



10.3 TensorFlow的自动求导机制



10.3.1 TensorFlow的可训练变量

梯度下降法

损失函数

$$Loss = \frac{1}{2} \sum_{i=1}^n (y_i - (wx_i + b))^2$$

损失函数求导数

$$\frac{\partial Loss}{\partial w} = \frac{1}{n} \sum_{i=1}^n x_i (wx_i + b - y_i)$$

$$\frac{\partial Loss}{\partial b} = \frac{1}{n} \sum_{i=1}^n (wx_i + b - y_i)$$

更新模型参数

$$w^{(k+1)} = w^{(k)} - \eta \frac{\partial Loss}{\partial w}$$

$$b^{(k+1)} = b^{(k)} - \eta \frac{\partial Loss}{\partial b}$$



- TensorFlow的**自动求导**机制
- **Variable**对象
 - 对Tensor对象的进一步封装
 - 在模型训练过程中**自动记录梯度信息**，由算法**自动优化**
 - **可以被训练**的变量
 - 在机器学习中作为**模型参数**

```
tf.Variable(initial_value, dtype)
```

数字

Python列表

ndarray对象

Tensor对象



■ 创建可训练变量

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import numpy as np
```

```
In [3]: tf.Variable(3)
```

```
Out[3]: <tf.Variable 'Variable:0' shape=() dtype=int32, numpy=3>
```

```
In [4]: tf.Variable([1, 2])
```

```
Out[4]: <tf.Variable 'Variable:0' shape=(2,) dtype=int32, numpy=array([1, 2])>
```

```
In [5]: tf.Variable(np.array([1, 2]))
```

```
Out[5]: <tf.Variable 'Variable:0' shape=(2,) dtype=int32, numpy=array([1, 2])>
```



10.3.1 TensorFlow的自动求导机制——可训练变量

```
tf.Variable(initial_value,dtype)
```

```
In [6]: tf.Variable(3.)
```

```
Out[6]: <tf.Variable 'Variable:0' shape=() dtype=float32, numpy=3.0>
```

```
In [7]: tf.Variable([1,2],dtype=tf.float64)
```

```
Out[7]: <tf.Variable 'Variable:0' shape=(2,) dtype=float64, numpy=array([1., 2.])>
```



□ 将张量封装为可训练变量

```
In [8]: tf.Variable(tf.constant([[1, 2], [3, 4]]))
```

```
Out[8]: <tf.Variable 'Variable:0' shape=(2, 2) dtype=int32, numpy=
        array([[1, 2],
               [3, 4]])>
```

```
In [9]: tf.Variable(tf.zeros([2, 3]))
```

```
Out[9]: <tf.Variable 'Variable:0' shape=(2, 3) dtype=float32, numpy=
        array([[0., 0., 0.],
               [0., 0., 0.]], dtype=float32)>
```

```
In [10]: tf.Variable(tf.random.normal([2, 2]))
```

```
Out[10]: <tf.Variable 'Variable:0' shape=(2, 2) dtype=float32, numpy=
        array([[1.113321, 0.79599154],
               [0.00233323, 0.33498392]], dtype=float32)>
```



■ 使用变量名

```
In [11]: x=tf.Variable([1,2])
```

```
In [12]: x
```

```
Out[12]: <tf.Variable 'Variable:0' shape=(2,) dtype=int32, numpy=array([1, 2])>
```

```
In [13]: print(x.shape, x.dtype)
```

```
(2,) <dtype: 'int32'>
```

```
In [14]: print(x.numpy())
```

```
[1 2]
```



■ trainable属性

```
In [15]: x.trainable
```

```
Out[15]: True
```

■ ResourceVariable

```
In [16]: type(x)
```

```
Out[16]: tensorflow.python.ops.resource_variable_ops.ResourceVariable
```



■ 可训练变量赋值

对象名.assign()

对象名.assign_add()

对象名.assign_sub()

```
In [17]: x=tf.Variable([1,2])
```

```
In [18]: x.assign([3,4])
```

```
Out[18]: <tf.Variable 'UnreadVariable' shape=(2,) dtype=int32, numpy=array([3, 4])>
```

```
In [19]: x.assign_add([1,1])
```

```
Out[19]: <tf.Variable 'UnreadVariable' shape=(2,) dtype=int32, numpy=array([4, 5])>
```

```
In [20]: x.assign_add([1,1])
```

```
Out[20]: <tf.Variable 'UnreadVariable' shape=(2,) dtype=int32, numpy=array([5, 6])>
```



10.3.1 TensorFlow的自动求导机制——可训练变量

```
In [21]: a=tf.constant(2)
```

```
In [22]: a.assign(3)
```

```
-----  
AttributeError                                Traceback (most recent call last)  
<ipython-input-22-22a29cba3fee> in <module>  
----> 1 a.assign(3)
```

```
AttributeError: 'tensorflow.python.framework.ops.EagerTensor' object has no attribute 'assign'
```

```
In [23]: a.trainable
```

```
-----  
AttributeError                                Traceback (most recent call last)  
<ipython-input-23-d74019e38028> in <module>  
----> 1 a.trainable
```

```
AttributeError: 'tensorflow.python.framework.ops.EagerTensor' object has no attribute 'trainable'
```



■ `isinstance()` 方法

```
In [22]: a=tf.range(5)
```

```
In [23]: x=tf.Variable(a)
```

```
In [24]: isinstance(a, tf.Tensor), isinstance(a, tf.Variable)
```

```
Out[24]: (True, False)
```

```
In [25]: isinstance(x, tf.Tensor), isinstance(x, tf.Variable)
```

```
Out[25]: (False, True)
```





10.3.2 TensorFlow的自动求导

■ 自动求导——GradientTape

```
with GradientTape() as tape:  
    函数表达式  
grad=tape.gradient(函数,自变量)
```

$$y = x^2 \Big|_{x=3}$$

$$y = 3^2 = 9$$
$$\frac{dy}{dx} = 2x = 6$$

```
In [26]: x = tf.Variable(3.)  
  
In [27]: with tf.GradientTape() as tape:  
          y = tf.square(x)  
  
In [28]: dy_dx = tape.gradient(y, x)  
  
In [29]: print(y)  
          print(dy_dx)  
  
tf.Tensor(9.0, shape=(), dtype=float32)  
tf.Tensor(6.0, shape=(), dtype=float32)
```



GradientTape(persistent, watch_accessed_variables)

```
In [30]: x = tf.Variable(3.)

with tf.GradientTape() as tape:
    y = tf.square(x)
    z = pow(x, 3)

dy_dx = tape.gradient(y, x)
dz_dx = tape.gradient(z, x)

print(y)
print(dy_dx)
print(z)
print(dz_dx)
```



```
RuntimeError                                Traceback (most recent call last)
<ipython-input-30-5ace6fa66e0a> in <module>
      6
      7 dy_dx = tape.gradient(y, x)
----> 8 dz_dx = tape.gradient(z, x)
      9
     10 print(y)

D:\Anaconda3\lib\site-packages\tensorflow_core\python\eager\backprop.py in gradient(self, target, sources, output_gradients, unconnected_gradients)
     963     """
     964     if self._tape is None:
--> 965         raise RuntimeError("GradientTape.gradient can only be called once on "
     966                             "non-persistent tapes.")
     967     if self._recording:
```

RuntimeError: GradientTape.gradient can only be called once on non-persistent tapes.



10.3.2 TensorFlow的自动求导

```
In [31]: x = tf.Variable(3.)

with tf.GradientTape(persistent=True) as tape:
    y = tf.square(x)
    z = pow(x, 3)

dy_dx = tape.gradient(y, x)
dz_dx = tape.gradient(z, x)

print(y)
print(dy_dx)
print(z)
print(dz_dx)

del tape
```

$$z = 3^3 = 27$$
$$\frac{dz}{dx} = 3x^2 = 27$$

```
tf.Tensor(9.0, shape=(), dtype=float32)
tf.Tensor(6.0, shape=(), dtype=float32)
tf.Tensor(27.0, shape=(), dtype=float32)
tf.Tensor(27.0, shape=(), dtype=float32)
```



`GradientTape(persistent,watch_accessed_variables)`

```
In [32]: x = tf.Variable(3.)

with tf.GradientTape(watch_accessed_variables=False) as tape:
    y = tf.square(x)

dy_dx = tape.gradient(y, x)

print(y)
print(dy_dx)

tf.Tensor(9.0, shape=(), dtype=float32)
None
```



添加监视——watch()

```
In [33]: x = tf.Variable(3.)

with tf.GradientTape(watch_accessed_variables=False) as tape:
    tape.watch(x)
    y = tf.square(x)

dy_dx = tape.gradient(y, x)

print(y)
print(dy_dx)

tf.Tensor(9.0, shape=(), dtype=float32)
tf.Tensor(6.0, shape=(), dtype=float32)
```



监视非可训练变量

```
In [34]: x = tf.constant(3.)

with tf.GradientTape(watch_accessed_variables=False) as tape:
    tape.watch(x)
    y = tf.square(x)

dy_dx = tape.gradient(y, x)

print(y)
print(dy_dx)

tf.Tensor(9.0, shape=(), dtype=float32)
tf.Tensor(6.0, shape=(), dtype=float32)
```



■ 多元函数求偏导数

$$f(x, y) = x^2 + 2y^2 + 1$$

$$f(3, 4) = 42$$

$$\left. \frac{\partial f(x, y)}{\partial x} \right|_{x=3} = 2x = 6$$

$$\left. \frac{\partial f(x, y)}{\partial y} \right|_{y=4} = 4y = 16$$

tape.gradient(函数, 自变量)

```
In [35]: x = tf.Variable(3.)  
         y = tf.Variable(4.)  
  
         with tf.GradientTape() as tape:  
             f = tf.square(x) + 2 * tf.square(y) + 1  
  
         df_dx, df_dy = tape.gradient(f, [x, y])  
  
         print(f)  
         print(df_dx)  
         print(df_dy)  
  
tf.Tensor(42.0, shape=(), dtype=float32)  
tf.Tensor(6.0, shape=(), dtype=float32)  
tf.Tensor(16.0, shape=(), dtype=float32)
```



```
In [36]: x = tf.Variable(3.)  
         y = tf.Variable(4.)  
  
         with tf.GradientTape() as tape:  
             f = tf.square(x)+2*tf.square(y)+1  
  
         first_grads= tape.gradient(f, [x, y])  
  
         print(first_grads)  
  
[<tf.Tensor: id=223, shape=(), dtype=float32, numpy=6.0>,  
 <tf.Tensor: id=228, shape=(), dtype=float32, numpy=16.0  
>]
```



10.3.2 TensorFlow的自动求导

```
In [37]: x = tf.Variable(3.)  
         y = tf.Variable(4.)  
  
         with tf.GradientTape(persistent=True) as tape:  
             f = tf.square(x)+2*tf.square(y)+1  
  
         df_dx = tape.gradient(f, x)  
         df_dy = tape.gradient(f, y)  
  
         print(f)  
         print(df_dx)  
         print(df_dy)  
  
         del tape  
  
tf.Tensor(42.0, shape=(), dtype=float32)  
tf.Tensor(6.0, shape=(), dtype=float32)  
tf.Tensor(16.0, shape=(), dtype=float32)
```



■ 求二阶导数

```
In [38]: x = tf.Variable(3.)
         y = tf.Variable(4.)

         with tf.GradientTape(persistent=True) as tape2:

             with tf.GradientTape(persistent=True) as tape1:
                 f = tf.square(x)+2*tf.square(y)+1

                 first_grads = tape1.gradient(f, [x,y])

             second_grads=[tape2.gradient(first_grads, [x,y])]

         print(f)
         print(first_grads)
         print(second_grads)

         del tape1
         del tape2
```

```
tf.Tensor(42.0, shape=(), dtype=float32)
[<tf.Tensor: id=296, shape=(), dtype=float32, numpy=6.0>,
 <tf.Tensor: id=301, shape=(), dtype=float32, numpy=16.0>]
[[<tf.Tensor: id=308, shape=(), dtype=float32, numpy=2.0>,
  <tf.Tensor: id=309, shape=(), dtype=float32, numpy=4.0>]]
```

$$f(x, y) = x^2 + 2y^2 + 1$$

$$\frac{\partial f(x, y)}{\partial x} \Big|_{x=3} = 2x = 6$$

$$\frac{\partial f(x, y)}{\partial y} \Big|_{y=4} = 4y = 16$$

$$\frac{\partial^2 f(x, y)}{\partial x^2} \Big|_{x=3} = 2$$

$$\frac{\partial^2 f(x, y)}{\partial y^2} \Big|_{y=4} = 4$$



■ 对向量求偏导

```
In [39]: x = tf.Variable([1., 2., 3.])
          y = tf.Variable([4., 5., 6.])

          with tf.GradientTape() as tape:
              f = tf.square(x)+2*tf.square(y)+1

          df_dx, df_dy = tape.gradient(f, [x, y])

          print(f)
          print(df_dx)
          print(df_dy)

          tf.Tensor([34. 55. 82.], shape=(3,), dtype=float32)
          tf.Tensor([2.  4.  6.], shape=(3,), dtype=float32)
          tf.Tensor([16. 20. 24.], shape=(3,), dtype=float32)
```

