

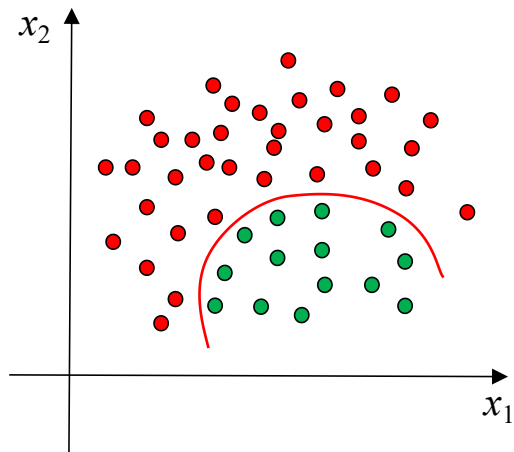
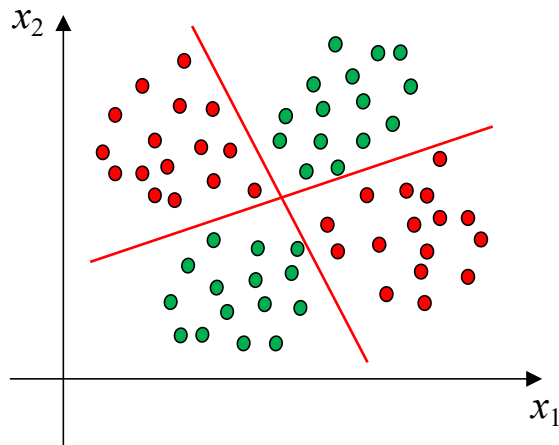


11.4 实例：实现多元逻辑回归



11.4.1 实现多元逻辑回归

■ 线性不可分



Iris数据集

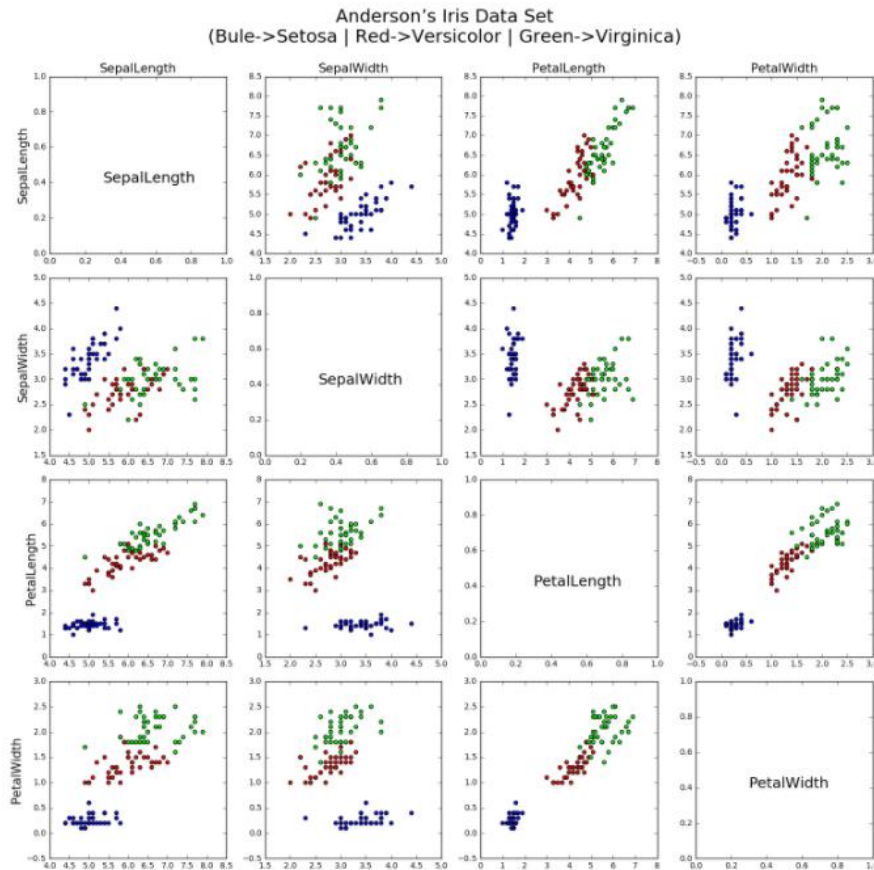
□ 150个样本

□ 4个属性

- 花萼长度 (Sepal Length)
- 花萼宽度 (Sepal Width)
- 花瓣长度 (Petal Length)
- 花瓣宽度 (Petal Width)

□ 1个标签

- 山鸢尾 (Setosa)
- 变色鸢尾 (Versicolour)
- 维吉尼亚鸢尾 (Virginica)



■ 加载数据

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import pandas as pd  
import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: TRAIN_URL = "http://download.tensorflow.org/data/iris_training.csv"  
train_path = tf.keras.utils.get_file(TRAIN_URL.split('/')[-1], TRAIN_URL)
```

```
In [4]: df_iris = pd.read_csv(train_path, header=0)
```



■ 处理数据

- 转化为NumPy数组
- 提取属性和标签
- 提取山鸢尾和变色鸢尾

```
In [5]: iris=np.array(df_iris)
```

```
In [6]: iris.shape
```

```
Out[6]: (120, 5)
```

```
In [7]: train_x=iris[:,0:2]  
train_y=iris[:,4]
```

```
In [8]: train_x.shape, train_y.shape
```

```
Out[8]: ((120, 2), (120,))
```

```
In [9]: x_train = train_x[train_y < 2]  
y_train = train_y[train_y < 2]
```

```
In [10]: x_train.shape, y_train.shape
```

```
Out[10]: ((78, 2), (78,))
```

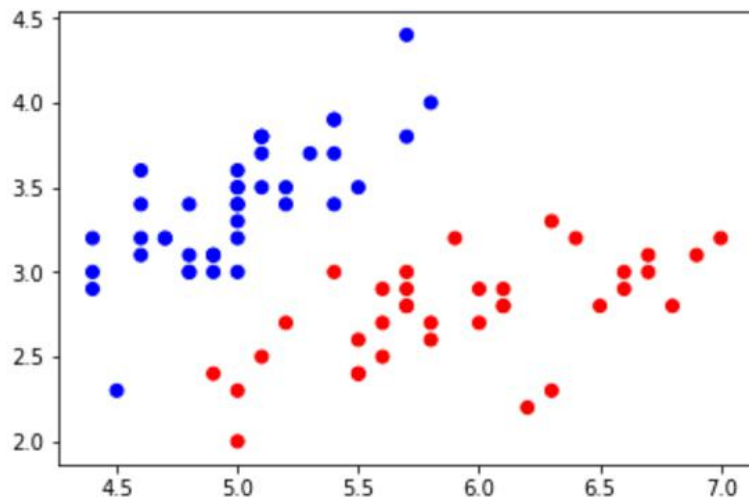
```
In [11]: num=len(x_train)
```



■ 处理数据

□ 可视化样本

```
In [12]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])  
plt.scatter(x_train[:,0], x_train[:,1], c=y_train, cmap=cm_pt)  
plt.show()
```

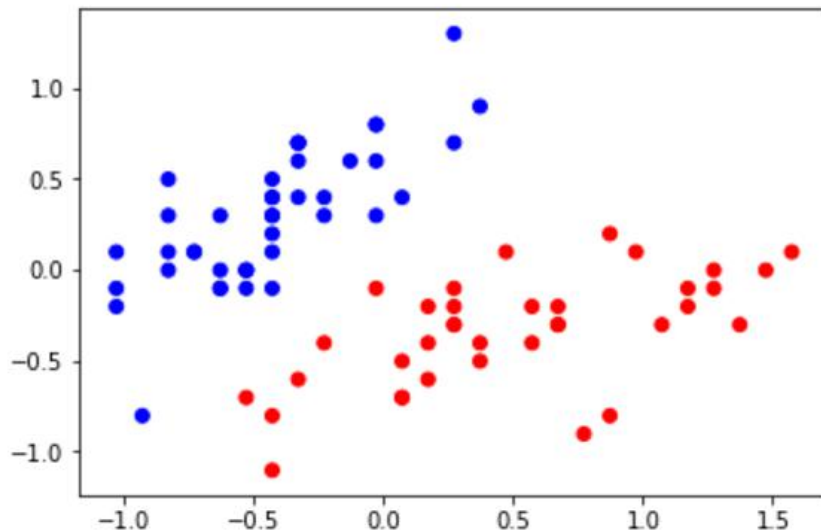


■ 处理数据

□ 属性中心化

```
In [13]: x_train=x_train-np.mean(x_train,axis=0)
```

```
In [14]: plt.scatter(x_train[:,0],x_train[:,1],c=y_train,cmap=cm_pt)  
plt.show()
```



■ 处理数据

□ 生成多元模型的属性矩阵和标签列向量

```
In [15]: x0_train = np.ones(num).reshape(-1, 1)
```

```
In [16]: X = tf.cast(tf.concat((x0_train, x_train), axis = 1), tf.float32)  
Y = tf.cast(y_train.reshape(-1, 1), tf.float32)
```

```
In [17]: X.shape, Y.shape
```

```
Out[17]: (TensorShape([78, 3]), TensorShape([78, 1]))
```



■ 设置超参数

```
In [18]: learn_rate=0.2  
         iter=120  
  
         display_step=30
```

■ 设置模型参数初始值

```
In [19]: np.random.seed(612)  
         W=tf.Variable(np.random.randn(3,1), dtype=tf.float32)
```



■ 训练模型

```
In [20]: ce=[]
         acc=[]

         for i in range(0, iter+1):
             with tf.GradientTape() as tape:
                 PRED = 1/(1+tf.exp(-tf.matmul(X,W)))
                 Loss = -tf.reduce_mean(Y*tf.math.log(PRED)+(1-Y)*tf.math.log(1-PRED))

                 accuracy = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED.numpy() < 0.5, 0., 1.), Y), tf.float32))
                 ce.append(Loss)
                 acc.append(accuracy)

                 dL_dW = tape.gradient(Loss, W)
                 W.assign_sub(learn_rate*dL_dW)

                 if i % display_step == 0:
                     print("i: %i, Acc:%f, Loss: %f" % (i, accuracy, Loss))
```

```
i: 0, Acc:0.230769, Loss: 0.994269
i: 30, Acc:0.961538, Loss: 0.481892
i: 60, Acc:0.987179, Loss: 0.319128
i: 90, Acc:0.987179, Loss: 0.246626
i: 120, Acc:1.000000, Loss: 0.204982
```

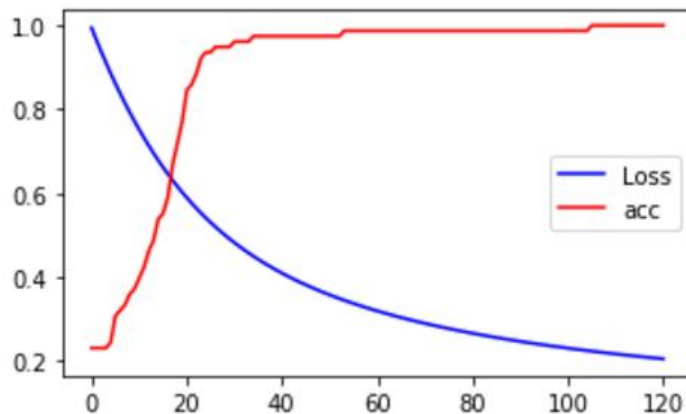


■ 可视化

□ 绘制损失和准确率变化曲线

```
In [21]: plt.figure(figsize=(5, 3))  
plt.plot(ce, color="blue", label="Loss")  
plt.plot(acc, color="red", label="acc")  
plt.legend()  
plt.show()
```

```
i: 0, Acc:0.230769, Loss: 0.994269  
i: 30, Acc:0.961538, Loss: 0.481892  
i: 60, Acc:0.987179, Loss: 0.319128  
i: 90, Acc:0.987179, Loss: 0.246626  
i: 120, Acc:1.000000, Loss: 0.204982
```



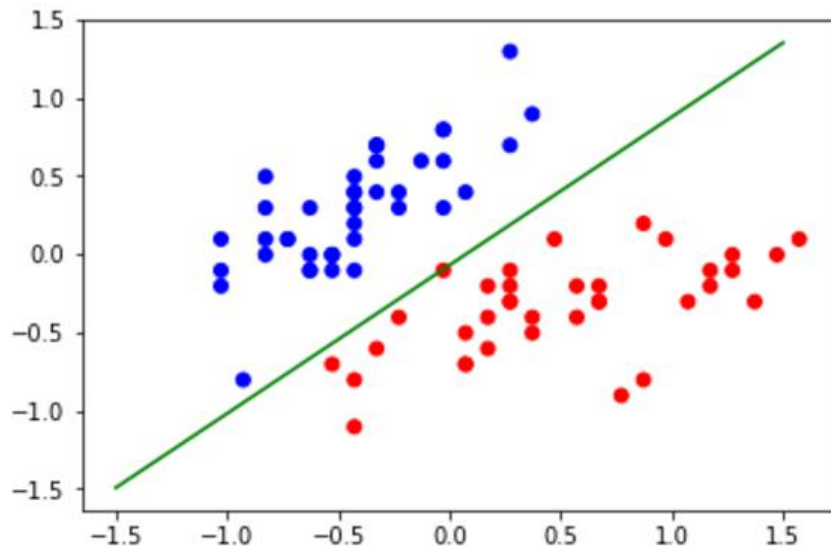
■ 可视化

□ 绘制决策边界

$$w_1x_1 + w_2x_2 + w_0 = 0$$

$$x_2 = -\frac{w_1x_1 + w_0}{w_2}$$

```
In [22]: plt.scatter(x_train[:,0],x_train[:,1],c=y_train,cmap=cm_pt)
x_=[-1.5, 1.5]
y_=- (W[1]*x_+W[0])/W[2]
plt.plot(x_, y_, color="g")
plt.show()
```



■ 可视化

□ 在训练过程中绘制决策边界

```
In [19]: np.random.seed(612)
          W=tf.Variable(np.random.randn(3,1), dtype=tf.float32)
```

```
In [20]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])
```

```
In [21]: x_=[-1.5, 1.5]
          y_=- (W[0]+W[1]*x_)/W[2]
```



训练模型

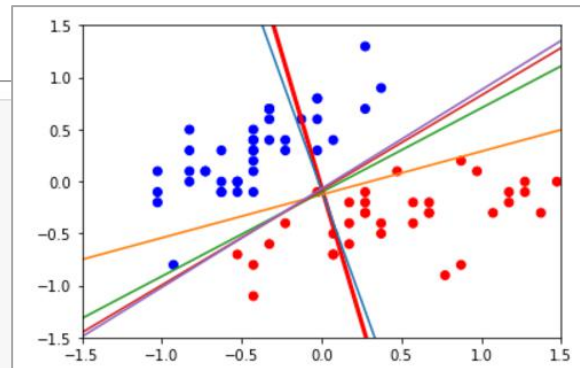
```
In [22]: plt.scatter(x_train[:,0],x_train[:,1],c=y_train,cmap=cm_pt)
plt.plot(x_,y_,color="red",linewidth=3)
plt.xlim([-1.5,1.5])
plt.ylim([-1.5,1.5])

ce=[]
acc=[]
for i in range(0, iter+1):
    with tf.GradientTape() as tape:
        PRED =1/(1+tf.exp(-tf.matmul(X,W)))
        Loss =-tf.reduce_mean(Y*tf.math.log(PRED)+(1-Y)*tf.math.log(1-PRED))

    accuracy = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED.numpy())<0.5,0.,1.), Y), tf.float32))
    ce.append(Loss)
    acc.append(accuracy)

    dL_dW= tape.gradient(Loss,W)
    W.assign_sub(learn_rate*dL_dW)

    if i % display_step == 0:
        print("i: %i, Acc:%f, Loss: %f" % (i, accuracy, Loss))
        y_=- (W[0]+W[1]*x_)/W[2]
        plt.plot(x_,y_)
```



```
i: 0, Acc:0.230769, Loss: 0.994269
i: 30, Acc:0.961538, Loss: 0.481892
i: 60, Acc:0.987179, Loss: 0.319128
i: 90, Acc:0.987179, Loss: 0.246626
i: 120, Acc:1.000000, Loss: 0.204982
```



■ 使用测试集

□ 加载数据集

```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import pandas as pd  
import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: TRAIN_URL = "http://download.tensorflow.org/data/iris_training.csv"  
train_path = tf.keras.utils.get_file(TRAIN_URL.split('/')[-1], TRAIN_URL)  
  
TEST_URL = "http://download.tensorflow.org/data/iris_test.csv"  
test_path = tf.keras.utils.get_file(TEST_URL.split('/')[-1], TEST_URL)
```



□ 数据处理

```
In [4]: df_iris_train = pd.read_csv(train_path, header=0)
df_iris_test = pd.read_csv(test_path, header=0)
```

```
In [5]: iris_train=np.array(df_iris_train)
iris_test=np.array(df_iris_test)
```

```
In [6]: iris_train.shape, iris_test.shape
```

```
Out[6]: ((120, 5), (30, 5))
```



□ 数据处理

```
In [7]: train_x=iris_train[:,0:2]
        train_y=iris_train[:,4]

        test_x=iris_test[:,0:2]
        test_y=iris_test[:,4]
```

```
In [8]: train_x.shape, train_y.shape
```

```
Out[8]: ((120, 2), (120,))
```

```
In [9]: test_x.shape, test_y.shape
```

```
Out[9]: ((30, 2), (30,))
```



□ 数据处理

```
In [10]: x_train = train_x[train_y < 2]
         y_train = train_y[train_y < 2]
```

```
In [11]: x_train.shape, y_train.shape
```

```
Out[11]: ((78, 2), (78,))
```

```
In [12]: x_test = test_x[test_y < 2]
         y_test = test_y[test_y < 2]
```

```
In [13]: x_test.shape, y_test.shape
```

```
Out[13]: ((22, 2), (22,))
```

```
In [14]: num_train=len(x_train)
         num_test=len(x_test)
```

```
In [15]: num_train, num_test
```

```
Out[15]: (78, 22)
```



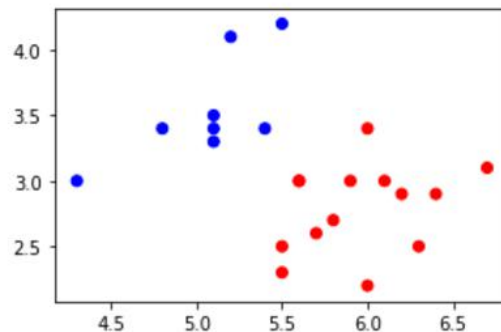
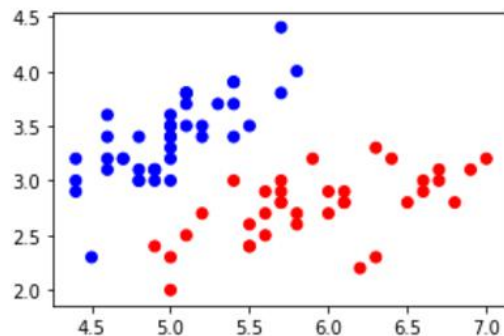
11.4.1 实现多元逻辑回归

```
In [16]: plt.figure(figsize=(10,3))
cm_pt = mpl.colors.ListedColormap(["blue", "red"])

plt.subplot(121)
plt.scatter(x_train[:,0],x_train[:,1],c=y_train,cmap=cm_pt)

plt.subplot(122)
plt.scatter(x_test[:,0],x_test[:,1],c=y_test,cmap=cm_pt)

plt.show()
```



□ 数据处理——中心化

```
In [17]: print(np.mean(x_train, axis=0))  
         print(np.mean(x_test, axis=0))
```

```
[5.42692308 3.1025641 ]  
[5.62727273 3.06363636]
```

```
In [18]: x_train=x_train-np.mean(x_train, axis=0)  
         x_test=x_test-np.mean(x_test, axis=0)
```



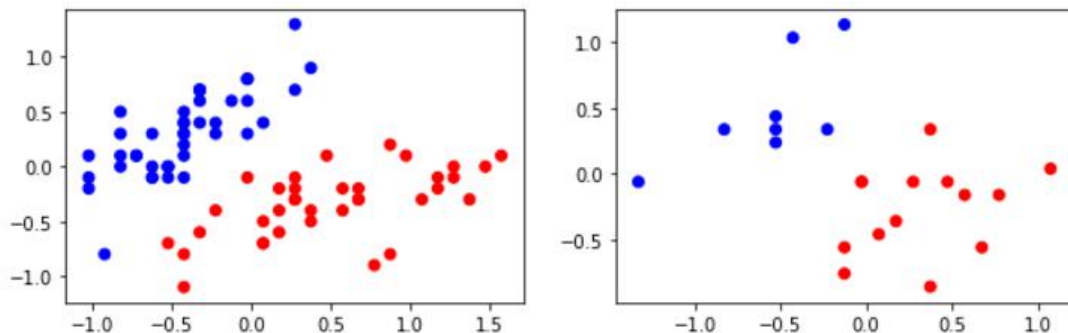
11.4.1 实现多元逻辑回归

```
In [19]: plt.figure(figsize=(10,3))

plt.subplot(121)
plt.scatter(x_train[:,0],x_train[:,1],c=y_train,cmap=cm_pt)

plt.subplot(122)
plt.scatter(x_test[:,0],x_test[:,1],c=y_test,cmap=cm_pt)

plt.show()
```



11.4.1 实现多元逻辑回归

```
In [20]: x0_train = np.ones(num_train).reshape(-1,1)
X_train = tf.cast(tf.concat((x0_train,x_train), axis = 1),dtype=tf.float32)
Y_train = tf.cast(y_train.reshape(-1,1),dtype=tf.float32)
```

```
In [21]: X_train.shape, Y_train.shape
```

```
Out[21]: (TensorShape([78, 3]), TensorShape([78, 1]))
```

```
In [22]: x0_test = np.ones(num_test).reshape(-1,1)
X_test = tf.cast(tf.concat((x0_test,x_test), axis = 1),dtype=tf.float32)
Y_test = tf.cast(y_test.reshape(-1,1),dtype=tf.float32)
```

```
In [23]: X_test.shape, Y_test.shape
```

```
Out[23]: (TensorShape([22, 3]), TensorShape([22, 1]))
```



□ 设置超参数、设置模型参数初识值

```
In [24]: learn_rate=0.2  
         iter=120  
  
         display_step=30
```

```
In [25]: np.random.seed(612)  
         W=tf.Variable(np.random.randn(3,1), dtype=tf.float32)
```



11.4.1 实现多元逻辑回归

```
In [26]: ce_train=[]
         ce_test=[]
         acc_train=[]
         acc_test=[]

         for i in range(0, iter+1):
             with tf.GradientTape() as tape:
                 PRED_train = 1/(1+tf.exp(-tf.matmul(X_train,W)))
                 Loss_train = -tf.reduce_mean(Y_train*tf.math.log(PRED_train)+(1-Y_train)*tf.math.log(1-PRED_train))
                 PRED_test = 1/(1+tf.exp(-tf.matmul(X_test,W)))
                 Loss_test = -tf.reduce_mean(Y_test*tf.math.log(PRED_test)+(1-Y_test)*tf.math.log(1-PRED_test))

                 accuracy_train = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED_train.numpy()<0.5,0,1.), Y_train), tf.float32))
                 accuracy_test = tf.reduce_mean(tf.cast(tf.equal(tf.where(PRED_test.numpy()<0.5,0,1.), Y_test), tf.float32))

                 ce_train.append(Loss_train)
                 ce_test.append(Loss_test)
                 acc_train.append(accuracy_train)
                 acc_test.append(accuracy_test)

                 dL_dW= tape.gradient(Loss_train,W)
                 W.assign_sub(learn_rate*dL_dW)

                 if i % display_step == 0:
                     print("i: %i, TrainAcc:%f, TrainLoss: %f ,TestAcc:%f, TestLoss: %f" % (i, accuracy_train, Loss_train, accuracy_test, Loss_test))
```

i: 0, TrainAcc:0.230769, TrainLoss: 0.994269 ,TestAcc:0.272727, TestLoss: 0.939684
i: 30, TrainAcc:0.961538, TrainLoss: 0.481892 ,TestAcc:0.863636, TestLoss: 0.505456
i: 60, TrainAcc:0.987179, TrainLoss: 0.319128 ,TestAcc:0.863636, TestLoss: 0.362112
i: 90, TrainAcc:0.987179, TrainLoss: 0.246626 ,TestAcc:0.863636, TestLoss: 0.295611
i: 120, TrainAcc:1.000000, TrainLoss: 0.204982 ,TestAcc:0.863636, TestLoss: 0.256212



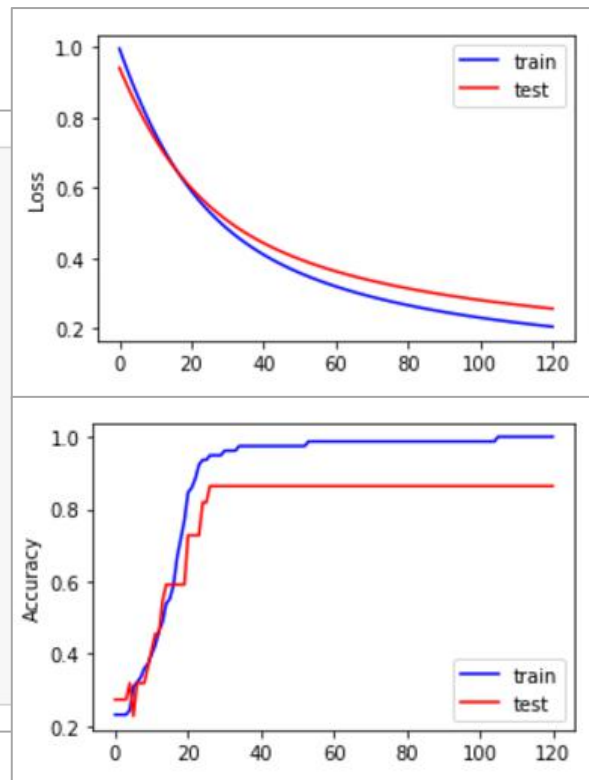
可视化

```
In [27]: plt.figure(figsize=(10,3))

plt.subplot(121)
plt.plot(ce_train,color="blue",label="train")
plt.plot(ce_test,color="red",label="test")
plt.ylabel("Loss")
plt.legend()

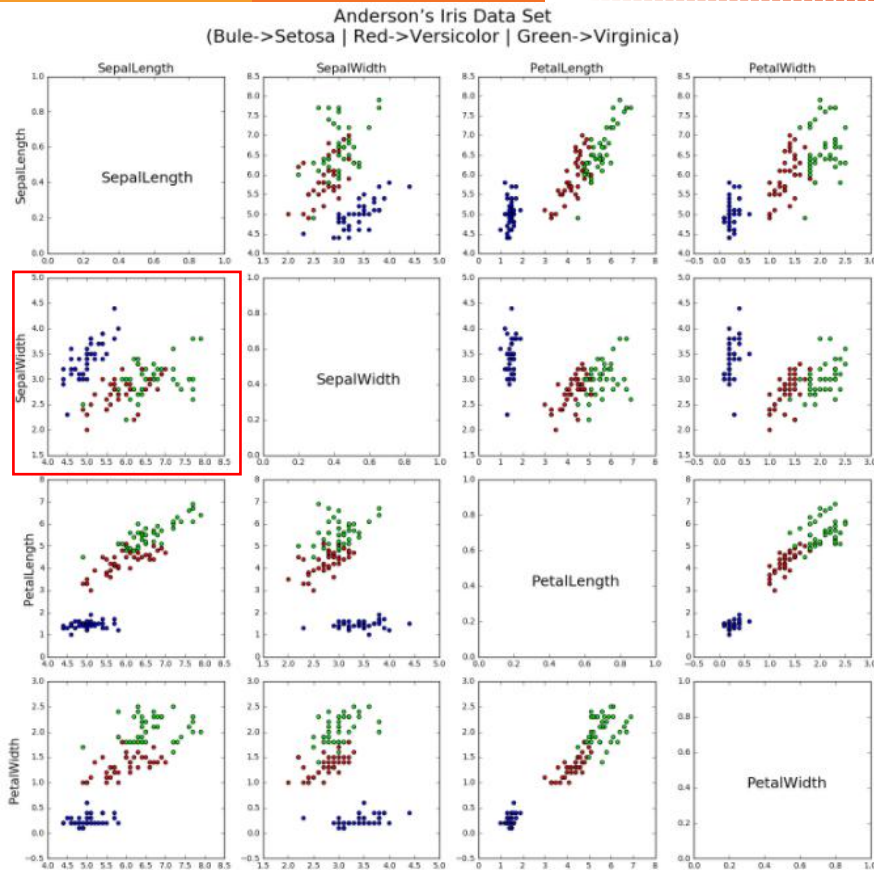
plt.subplot(122)
plt.plot(acc_train,color="blue",label="train")
plt.plot(acc_test,color="red",label="test")
plt.ylabel("Accuracy")

plt.legend()
plt.show()
```



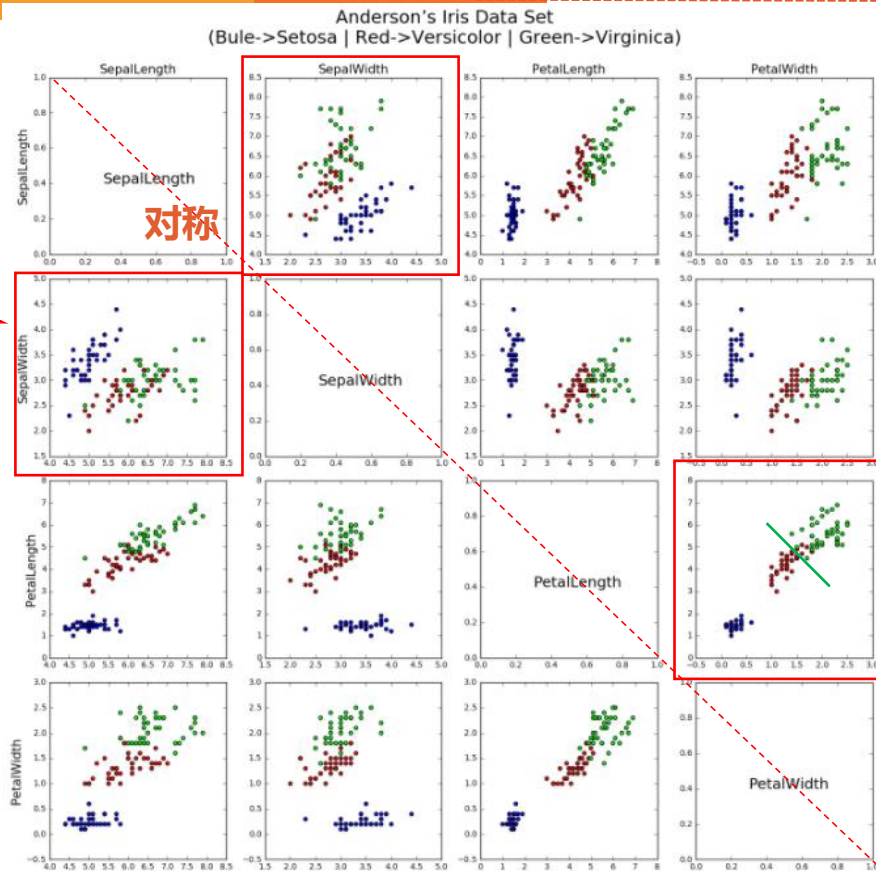
11.4 .1 实现多元逻辑回归

例程中的散点图



11.4.1 实现多元逻辑回归

例程中的散点图

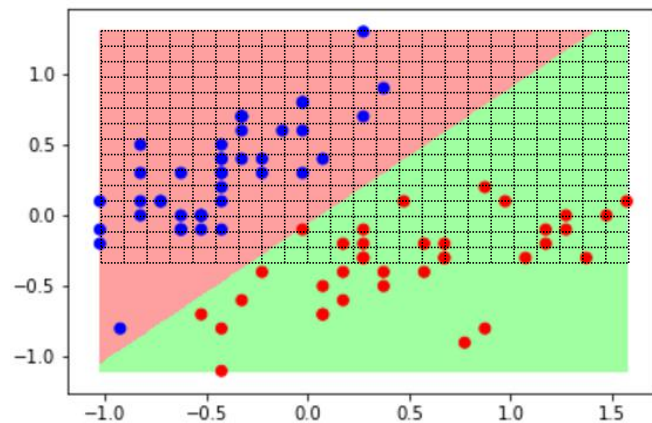
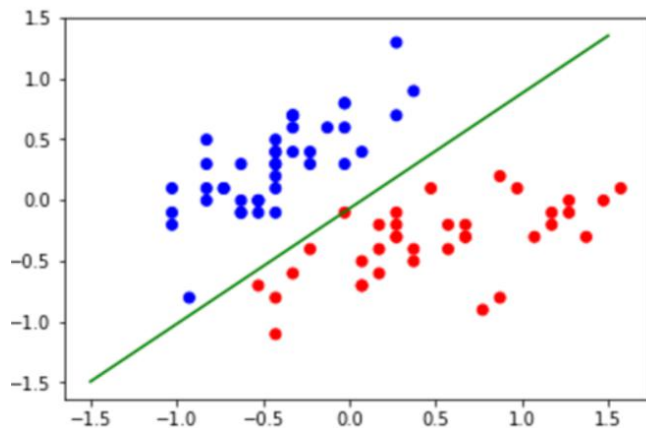




11.4.2 绘制分类图

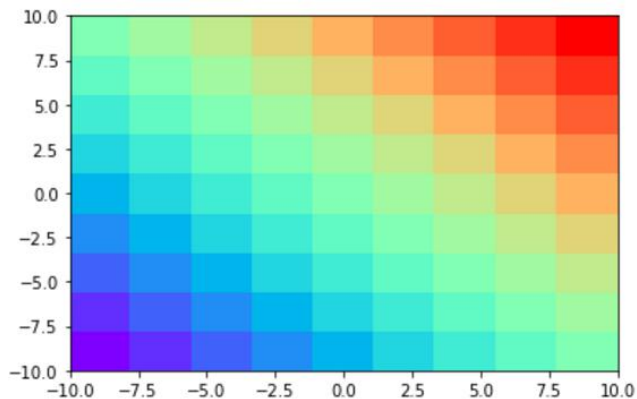
11.4.2 绘制分类图

- 线性分类器
- 决策边界



11.4.2 绘制分类图

生成网格坐标矩阵: `np.meshgrid()`
填充网格: `plt.pcolormesh()`



```
In [1]: import tensorflow as tf  
print("TensorFlow version:", tf.__version__)
```

TensorFlow version: 2.0.0

```
In [2]: import numpy as np  
import matplotlib as mpl  
import matplotlib.pyplot as plt
```

```
In [3]: n = 10
```

```
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)
```

```
X, Y = np.meshgrid(x, y)  
Z = X+Y
```

meshgrid()详见9.6小节

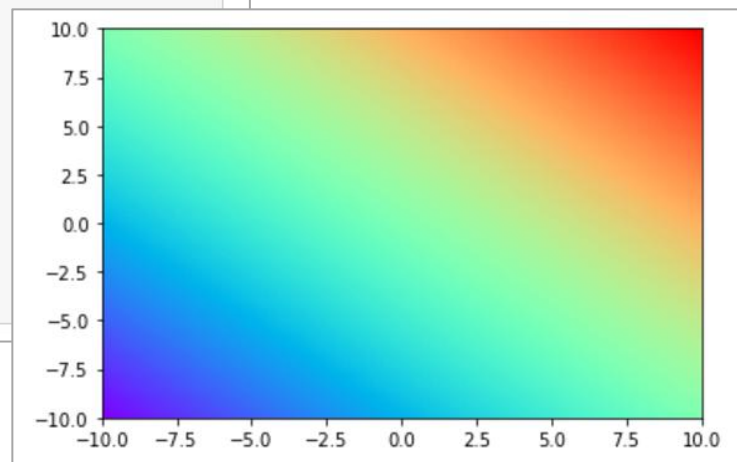
```
plt.pcolormesh(X, Y, Z, cmap="rainbow")
```

```
plt.show()
```



11.4.2 绘制分类图

```
In [4]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X+Y  
  
plt.pcolormesh(X, Y, Z, cmap="rainbow")  
  
plt.show()
```



11.4.2 绘制分类图

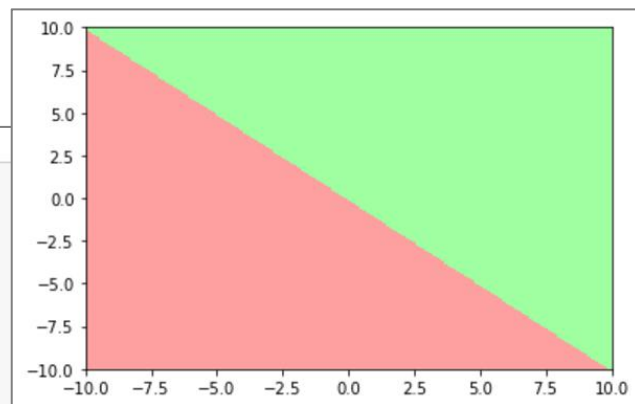
```
In [5]: n = 200

x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)

X, Y = np.meshgrid(x, y)
Z = X+Y

cm_bg = mpl.colors.ListedColormap(["#FFA0A0", "#A0FFA0"])
plt.pcolormesh(X, Y, Z, cmap=cm_bg)

plt.show()
```



11.4.2 绘制分类图

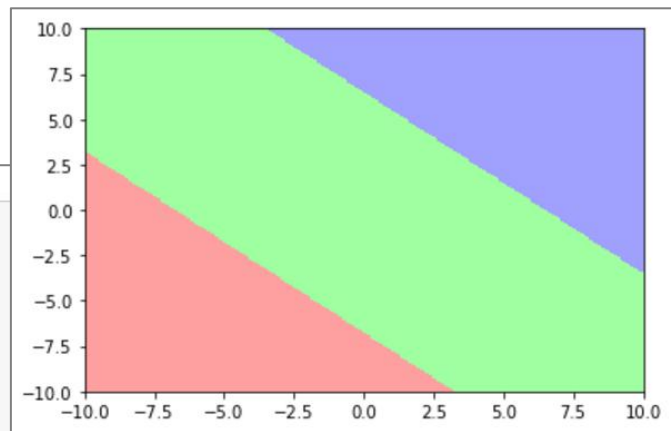
```
In [6]: n = 200

x = np.linspace(-10, 10, n)
y = np.linspace(-10, 10, n)

X, Y = np.meshgrid(x, y)
Z = X+Y

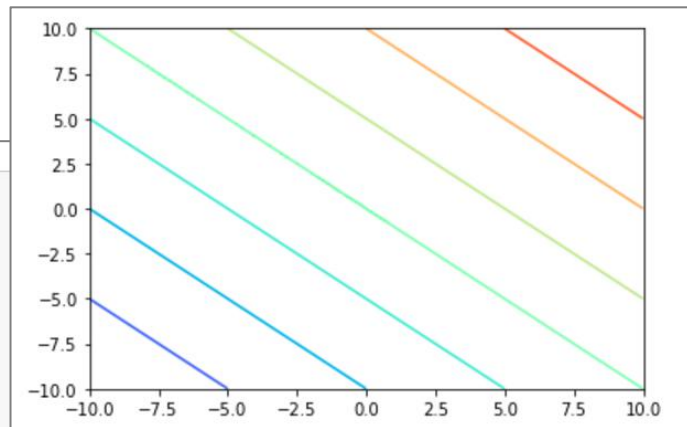
cm_bg = mpl.colors.ListedColormap(["#FFA0A0", "#A0FFA0", "#A0A0FF"])
plt.pcolormesh(X, Y, Z, cmap=cm_bg)

plt.show()
```



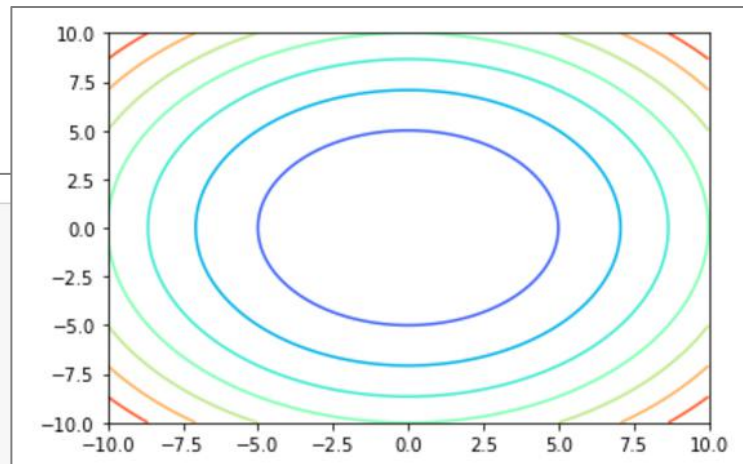
绘制轮廓线: `plt.contour()`

```
In [7]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X+Y  
  
plt.contour(X, Y, Z, cmap="rainbow")  
  
plt.show()
```



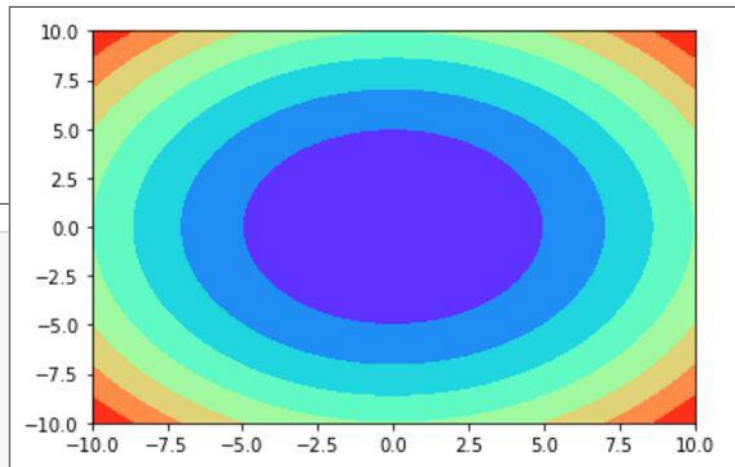
绘制轮廓线: `plt.contour()`

```
In [8]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X**2 + Y**2  
  
plt.contour(X, Y, Z, cmap="rainbow")  
  
plt.show()
```



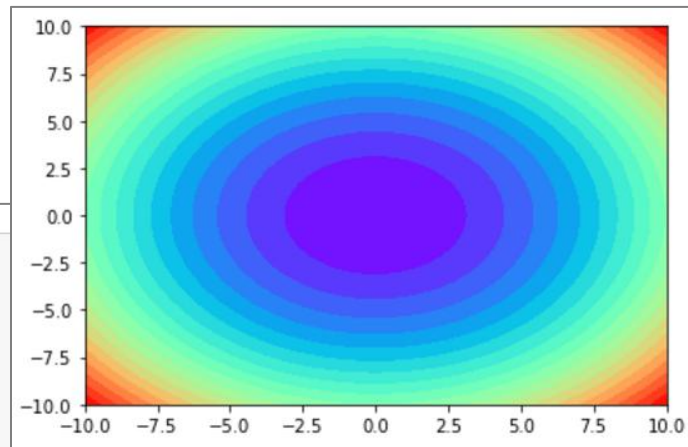
填充分区: `plt.contourf()`

```
In [9]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X**2+Y**2  
  
plt.contourf(X, Y, Z, cmap="rainbow")  
  
plt.show()
```



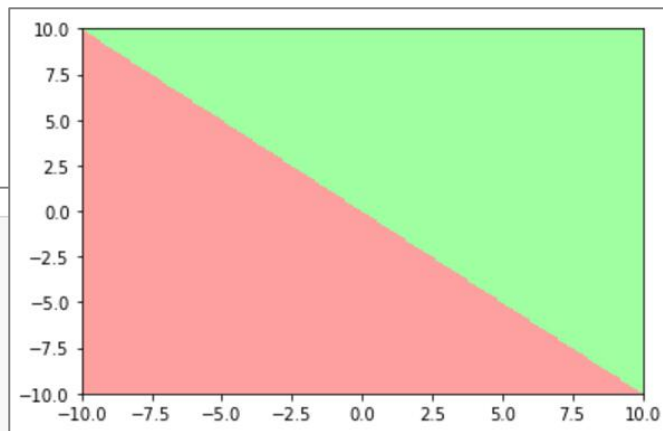
填充分区: `plt.contourf()`

```
In [10]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X**2 + Y**2  
  
plt.contourf(X, Y, Z, 20, cmap="rainbow")  
  
plt.show()
```



生成网格坐标矩阵: `np.meshgrid()`
绘制分类图: `pcolormesh()/plt.contourf()`

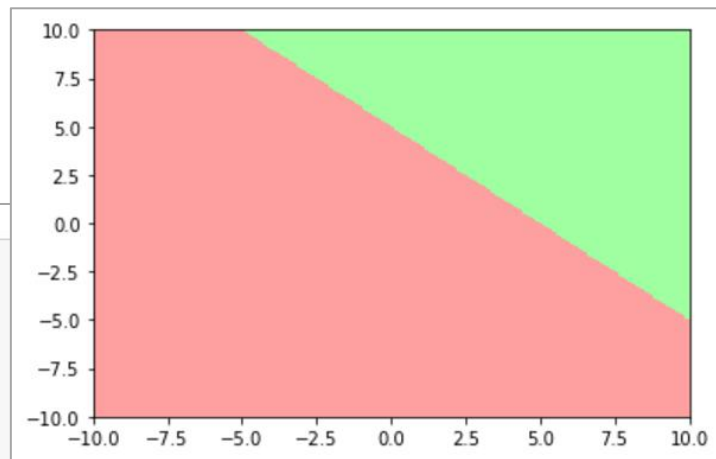
```
In [11]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X+Y  
  
cm_bg = mpl.colors.ListedColormap(["#FFA0A0", "#A0FFA0"])  
Z=tf.where(Z<0, 0, 1)  
plt.pcolormesh(X, Y, Z, cmap=cm_bg)  
  
plt.show()
```



11.4.2 绘制分类图

生成网格坐标矩阵: `np.meshgrid()`
绘制分类图: `pcolormesh()/plt.contourf()`

```
In [12]: n = 200  
  
x = np.linspace(-10, 10, n)  
y = np.linspace(-10, 10, n)  
  
X, Y = np.meshgrid(x, y)  
Z = X+Y  
  
cm_bg = mpl.colors.ListedColormap(["#FFA0A0", "#A0FFA0"])  
Z=tf.where(Z<5, 0, 1)  
plt.pcolormesh(X, Y, Z, cmap=cm_bg)  
  
plt.show()
```



■ 根据鸢尾花分类模型，绘制分类图

```
In [29]: M=300
x1_min, x2_min = x_train.min(axis=0)
x1_max, x2_max = x_train.max(axis=0)
t1 = np.linspace(x1_min, x1_max, M)
t2 = np.linspace(x2_min, x2_max, M)
m1,m2 = np.meshgrid(t1, t2)

In [30]: m0=np.ones(M*M)
X_mesh = tf.cast(np.stack((m0,m1.reshape(-1),m2.reshape(-1)), axis=1),dtype=tf.float32)
Y_mesh =tf.cast(1/(1+tf.exp(-tf.matmul(X_mesh,W))), dtype=tf.float32)
Y_mesh=tf.where(Y_mesh<0.5, 0, 1)

In [31]: n=tf.reshape(Y_mesh, m1.shape)
```

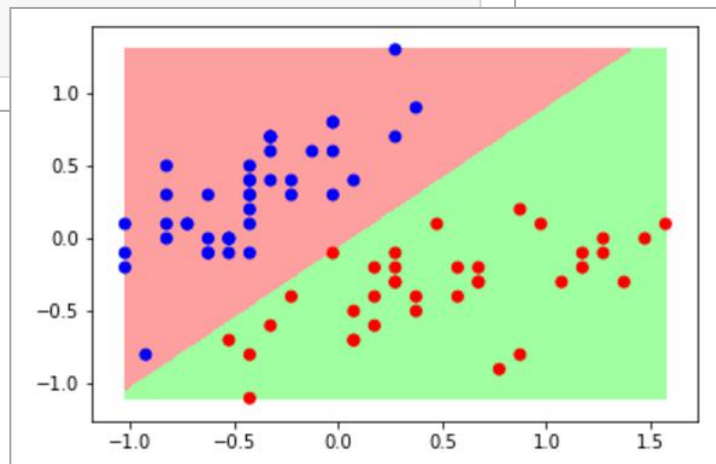


11.4.2 绘制分类图

```
In [32]: cm_pt = mpl.colors.ListedColormap(["blue", "red"])
cm_bg = mpl.colors.ListedColormap(["#FFFA0A0", "#A0FFA0"])

plt.pcolormesh(m1, m2, n, cmap=cm_bg)
plt.scatter(x_train[:,0], x_train[:,1], c=y_train, cmap=cm_pt)

plt.show()
```



11.4.2 绘制分类图

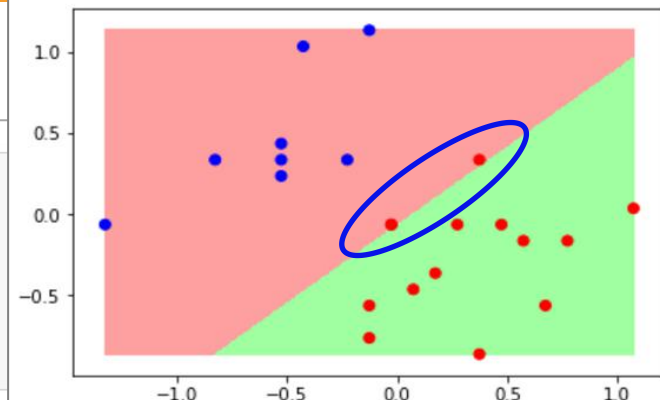
0.863636=19/22

```
In [33]: M=300
x1_min, x2_min = x_test.min(axis=0)
x1_max, x2_max = x_test.max(axis=0)
t1 = np.linspace(x1_min, x1_max, M)
t2 = np.linspace(x2_min, x2_max, M)
m1,m2 = np.meshgrid(t1, t2)
```

```
In [34]: m0=np.ones(M*M)
X_mesh = tf.cast(np.stack((m0,m1.reshape(-1),m2.reshape(-1))), axis=1),dtype=tf.float32)
Y_mesh =tf.cast(1/(1+tf.exp(-tf.matmul(X_mesh,W))), dtype=tf.float32)
Y_mesh=tf.where(Y_mesh<0.5, 0, 1)
```

```
In [35]: n=tf.reshape(Y_mesh,m1.shape)
```

```
In [36]: plt.pcolormesh(m1, m2, n, cmap=cm_bg)
plt.scatter(x_test[:,0],x_test[:,1],c=y_test,cmap=cm_pt)
plt.show()
```



13	5.5	2.5	4	1.3	1
14	5.6	2.8	4.9	2	2
15	5.5	4.2	1.4	0.2	0
16	5.5	2.3	4	1.3	1
17	5.6	3	4.1	1.3	1
18	5.6	3	4.5	1.5	1
19	5.7	2.6	3.5	1	1
20	5.8	2.7	3.9	1.2	1
21	5.7	2.5	5	2	2
22	5.9	3	4.2	1.5	1



11.4.2 绘制分类图

- 分别绘制**训练集**和**测试集**的分类图

