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TensorFlow: Open sou
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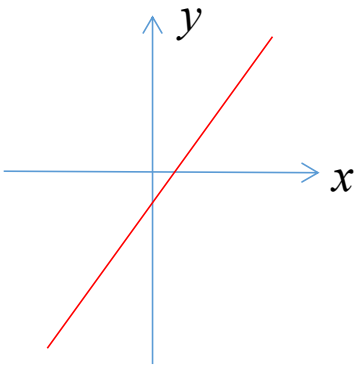
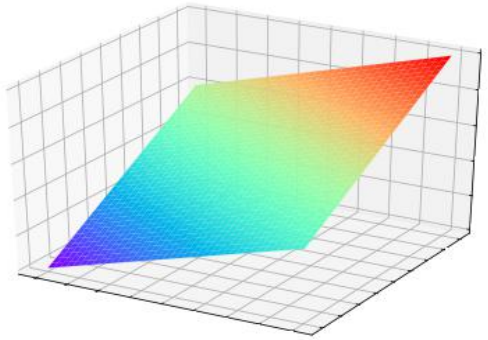
9.4 多元线性回归

- **多元回归**（Multivariate Regression）：回归分析中包括**两个或两个以上**的**自变量**
- **多元线性回归**（Multivariate Linear Regression）：因变量和自变量之间是**线性**关系

$$y = wx + b$$



$$y = w_1x_1 + w_2x_2 + b$$



$$y = w_1x_1 + \ldots + w_mx_m + b$$

超平面 (Hyperplane)：

直线在**高维空间**中的推广

模型：$\hat{y} = w_1 x^1 + \ldots + w_m x^m + b$

令$b=w_0$，$x^0=1$

$x^1, x^2, \ldots, x^m$	样本属性
$w_1, w_2, \ldots, w_m$	权值

x^1: 面积，x^2: 房间数，x^3: 楼层数

w_1=0.6，w_2=0.3，w_3=0.1

向量形式：$\hat{y} = w_0 x^0 + w_1 x^1 + \ldots + w_m x^m = W^T X$

$$W = (w_0, w_1, \ldots, w_m)^T$$

$$X = (x^0, x^1, \ldots, x^m)^T$$

n 个样本 (X_i, y_i) （$i=1, 2, \ldots, n$）

$$\hat{y}_i = W^T X_i$$

损失函数：$Loss = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - W^T X_i)^2$

损失函数：$$Loss=\sum_{i=1}^{n}(y_i-\hat{y}_i)^2=\sum_{i=1}^{n}(y_i-W^TX_i)^2$$

$$Loss=(Y-XW)^T(Y-XW)$$

$$X=(X_1,X_2,...,X_n)^T \quad Y=(y_1,y_2,...,y_n)^T$$

$$X_i=(x_i^0,x_i^1,...,x_i^m)$$

极值问题：参数向量W取何值时，*Loss*函数达到最小？

函数$f(x)$最小化时自变量x的取值 $\arg\min_x f(x)$

函数$f(x)$最大化时自变量x的取值 $\arg\max_x f(x)$

$$\arg\min_w (Y-XW)^T(Y-XW)$$

求解模型参数

$$Loss = (Y - XW)^T (Y - XW)$$

$$\frac{\partial Loss}{\partial W} = \frac{\partial((Y - XW)^T (Y - XW))}{\partial W}$$

$$= 2X^T (XW - Y)$$

$$= 2X^T XW - 2X^T Y = 0$$

$$\Rightarrow X^T XW = X^T Y$$

$$\Rightarrow W = (X^T X)^{-1} X^T Y \quad (X^T X\text{为满秩矩阵})$$

线性方程组

$$
\begin{aligned}
&w_0 + w_1 x_1^1 + \ldots + w_j x_1^j + \ldots + w_m x_1^m = \hat{y}_1 \\
&w_0 + w_1 x_2^1 + \ldots + w_j x_2^j + \ldots + w_m x_2^m = \hat{y}_2 \\
&\ldots \\
&w_0 + w_1 x_i^1 + \ldots + w_j x_i^j + \ldots + w_m x_i^m = \hat{y}_i \\
&\ldots \\
&w_0 + w_1 x_n^1 + \ldots + w_j x_n^j + \ldots + w_m x_n^m = \hat{y}_n
\end{aligned}
$$

n 个样本

m 个属性

$$
\begin{bmatrix} 1 & x_1^1 & \ldots & x_1^m \\ 1 & x_2^1 & \ldots & x_2^m \\ \ldots & \ldots & \ldots & \ldots \\ 1 & x_n^1 & \ldots & x_n^m \end{bmatrix} \cdot \begin{bmatrix} w_0 \\ w_1 \\ \ldots \\ w_m \end{bmatrix} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \ldots \\ \hat{y}_n \end{bmatrix}
$$

矩阵形式： $XW = \hat{Y}$

损失函数：

$$Loss = (Y - \hat{Y})^2 = (Y - XW)^2$$

$$\frac{\partial Loss}{\partial W} = 2X^T(XW - Y) = 0$$

$$W = (X^T X)^{-1} X^T Y$$

n维向量：向量中的**元素个数**为n

$(1,2,3)^T$：3维向量

$(1,2,3,4,5)^T$：5维向量

$(1,2,...,n)^T$：n维向量

$X=(x^0,x^1,...,x^m)^T$ ：m+1维向量

m×n矩阵：由 **m×n 个数**排成的m行n列的数表

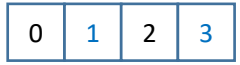
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\begin{bmatrix} 1 & x_1^1 & \cdots & x_1^m \\ 1 & x_2^1 & \cdots & x_2^m \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_n^1 & \cdots & x_n^m \end{bmatrix}$$

$n\times(m+1)$

多维数组 (TensorFlow/NumPy...)

rank=1



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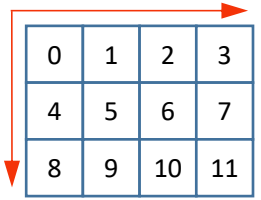

rank=3