infinite bandwidth with zero phase shift
- Infinite slew rate

- Infinite input impedance
- Zero output impedance
- Zero input bias and offset current
- Zero input bias and offset voltage
- Zero noise, etc. . . .

None of these ideal properties can exist perfectly in a real op-amp. In a real op-amp, these properties should be non-infinite or non-zero, which could be modeled with equivalent resistors, capacitors, voltage sources, and current sources in the op-amp model. Some parameters may eventually have negligible effect on the final design while others limit the final performance of the design that must be evaluated. The following parameters must be carefully considered in this design.

### Gain-bandwidth Product

The gain-bandwidth product (GBW or GBP) for an op-amp is the product of the amplifier circuit's bandwidth and the closed-loop gain at the bandwidth. This parameter is not infinite but fixed in a real op-amp, and it determines the maximum bandwidth that can be extracted from the amplifier circuit for a given gain and vice versa. Thus, op-amp applications must balance the tradeoff between two important parameters gain and bandwidth.

For proper filter functionality, gain-bandwidth product is an important op-amp parameter. In general, the open-loop gain ( $A_{OL}$ ) should be 100 times (40 dB) above the maximum closed-loop gain ( $A_{PEAK}$ ) of a filter section to allow a maximum gain error of 1%, as Figure 2.8 shows.

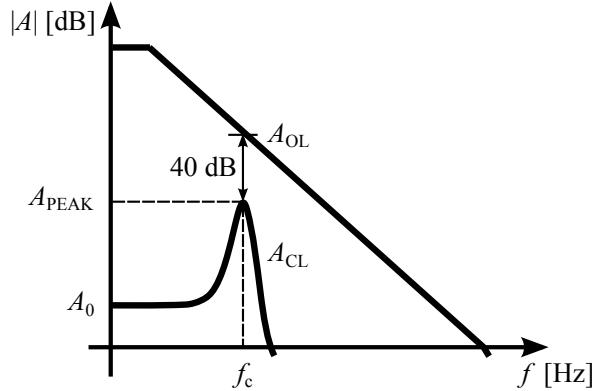


Figure 2.8: Open-loop gain  $A_{OL}$  and the filter response (closed-loop gain)  $A_{CL}$ .

A general rule is that

$$GBW = 100 \cdot \text{Gain} \cdot f_c \quad (2.9)$$

where gain is the maximum closed-loop gain,  $f_c$  is the cut-off frequency (low-pass filter) or maximum frequency needed to operate (high-pass filter). Equation (2.9) is a good design guide to determine the necessary gain-bandwidth product of an op-amp for an individual first-order and second-order ( $A_{PEAK} < 1$ ) filter [40].

## Slew Rate

An important parameter that determines the speed of an op-amp is the slew rate (SR). A real op-amp has internal capacitors that are charged and discharged during normal op-amp operations. With its internal resistance, a non-zero time constant could be calculated, which determines the maximum rate of signal change (slew rate) without distortion. In other words, slew rate is the maximum transient slope at any point of a signal in a circuit. An op-amp that is operated beyond the nominal slew rate could create non-linear effects. For adequate full power response, the slew rate (in volts per microsecond) of an op-amp at all points of a signal must be greater

than [40]

$$\text{SR} = \pi \cdot V_{\text{p-p}} \cdot f_c, \quad (2.10)$$

where  $V_{\text{p-p}}$  is the signal peak-to-peak voltage and  $f_c$  is the cut-off frequency (low-pass filter) or maximum frequency needed to operate (high-pass filter).

### **Input Offset Voltage**

The input offset voltage parameter, a DC characteristic, is defined as the DC offset voltage that must be applied between the two input terminals to keep output DC voltage zero within the op-amp. It is expressed in units of volts.

Due to the manufacturing process, the transistors of the two input terminals in real op-amps may not be exactly matched, thus zero differential input produces a non-zero output. In order to cancel that output offset, all op-amps require a small voltage difference between their inverting and noninverting inputs to balance the mismatch. The required voltage, known as the input offset voltage,  $V_{\text{IO}}$ , is normally modeled as a voltage source driving the noninverting input [37].

Input offset voltage is always multiplied by the noninverting gain of the amplifier circuit and added to (or subtracted from) the signal gain of the circuit. In large-gain DC-coupled circuits,  $V_{\text{IO}}$  may be significant and may need to be reduced through offset adjustment techniques, if the DC accuracy is important [41].

### **Input Bias Current**

Bias current is required by the input circuit of all op-amps for proper operation. The input bias current  $I_{\text{IB}}$ , a DC characteristic, is computed as the average of the two input bias currents  $I_+$  and  $I_-$ .

Bias current is a problem for op-amps because it flows in external impedances and produces voltages. In transimpedance amplifiers, the input bias current generates an additional output offset voltage with the large feedback resistor. This output offset voltage may send the output signal into saturation, depending on the op-amp power supply operation [42]. The best solution is to use an op-amp with either a CMOS or JFET input due to its very low input bias current [37].

## Other

Some op-amps are unity-gain stable, suitable for voltage buffers, while some other op-amps are optimized for higher closed-loop gains. Using those non-unity gain stable op-amps in buffer applications will cause problems.

The voltage supply range should be wide to leave enough margins for the amplitude of the output signals of amplifier applications. This aims to avoid saturation of output signals.

This circuit processes quadrant photodiode signals, which needs four parallel channels. It is better to utilize 4-channel chips (four op-amps in single chip) to implement every stage, which uses the least number of chips and keep the performance of every channel similar. By selecting each op-amp at each stage, the op-amp can be tailored to the specific application at that stage.

## Devices

The TI OPA4140, OPA4228, and OPA4227 are selected for the transimpedance amplifiers, filters, and buffer amplifiers, inverting amplifiers, respectively.

The OPA4140 is a high-precision, low-noise, rail-to-rail output, 4-channel, JFET op-amp. It has [43]:

- 11 MHz Gain Bandwidth Product
- 20 V/ $\mu$ s Slew Rate
- 30  $\mu$ V Input Offset Voltage
- $\pm 0.5$  pA Input Bias Current
- $\pm 2.25$  V to  $\pm 18$  V Voltage Supply Range etc . . .

It is suitable for the transimpedance amplifier in this design. This circuit is expected to process a 70 kHz split frequency plus varied Doppler frequency signal, whose frequency must be less than 200 kHz. Thus, the cut-off frequency  $f_c$  of the filter, consisting of feedback resistor  $R_f$  and feedback capacitor  $C_f$  (Figure 2.3), is set at 200 kHz. Thus,  $R_f$  and  $C_f$  are chosen to be 100 k $\Omega$  and 8 pF, respectively, which will be discussed in the following section. From terminal capacitance versus reverse voltage diagram in the S5981 photodiode, when the reverse voltage is 0.1 V, the terminal capacitance  $C_p$  is 140 pF. This assumes that in zero-bias, the photodiode has the same 140 pF  $C_p$ . A general guide (different from Equation (2.9)) to determine the minimum GBW requirement for the transimpedance amplifier is [44]

$$\begin{aligned} \text{GBW} &= 2\pi \cdot f_c^2 \cdot R_f \cdot (C_f + C_p) \\ &= 2\pi \cdot (200 \text{ kHz})^2 \cdot 100 \text{ k}\Omega \cdot (8 \text{ pF} + 140 \text{ pF}) \\ &= 3.7 \text{ MHz.} \end{aligned} \tag{2.11}$$

The OPA4140 has an 11 MHz GBW, which is higher than the required 3.7 MHz and is sufficient for the transimpedance amplifier in this design.

The current from the photodiode is 0.43 to 21.5  $\mu$ W and produces 0.043 to 2.15 V flowing through the 100 k $\Omega$  feedback resistor. The slew rate (according

to Equation (2.10)) for this transimpedance amplifier must be greater than

$$SR = \pi \cdot 2.25 \text{ V} \cdot 200 \text{ kHz} = 1.4 \text{ V}/\mu\text{s}. \quad (2.12)$$

The OPA4140 has a  $20 \text{ V}/\mu\text{s}$  slew rate, which is much greater than  $1.4 \text{ V}/\mu\text{s}$ . This amplifier meets the slew rate requirement.

Considering the transimpedance amplifier has a current input configuration, the input bias current and input current noise will impact the quality of the output signal. The OPA4140 has a low input bias current ( $\pm 0.5 \text{ pA}$ ), so this impact is low. The other following stages provides strategies to eliminate this impact, for example, the following high-pass filter removes the DC offset voltage produced by input bias current flowing through feedback resistor.

The OPA4228 is a high-precision, low-noise, wide-bandwidth, high-speed 4-channel op-amp. It has [45]:

- 33 MHz Gain Bandwidth Product
- $11 \text{ V}/\mu\text{s}$  Slew Rate
- $10 \mu\text{V}$  Input Offset Voltage
- $\pm 2.5 \text{ pA}$  Input Bias Current
- $\pm 5 \text{ V}$  to  $\pm 18 \text{ V}$  Voltage Supply Range etc ...

It is suitable for the active filters in this design. There are one first-order, high-pass filter and one second-order, Sallen-Key low-pass filter in this design. Based on their design and applying Equation (2.9), the necessary gain-band product of this op-amp meets this specification. Both filters have a unity-gain, and their cut-off frequency

(low-pass filter) or maximum operating frequency (high-pass filter) is 200 kHz. Thus, for filters, the GBW of the op-amps should at least be

$$\text{GBW} = 100 \cdot 1 \cdot 200 \text{ kHz} = 20 \text{ MHz}. \quad (2.13)$$

The OPA4228 has a 33 MHz GBW, which is higher than the 20 MHz required and enough for the active filters in this design.

The high-pass filter just blocks the DC voltage and the peak-to-peak voltage of its AC signal is almost same as that of the transimpedance amplifier. The gain of high-pass filter is unity. So the minimum slew rate of the op-amp for the high-pass filter is 1.4 V/ $\mu$ s.

The inverting amplifier before the low-pass filter adjusts the amplitude of signal to match the amplitude of input signal of the ADC, which is at most 2 V<sub>p-p</sub>. The minimum slew rate of the low-pass filter needed is

$$\text{SR} = \pi \cdot 2 \text{ V} \cdot 200 \text{ kHz} = 1.26 \text{ V}/\mu\text{s}. \quad (2.14)$$

The OPA4228 has an 11 V/ $\mu$ s slew rate, which is much greater than the required 1.4 V/ $\mu$ s. This amplifier meets the slew rate requirement.

The OPA4228 has a low DC offset (10  $\mu$ V), which determines the quality of the output voltage signal. In practice, this whole circuitry does not need very high DC precision, because the back-end ADC is AC-coupled. That means one -1 to 1 V sine signal is the same as one 0 to 2 V sine signal at same frequency for the ADC.

The OPA4227 and OPA4228 are in same series. The differences between them are that the OPA4227 is unity-gain stable, OPA4228 is optimized for closed-loop gains of 5 or higher. While the OPA4227 has a relatively narrow bandwidth (8 MHz) and

slower slew rate ( $2.3 \text{ V}/\mu\text{s}$ ), they are still suitable as voltage buffers and as inverting amplifiers. The remaining characteristics are the same between op-amps [45].

The main reason to choose OPA4227 is its unity-gain stability. Voltage buffers and inverting amplifiers in this circuitry have unity-gain or very low gain. At first, the OPA4228 were used as voltage buffers and inverting amplifiers on the actual PCB board, which caused self-exitations at several megahertz because the OPA4228 needs a closed-loop gain greater than five to be stable. However, it performs well as a filter amplifier in this design. Replacing the OPA4228 with the OPA4227, the voltage buffers and inverting amplifiers work as intended.

### 2.2.3 Capacitors and Resistors Selection

After selecting the op-amps, the capacitors and resistors also must be carefully selected. Because those op-amps are not ideal, external capacitance and resistance will interact with the op-amp internal impedance and alter the performance of the whole system.

For filters, capacitor values can range from  $1 \text{ nF}$  to several microfarads. The lower limit avoids coming too close to parasitic capacitances of other components. Resistor values should stay within the range of  $1 \text{ k}\Omega$  to  $100 \text{ k}\Omega$ . The lower limit avoids excessive current draw from the op-amp output, which is particularly important for single supply op-amps in power-sensitive applications. The upper limit avoids excessive resistor noise [37].

Thus,  $C$  and  $R$  are  $10 \text{ nF}$  and  $15.8 \text{ k}\Omega$  in the high-pass filter. In the Sallen-Key low pass filter,  $C_1$  and  $C_2$  are  $2.7 \text{ nF}$  and  $1.2 \text{ nF}$ , and  $R_1$  and  $R_2$  are  $316 \Omega$  and  $619 \Omega$ . Because this application is not power-sensitive and the op-amp is not in single supply mode,  $R_1$  and  $R_2$  are a slightly smaller than the  $1 \text{ k}\Omega$  lower limit guidelines but it

can be overlooked in this case.

In the transimpedance amplifier circuit, the feedback resistor  $R_f$  should be as large as possible to minimize noise (signal-to-noise ratio improves by  $\sqrt{R}$ ). However, it should be consistent with the bandwidth requirement and keep the voltage output within the op-amp voltage supply, otherwise it will cause signal saturation. The feedback resistor  $R_f$  is selected to be  $100 \text{ k}\Omega$  in this circuit. For a  $200 \text{ kHz}$  bandwidth,  $C_f$  is selected to be  $8 \text{ pF}$ .

In the inverting amplifier, the feedback resistor  $R_f$  is selected to be  $5 \text{ k}\Omega$ , and the input resistor  $R_{in}$  is selected to be a potentiometer, whose range is  $10 \Omega$  to  $10 \text{ k}\Omega$ . According to Equation (2.6), this circuit has a capability to adjust amplitude of input signal from  $0.043$  to  $2.15 \text{ V}$  to ultimately interface to the  $2 \text{ V}_{\text{p-p}}$  ADC.

The tolerance of the selected capacitors and resistors depends on the circuit sensitivity and on the circuit performance. The whole circuitry does not need high gain accuracy. Since the gain is adjustable, any gain difference between theory and practice could be eliminated by compensation. Meanwhile, it also does not need very high cut-off frequency accuracy. The filters in this design are just for blocking DC bias and high frequency noise. The  $1 \text{ kHz}$  to  $200 \text{ kHz}$  passband is very flexible and even a  $10 \text{ kHz}$  to  $190 \text{ kHz}$  is acceptable for this circuitry. Thus, common tolerance components,  $1\%$  resistors and  $5\%$  capacitors are selected in this system.

#### 2.2.4 Linear Regulator Selection

Linear regulators are used to maintain and provide a steady positive and negative voltage to power the op-amp based and signal processing circuitry. In this design, op-amps are in dual-supply mode, which need positive and negative voltage supplies. The quality of the power supply determines the quality of output signal op-amp

circuit as well. Thus, the TI TPS7A4901 and TPS7A3001 are selected as the voltage regulators. They are positive and negative high-voltage ( $+36\text{ V}$  and  $-36\text{ V}$ ), ultralow-noise ( $15.4\text{ }\mu\text{V}_{\text{rms}}$  and  $15.1\text{ }\mu\text{V}_{\text{rms}}$ , 72 dB PSRR (Power-Supply Ripple Rejection)), capable to sourcing a maximum load of 150 mA and 200 mA [46; 47]. Figure 2.9 shows the typical application circuits of TPS7A4901 and TPS7A3001 [48].

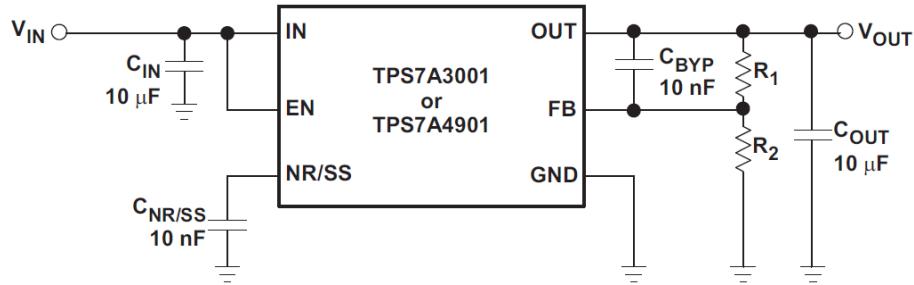


Figure 2.9: TPS7A4901 and TPS7A3001 typical application circuits.

These two linear regulators have the capability to adjust the output voltage by using external resistors with specific values. In this design, the input voltages from the supply are  $\pm 12\text{ V}$ , and linear regulator output DC voltages (for op-amp voltages supply) are set as  $\pm 5\text{ V}$ .

Another important characteristic of the linear regulators is the capability to supply power. Theoretically, these two regulators could provide up to 1.5 W of power and the six op-amps, the main part in circuit, consume 0.84 W at most. Thus, these two regulators provide enough power for op-amp operation.

## 2.3 Printed Circuit Board Design

After investigating the theory and designing the circuit schematic, the printed circuit board (PCB) must be designed. In this design, Cadence® Allegro® software was

used to build the schematics, perform analog simulations, lay out the PCB design, and prepare the whole design flow from front to back.

### 2.3.1 Schematic Design

The circuit schematic was completed before using the electronic design automation (EDA) tools, so the initial process is to transcribe the schematic diagram from the concept sketch to the EDA tool. The EDA tool used in this phase is Cadence® Allegro® Design Entry Capture CIS, which is an industry standard in schematic design entry.

Besides the circuits introduced previously, two LEDs and several bypass capacitors are also added into the whole schematic. The two LEDs indicate power supply status, which are helpful for debugging and protecting on-board chips. If  $\pm 5$  V are supplied, the LEDs will illuminate. The bypass capacitors reside across the op-amp power supplies to ground, conduct the alternating current around op-amp to ground. Therefore, it decreases the impact on the performance of op-amp caused by the surrounding noise.

Figures A.1 and Figures A.2 are the two parts of the entire schematic diagram of this quadrant photodiode detection and processing circuitry. Figures A.1 is a schematic of four channels of high-pass filters, inverting amplifiers, Sallen-Key low-pass filters, buffers and SMA connectors. Figures A.2 is a schematic of two linear regulators, header, two DB9 connectors, two LEDs, quadrant photodiode, four channels of transimpedance amplifiers and buffers and several bypass capacitors.

### 2.3.2 PSpice Simulation

After designing the schematic and before building the PCB, a simulation is needed to verify whether the circuit is functionally correct. The simulation software used in this design is PSpice®, which is integrated in Cadence Allegro.

PSpice® delivers complete analog and mixed-signal circuits simulation and verification solution. In this design, the function mainly used is that ensuring functional correctness of schematic designs by verifying the analog portions for node voltages, branch currents and device power with resources such as models from components vendors and built-in mathematical functions.

In this design, it simulated the circuit just after the photodiode, because the S5981 quadrant photodiode model is not provided by vendor, and its parameters are not complete in the datasheet for building an equivalent model. Thus, a current source is introduced to replace photodiode to generate the photocurrent. With the remaining components, two types of analysis have been performed, which are Transient and AC Sweep/Noise analysis.

The transient analysis simulates the transient response of the circuit. In this case, the responses to the minimum ( $1 \mu\text{W}$ ) and maximum ( $50 \mu\text{W}$ ) incident optical power were simulated. Setting the outputs of current source as  $0.43 \mu\text{A}$  and  $21.5 \mu\text{A}$  amplitude,  $70 \text{ kHz}$  frequency sine waves, the outputs of every stage – transimpedance amplifiers, high-pass filters, inverting amplifiers and Sellan-Key low-pass filters – are shown in Figure 2.10. As shown in Figure 2.10, every stage works as expected and the output of the whole circuitry can maintain approximately  $2 \text{ V}_{\text{p-p}}$ , which matches the input voltages of the following ADCs.

The AC sweep analysis determines the frequency response of circuit system.

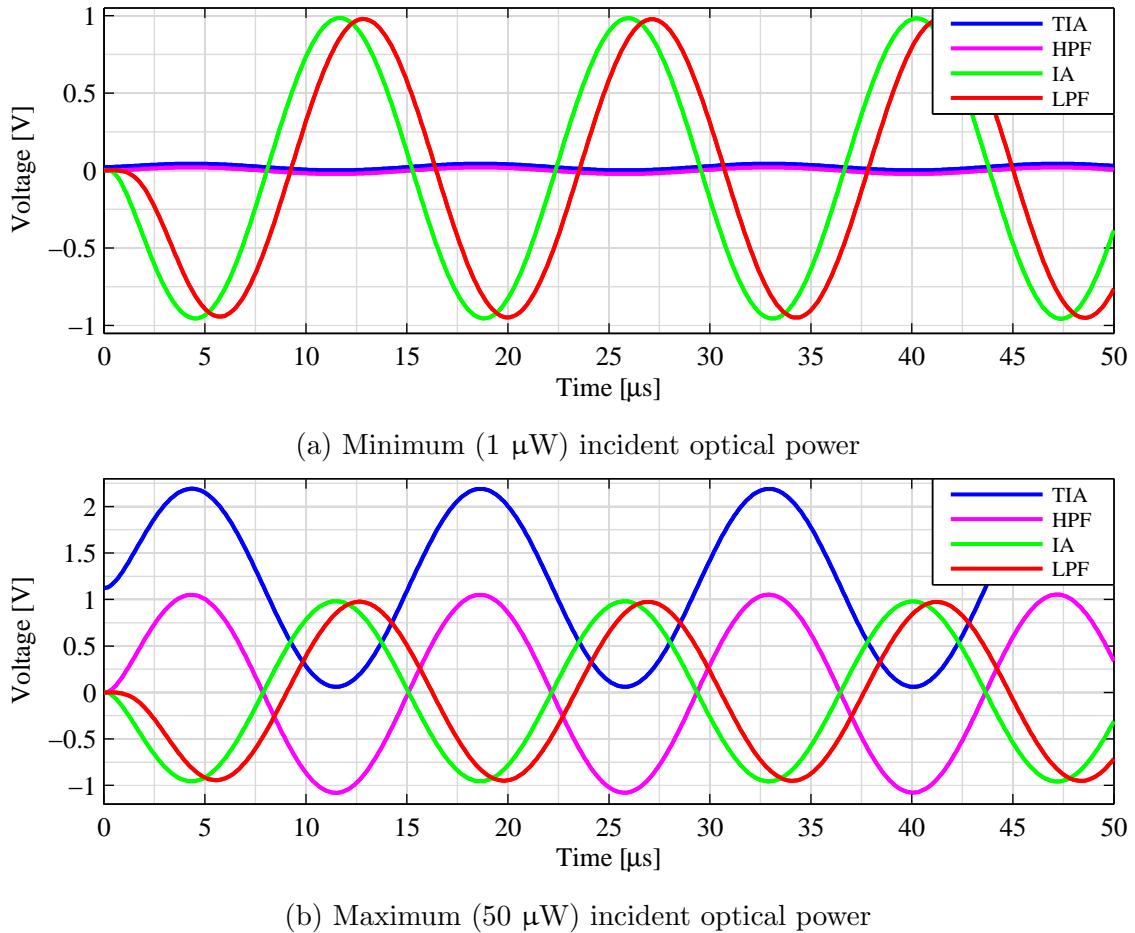


Figure 2.10: Transient analysis of the response to minimum and maximum incident optical power. The subfigure (a) shows the outputs of the transimpedance amplifier (TIA), high-pass filter (HPF), inverting amplifier (IA) and low-pass filter (LPF) when minimum optical power is incident. The subfigure (b) shows outputs when the optical power is maximum. Because the inverting amplifier has an adjustable gain, the output voltage of whole system can maintain approximately 2 V<sub>p-p</sub> by adjusting the potentiometer.

Figure 2.11 shows a Bode plot of system. The system has a passband from 1 kHz

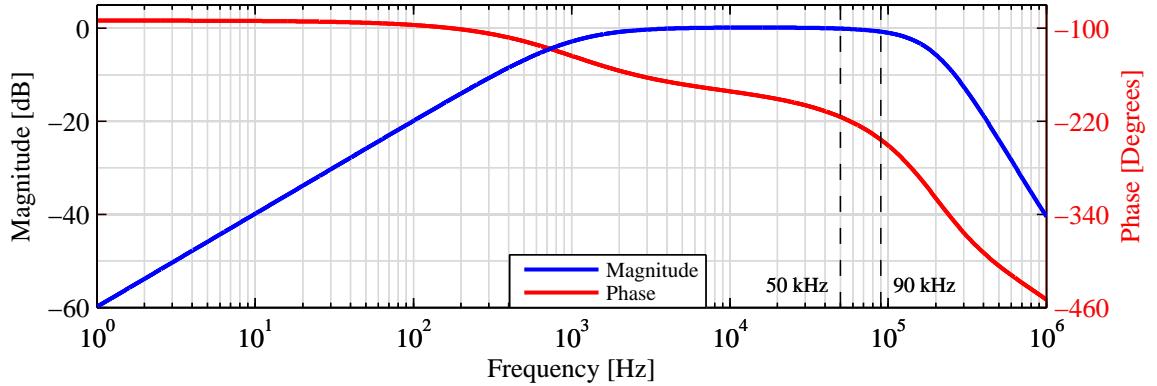


Figure 2.11: Bode plot produced by AC sweep analysis.

to 200 kHz, which are the cut-off frequencies of the high-pass and low-pass filters. It ensures that the system could pass 70 kHz split frequency plus varied Doppler frequencies, and block DC voltage and high frequency noise. However, in phase response subfigure, this system has a nonuniform phase shift in the passband, which causes a precision problem when measuring a varied velocity target. That will be discussed in detail in the Future Work section.

A noise analysis also has been performed. Within the passband, the simulation shows that the entire processing circuitry has a 63 dB signal-to-noise ratio (SNR), which will be worse in practice. Thus, the noise level still needs to be quantified in real test. Figure 2.12 shows the SNR produced by the noise analysis.

### 2.3.3 PCB Layout

After verifying the function of the circuit, Cadence® Allegro® PCB Designer is used to create a PCB layout. It provides a complete placement and routing environment – from basic floorplanning, placement, and routing to placement replication, advanced interconnect planning – for simple to complex PCB designs.

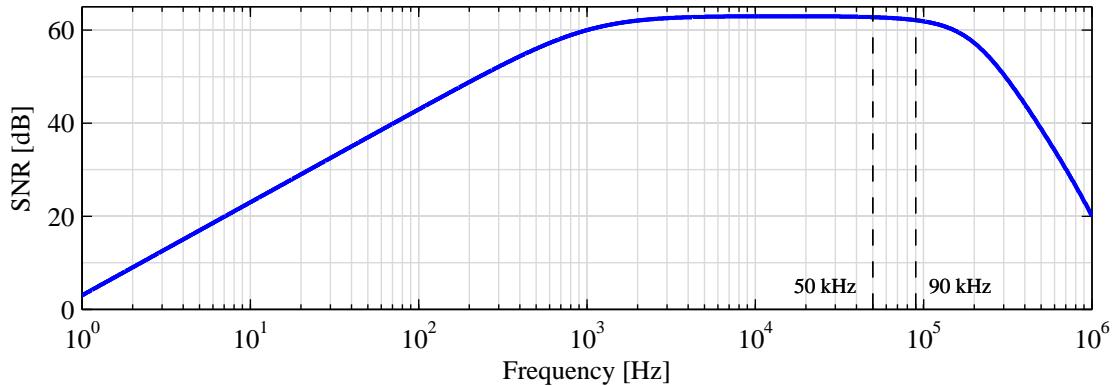


Figure 2.12: SNR of the entire processing circuitry produced by noise analysis.

This design is relatively simple because there are minimal components and chips and the on-board signal is at most 200 kHz, which is regarded as low speed. However, as this device is a part of precision instrument, is expected to have high performance and high output signal quality. Meanwhile, as a detection device, it is expected to be compact. Thus, it still must be carefully designed.

This detection and processing system is an analog circuit, which is vulnerable to noise and drift. Considering the signal integrity (SI) and electromagnetic compatibility (EMI), this circuit is designed as 4-layer PCB with internal ground and power layers.

In order to keep performance differences among these four parallel processing channels minimum, the length of the signal routes in four channels are maintained identical, relying on Constraint Management, which is a feature of this software.

In practice, the linear regulators are huge thermal sources. The heat produced by them influences several parameters of the photodiode, such as dark current, which will decrease the precision and predictability of the circuit. Therefore, the system is split into two PCBs. The separate detection and processing parts insulate the detector from the thermal source.

Several other techniques are also involved in this design. Figure A.3 and Figure A.4 shows the PCB layout and routes. Figure 2.13 shows final PCBs with components and chips soldered on them.

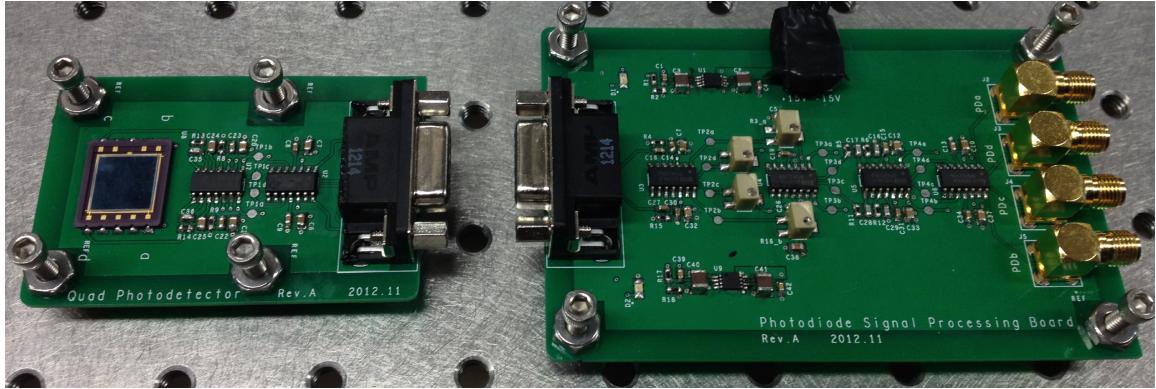


Figure 2.13: The soldered PCBs of detection and processing circuitries.

## 2.4 Verification Measurement

The magnitude and phase response, background noise of each channel have been measured. The aim of the verification measurements is to obtain the real response characteristic of this circuitry, to judge its performance, and then to calibrate it.

### 2.4.1 Setup

To simulate the interference in a real measurement, a function generator drives an acousto-optic modulator (AOM) to modulate output power of a laser source and makes the intensity of the laser light vary at the same frequency as the function generator's output. Then a lock-in amplifier is used to compare the phase difference between input and output signals of the detection and processing board, and to provide a voltage directly proportional to the voltage of board's output. The ideal

instrument to measure the frequency response of a system is the network analyzer, which was not available for this test. A potential problem of switching to a lock-in amplifier is that this instrument may have a non-uniform frequency response, which is not specified by the manufacturer. Therefore, the measurement result could be the frequency response of the board superimposing on that of the lock-in amplifier.

### 2.4.2 Measurement

Utilizing the function generator to generate a chirp signal with the frequency sweeping from 1 kHz to 100 kHz<sup>1</sup>, the output waves recording the phase differences and voltages at every frequency could be regarded as phase response and magnitude response. Two points must be mentioned: 1. for convenience, the output signal of the function generator replaces the optical power as the input signal in the calculation, ignoring any phase delay in the AOM, which has a 10 ns response rate. 2. The magnitude response is just the normalized output voltages, which does not indicate the actual gains between output and input signals, but still represents trend of gain changing as a function of frequency.

First, the influence on phase shift from the lock-in amplifier itself must be tested. The time constant is one of lock-in amplifier's parameters and adjusts the cutoff frequency of the internal low-pass filters, which will introduce phase shifts in the signals. In this verification measurement, time constants 100  $\mu$ s, 300  $\mu$ s, 1 ms, and 3 ms are each used when verifying each of the four channels.

Figure 2.14 shows the frequency response of Channel A with each time constant, and the simulation result from PSpice. The other three channels have similar shapes and trends for their responses. From Figure 2.14, the shape and trend of each response

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<sup>1</sup>The frequency of 102.4 kHz is the upper level for the SR830 lock-in amplifier to lock a signal [25].

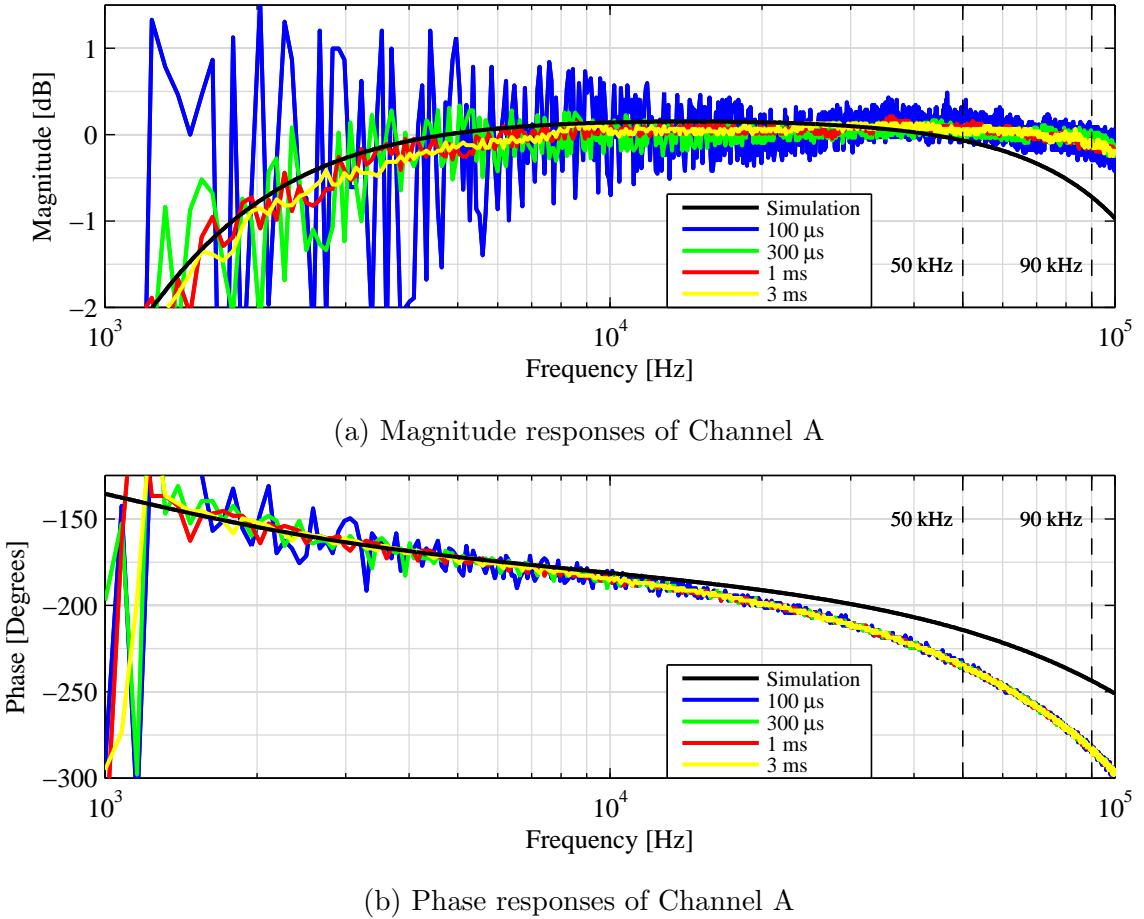


Figure 2.14: Frequency responses of Channel A with 4 different time constants and the simulation result from PSpice.

has a similar profile to the simulation. Changing the time constant changes the noise level, as expected, but not the overall trend. Also, the measured response matches the simulation results well in the low frequency regime but shift at the high frequency part regime. This circuitry deals with the 70 kHz split frequency and maximum  $\pm 20$  kHz Doppler frequency, thus the band from 50 kHz to 90 kHz in frequency response must be carefully examined.

Swing of the magnitude response (Figure 2.14(a)) in this band does not impact on the following phase calculation in FPGA board, because the algorithm is not related

with the magnitude of the signals.

The phase responses (Figure 2.14(b)) in this band are stable and identical with any time constants. That means the differences among time constants of the lock-in amplifier do not cause differences among the phase response in this band. However, the difference between the measured response and simulation result does exist. There are two potential sources of this difference, one is the superimposed phase response of lock-in amplifier, and another is the difference between real values and nominal values of the electrical components.

Then, the difference in performance among four channels must be determined. Theoretically, these four channels should be exactly same. However, due to the manufacturing process, different elements in the same components, different components in same model may cause slight differences in performance of the entire circuitry. If that exists, quantification and calibration processes are required to eliminate it.

Figure 2.15 shows the frequency responses of each channel with a 3 ms time constant. The phase responses (Figure 2.15(b)) are identical in the 50 kHz to 90 kHz band, which means these four channels have good consistency in the phase characteristic. However, the magnitude response has more differences. One reason is the resistance of the potentiometers in the inverting amplifier is always drifting, which means the gain is drifting as well. Even with same time constant in same channel, the magnitude responds are changing test by test. Thus, it is the bad repeatability that causes the magnitude responds to not match each other.

From both the simulation results and practical measurements, in the 50 kHz to 90 kHz band, the entire circuitry does not have a uniform phase shift. There is a maximum 50° phase shift difference when measuring 50 kHz and 90 kHz signals. The

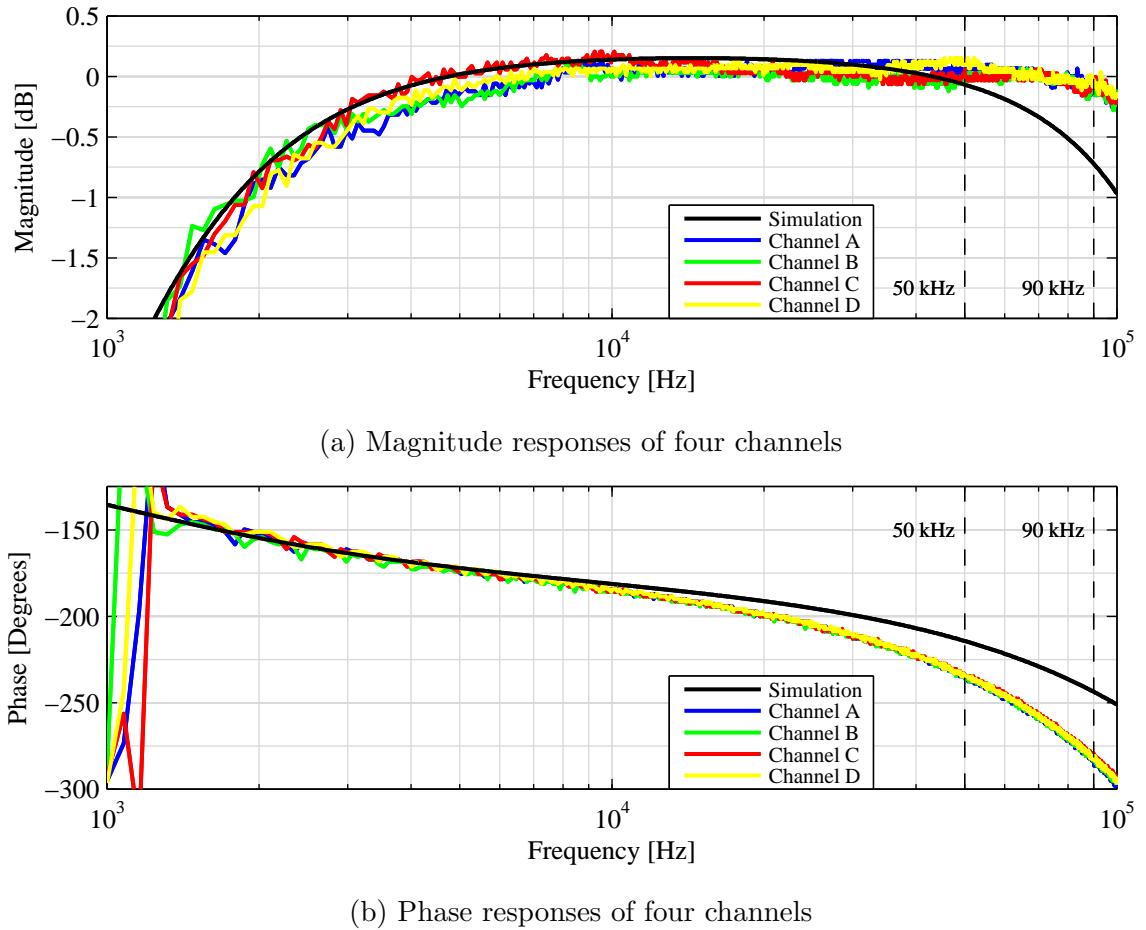


Figure 2.15: Magnitude and phase responses of four channels with 3 ms time constants and the simulation result from PSpice.

functionality of phasemeter is to measure the phase difference between two signals, however, the system inherently introduces a large systematic error. This error will decrease the precision of measurement result dramatically if not corrected. This will be discussed in the Future Work section.

The background noises of the four channels are also measured. Figure 2.16 show the background noise of Channel A. This noise level has been measured in case of the potentiometer in certain status that the output signal's peak-to-peak voltage is 1.6 V.

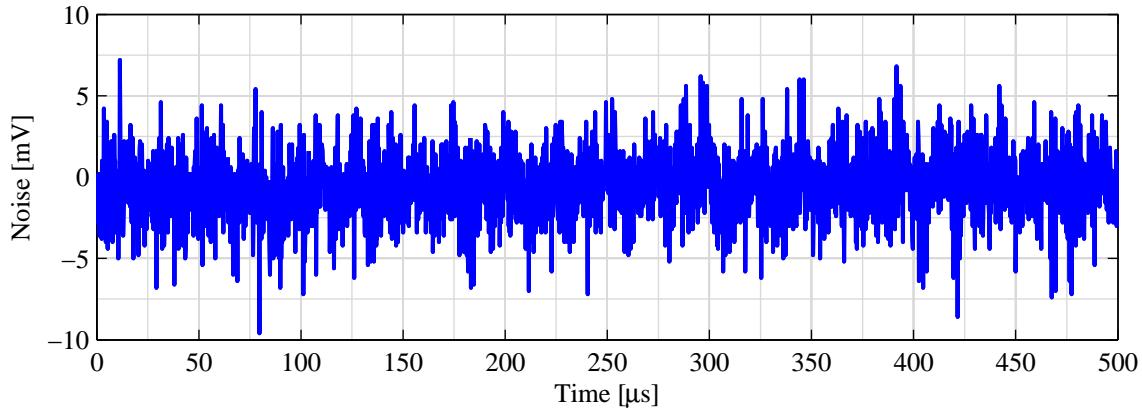


Figure 2.16: The background noise of Channel A.

The RMS value of the noise is 2.08 mV, for Channel A, the signal-to-noise ratio is

$$\text{SNR} = 20\log_{10} \frac{1.6/\sqrt{2}}{2.08 \times 10^{-3}} = 54.7 \text{ dB.} \quad (2.15)$$

The other channels also have the same SNR levels. Although that is approximately 10 dB lower than the simulation results in Figure 2.12, it is still acceptable.

# 3 Digital Signal Processing

## Module Design

In this project, a Field-Programmable Gate Array (FPGA) is the hardware used to demonstrate and implement the phasemeter signal processing algorithms, which are phase-locked loop based lock-in detection algorithm, and single-bin discrete Fourier transform algorithm.

### 3.1 FPGA Introduction

An FPGA is a semiconductor device whose hardware is reconfigurable, which means through programming, its internal logic components' physical connection can be adjusted for specific applications, achieving specific features and functions.

FPGA development is an integrated and advanced way to design digital circuits. In an FPGA chip, the resource contains large scale logic components, dedicated multiplier-accumulators (MACs), routing, embedded SRAM, high-speed transceivers, high-speed I/Os, and etc. Through configuring their interconnection, it can perform any level of digital circuits, from merely simple logic gates like AND and XOR, to complex combinational and sequential functions, such as custom digital signal

processing, dynamic control and communication blocks, or even soft embedded processors, which transforms the devices into systems on a chip (SoC) [49].

### 3.1.1 Comparison between FPGAs and DSP Processors

The phasemeter contains a digital signal processing system, which has a complex algorithm. It requires mathematical operations on a high-speed flow of data samples. There are two mainstream solutions for digital signal processing applications, one is based on digital signal processors, and another is based on FPGAs.

A digital signal processor (DSP) is a type of microprocessor specifically for digital signal processing applications, whose architecture is optimized, for example, for the mathematical operations. DSP processors are software-based processors, which are programmable through software, but their hardware architecture is not flexible as an FPGA. They are typically programmed in C, sometimes with assembly code for performance, however, the software programs just control the ways and orientations of how data flows between each hardware block and cannot reconfigure the interconnection of the hardware. Therefore, its architecture such as the number of MAC blocks, memory, hardware accelerator blocks, and bus widths all are fixed [50].

DSPs execute mathematical operations based on instruction, not clock. Typically, a mathematical operation on a single sample requires three to four instructions. A complete processing of a function, such as a single FFT or digital filter, requires dozens of iterative mathematical operations. Every instruction shares the fixed hardware cyclically, thus the output must wait until every instruction has finished before it can be released. The process flow for a DSP is shown in Figure 3.1(a).

Obviously, when handling extremely math-intensive tasks, a DSP's performance is limited by the clock rate and the number of instructions it can execute or

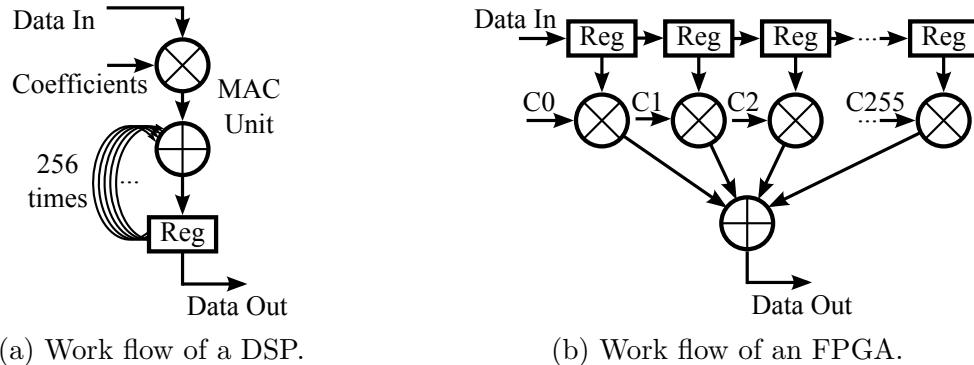


Figure 3.1: Work flow of a DSP and an FPGA to implement a 256-tap FIR filter. In (a), because the MAC unit in DSP is time-shared, and a 256-tap FIR filter algorithm needs 256 times MAC operations, it takes 256 clock cycles to execute these loops. That means this serial processing products one output every 256 clock cycles. The rate to process incoming data is dramatically slower than its clock rate. In (b), because every MAC operation could have its own dedicated MAC, there is no need to share it with others, the 256 MAC operations can be executed in one clock cycle. That means this parallel processing products one output every clock cycle. The rate to process incoming data is the same as clock rate, which is very efficient.

useful operations it can do per clock cycle. However, simply increasing clock rate cannot increase the performance dramatically because it creates many difficult system challenges, such as signal integrity issues.

In contrast, FPGAs are a form of highly configurable hardware, which could be considered as a blank breadboard with a large quantity of unconnected gates on it. The device is programmed by connecting the gates together to form adders, multipliers, and more complex operation models. Generally, FPGAs are programmed using a hardware description language such as Verilog or VHDL (Very High Speed Integrated Circuit Hardware Description Language).

An FPGA has flexible hardware. It can allocate each operation its own hardware resources. In this case, the operations can run independently with no need to wait for idle hardware. They constitute an assembly line-like processing chain to process a continuously streaming signal with their dedicated resources for each step, and have

the potential ability to perform a mathematical operation on the incoming data every clock cycle because the FPGA is clock based. The FPGA process flow is shown in Figure 3.1(b). Its performance is limited by the clock rate and the delay of signal propagation between gates.

When sample rates (input rates) grow above a few megahertz or the system requires numerous parallel channels, FPGAs outperform DSPs in extremely complicated algorithm applications such as dynamic control strategy [51], real-time signal processing [52], and imaging processing [53].

Most digital signal processing systems designs begin with a block diagram design. Actually translating the block diagram in an FPGA development software is simpler than converting it to C code for the DSP.

In this design, the phasemeter must process incoming data at 50 MSPS, and the important attributes are high-speed, real-time, and precise performance. To some degree, the price and power consumption are ignored, because this is a prototype used in a laboratory and not a mobile or battery-powered device. Therefore, an FPGA is chosen as the platform to implement the phasemeter digital signal processing.

## 3.2 Hardware Introduction

In this project, the phasemeter algorithm was implemented in an Altera DE2-115 board, which is a development and education FPGA board. It contains many interfaces and peripherals to accommodate various application needs, as Figure 3.2 shows.

For this research, the desirable attributes of this system are [54]:

- A Cyclone IV EP4CE115F29 FPGA features 114,480 logic elements (LEs),

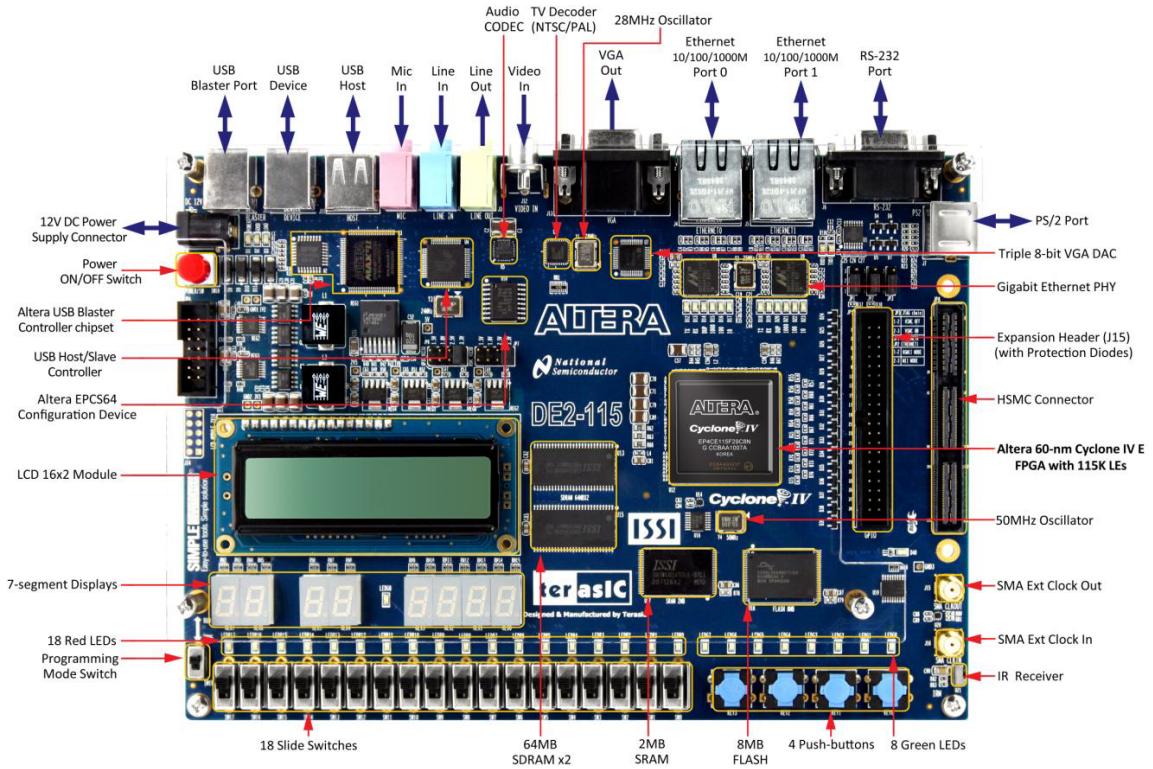


Figure 3.2: Altera DE2-115 FPGA Board [54].

266 18 bit $\times$ 18 bit multipliers, 432 M9K memory blocks and up to 3.9 Mbits embedded memory. It has enough on-chip resource for heavy digital signal processing.

- A High-Speed Mezzanine Card (HSMC) connector supports additional functionality and connectivity via HSMC daughter cards and cables. A high-speed AD/DA card connects DE2-115 FPGA board in this design, as Figure 3.3 shows, which has dual AD channels with 14-bit resolution and data rate up to 65 MSPS and provides samples precisely and rapidly for following digital signal processing [55].
- 128 MB (32 M $\times$ 32 bit) SDRAM to store measurement result for verification.
- Two Marvell 88E1111 Gigabit Ethernet PHY with RJ45 connectors are

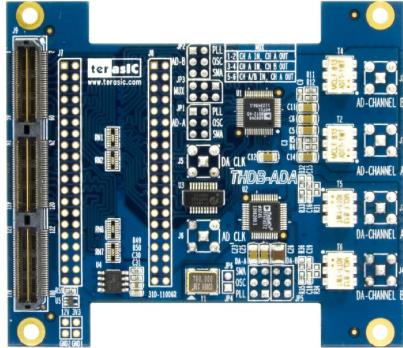


Figure 3.3: High-Speed AD/DA Daughter Card [55].

equipped on the board, which integrate 10/100/1000 Mbps Gigabit Ethernet transceiver support MII/RGMII MAC interfaces. Gigabit Ethernet interface is applied to communicate with host PC.

### 3.3 Software Introduction

In this project, Matlab<sup>®</sup>/Simulink<sup>®</sup> and its toolbox DSP Builder, Altera<sup>®</sup> Quartus<sup>®</sup> II are used to implement the algorithm into FPGA.

#### 3.3.1 Simulink

Simulink is a block diagram environment for Model-Based Design. It supports system-level design, simulation, and automatic code generation, which are three key features needed by this project.

In this project, a rapid, effective process is needed to model and verify the algorithm of the digital signal processing system. The Model-Based Design in Simulink is that process. Simulink provides a graphical user interface (GUI) for

building models for system-level designs as block diagrams, which simplifies the modeling process.

### **3.3.2 DSP Builder**

Simulink can also incorporate certain specific toolboxes developed by hardware vendors, such as Altera's DSP Builder toolbox. DSP Builder provides the hardware representations of common DSP function blocks, integrates the algorithm development, simulation, and verification capabilities of the MATLAB and Simulink with the Altera Quartus II software.

Automatically generating hardware description language (HDL) for DSP models is the most significant feature valued in this project, it shrinks the development cycle from algorithm to rapid prototyping, also avoids the introduction of manually coded errors.

### **3.3.3 Quartus II**

The Altera Quartus II design software is an FPGA development environment for analysis, synthesis, placement, routing, and assembly of HDL designs, which can compile Verilog/VHDL designs, perform timing analysis, simulate a design's performance, and download the configuration to the specific target device (FPGA) with the programmer.

In this project, the peripherals of the phasemeter digital signal processing part, analog-to-digital converters (ADCs) controller, SDRAM controller, etc., were also designed in this software. Importantly, design constraints and time analysis are also done in this software to ensure not only the functionality but also the timing performance meet the design requirements.

## 3.4 Model Design

### 3.4.1 Fixed-point Precision

This project's target hardware is an FPGA. Commonly, FPGAs use fixed-point numbers, which is a data type for a number that has a fixed number of digits. Because of the FPGA's structure, specifically the structure of logic cells (LE) and embedded multipliers, it is more straightforward to implement fixed-point operations, just like a normal digital circuit. In general, it can be assumed that fixed-point implementations use less resources (logic cells, embedded multipliers and routing resources) and in higher speed. Other data types, floating-point, have a speed, resource, and complexity penalty though it provides high resolution over a large dynamic range.

With the rapid development of FPGAs, the speed and resources are not generally a limitation anymore, which are enough for extremely complicated digital signal processing algorithms, dynamic control, and communication systems. However, to eliminate complexity as well, developers must purchase extra licenses of floating-point versions of development software and Intellectual Property (IP) cores for their floating-point implementations. That is one reason why fixed-point is chosen in this project. However, the essential reason is that fixed-point implementation can achieve sufficient resolution for this project, which can be proved by calculations and simulations.

The phasemeter is a part of a displacement interferometer that converts the interference signals to target displacement. The relationship between displacement  $\Delta x$  and phasemeter output  $\phi$  is given by Equation (1.3). Assuming the final resolution of the displacement interferometer should be 1 pm and 1 nm respectively,

$N$  is 2,  $\eta$  is 1, so the angular resolution for 1 pm and 1 nm displacement resolutions are

$$R_{\phi 1} = \frac{2\pi NR_{\Delta x 1}\eta f}{c} = \frac{2\pi \cdot 2 \cdot 1 \text{ pm} \cdot 1}{633 \text{ nm}} = 1.99 \times 10^{-5} \text{ rad, and} \quad (3.1)$$

$$R_{\phi 2} = \frac{2\pi NR_{\Delta x 2}\eta f}{c} = \frac{2\pi \cdot 2 \cdot 1 \text{ nm} \cdot 1}{633 \text{ nm}} = 1.99 \times 10^{-2} \text{ rad.} \quad (3.2)$$

In order to achieve these angular resolutions, the bit width after decimal point of phasemeter output are given by

$$n_1 = \log_2(R_{\varphi 1}) = -15.6, \text{ and} \quad (3.3)$$

$$n_2 = \log_2(R_{\varphi 2}) = -5.6. \quad (3.4)$$

That means fixed-point data type output of the phasemeter must be at least 16 bits or 6 bits after decimal point, respectively.

Meanwhile, the precision of the phasemeter also depends on the sampling rate, resolution of input signals, and bit width of internal data flow. The ADC daughter card has a defined sampling rate, which is up to 65 MSPS, and resolution of inputs is 14-bits. Thus, the bit width of internal data flow must consider these values to make sure that is wide enough for the required precision.

### 3.4.2 Fixed-point Model Design

The first step in programming is to represent every function block in the algorithm into a fixed-point Simulink model using the DSP Builder toolbox. The Altera DSP Builder toolbox has already provided sufficient blocks for basic and advanced digital signal processing operations, while specific functions must be built by designers using

Verilog HDL or VHDL. The following are some important functions and subsystem blocks designed in this project.

## Low-Pass Filter

There are two categories of digital filters: infinite impulse response (IIR) filters and finite impulse response (FIR) filters, or based on the structure, they are referred to as recursive filters and nonrecursive filters, respectively.

Compared to an FIR filter, an IIR filter can often be much more efficient for a given frequency response and for a given filter order, it requires few delay elements, adders, and multipliers. This is because the IIR filter incorporates feedback and is capable of realizing both zeros and poles of a transition function.

In this project, all digital filters are designed in the FPGA and these filters must occupy few resources and be implemented for real-time measurement and control. IIR filters were chosen for low-pass filters because they are highly selective filters that can be realized with low-order, can run at high speeds, and need fewer resources for the same tolerance compared with FIR filters [56].

However, there are also some disadvantages to IIR filters. For instance, the feedback can introduce instabilities. While the most significant one is its nonlinear phase response or nonconstant group delay, a nonconstant group delay means that not all frequencies experience the same delay. The delay causes phase shift. In that case, processing signals in different frequency would introduce non-uniform phase shift, which impact the measurement precision of the phasemeter system. Thus, not only linear but also uniform phase response in the passband is expected in this project. Future details about the solution will be discussed in the Future Work section.

The IIR filter in this project is based on a classic Butterworth model, which has a

maximally flat passband and stopband, but wide transition band [35]. The IIR filter structure in this project is biquadratic (biquad), specifically Direct II form. The structure of the IIR filter implemented in the FPGA is shown in Figure 3.4.

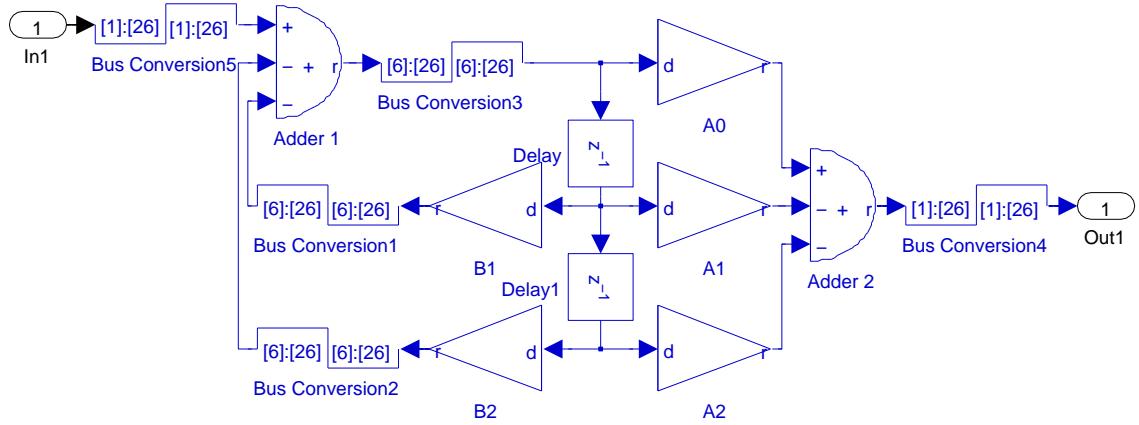


Figure 3.4: The structure of the IIR filter implemented in the FPGA. This digital biquad uses two three-input adders, two delays, five multipliers and several bus conversions. The multiplier coefficients are  $A_0$ ,  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$ . These coefficients are calculated during the filter design process by using Matlab. Bus conversions here control data width to be wide enough but not waste of resource (embedded multipliers), so they must be chosen carefully for each product, especially for the signals through gain  $B_1$  and  $B_2$ . Since these coefficients may be larger than 1, before filter goes into steady status, the amplitudes of these signal could be unpredictably high.

The transfer function of this biquad structure is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{A_0 + A_1 z^{-1} + A_2 z^{-2}}{1 - B_1 z^{-1} - B_2 z^{-2}}, \quad (3.5)$$

where  $A_0$ ,  $A_1$ ,  $A_2$ ,  $B_1$ , and  $B_2$  coefficients that determine the filter's response. In order to have better frequency response, fourth-order or sixth-order filters were used in this project by aligning two or three biquad stages in series. Innate high-order filters are highly sensitive to the values of their coefficients; even a slight difference between the actual value and the theoretic value could cause instability. Thus, higher-order filters

are usually designed by cascading biquad stages. Each biquad gives a second-order response. Figure 3.5 shows the frequency response of a fourth-order IIR filter, which are aligned by two stages of the second-order IIR filter in Figure 3.4.

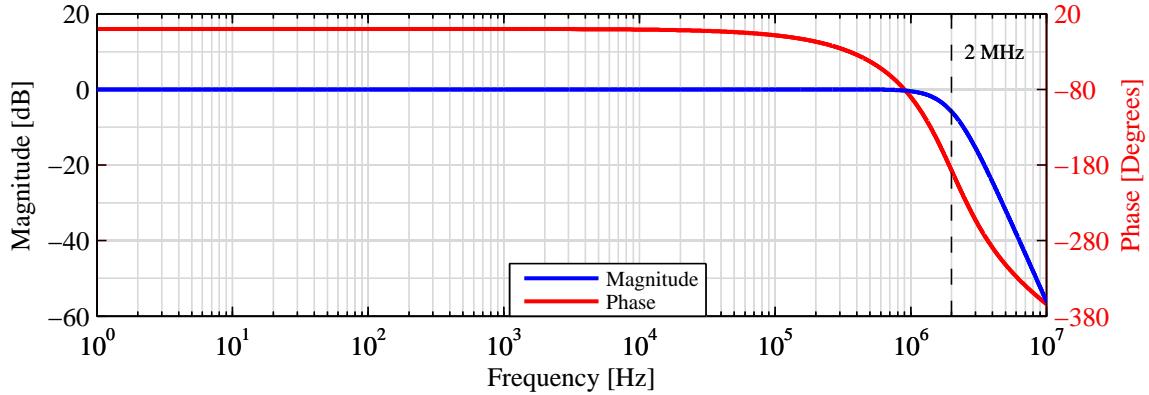


Figure 3.5: Bode plot of the fourth-order IIR filters. These filters are aligned by two stages of second-order IIR filters, and their cut-off frequency is 2 MHz.

## PLL

Basic principle of a PLL has been introduced in the previous chapter. While implemented in the FPGA, it is all digital PLL (ADPLL) [57], which needs more specific considerations during its design. The schematic diagram of an ADPLL is shown as Figure 3.6.

The multiplier acts as a phase detector, which compares the output of the numerically controlled oscillator (NCO) with input reference signal. The product of these two signals has two terms, which contain the sum and the difference of instantaneous phases of these two signals, respectively.

The loop filter consists of three second-order IIR filters, which has been introduced previously. This configuration has better filtering capability than just one or two in

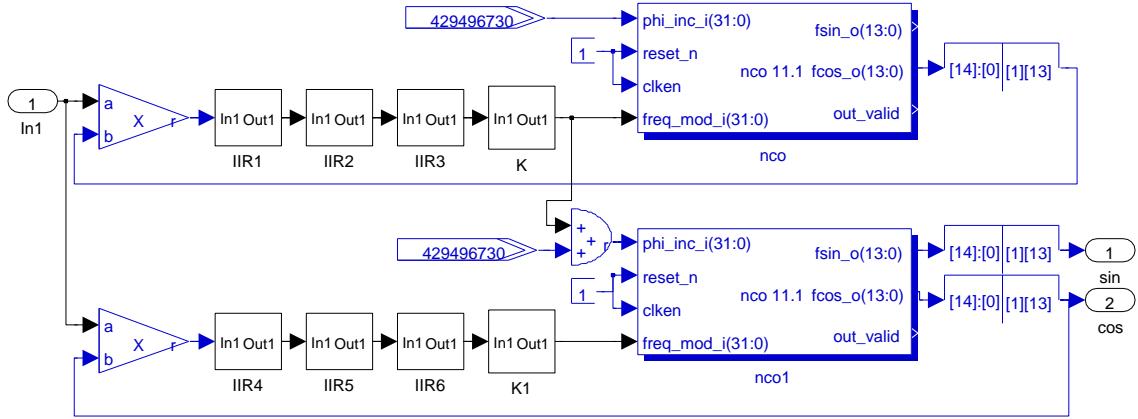


Figure 3.6: Schematic diagram of ADPLL. It has two phase-locked loops stacking, the upper one is for locking frequency; the lower one is for locking phase.

series, and it reduces the time for establishing the signal lock and makes the locked phase more precise.

In an ADPLL, the NCO plays the role of a VCO, which is a key part of designing the ADPLL in an FPGA. The DSP Builder toolbox provides the NCO Intellectual Property (IP) Core, which has complex configuration and functions. The critical function needed in an ADPLL is frequency modulation, which could adjust the oscillating frequency by input number *freq\_mod\_i*<sup>1</sup>. It is very similar to a VCO adjusting with input voltage. The input number of the NCO is the output of filter times a factor *K*. The factor *K* should be chosen carefully, since a large value makes it miss locking the signal easily and a small value make it too slow to achieve locking.

When implemented in practice in the FPGA, one ADPLL could not lock both the frequency and phase at the same time, when the frequency of the input signal is different from the quiescent frequency of the NCO. According to Equation (1.20), Equation (1.21), the relationship between the frequency modulation input (also

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<sup>1</sup>The *freq\_mod\_i* is frequency modulation input, one of NCO MegaCore input signals.

feedback signal)  $K((\omega_i - \omega_o)t + (\theta_i - \theta_o))$  and frequency of output signal  $\omega_o$  is

$$\omega_o = \omega_v + K((\omega_i - \omega_o)t + (\theta_i - \theta_o)). \quad (3.6)$$

When  $\omega_o$  approaches  $\omega_i$  or the frequency almost locked, the term  $(\omega_i - \omega_o)$  approaches zero and then only the term  $(\theta_i - \theta_o)$  contributes the frequency bias from  $\omega_v$  (the quiescent frequency of the NCO in the ADPLL). Hence, the phase different  $(\theta_i - \theta_o)$  cannot be zero, when the frequency is locked. Figure 3.7 shows the feedback signal of the first ADPLL from the initial to steady state.

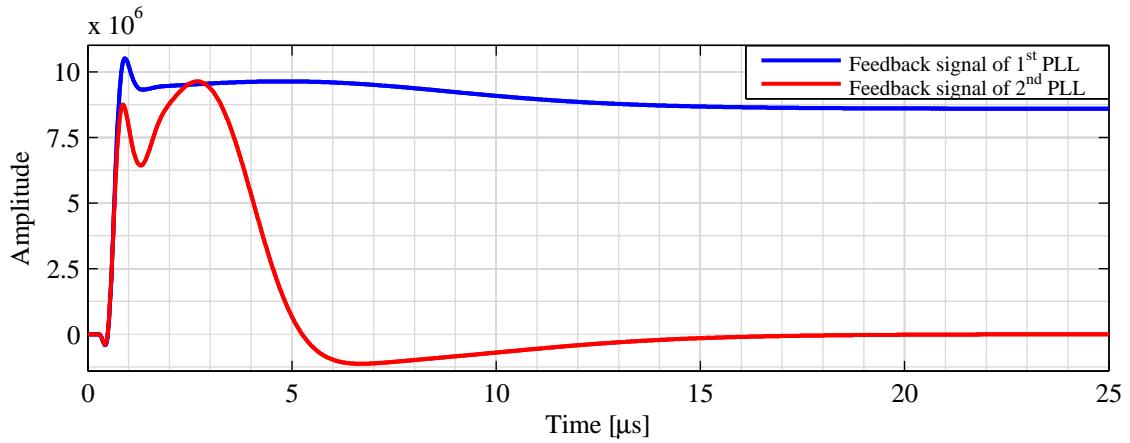


Figure 3.7: Feedback signals of two loops in the ADPLL. The input signal frequency here is 5.1 MHz and quiescent frequency of NCO in 1st loop is 5 MHz, and the output signal locks input signal at same frequency and phase finally. The feedback signal of the 1<sup>st</sup> PLL approaches to a constant value in steady state, and that of the 2<sup>nd</sup> PLL approaches.

The additional ADPLL is needed to lock the phase to achieve frequency and phase locking. Because the NCO block is highly customized, the additional NCO can be set to oscillate at the frequency locked in first stage, through modifying *phi\_inc\_i*<sup>2</sup>. Therefore,  $\omega'_v$  (the quiescent frequency of the NCO in the additional ADPLL) equals  $\omega_i$ . Similarly, when the frequency is locked,  $\omega_o$  equals to  $\omega_i$ , and so does  $\omega'_v$ . There is

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<sup>2</sup>The *phi\_inc\_i* is input phase increment, one of NCO MegaCore input signals.

no frequency bias, so it needs the term  $(\theta_i - \theta_o)$  to remain zero in the final. Hence, the feedback signal approaches to zero (Figure 3.7), which means frequency and phase are locked at same time.

The second ADPLL generates the in-phase  $f_{sin\_o}$ <sup>3</sup> and quadrature  $f_{cos\_o}$ <sup>4</sup> signal of input reference signal in real time. The frequency precision of the in-phase and quadrature signals influences their instantaneous phases and influences the precision of the final result. Figure 3.8 shows the stability of the frequencies of the in-phase and quadrature signals.

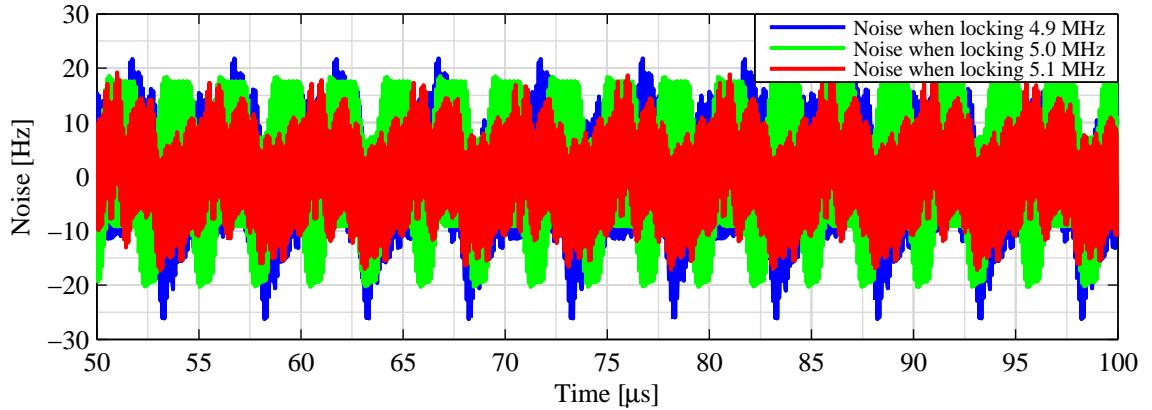


Figure 3.8: Stability of the frequencies of the PLL output signals. These are the noise of the output signal frequencies, when the two ADPLLs lock to 4.9 MHz, 5.0 MHz, and 5.1 MHz input reference signals.

The reference signal may vary within a range of  $\pm 100$  kHz around 5 MHz [24], so the ADPLL should have the capacity to lock to the frequencies in this range with a good performance. From Figure 3.8, the maximum noise is about 25 Hz, the relative error is about  $5 \times 10^{-4}$ . This frequency noise will be added to Doppler frequency in following calculation. According to Equation (1.3), a 25 Hz Doppler frequency error causes 7.9  $\mu\text{m}/\text{s}$  error in velocity of target mirror.

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<sup>3</sup>The  $f_{sin\_o}$  is output sine value, one of NCO MegaCore output signals.

<sup>4</sup>The  $f_{cos\_o}$  is output cosine value, one of NCO MegaCore output signals.

## Arctangent

Capturing the phase information from in-phase and quadrature signals needs an arctangent operation ( $\text{atan2}(Q, I)$ ). The methods to implement the arctangent operation are: coordinate rotation digital computer (CORDIC), lookup table (LUT) methods, and power series.

Trigonometric functions can be implemented by the CORDIC algorithm simply and efficiently. It can calculate the sine, cosine, arctangent, etc. to any precision, provided there is sufficient hardware space. The only operations it requires are addition, subtraction, bit shift and lookup, no hardware multiplication needed.

When performing an arctangent operation, it runs in vectoring mode CORDIC. In-phase ( $I$ ) and quadrature ( $Q$ ) signals are known from the previous block, the phase between the vector  $(I, Q)$  and positive X-axis is the result of arctangent operation. In brief, the vectoring mode CORDIC is an iterative process, it rotates successive constant phases  $\alpha_i$  in clockwise or counterclockwise angles

$$\alpha_i = \pm \arctan 2^{-i}. \quad (3.7)$$

That direction, clockwise or counterclockwise, which will be selected to force the angle to approach to the final rotation  $\arctan(Q/I)$  from the positive X-axis at each step, just like Figure 3.9 shows.

Hence, the phase  $\theta$  is the sum of all these rotating phases  $\alpha_i$ .

$$\theta = \sum_{i=0}^n \alpha_i \quad (3.8)$$

The approximation depends on the number,  $n$ , of successive rotations it takes. The

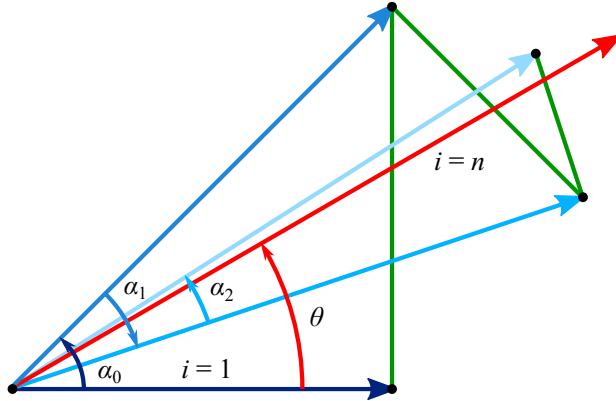


Figure 3.9: The first three rotations in the iterative process. The rotation here is pseudo-rotation, which produces a vector with the same direction but a different length, compared with the rotated vector. Each rotation is approaching to the desired final rotation. Consider the desired final rotation of  $30^\circ$  here,  $\theta = \tan^{-1}2^0 - \tan^{-1}2^{-1} + \tan^{-1}2^{-2} = 45^\circ - 26.57^\circ + 14.04^\circ = 32.47^\circ \approx 30^\circ$ .

multiplications are all power of two in the whole process, which can be implemented by bit shifts and adds in binary arithmetic. Therefore CORDIC needs no actual multiplier function.

The CORDIC algorithm is faster than other methods without using hardware multipliers, and occupies the fewest number of gates. Alternatively, when on-chip (FPGA) RAM and hardware multiplier resources are abundant for using, lookup tables and power series methods are generally faster than CORDIC functions.

In this project, the CORDIC is chosen because it is a relatively straightforward implementation by using the block in DSP Builder toolbox directly. The CORDIC block implements these iterative steps using a set of shift-add algorithms to perform a coordinate rotation. In conjunction with the other peripheral blocks (Figure 3.10), it is sufficient to perform the arctangent operation in FPGA.

Figure 3.11 shows the output of the CORDIC subsystem, which is a sawtooth wave. Because of the  $\text{atan2}$  with a principal value in the range  $(-\pi, \pi]$ , the output signal, which is the instantaneous phase, is wrapped to the interval  $(-\pi, \pi]$ .

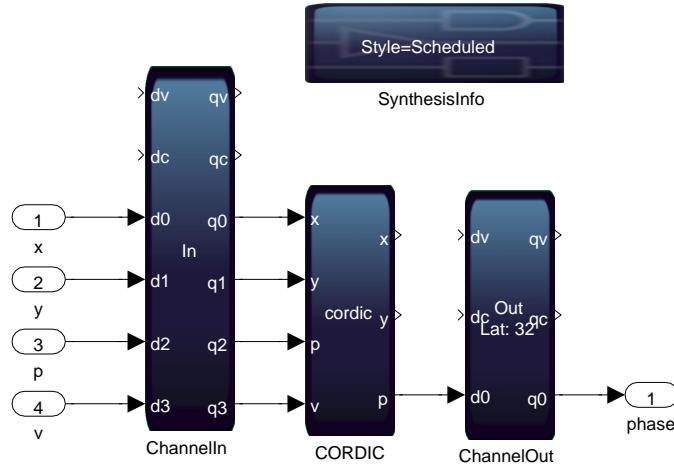


Figure 3.10: The schematic of the CORDIC subsystem. Besides the CORDIC block, other peripherals are also in this subsystem, including ChannelIn and ChannelOut blocks, which indicate to DSP Builder that these signals arrive from their source and leave to the destination synchronized, so that the synthesis tool can interpret them. And SynthesisInfo block shows the latency, port interface, and estimated resource utilization for the current primitive subsystem.

The latency is 32 clock cycles due to the iterative process. In order to reduce latency for further real-time optimization, methods employing hardware multipliers may be required to replace the CORDIC in the future.

## Unwrap

For displacement interferometers (Equation (1.3)), the unwrapped, continuous phase is required; otherwise, the displacement is limited in a small range and is continuously wrapped within the  $2\pi$  range. Hence, an unwrapping function is needed to remove the  $2\pi$  phase jumps. However, there is no specific block for this function in the DSP Builder toolbox library. Therefore, a customized block is required to be designed through HDL Import block, which imports existing blocks implemented in HDL into DSP Builder.

The algorithm of the atan2 described in HDL is adding  $2\pi$  whenever the jump

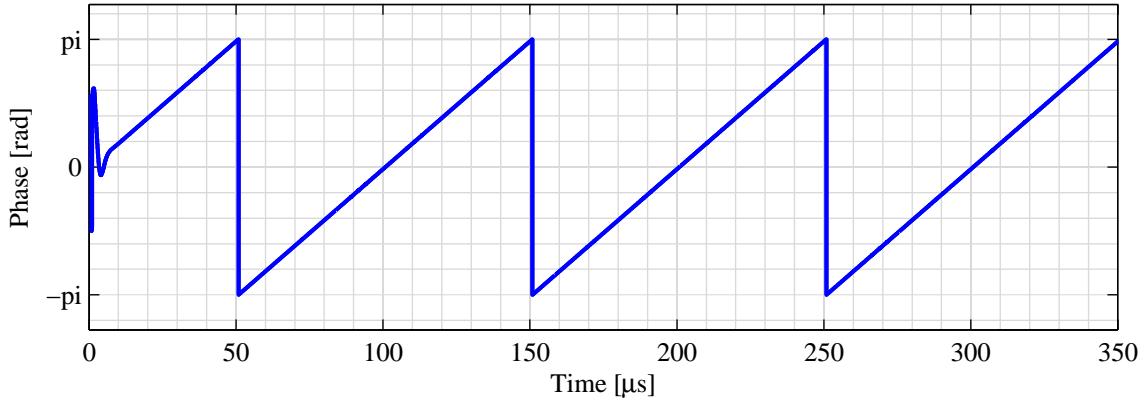


Figure 3.11: The output of the CORDIC subsystem. Input signals are sine and cosine waves with a 10 kHz frequency. Output signal is a sawtooth wave and every cycle has a  $2\pi$  jump.

smaller than  $-\pi$  and subtracting  $2\pi$  whenever the jump larger than  $\pi$  briefly. The flowchart of this process is shown in Figure 3.12. That allows the result of the

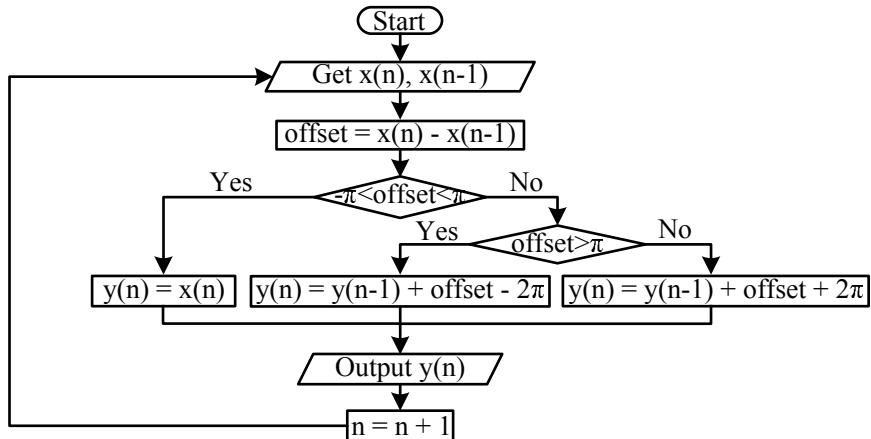


Figure 3.12: Flowchart of unwrapping process. The input  $x(n)$  is the raw phase data from previous function, which is wrapped into the range  $(-\pi, \pi]$ . The output  $y(n)$  is the modified, unwrapped instantaneous phase data.

atan2 to accumulate without limit and produces an unwrapped instantaneous phase. The sawtooth wave in Figure 3.11 is unwrapped in this block and converted to the continuous wave in Figure 3.13.

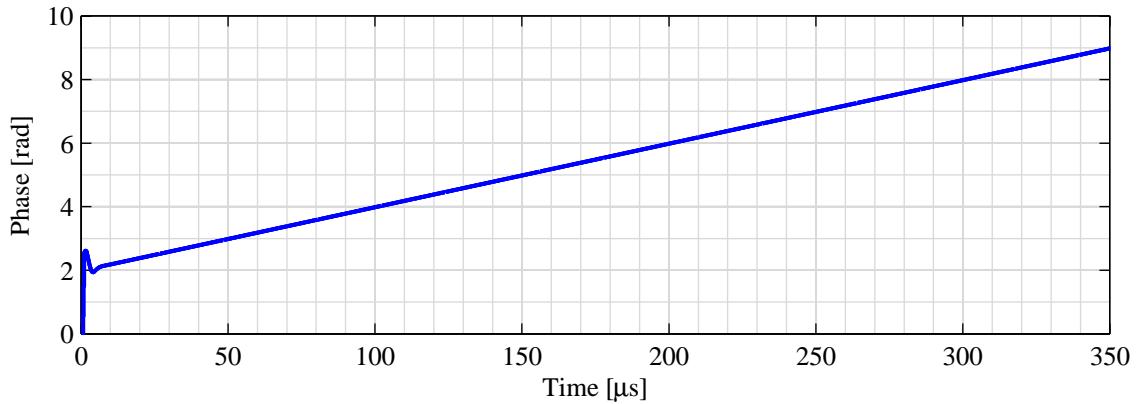


Figure 3.13: Unwrapped phase signal. Input signals with 10 kHz frequency are the same as Figure 3.11. Its amplitude is normalized into -1 to 1 rad first, which is easy for following process. Then it is unwrapped to a continuous signal, which is useful for calculating the displacement.

## 3.5 Simulink Simulation

The main subsystems, such as the ADPLL, IIR filter, arctan2, and unwrap, have been described previously. These subsystems cooperate with some simple blocks together, like input/output ports, products, bus conversions, binary point castings, to achieve the desired functionality of the algorithm. In the following section, some results are shown from the simulation of the entire digital signal processing algorithm.

### 3.5.1 Resource Usage

As discussed in the fixed-point section, the bit width of the internal data flow is one critical factor for determining the precision of this digital system. Particularly in this design, every multiplication doubles the data width, however, reserving that long data width of products especially after the decimal point is unnecessary and even a waste of embedded multipliers. Bus conversions and binary point castings are used to control the internal bus width. The final goal is achieving certain precision

with fewest resources possible. While the PLL and single-bin DFT (SBDFT) models here are prototypes of the phasemeter, in order to figure out the boundary of the precision, a wide enough bit width has been reserved. In future work, the precision and resources will be investigated to be more balanced.

Figure A.5 and Figure A.6 are the fixed-point and synthesizable models of phasemeter using the PLL and SBDFT, respectively. Comparing the structure of these two phasemeters using different methods, the model with PLL is more straightforward and concise. Because of the PLL, almost half of data path is streamlined in the model. However, it does not cause a reduction of the resource usage. As can be seen, only the logic usage (9% versus 11%) decreases slightly, which is from simplifying the structure. RAM usage (35% versus 4%) and multipliers (77% versus 62%) increase significantly because the ADPLL contains more IIR filters, costing more multipliers, and two NCOs with frequency modulation costing more RAM. What is more important is the RAM and multiplier are more precious and limited on the chip. Hence, RAM and multiplier usage must be considered in the design process. Fortunately, the usage here is before optimization, and optimizing the internal bus width will reduce the resource usage to some degree.

### 3.5.2 Bit Precision

These two phasemeter models can be treated as pure digital signal processing modules. Some simulations have been done to test the precision boundary of the digital signal processing modules. Figure 3.14 shows three sets of simulations. Each of the simulations compares the error levels between the PLL and SBDFT methods for different input measurement signal frequencies. These frequencies are 5 MHz, 5.01 MHz and 6.5 MHz, which correspond with target stage's static, low-velocity

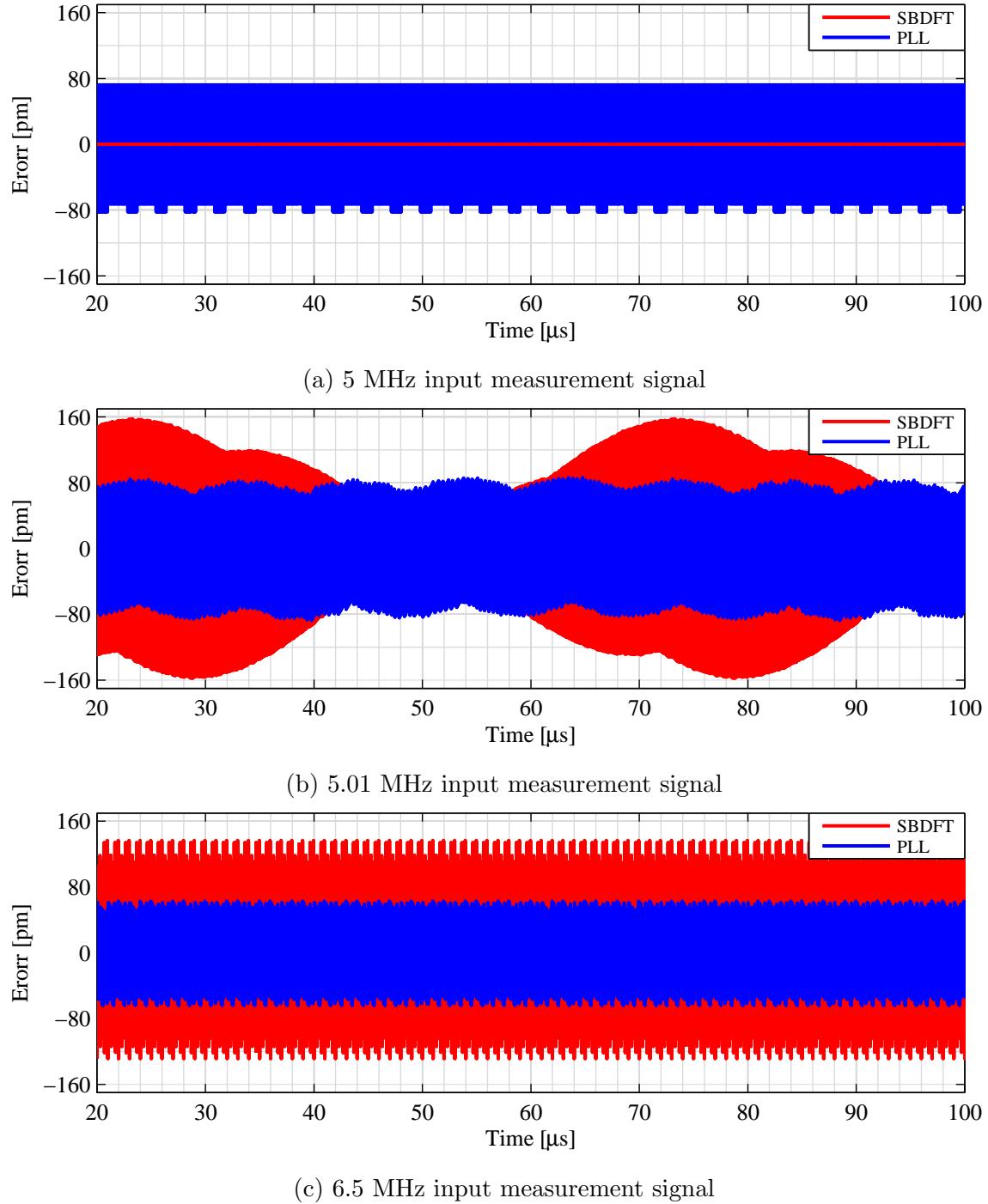


Figure 3.14: Displacement errors in simulations. Data in these three figures has converted from phase to displacement according to Equation (1.3). The frequencies of input measurement signals are 5 MHz, 5.01 MHz and 6.5 MHz in (a), (b), and (c), respectively.

movement and high-velocity movement states, respectively.

The two phasemeter models with PLL and SBDFT were configured to work within the same measurement range, the measurement signal frequency could vary from 3 MHz to 7 MHz ( $\pm 2$  MHz around 5 MHz). In the simulations, the reference signals were set as 5 MHz constantly. In practice, the reference signal may vary within a range of  $\pm 100$  kHz around 5 MHz [24]. Both phasemeter models were designed to deal with varied reference signals but for simplicity in the simulations, the reference signal frequency was constant in these cases.

In Figure 3.14(a), the measurement signal and the reference signal are the same, both 5 MHz. The error of the SBDFT algorithm is nearly zero while the PLL algorithm is within  $\pm 75$  pm. In Figure 3.14(b), the measurement signal is 5.01 MHz. The error of the SBDFT algorithm is within  $\pm 160$  pm, while the PLL algorithm is within  $\pm 85$  pm. In Figure 3.14(c), the measurement signal is 6.5 MHz. The error of the SBDFT algorithm is within  $\pm 140$  pm, while the PLL algorithm is within  $\pm 60$  pm. If these signals are analyzed further, there are some critical observations. The two channels that process the measurement and reference signals in the SBDFT algorithm are essentially the same. When the measurement and reference signals also are same, both 5 MHz, the difference between two processing results is zero. In the PLL algorithm, the IIR filter in ADPLL cannot remove the high frequency components entirely, even if the input measurement signal frequency is the same as the quiescent frequency of the NCO in the ADPLL. That imperfect 5 MHz signal that is generated by the ADPLL leads to the  $\pm 75$  pm error in the end. This can be filtered further to reduce this noise.

In Figures 3.14(b) and 3.14(c), the error levels in the PLL are lower than that in SBDFT by about a factor of two. However, the overall error levels at the higher

Doppler frequencies are generally slightly lower. One possible reason for that may be the filters. The low-pass filters in phasemeters remove the high frequency components and pass the frequency difference between measurement and reference signals. The cut-off frequency of the low-pass filters are set at 2 MHz. For the 5.01 MHz measurement signal, the frequency difference 10 kHz ( $5.01\text{ MHz} - 5\text{ MHz}$ ) is much less than 2 MHz. The filters pass the 10 kHz signal as well as its multiple harmonics at same time, which may influence the error level. For the 6.5 MHz measurement signal, the frequency difference is 1.5 MHz, much closer to 2 MHz. The multiple harmonic problem is significantly reduced at the higher frequencies. Narrowing the passband of the filters may reduce the multiple harmonic problem, however, it narrows the measurement range as well. That is a tradeoff between measurement range and precision.

### 3.5.3 Frequency Response

Frequency response is a dynamic characteristic of the system, including the magnitude response and phase response, which measures the magnitude gain and phase difference between output and input signals as a function of frequency.

In this project, the phase response is more important than the amplitude response. The whole system aims to measure the phase difference or phase shift of two incoming signals precisely; any non-uniform phase delay caused by the system will influence the measurement precision of the phase. Therefore, the frequency response, and especially the phase response, must be characterized to avoid or compensate the impact from the system itself.

However, these phasemeter models contain many complex blocks or functions, which mean that the whole system is not a linear, time-invariant (LTI) system. For

instance, the arctan2 is a complex function that is not LTI. It may not practical to measure a frequency response or derive a transfer function of a non-LTI system. Fortunately, only the filters in the system influence phase of output signals, and they are LTI systems. Theoretically, the phase response of filters provides the information how the entire phasemeter system effects the phase of the output signals. The Bode plot of the fourth-order IIR filters is shown in Figure 3.5.

The filters cause nonuniform phase delay, which is significant in range of 100 kHz to 2 MHz. That nonuniform phase delay impacts measurement precision, especially when the frequency of input signals varied in a wide range. The Doppler frequency corresponds with the velocity of target stage. Any variable motions will lead errors of measuring the phase, in order words, the displacement. This problem will be addressed in the future.

In conclusion, the PLL algorithm has worse static characteristics but better precision than SBDFT algorithm in dynamic cases. However, the PLL algorithm costs more resources. Furthermore, both of them lead non-uniform phase delay, which must be solved in the future. Besides the simulations, some verifications of these two algorithms in practice also have been done in following section.

## 3.6 FPGA Implementation

In order to verify this model in practice, the whole model as a digital signal processing module must be converted into HDL and downloaded into the FPGA chip. The DSP Builder toolbox provides a block Signal Compiler, which can export synthesizable HDL to a directory.

In the Quartus II software, the DSP module represented by the HDL files is

instantiated as a symbol in the schematic design mode. This DSP module would not work without inputs signals from an external source, a clock, and a reset signal. Meanwhile, memory is also need to store the computed results. Hence, an ADC controller, a SDRAM controller, and some control signals were added into the schematic to access other on-board resource, as Figure 3.15 shows.

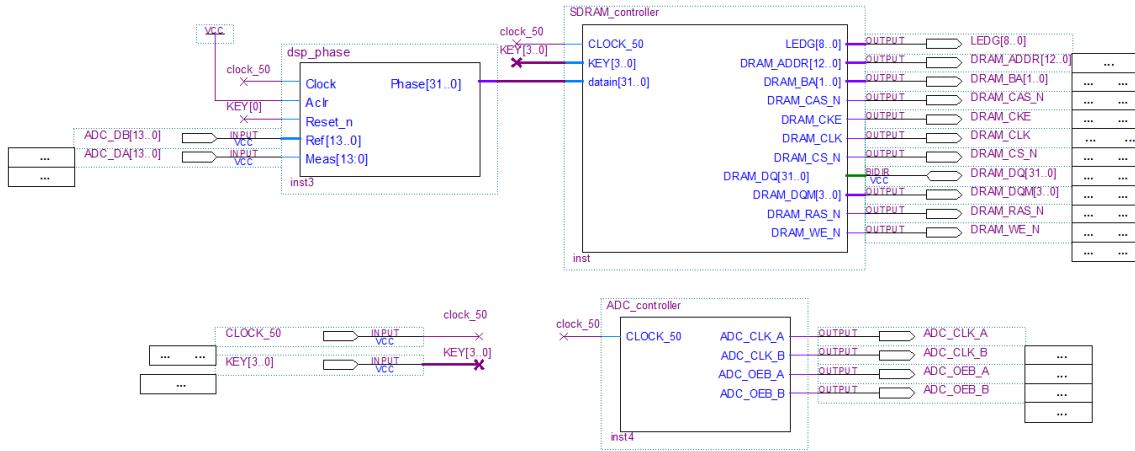


Figure 3.15: The schematic of the design with all necessary modules in Quartus II

In this design, the global clock driving the whole system is 50 MHz, which is produced by an on-board oscillator. As previously discussed, the ADCs utilized in this project can sample up to 65 MSPS. For simplicity, the dual ADC channels are clocked by the global clock directly, thus, the sampling rate is 50 MSPS for the inputs and outputs.

At this stage, the goal is to verify the performance of this digital system but with a long measurement time. The SDRAM was chosen as an easy-access but capacity-limited media to log this data. For verification measurements, the data flow can be as fast as 200 MB/s ( $50 \text{ MSPS} \times 32 \text{ bit}$ ) which requires special consideration. Hence, a downsampling rate at 94.7 KSPS is applied when logging data into SDRAM. Therefore, the 128 MB SDRAM could store a 338 s measurement data set, which is

enough for this verification.

After designing the logic, other steps like device specifying, pin assignment, timing constraining are significant as well. After analysis & synthesis, placement & routing, and assembly, a bitstream file is generated, which describes the configuration inside the FPGA chip. Downloading the bitstream file via JTAG is the last step of design. After finishing all of these, the FPGA board executes the phasemeter algorithm in hardware.

Some measurements similar to the previous simulations have also been done. These measurements test the error level of the phasemeter when measuring constant Doppler frequencies, 0, 10 kHz and 1.5 MHz, in other words, the frequencies of measurement signals were 5 MHz, 5.01 MHz and 6.5 MHz, and the frequency of the reference signal was 5 MHz. All of the conditions were same, except the measurement and reference signals were generated by the function generator and converted by ADCs from analog to digital signal. The measurement results were stored in SDRAM in the end.

Figure 3.16 shows the error level of each measurement. The error levels in practice have the same approximate order of magnitude as the simulations. The error levels of the SBDFT version do not change, except in the static state, but they still remain within the  $\pm 150$  pm range. The error levels of the PLL version increase by a factor of two and are slightly less than that of the SBDFT version. These measurements indicate that the quality of the signals generated by function generator and the performance of ADCs are also factors to influence and maybe determine the error level of the measurement.

In conclusion, both the PLL and SBDFT algorithms have an error level less than  $\pm 150$  pm in practice. The PLL algorithm has a little better dynamic precision, and

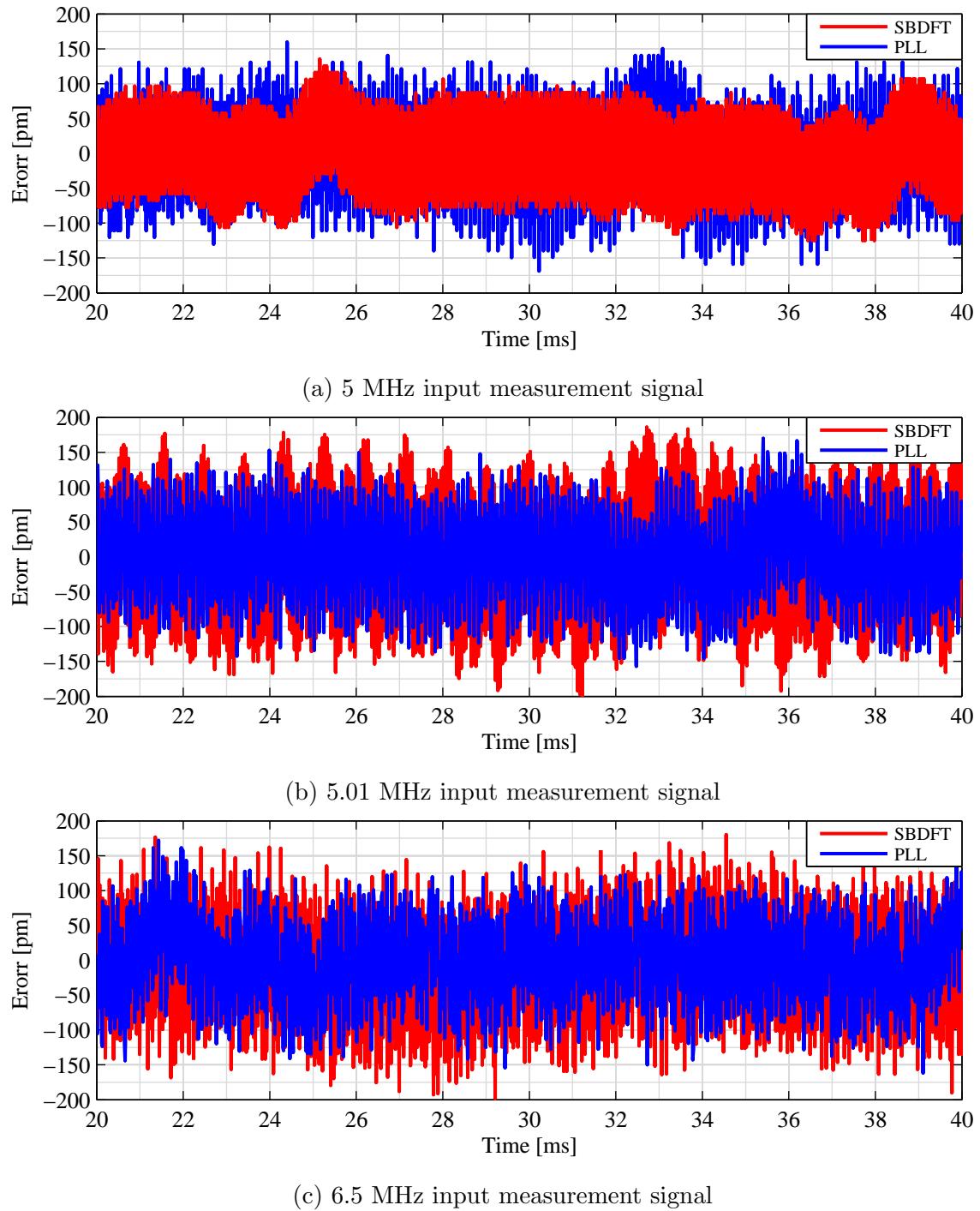


Figure 3.16: Displacement errors in practical measurements. Data in these three figures has been converted from phase to displacement according to Equation (1.3). The frequencies of the input measurement signals are 5 MHz, 5.01 MHz, and 6.5 MHz in (a), (b), and (c), respectively.

the SBDFT algorithm has a little better static precision. Considering the resource usage, SBDFT algorithm is preferred to implement in FPGA.

### 3.7 Verification

Velocity verification measurements of a high speed piezo stage also have been done using a displacement interferometry system and the phasemeter board with colleagues in the research group. According to Equation (1.3), displacement is determined from phase and velocity is the first derivative of displacement. Thus, measuring the phase will provide the information of displacement and velocity. Figure 3.17 shows the velocities and displacements of a piezo stage, which was driven at different velocities. Each drive velocity corresponds to an estimated velocity based on the specifications

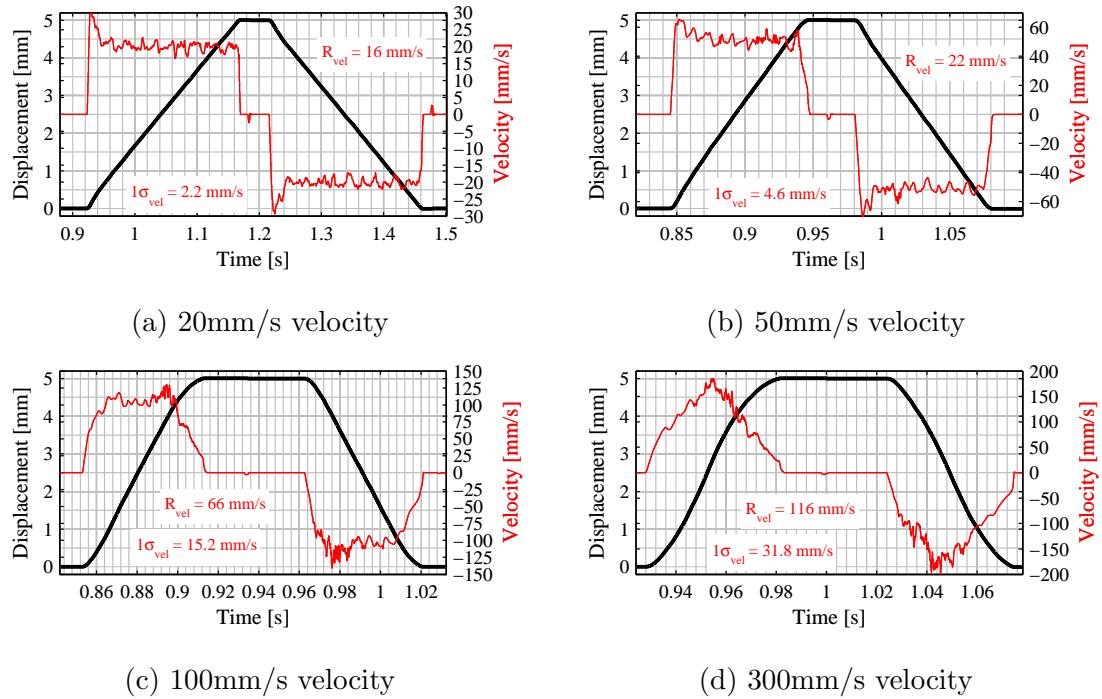


Figure 3.17: Velocity verification measurements of a high speed piezo stage with different drive velocities.

for the stage [58]. These measurements were performed to verify the performance of the stage in an operation that highly velocity dependent. When velocities are low (Figure 3.17(a) and Figure 3.17(b)), the stage velocity does not fluctuate much and there is only a minor deviation between the stated and measured velocities. However, as the velocity increases (Figure 3.17(c) and Figure 3.17(d)), the stated stage velocity differs greatly from the measured value. Thus, with this phasemeter system, high speed measurements can be taken to verify the velocity with high repeatability.

# 4 Measurement Data Transmission

In order to further process and apply, the computed results of phasemeter need high-volume memory to store or efficient interface to transmit to host PC. Considering the ease of further processing and real-time control, using an interface to transmit data is preferred. In this design, UDP was chosen as a protocol of Ethernet interface to transmit data, transmitting end was implemented in FPGA based on SOPC technique; receiving end was implemented in PC based on xPC target, which is a Matlab real time system.

## 4.1 User Datagram Protocol

### 4.1.1 User Datagram Protocol Introduction

The User Datagram Protocol (UDP) is one protocol in the Internet protocol suite for networking. With a UDP, computer applications (software) can send data encapsulated in packets to other hosts (computers) on an Internet Protocol (IP) network with no need to establish a special communication channel first. UDP

assumes that the Internet Protocol (IP) is used as the underlying protocol, so sometimes it is also called UDP/IP, just like TCP/IP.

UDP delivers packets in a connectionless and unreliable way because it does not use acknowledgements (handshaking dialogues) to make sure messages arrive. There is no guarantee of delivery, ordering or duplicate protection. UDP only provides checksums, one mechanism for data integrity. Thus, UDP needs a minimum of protocol mechanisms [59].

However, UDP is still competent for certain applications. When error checking and correction is not necessary in an application, using UDP could save network resources through avoiding such processing. When latency is a sensitive factor in an application, such as real-time applications, using UDP could drop packets rather than wait for delayed packets, which could decrease the retransmission delays, thus, decrease the latency. Obviously, UDP sacrifices the reliability to reduce latency and simplify the process. If an application emphasizes reliability over latency, requires reliable and ordered delivery of streams of data, it should use the Transmission Control Protocol (TCP).

In this design, both reliability and reduced latency must be addressed. Since this phasemeter is a part of a displacement measuring interferometer, which is a precision instrument, it certainly requires a reliable way to transmit measurement data. The so-called UDP unreliable delivery occurs when it is used in large networks such as the Internet, while it has been proved to work well in a local environment. The phasemeter connects the Ethernet card of a host PC directly through a crossover Category 5 cable (Cat 5), that constitutes the local environment. Hence, reliability should not be a problem. Meanwhile, this design may add potential real-time control model in it, which needs the latency as low as possible, so UDP is chosen in this

design.

#### 4.1.2 Mechanism

UDP is a transport protocol, which lies in the transport layer of the TCP/IP 5-layer reference model, which is one of two major layer models to describe the network architecture and organize the protocols. The other is the OSI 7-layer reference model. The model partitions the networking task and organizes the protocol suite into layers, and allocates the subtask to each layer. In order to figure out how to transport data based on UDP, it is necessary to understand the protocol layer of the 5-layer model. Figure 4.1 shows the layers as well as the form of the data as it passes between them [60].

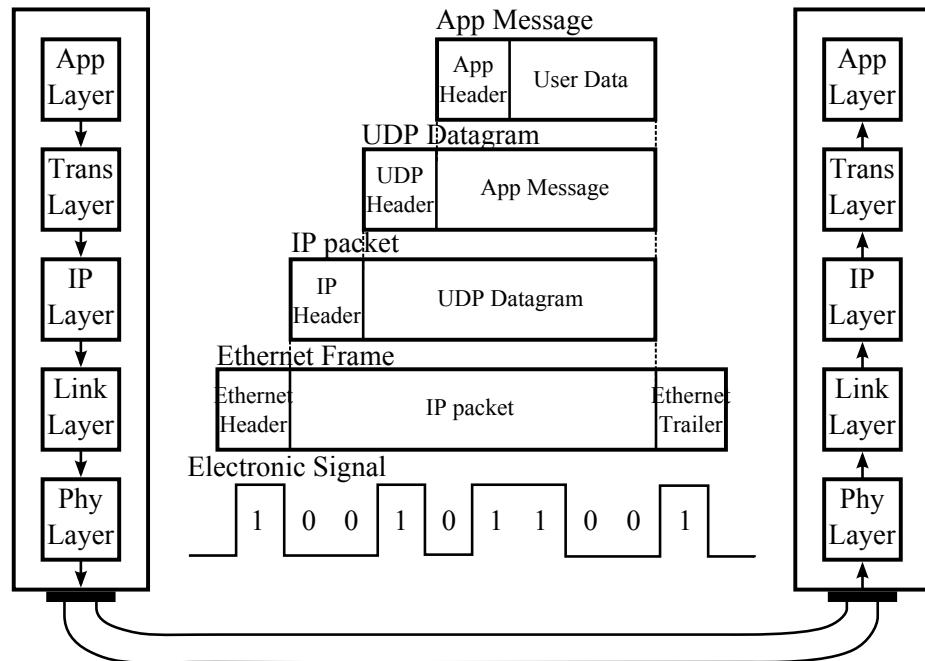


Figure 4.1: TCP/IP 5-layer reference model.

- *Application Layer.* At the highest layer, users access to internet services through application programs, the applications create user data and aim to

communicate this data with other applications on another or the same host PC. The applications create and encode user data according to application protocols in this layer, such as Simple Mail Transfer Protocol (SMTP), File Transfer Protocol (FTP), Secure Shell (SSH), Hypertext Transfer Protocol (HTTP), etc. Then applications layers transport data in the required format to the transport layer for delivery.

- *Transport Layer.* The transport layer provides a transport mechanism. It establishes the connection between the ports, which could be treated as data channels that an application uses to exchange data. The protocols in this layer deal with opening and maintaining connections (channels) between two ports. Meanwhile, it encapsulates the message from application layer into a datagram. The datagram contains not only the data from upper layer but also an identification of the source port and destination port the channel occupies. The form of encapsulation is based on which transport protocol used. In this design, UDP protocol is used, so transport layer prepends a UDP header to the message from application layer and passes it to lower layer. The format UDP header is shown as Figure 4.2. Thus, UDP layer only identifies the ports of source or destination.

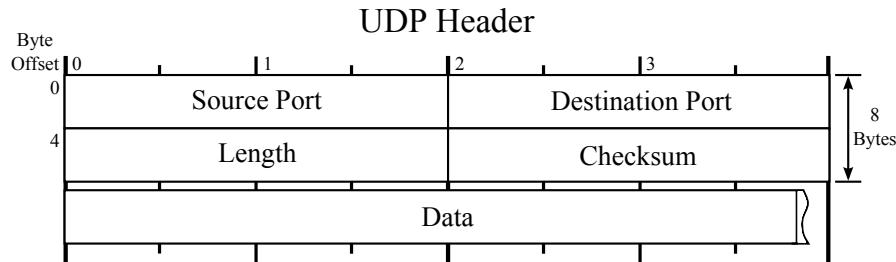


Figure 4.2: The format of the UDP header. It is divided into four 16-bit fields that specify the port from which the message was sent (Source Port), the port to which the message is destined (Destination Port), the message length (Message Length), and a UDP checksum (Checksum).

- *Internet Layer.* The internet layer provides a communication mechanism between two PCs; the protocol that defines this delivery mechanism is called Internet Protocol (IP). It provides an unreliable, best-effort, connectionless packet delivery. It encapsulates the datagram from transport layer in a packet, fills in the IP datagram header, which contains an identification of source and destination addresses and a type field that identifies the contents of the packet. Then the internet layer uses certain algorithms to determine whether to deliver the packet directly or send it to a router, and passes the packet to the appropriate network interface (lower layer) for transmission. The format of the IP header is shown in Figure 4.3. Thus, the IP layer only identifies the IP addresses of source and destination.

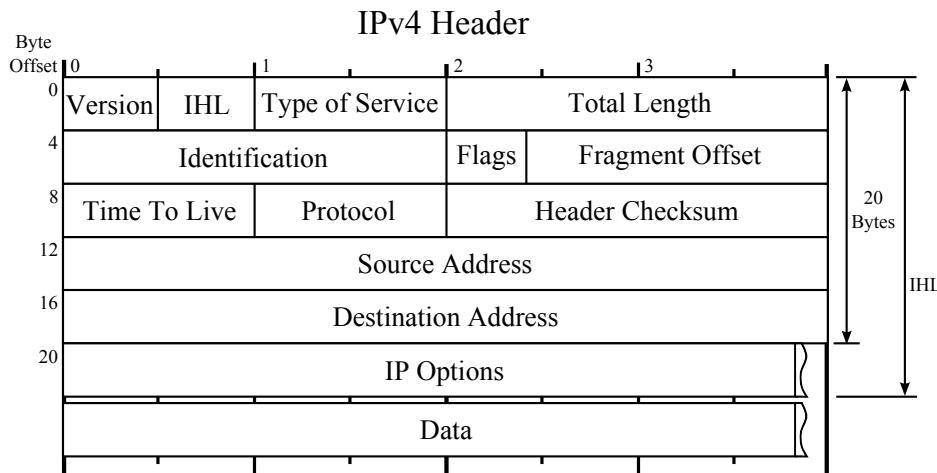


Figure 4.3: The format of the IPv4 header. IP protocol has two versions, IPv4 and IPv6. In this design, IPv4 is used. The IPv4 header consists of 14 fields, of which 13 are required. In these fields, it specifies the IP address from which the packet was sent (Source Address) and the IP address to which the packet is destined (Destination Address).

- *Link Layer.* The Link layer or media-access layer protocol is the lowest software layer in the TCP/IP model. The protocols on this layer access and control the hardware devices and media that make up the network, accept packets from

internet layer, and transmit them to lower hardware layer. Depending on the different network types, there is a wide variety of link layer protocols. Ethernet that used in this design is one of them. The link layer encapsulates the IP packet into an Ethernet frame, adding an Ethernet header and trailer. Figure 4.4 shows that the Ethernet frame format identifies the physical (MAC) address of source as well as destination.

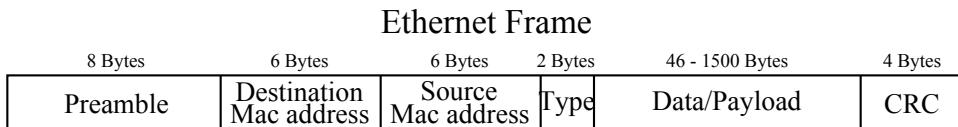


Figure 4.4: The format of the Ethernet frame. An Ethernet frame begins with a Preamble and Start Frame Delimiter, and then MAC Addresses of the Destination and Source. The middle section of the frame is payload data including any headers and data from upper layers. The frame ends with a 32-bit Cyclic Redundancy Check (CRC), which is used to detect any corruption of data in transit.

- *Physical Layer.* The Physical layer (Hardware layer) is the lowest layer in the TCP/IP model, which provides an electrical and mechanical interface between networking hardware and transmission medium. The protocol in the physical layer defines the basic technologies of network hardware transmission. It encodes binary data of the frame to electronic signals, and transmits the signals over a hardware transmission medium. In this design, Ethernet 100BASE-TX protocol was used. The hardware of this layer is usually PHY, which connects a link layer (or Media Access Control (MAC)) to a transmission medium such as an optical fiber or copper cable.

## 4.2 Transmitting End Implementation

Two solutions are considered to implement and transmit data from the FPGA board to the host PC by UDP. One is to build an all-hardware model for the protocol layers, which means from upper application layer to lower link layer all protocols are written in HDL and synthesized in logic circuits. Another one is to build a link layer model in hardware, while to implement application, transport, and internet layers in software, which means all protocols in these three layers are written in C code, and an embedded processor executes the C code to communicate. The latter is relatively common and effective for development, so in this design, it is chosen as initial design solution.

### 4.2.1 Hardware

For processing digital signal, the phasemeter has employed an FPGA in the system, but it cannot execute C code inherently. In order to achieve the goal, a discrete microprocessor dedicated to transmit data is needed, or a soft-core processor is embedded in FPGA, which helps the system be more compact.

The technology embedding a processor in an FPGA is named System-on-Programmable-Chip (SOPC). SOPC enables a complete embedded system to be defined and generated on a FPGA chip in much less time and in a more flexible manner. The most significant part of the embedded system is the embedded processor. The processor in the SOPC is a true soft-core processor. Unlike a discrete microprocessor that is fixed in silicon, the soft-core processor is just a specific logic design that can be configured in an FPGA [61]. It is flexible and changes to functionality and performance of the soft-core processor can be made by modifying the

specific logic design. Meanwhile, peripherals in the SOPC are also easily customized for expansion or for removal. Any new embedded system design can be easily tested by reconfiguring the FPGA using system's JTAG interface. After the configuring the hardware, the software development flow is similar to that of discrete microcontroller designs [62].

Figure 4.5 shows the hardware architecture of the FPGA design, which includes the SOPC embedded system. The core of this system is the Nios II processor, which

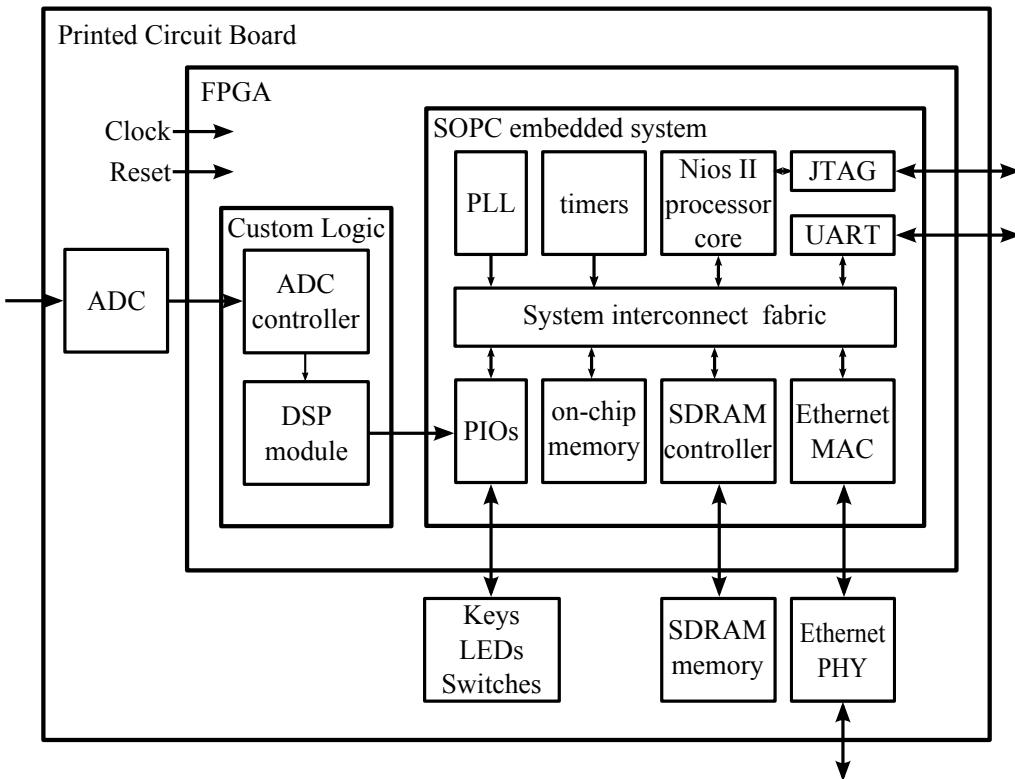


Figure 4.5: The hardware architecture of this FPGA design. It includes the on-chip SOPC embedded system, custom logic, and the peripherals outside the FPGA chip.

executes the instructions in programs and controls and communicates with other peripherals. The system interconnect fabric is based on an Avalon interface, which connects all of the components and exchange addresses, data and control signals among them. In this design, the peripherals include a clock, PLL, JTAG, UART,

on-chip memory, SDRAM controller, timers, PIO, Ethernet MAC, etc. Some of them are just logical designs based on on-chip hardware resources in the FPGA, such as PLL and on-chip memory; and others are controller or interface to control or access to off-chip devices, such as Ethernet MAC to control off-chip Ethernet PHY, SDRAM controller to control off-chip SDRAM chip, and PIO interface to access to keys, switches, and LED. The SOPC embedded system also can connect to custom logic outside the system but inside the FPGA through a PIO interface. In this design, the custom logic has been shown in Figure 3.15, which includes an ADC controller and DSP module, the SDRAM controller switches to integrate into the SOPC system.

Until now, all components and devices are physically connected, which build the hardware foundation of the whole design. The DSP model calculates the phase difference and outputs the results clock by clock. The Nios II processor controls the PIO and reads the results through Avalon bus. Meanwhile, it also reads a time stamp from a timer for each result. Then, relying on Ethernet MAC to access to off-chip PHY, the date is transmitted to PHY and encoded, then sent out through the Ethernet cable. This procedure describes how the data flows inside the SOPC embedded system hardware, however, it still need software to control the flows step by step. The following sections will discuss how the software commands the hardware to finish the data transmission tasks, including operating system section and application section.

#### 4.2.2 Operating System and Drivers

Transmitting data through the Ethernet is a networking task, which is relatively complex. To deal with the complex and multiple tasks, only hardware is not enough, the SOPC needs an operating system and drivers lying on hardware. They both are

critical parts to embedded system, just like body and nervous system to human being.

The model in Figure 4.6 shows the architectural layers of this SOPC embedded system. Each layer implements the specific functionality in a different hierarchy and

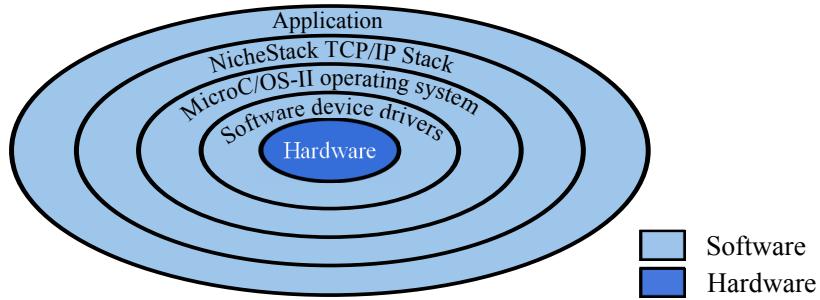


Figure 4.6: Layered software model of this SOPC embedded system [63].

provides functions supporting the next outer layer. Thus, each layer only calls services that the adjacent inner layer provides, regardless of how they are implemented in other layers.

The following list describes each layer [63]:

- *Hardware*. The core of this model represents the SOPC embedded system and custom logic implemented in the FPGA, and other peripherals off-chip. The details about the hardware have been introduced in previous section.
- *Software device drivers*. The drivers layer lies on the hardware layer. It manages and controls the data, address, and control signals to feed into or obtain from hardware peripherals in a hardware-specific sequence. It abstracts these procedures from the outer layers. The outer layers do not need to know much detail about hardware, but interact with drivers through the interfaces they provide.
- *MicroC/OS-II operating system*. MicroC/OS-II is a real-time, multi-threaded operating system. In this design, MicroC/OS is used because it is a popular real-

time kernel and can support networking tasks, which is based on its multitasking and intertask communication services.

- *NicheStack® TCP/IP Stack.* The Nios® II Edition is used in this design. It is a light implementation of the TCP/IP suite with small memory footprints, specifically for the embedded systems. It provides networking services by simplifying all mechanism of the TCP/IP suite to the sockets application programming interface (API), which could be easily called by networking applications, regardless of how the TCP/IP works in the background.
- *Application.* The application layer is user customized, depending on the expected task to be executed. In this design, the application is a network transmission task based on UDP protocol, which will be introduced in detail in the following section.

Software device drivers, the operating system, and internet stack must be properly configured, and the application must be designed and programmed for a specific project.

### 4.2.3 Application

An operating system, hardware, and drivers provide a hardware and software foundation, a program is still needed to call these functions and utilize these resources to achieve an application.

Figure 4.7 shows the work flowchart of this application program. Almost every step has ready-made functions provided by lower layer to call, thus, the program design is to call different functions in an appropriate sequence and timing, and to keep the data flowing among these functions with a correct orientation and format.

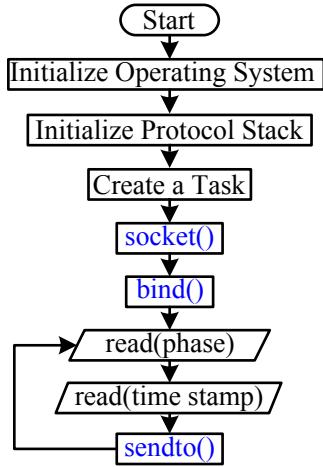


Figure 4.7: The work flowchart of this application program. First, the MicroC/OS-II operating system and NicheStack TCP/IP Stack are initialized. Then, a networking task is created, a UDP connection through Socket API (blue) is built, data from custom logic and peripherals is obtained. Finally, the data is sent out and the process is looped for repeated operation.

The initialization of the operating system and protocol stack is necessary and common procedure for every application, thus, the focus of this section is not on these steps. The main aspect is socket programming, which creates the network connection and exchanges data through the connection.

A socket is a mechanism providing an endpoint for networking communication, which abstracts networking communication to handling network I/O. Three functions of the socket API, *socket()*, *bind()*, and *sendto()*, are used in this program. The *socket()* function creates an endpoint for UDP communication. It returns a descriptor *fd*

$$fd = \text{socket}(pf, type, protocol).$$

The argument *pf* identifies the protocol family, whose value equals to AF\_INET in this socket, representing IPv4. The argument *type* identifies the type of communication to be used with the socket, whose value equals to SOCK\_DGRAM, representing

connectionless packet delivery service. The third argument *protocol* is set to 0, because UDP protocol is the only protocol meeting the given protocol family and type.

The *bind()* function associates a socket created by *socket()* to a local port and IP address, which has following form:

```
bind(socket, localaddr, addrlen).
```

The argument *socket* is the descriptor *fd* of the socket expected to be bound. The argument *localaddr* is a structure identifying the port and address that the socket needs to bind. The argument *addrlen* identifies the length of the structure.

Because UDP is a connectionless packet delivery, the *listen()*, *connect()*, etc., for connection delivery is unnecessary. Now, the channel of the UDP communication is built. The program runs into a loop, reads the output port of the DSP module at certain rate, then saves the sample of phase difference into a register. Next, it reads and saves the time stamp from the timer, which indicates the relative sample time between two samples, which is useful for synchronizing data during post-processing.

In the end, the *sendto()* function sends a packet through the connectionless socket, which has following form:

```
sendto(socket, packet, length, flags, destaddr, addrlen).
```

The argument *socket* is the descriptor of the socket which is the same as above. The argument *packet* is the address of the data to be sent, specifically, the address of the register storing the phase difference and time stamp. The argument *length* identifies the length of the data to be sent in bytes. The argument flag is set to 0. The argument *destaddr* identifies the destination address and the argument *addrlen* identifies the

length of the address [60].

After sending one packet, the program goes back to the beginning of the loop, obtains the data and sends it out repeatedly. Thus, the UDP transmitting end in the design has been finished from bottom hardware to top application software.

### 4.3 Receiving End Implementation

The receiving end of the communication through UDP can be designed in several solutions. One straightforward way is using a workstation computer to receive the packet directly. However, the workstation computer runs an operating system, which distributes the computing power among multiple tasks with limited performance. The resource of the workstation computer is not expected to be occupied by the packet receiving application, which may slow down other important tasks. In turn, the packet receiving application expects to have its own dedicated hardware with predictable performance that runs in real time. Thus, a separate target PC is required to execute packet receiving tasks independently, and controlled by workstation computer, or host PC.

In this design, xPC Target is used to implement the latter way to receive packets. The xPC Target is a toolbox of Matlab/Simulink, which provides a real-time environment to test and execute Simulink models. It implements a real-time kernel in a standard desktop PC<sup>1</sup> and executes real-time applications in standard PC architecture. Though its original usage is for rapid real-time prototype verification, it still could be used as a robust part of the measurement instrument, particularly for research applications.

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<sup>1</sup>The desktop PC is the target PC, which can be a 32-bit Intel® or AMD® processor (386 compatible or higher) [64].

### 4.3.1 Hardware

The hardware of this real-time environment consists of a host PC, a target PC, and the network connection between them. The configuration of the environment is shown in Figure 4.8.

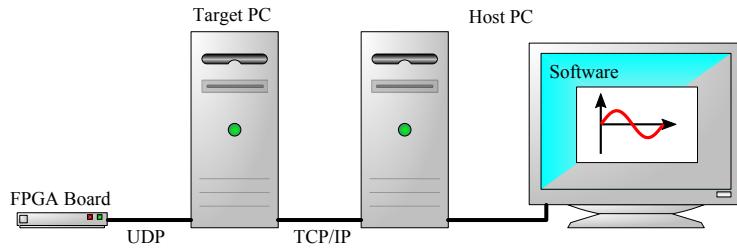


Figure 4.8: The configuration of the real-time environment, including a host PC, a target PC, and the network connection between them.

#### Host PC

The host PC in this design is the workstation computer used for programming code, processing data, controlling other equipment, etc., but is not involved in real-time applications directly. It runs a Windows operating system and runs Matlab/Simulink on it. The xPC Target toolbox in Matlab/Simulink provides the real-time UDP Network Configuration and Receive functional blocks to build Simulink models and can create real-time applications from the Simulink model. Then the host PC downloads the real-time application to target PC through Ethernet adapter and cable. It also harvests the acquired data in the storage media of target PC in the end.

#### Target PC

The target PC in this design is an old computer with Intel Pentium III processor, but its performance is enough for this application. It runs a minimal operating system

booted from a special target boot disk created by the xPC Target software, and has few peripherals. Thus, it can allocate most of its resources to the real-time application. In this design, the target PC receives the UDP packet from FPGA board in real time, and stores the data in its hard drive. It is dedicated to this only task, however, the host PC could participate in other tasks without influence.

### **Host-Target Connection**

The host and target PC are connected directly by a crossover Ethernet cable and Ethernet adapters, using TCP/IP protocol for communication, specifically in the design, downloading real-time application code to target PC and transmit data from storage media of target PC to host PC. Both the host and target PC have two Ethernet adapters, which are used for accessing to Internet and connecting to target PC, receiving UDP packet from FPGA board and connecting to host PC respectively.

#### **4.3.2 Software**

The software of the receiving end includes the Simulink model of UDP receiving end, the scripts converting it to a real-time application, downloading to the target PC, initializing it, and obtaining the data from the target PC.

##### **Simulink Model**

The xPC Target toolbox provides comprehensive functional blocks to implement the real-time UDP communication. Unlike in the transmitter end, the socket programming needs to be done by the developer. These functional blocks in the receiving end have integrated the ready-made socket programming and only need to

be configured. The Simulink model of the UDP receiving end is shown in Figure 4.9.

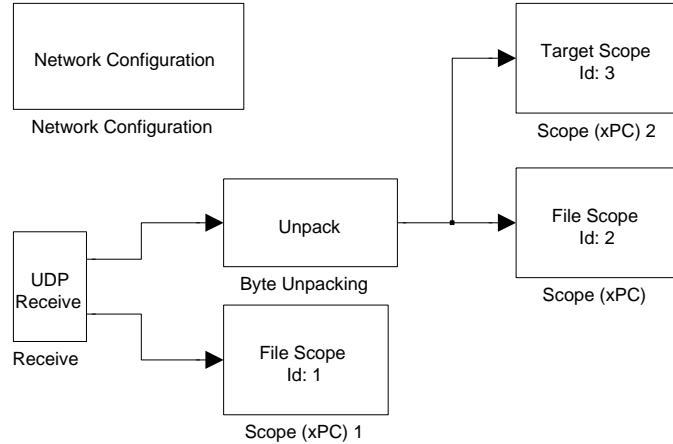


Figure 4.9: The Simulink model of the UDP receiving end.

The Network Configuration block sets the parameters of the Ethernet adapter of target PC for UDP communication (not for host-target connection), including IP address, subnet mask, gateway, and its location on the motherboard – PCI bus and slot. The UDP Receive block sets parameters of UDP communication, including source IP address, destination port, and data width. The File Scopes store the received data to the hard disk of the target PC.

## Scripts

There are two scripts used in the receiving end. One is to convert the UDP receiving end Simulink model to real-time application, download it to target PC, configure the parameters of the task, including sample time, sampling rate and etc. Another is to harvest the data received through the UDP, stored in target PC, and convert the endian of data<sup>2</sup>.

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<sup>2</sup>The order of the data stored in Intel CPU system is little-endian; the order of the data transmitted through UDP is big-endian

## 4.4 New-built Phasemeter System Test

The test about the performance of the UDP transmission has been done. The result shows the speed to send UDP package from the FPGA board is about 7,000 packages per second, however, the maximum speed to send UDP package with the same package size in this design is about 150,000 packages per second in theory. There is huge space to increase the speed of transmission in future work, otherwise, the current speed is not sufficient for real-time control purpose.

To test the performance of collaboration among all parts in the phasemeter system, some comparisons have been done between the currently-used, commercial phase measurement solution in laboratory and the new-built phasemeter system.

For comparison, the phase measurement solution currently used included a commercial single element photodetector, a commercial quadrant photodetector, a lock-in amplifier instrument, a NI USB data acquisition card, and a target PC. The new-built phasemeter system included a designed quadrant photodetector with larger active area, a commercial single element photodetector, a FPGA board, and a target PC, its setup is depicted in Figure 4.10.

A comparison was performed between these two solutions with an interferometer to measurement a uniform motion and a variable motion of a piezo stage. The stage was driven by a ramp square signal and a chirp sine signal from function generator to form the motions. Figure 4.11 are the measurements of the displacements of the stage.

The displacements of the piezo stage are controlled by the driving voltages. In this measurement, the range of the displacements are from  $-1.25 \mu\text{m}$  to  $1.25 \mu\text{m}$ . The stage is open-loop controlled, and its movement repeatability is not good. So it is

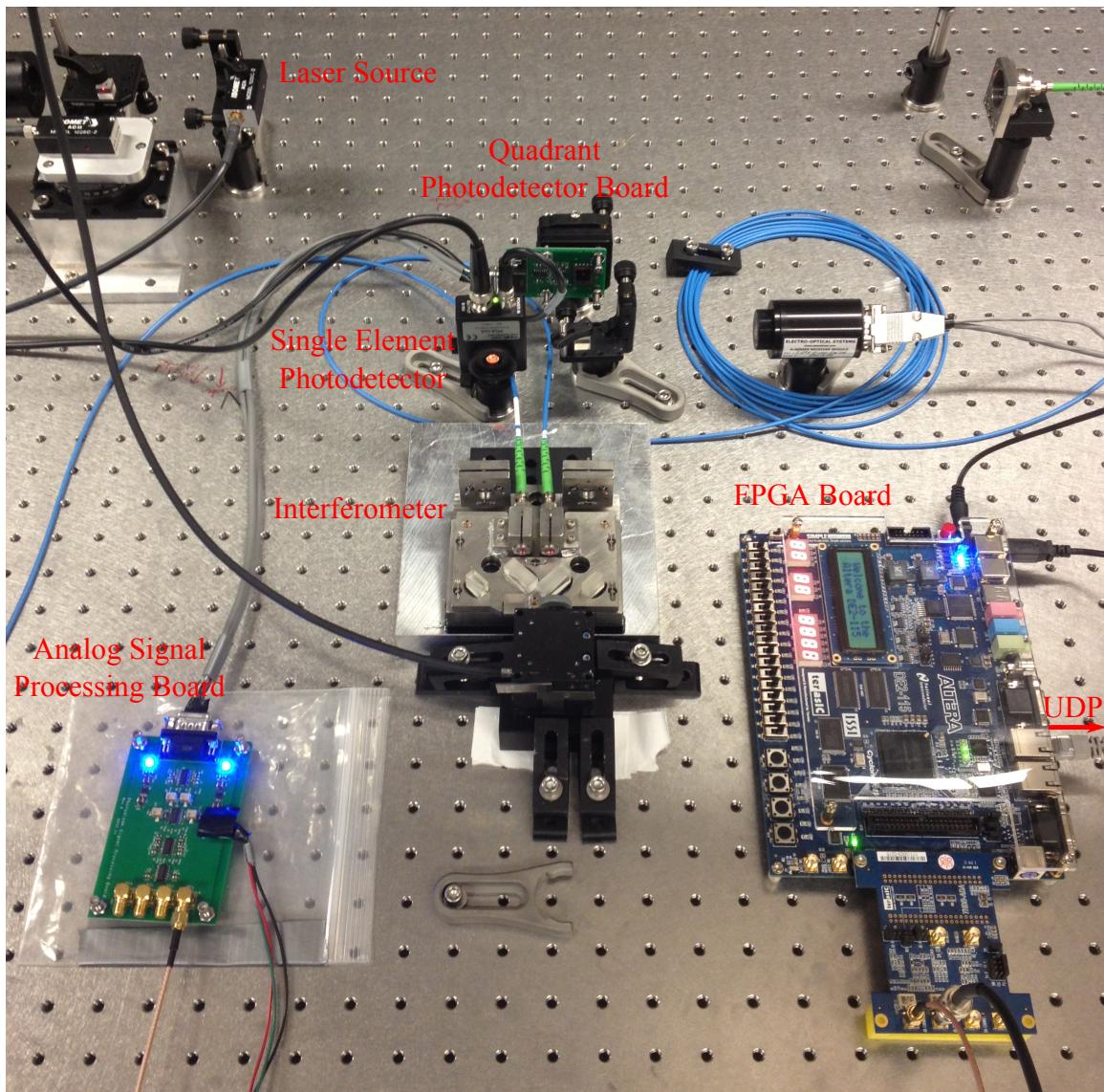


Figure 4.10: The setup of new-built phasemeter system with interferometer. It includes laser source, single element and quadrant photodetectors, analog signal processing board, and FPGA board, the target PC is out of the picture.

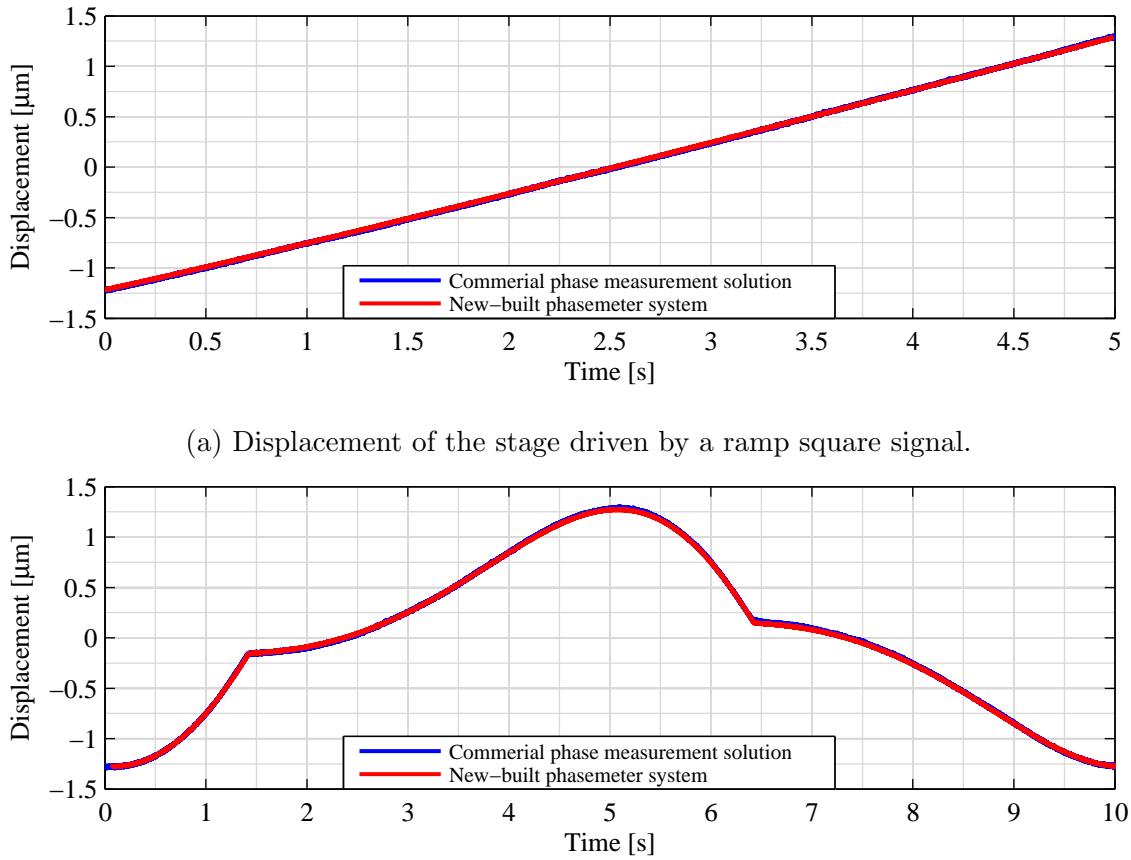


Figure 4.11: Displacements of the stage driven by two signals from function generator. One is a ramp square signal with 100 mHz frequency, 200 mV<sub>p-p</sub> amplitude, and 500 mV offset. Another is a chirp sine signal with a frequency from 0 to 200 mHz, 200 mV<sub>p-p</sub> amplitude, 500 mV offset, and 5 s sweep time.

hard to know the theoretical displacement to verify the measurements taken by the two solutions. If just comparing between these two measurements, the result from currently-used solution and the result from new-built phasemeter system match each other approximately. Figure 4.12 shows the low-frequency part of Figure 4.11(b) in detail.

Due to the open-loop control, the displacements of the stage are not strict smooth line. The 10 nm to 20 nm swings are the characteristic of the stage rather than the

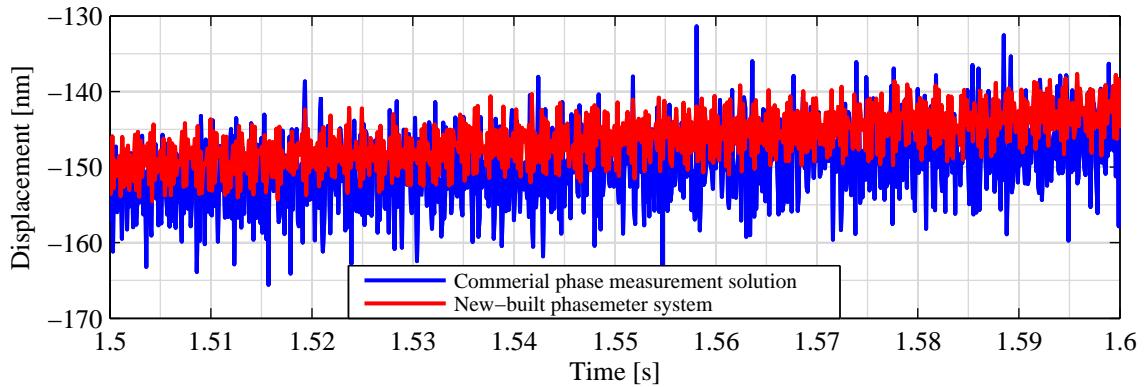


Figure 4.12: The low-frequency portion of the displacement of the stage driven by chirp sine signal.

noise introduced from phase measurement tools. Meanwhile, the tiny offset between two displacements is due to the movement repeatability. Because in tens of cycles, there is no displacement in one cycle exactly matching the displacement in other cycles.

Hence, to verify the performance of the new-built phasemeter system, a close-loop controlled stage must be applied in future. However, from the comparisons, the new-built phasemeter system is at least not worse than the currently-used one, which has the capability to replace the current one.



# 5 Conclusions and Future Work

This design has achieved a high-speed, high-precision, economical, small-volume, and user-friendly interface phasemeter prototype using a custom analog processing board and commercially available FGPA. This prototype also has several aspects that must be improved in future work. The following are the conclusions and highlights of current phasemeter prototype firstly.

## 5.1 Detection and Processing Board

This detection and processing board was designed for wavefront sensing a 70 kHz interference signal to detect displacement, pitch, and yaw of a target mirror. The photodetector employed in this board is a large active area ( $100 \text{ mm}^2$ ) quadrant photodiode, which has a high spatial sensitivity to measure pitch and yaw in theory.

The processing circuitry can adjust the output voltage for different incident optical powers, whose gain is 0.5 to 500. Thus, the output voltage could remain a constant value for the ADC to digitalize.

It has a 1 kHz to 100 kHz passband, which is sufficient for a 70 kHz split frequency

plus varied Doppler frequency. In this band, the stability and repeatability of the magnitude response of the four channels must be improved. This may be caused by the quality of the potentiometers used in inverting amplifier circuit. While the magnitude characteristic does not significantly impact the phase measurement, in the 50 kHz – 90 kHz band, the circuitry has a maximum 50° system phase shift, which will introduce errors when measuring changing target velocities. This must be solved in future iterations.

The entire board has a background noise with an RMS value of 2.08 mV when the output signal is 1.6 V<sub>p-p</sub>. The signal-to-noise ratio is 54.7 dB. This noise level is generally considered low, which is good for the precision of the phasemeter. The detector is sensitive to stray light in the laboratory, thus it is better to use a black box to isolate the detector in practical applications.

## 5.2 Digital Signal Processing based on FPGA

A digital signal processing module was designed as the core processing system to implement the phasemeter algorithm in an FPGA. It is designed for a 5 MHz split frequency plus varied Doppler frequency with a 50 MSPS processing rate. In practice, it is also compatible at 70 kHz split frequency signals. Currently, only one channel (processing one measurement and one reference signal) has been implemented in the FPGA due to the resource limitation of the current hardware. Expanding this system to three more channels simply requires a larger FPGA and is not a technological limitation.

In this design, both the PLL and SBDFT algorithms for the phasemeter are implemented in the FPGA. The PLL algorithm has a straightforward and concise

structure, but it costs more on-chip hardware resources, such as RAM and multipliers. Meanwhile, the noise levels of the PLL and SBDFT algorithm do not differ too much. Both have a noise level around 150 pm (in displacement), when using a function generator to simulate measurement signal in static, low velocity, and high velocity states (0, 10 kHz, and 1.5 MHz in Doppler frequency). Because fewer resources are used, the SBDFT is better suited for a single channel. When processing four channels, however, the resource difference between the two algorithms may change because some components are shared. Thus, for four channels, the PLL and SBDFT should be compared to determine which algorithm is more efficient.

The digital signal processing module also suffers from the inherent system phase shift due to filters. In the 100 kHz to 2 MHz band in the phase response, the maximum phase shift is about  $180^\circ$ . For a variable velocity target, measuring its motion will introduce extra phase shifts coming from the phasemeter system itself, which impacts the precision of the phasemeter.

### 5.3 UDP Transmission

A soft microprocessor was built in the FPGA chip, which ran a network task on MicroC/OS-II to pack the computed results and the time stamps, and then send them as UDP packets through Ethernet interface. A target PC was configured as the receiving end in a Matlab/Simulink xPC target system. The communication between the FPGA board and the target PC for logging continuous data sets worked as an alternative to other communication protocols such as VME or USB. One limitation is that the bandwidth is only 7,000 packages per second, while it should be 150,000 packages per second in theory.

## 5.4 Future Work

This initial phasemeter prototype achieved high performance given the relative simplicity of the components used. To improve it to overall phasemeter, following aspects must be addressed in future work.

1. The inherent phase shift due to filters must be eliminated or compensated. The inherent phase shift from filters occurs in both the analog and digital circuits when measuring variable target motions. The reason is that the filter has a non-uniform phase responses to difference frequency signals, which has been illustrated in Figures 2.11 and 3.5. Figure 5.1 shows simulations when target mirror moving at a constant and a varied velocity. From the figure, the relative displacement between any two time points is not impacted by the constant error, however, it is impacted by the varied error, which would cause uncertainty. Hence, an elimination or compensation method must be devised to improve the precision of the measurement.

The primary thought is compensating the inherent phase shift due to both analog and digital filter in the FPGA. There are two potential ideas, one is measuring their practical phase responses firstly, make a look-up table for these responses in FPGA, then calculating the instantaneous frequency during phase measurement, checking the look-up table, and compensating the phase shift in real time. Another is designing a digital circuit like an all-pass filter after the filter, which has an advance phase response (the filters has delay phase response). The phase response of the circuit can complement that of filter at every point, make the entire phase shift uniform.

2. The photodiode on detection and processing board must be redesigned to work

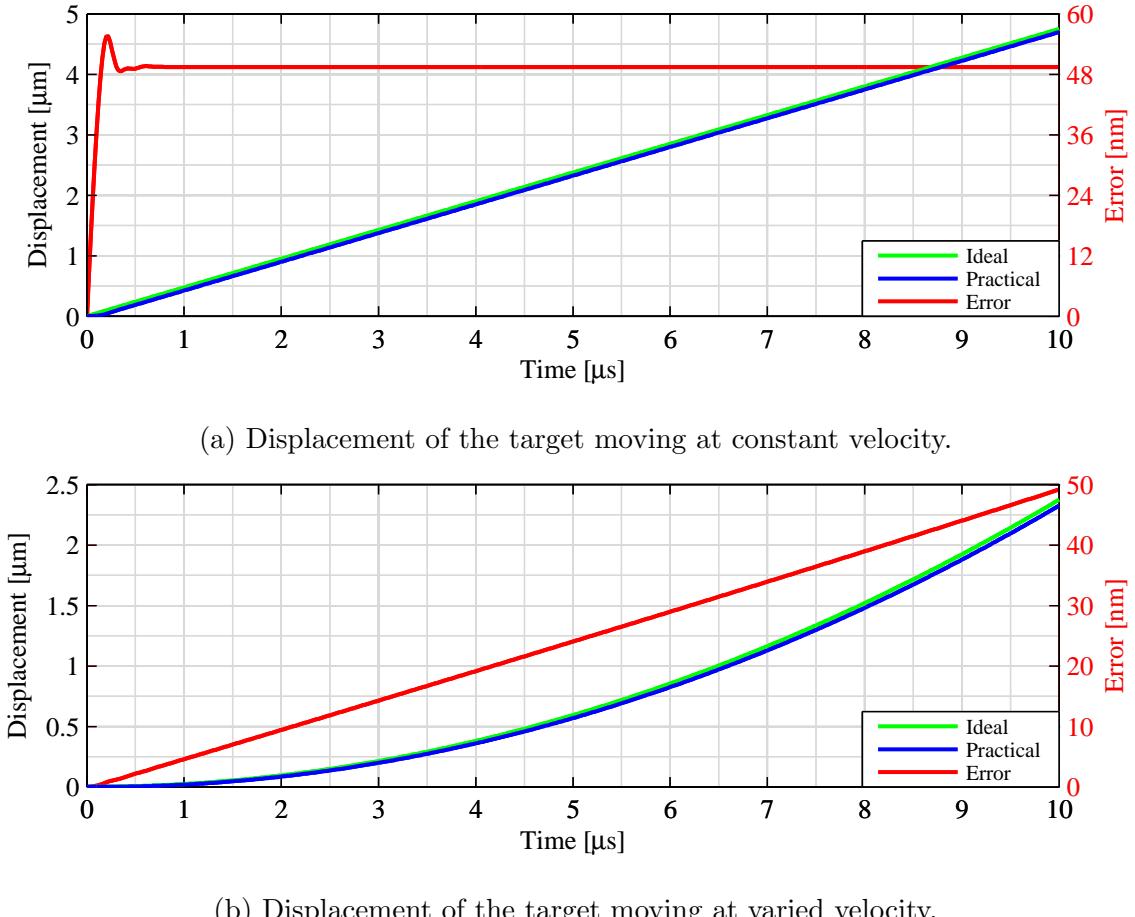


Figure 5.1: The simulations of the target moving at a constant and a varied velocity. The green lines are the ideal displacements, which are calculated by the signals free from the phase shift due to filters. The blue lines are the practical displacements, which suffers from the non-uniform phase shift. The red lines are the errors between ideal and practical displacements. (a) shows the displacements of the target moving at a constant velocity 0.475 m/s (according to Equation (1.3), the Doppler frequency is 1.5 MHz). Because the Doppler frequency is constant, the signals with this frequency have constant phase shift in the phase response plot, the ideal and practical displacement have a constant error about 50 nm in stable state. (b) shows the displacements of the target moving at varied velocity from 0 to 0.475 m/s (Doppler frequency from 0 to 1.5 MHz). The filters introduce different phase shifts for the signals with different frequencies, which lead non-uniform error (from 0 to 50 nm) of the displacements when the target moves at different velocities.

in photoconductive (reverse bias) mode. Photodiodes working in that mode have a high response speed, thus it can be designed for sensing 5 MHz split frequencies plus varied Doppler frequency. The PCB board also must be redesigned with considering the signal integrity issues, to ensure it can work with high speed signal. Currently, the photodiode works in photovoltaic (zero bias) mode, which has higher sensitivity but lower response speed. Thus, it is designed for sensing a 70 kHz signal.

3. The digital signal processing module in the FPGA must be expanded to four channels (processing four measurement signals from quadrant photodiode and one reference signal). In that case, the phasemeter can measure pitch and yaw of target mirror. The four channels occupy a larger area or more resource on FPGA chip, so a larger and more advanced FPGA will be employed, three more digital signal processing channels will be multiplied in the FPGA chip. Also, more ADC daughter cards will be employed to digitalization extra three measurement signals. At current stage, due to the limited resource of FPGA hardware, only one channel is implemented in FPGA, which can only measure the displacement of target mirror.
4. The data logging transmission speed must increase. Theoretically, the maximum speed to send a UDP package with the same package size in this design is about 150,000 packages per second. One package contains a computed result and a time stamp, thus, the maximum sampling rate at the terminal is 150 kHz. Currently, the transmission is only about 7 kHz, which is far slower than theoretical value, and not sufficient for real-time control. Some optimizations in hardware and software for this Ethernet system have been proposed, such as using a DMA engine for exchanging data to and from the

Ethernet device, increasing the processor and memory frequency, using low-latency memory for Nios II software execution, using fast packet memory to store Ethernet data [65].

The Fast Ethernet, what is used in this design now, has a 12.5 MB/s bandwidth and 1000  $\mu$ s latency. If possible, a wider bandwidth and lower latency bus should be applied for high-speed, real-time control in future applications. PCI Express is a potential candidate with 250 MB/s bandwidth (PCIe x1) and 0.7  $\mu$ s latency.



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# A Appendix

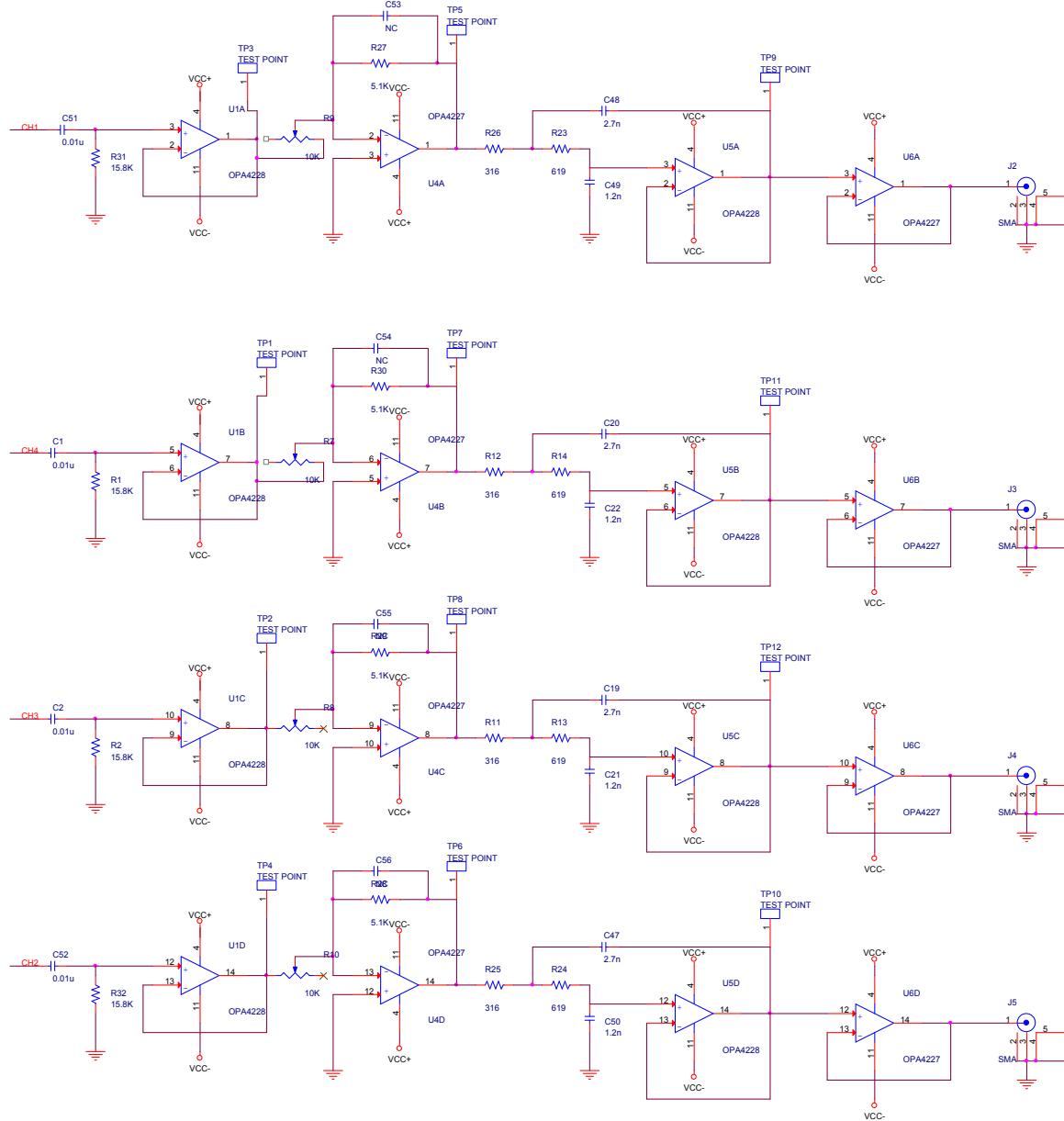


Figure A.1: The schematic diagram of this quadrant photodiode detection and processing circuitry Part 1. It consists of four channels of high-pass filters, inverting amplifiers, Sallen-Key low-pass filters, buffers and SMA connectors.

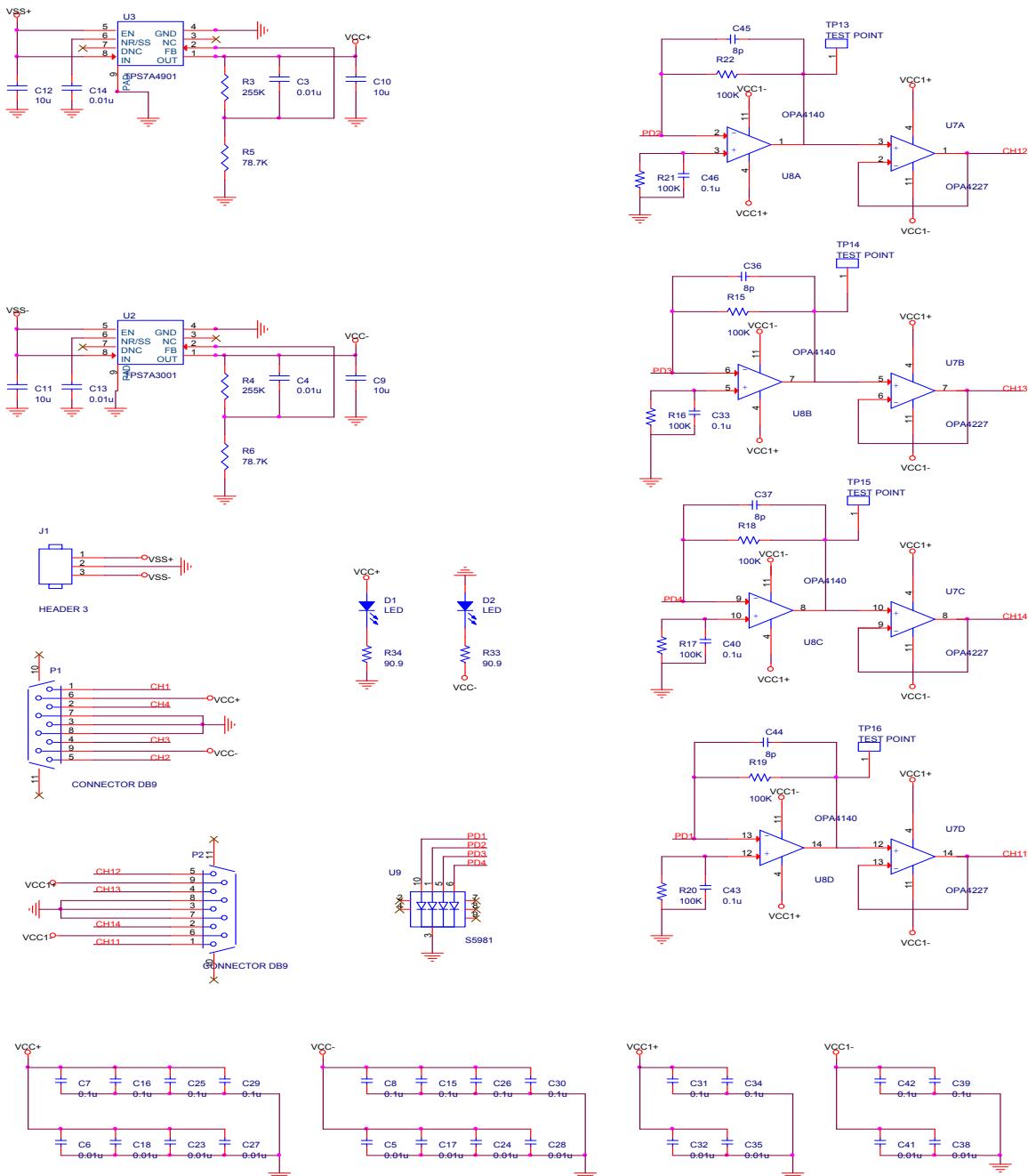


Figure A.2: The schematic diagram of this quadrant photodiode detection and processing circuitry Part 2. It consists of two linear regulators, header, two DB9 connectors, two LEDs, quadrant photodiode, four channels of transimpedance amplifiers and buffers and several bypass capacitors.

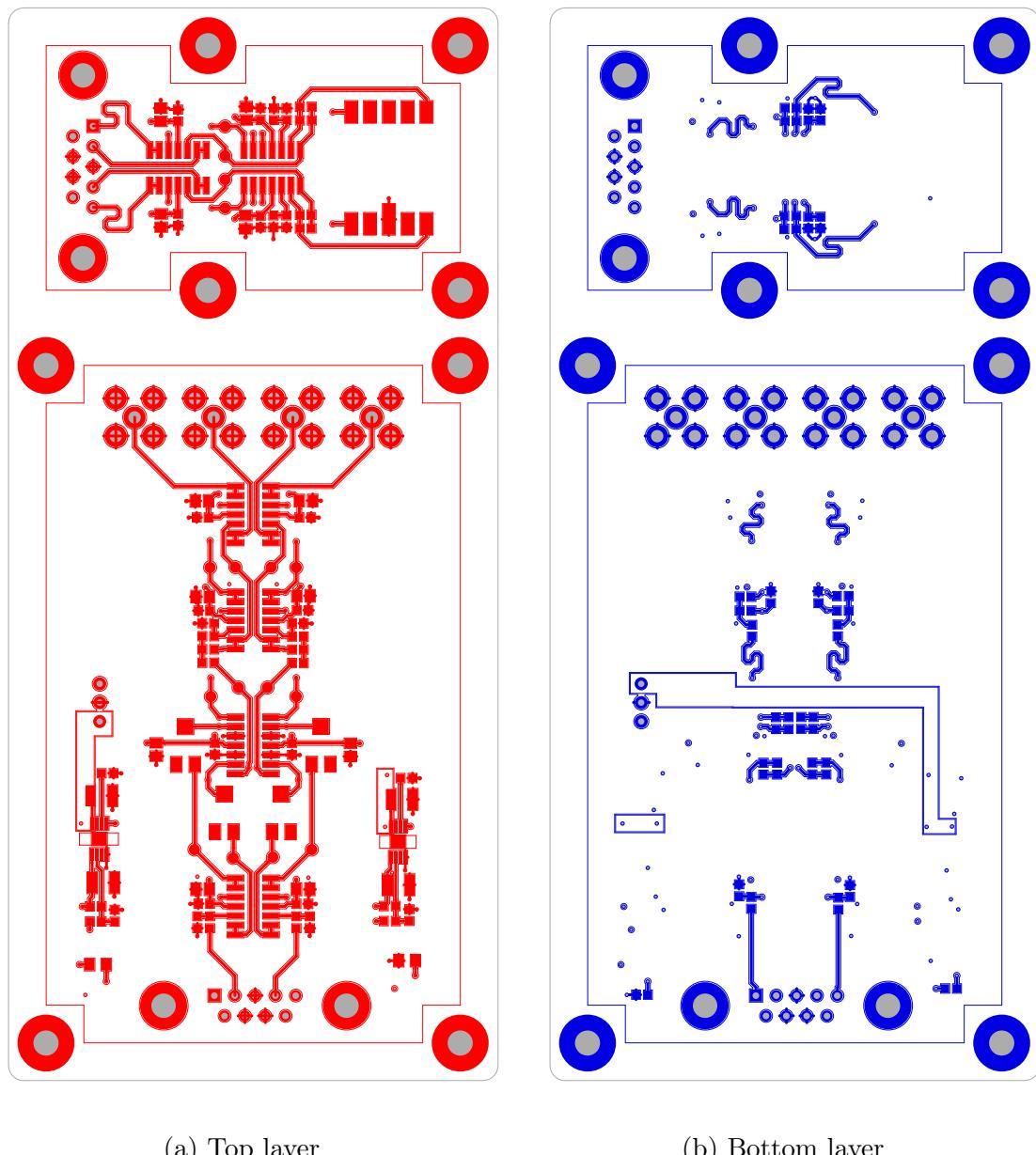


Figure A.3: The detection and processing circuitry PCB layout and routes Part 1, the first (top) and fourth (bottom) layers.

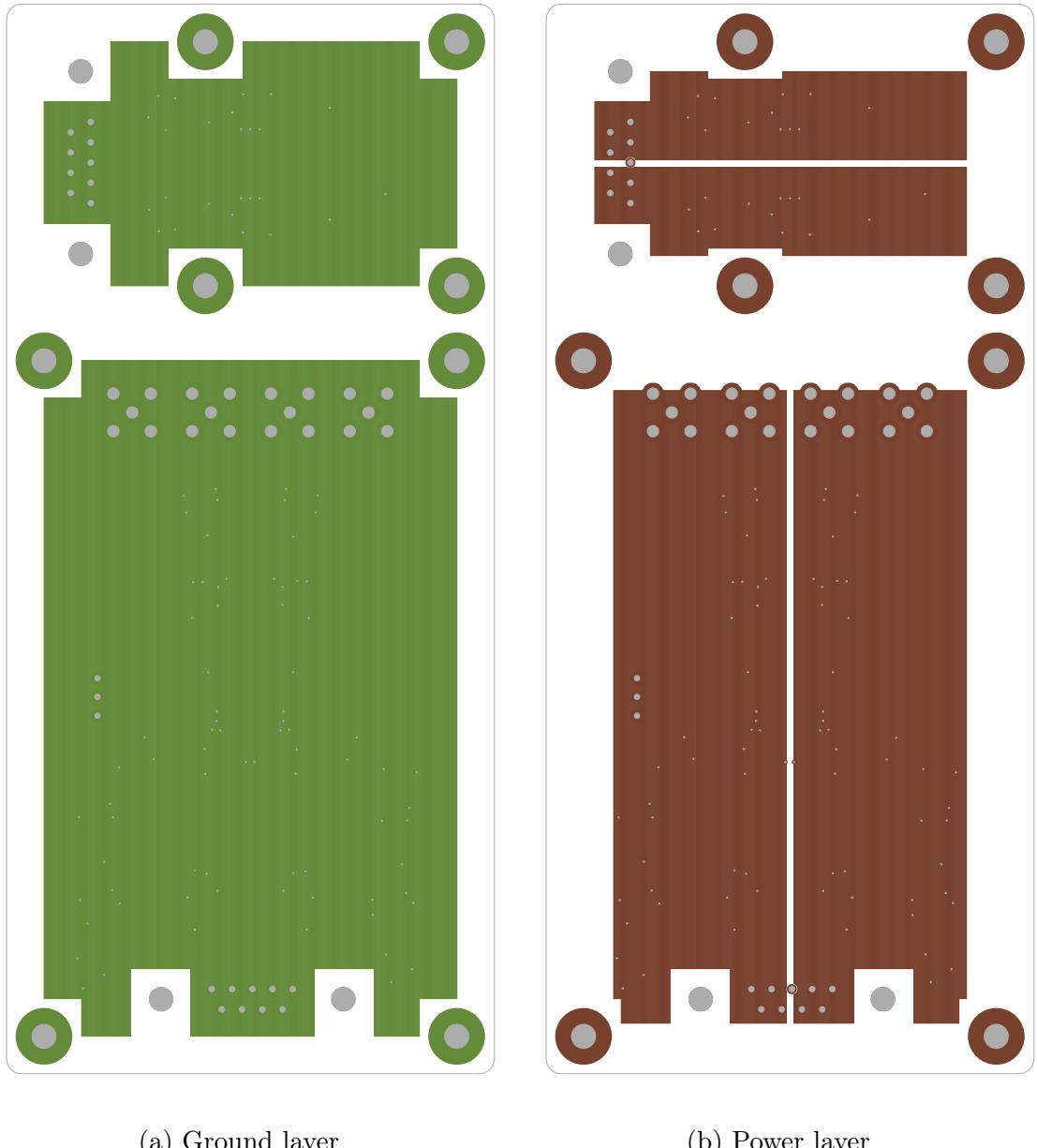


Figure A.4: The detection and processing circuitry PCB layout and routes Part 2, the second (ground) and third (power) layers

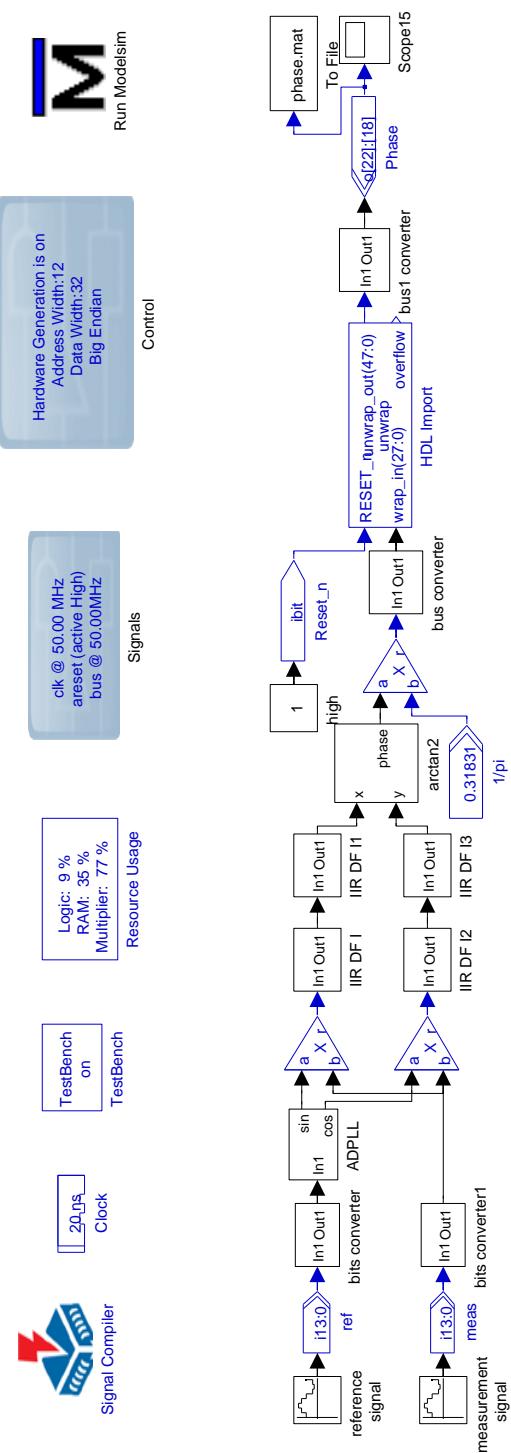


Figure A.5: The fixed-point and synthesizable models of PLL algorithm.

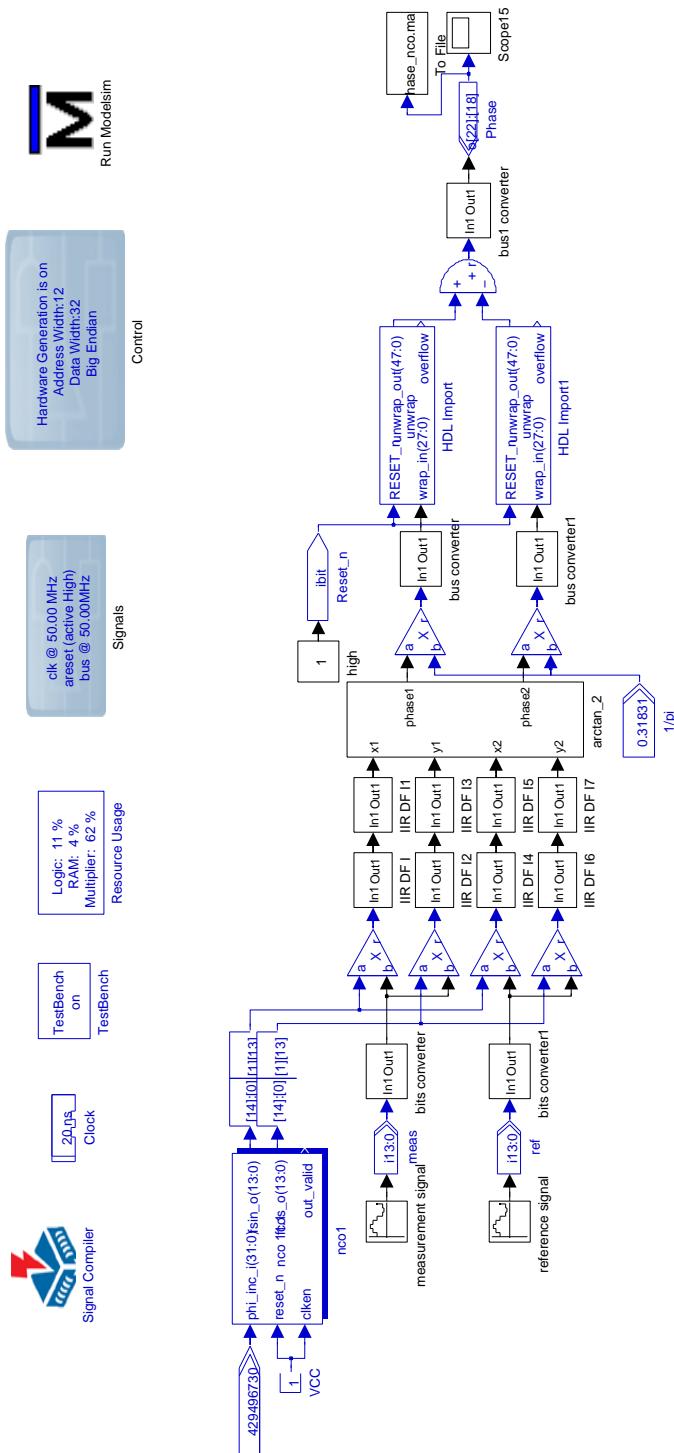


Figure A.6: The fixed-point and synthesizable models of SBDFT algorithm.