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DYNAMICS OF THE PETER PRINCIPLE*

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In this paper, a realistic Markovian model of hierarchies is considered which reveals that under suitable conditions *The Peter Principle* applies. That is, above a certain critical hierarchical level, performance decreases slowly, but steadily, with increasing level. This can be true even if there is effective screening and promotion is by merit, rather than seniority. Screening procedures which are uniform, realistic and selective can actually decrease *relative* performance after promotion. Criteria for the manifestation of this phenomenon will be presented. Basically, it is more likely to manifest itself in bureaucracies having *low* internal mobility and appears when people who are passed over for promotion improve more with another year's experience than those promoted to new jobs which are unfamiliar and more challenging. Mathematically, the controlling parameter is the ratio of two eigenvalues, each the largest eigenvalue of a 3×3 matrix. These eigenvalues are respectively those of two transition matrices, one describing reclassification of successful candidates after promotion, and the other, the reclassification of unsuccessful candidates.

Introduction

"A man rises until he reaches his level of incompetence."

Satire is an effective means of dramatizing the foibles of the human condition. Yet the very exaggerations of satire somewhat blunts its message. Supposedly the situation is not as bad as described, and in any event, the Peter Principle does not plague competent people or competent hierarchies—if there are such things. Presumably, with effective screening and selective promotion any organization should have better people at its top ranks.

Will they?

The purpose of this note is to consider this question mathematically. By analyzing a Markov model of a bureaucracy, it will be shown that the answer is no, not always, *even if promotion is by merit rather than seniority*. Screening procedures which are uniform, realistic and selective can actually *decrease* relative performance at higher levels in a manner characteristic of the Peter Principle. Criteria for the manifestation of this phenomenon will be presented. It is more likely to manifest itself in bureaucracies having low internal mobility and depends primarily upon the relative efficiency of people who are passed over and given the benefit of another year's experience on the learning curve as compared to those promoted, in their new job.

Markovian models of hierarchical processes have been utilized with success by Seal [4], Young and Almond [6], Gani [2], and White [5]. Our model is somewhat different in that the occupants of each hierarchical level are ranked into three competence classes a, b, and c. Furthermore, we include exit from the hierarchy, whether by mandatory retirement, resignation, or death. Without this important feature, all men would eventually reach the top level—a highly unrealistic conclusion, Bartholomew [1].

Formulation

We shall consider an n-level hierarchy. At each level we suppose that we can make a good—but not perfect—discrimination between three types of individuals, labelled

* Received November 1969; revised February 1970.

TABLE 1

Transition probabilities for promotion ($i \to i+1$), recycling ($i \to i$), or termination ($i \to T$) at each level of a hypothetical hierarchy which maintains a uniform review procedure at all levels.

from \ to	→	a_{i+1}	b_{i+1}	c_{i+1}	a_i	b_i	c_i	T
a_i		.10	.06	.01	.55	.14	—	.14
b_i		.01	.05	.03	.22	.57	.06	.06
c_i		—	—	.02	—	.19	.60	.19

a_i, b_i, and c_i according to performance and hierarchal level i. We shall assume that performance can be gauged only in a probabilistic sense, that is, even if a man is truly an a_i, how will we recognize him? Thus we shall suppose that people are promoted only if they are *thought* to be of prime calibre. Arbitrarily we shall give an a_i individual an efficiency rating of 100%, a b_i—80%, and 60% for any c_i. These figures have been chosen for convenience of illustration—any other monotonic assignment would do just as well. For a more sophisticated analysis we can also make the efficiencies functions of i, the hierarchical level as well. This would be desirable in weighting the importance of top management. For numerical convenience this feature will be omitted from the succeeding analysis. Its effect, however, could easily be incorporated. If so, nonlinear weighting of top management's importance would merely skew the figures that are presented, and if anything, the distortion would dramatize more effectively the unhappy manifestation of the Peter Principle.

Within our hierarchical model, all personnel are reviewed and considered for promotion at periodic intervals, say annually. Promotion and reclassification within levels is characterized by an array of transition probabilities such as the one given in Table 1. This hierarchy promotes 17% of its a_i's, 9% of its b_i's and 2% of its c_i's each year. It would like to be more selective, but the vagaries of human assessment prevent its so doing. None the less, by any criterion, its selectivity, 17:9:2, would be judged to be quite favorable. Notice that after promotion, an individual's category is likely to change. For example, 6/17 of the a_i's that are promoted are unable to do as well in their more demanding job and become b_{i+1}'s. We shall have considerable use for these promotion indices as a separate entity and shall arrange them into a transition matrix

$$\mathsf{P} = \begin{pmatrix} .10 & .01 & 0.0 \\ .06 & .05 & 0.0 \\ .01 & .03 & .02 \end{pmatrix}.$$

It will also be of value to observe the reclassification of personnel that are *not* promoted. Some a_i's, resentful of being passed over for promotion either slacken off and become b_i's (14%), or leave for better prospects (14%). On the other hand, with more job experience some b_i's become a_i's (22%). Fewer b_i's leave than a_i's because they have a tendency to keep plugging away. Finally, the poor performance of a fair number (19%) of c_i's is recognized, and they are dismissed. A few c_i's (2%) are either very lucky or ingratiating and are promoted. A larger percentage (19%) learns enough to improve and become b_i's without being promoted. Recycled personnel will be described by the square matrix

$$\mathsf{R} = \begin{pmatrix} .55 & .22 & 0.0 \\ .14 & .57 & .19 \\ 0.0 & .06 & .60 \end{pmatrix}$$

and the terminations T by the rectangular matrix

$$\mathsf{T} = (.14 \quad .06 \quad .19).$$

It will be useful to generalize the above notions to distinguish between firms with varying degrees of internal mobility. Loosely speaking,[1] we can identify faster growing firms with faster promoting firms. Whatever the rate, let us assume that the promotion ratios are maintained in the proportion 17:9:2 and that the entries of R and T will be adjusted accordingly to keep the totals to unity. That is, if we introduce a mobility parameter α (which we shall sometimes call, somewhat inaccurately, a growth parameter), then

(1a)
$$\mathsf{P}(\alpha) = \alpha \begin{pmatrix} 1. & .1 & 0. \\ .6 & .5 & 0. \\ .1 & .3 & .2 \end{pmatrix}$$

and

(1b)
$$\mathsf{R}(\alpha) = \begin{pmatrix} 4r & 2s & 0. \\ r & 5s & 2t \\ 0. & s/2 & 6t \end{pmatrix}$$

and

(1c)
$$\mathsf{T}(\alpha) = (r \quad s/2 \quad 2t)$$

where

$$r = (1. - 1.7\alpha)/6.$$
$$s = (1. - .9\alpha)/8.$$
$$t = (1. - \alpha/4)/10.$$

In this fashion, rapidity of promotion within a hierarchy is easily described by the parameter α. Doubling it doubles the rate of promotion for any category.

By varying this one parameter α we shall obtain a wide variety of results which could only otherwise be obtained by manipulating 42 independent parameters (we have a 7×7 basic matrix with 7 constraints). Accordingly, while we are keeping the ratio 17:9:2 constant, this will not be an important feature of the subsequent analysis. As will subsequently materialize, the key parameter will be the ratio of two eigenvalues, each the largest eigenvalue of a 3×3 matrix.

While it is necessary to have numerical values for a_i, b_i, and c_i for purposes of matrix manipulation, numerical population statistics are difficult to interpret. Accordingly, in tables and graphs we shall prefer to use the *percentages*

$$A_i = 100.\, a_i/T_i,$$
$$B_i = 100.\, b_i/T_i,$$
$$C_i = 100.\, c_i/T_i,$$

where $T_i = a_i + b_i + c_i$.

[1] Strictly speaking growth is described by non-Markovian processes inasmuch as totals are not conserved. However, we are primarily concerned with ratios and relative distributions. For such internal parameters, the Markov model is more than adequate.

While we are describing a 40 level hierarchy, it might be asked whether 40 levels are unrealistically high. The answer is, no. Having 40 promotion levels does not mean we need have 40 *explicit* hierarchical levels. Often there are subpromotions within grade, *e.g.* a man may be selected for staff college or given an appointment which is generally regarded as prerequisite for promotion without actually being given a title or salary change. Such selections are tantamount to subpromotion with definite psychic rewards. Accordingly the data pertaining to the 40 levels can be regrouped into subgroups in many ways, giving considerable flexibility in modelling the promotion characteristics of a rather wide variety of hierarchies. Alternatively, levels can be collapsed, identifying a number of adjacent levels by summing components.

We have introduced specific entries in **P**, **R**, and **T** for purposes of example, and the parameters, of course, can be generalized to describe any hierarchy. Also, if desired, the transition probabilities can be made to vary from level to level. While easily introduced, such complications make the calculus more complex without revealing any new insights. Furthermore, one can question the practical significance of such varying selectivity. Why should generals be better at selecting colonels, than captains—lieutenants?

With these ideas in mind, promotion within our stylized hierarchy can be described as a finite Markov chain. For example, for an annual review described by a four level hierarchy, we would have the matrix equation:

$$(2)\qquad \begin{pmatrix} a'_1\\ b'_1\\ c'_1\\ a'_2\\ b'_2\\ c'_2\\ a'_3\\ b'_3\\ c'_3\\ a'_4\\ b'_4\\ c'_4\\ T' \end{pmatrix} = \left(\begin{array}{c|c|c|c|c} \mathsf{R}(\alpha) & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}.1\\.7\\.2\end{matrix} \\ \hline \mathsf{P}(\alpha) & \mathsf{R}(\alpha) & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \mathsf{P}(\alpha) & \mathsf{R}(\alpha) & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \mathsf{P}(\alpha) & \begin{matrix}0&0&0\\0&0&0\\0&0&0\end{matrix} & \begin{matrix}0\\0\\0\end{matrix} \\ \hline \mathsf{T}(\alpha) & \mathsf{T}(\alpha) & \mathsf{T}(\alpha) & \begin{matrix}1. & 1. & 1.\end{matrix} & 0 \end{array}\right) \cdot \begin{pmatrix} a_1\\ b_1\\ c_1\\ a_2\\ b_2\\ c_2\\ a_3\\ b_3\\ c_3\\ a_4\\ b_4\\ c_4\\ T \end{pmatrix}$$

In this cycling (easily generalized for any n), a man is retired after he reaches the top level. Furthermore, as many new recruits enter the firm as those who leave. We assume that the calibre of new recruits is such that a_0, b_0 and c_0 are introduced in the ratio 1:7:2. Because of repeated annual review, the statistics of the input population is largely unimportant inasmuch as any initial quality distribution will be rapidly forced into an ability pattern determined by the selectivity of **P**, **R** and **T**.

For numerical purposes we have considered a 40 level hierarchy corresponding to the 40 years' work span of the average individual. For added realism, we have included mandatory retirement after 40 years' service. This is very important. Without mandatory retirement at some age, eventually everybody would reach the top level. While the resulting Markov matrix is of rather large order, 1601^2, there is neither need to display it in the text nor to store it in the computer inasmuch as any input vectors can easily be updated by suitable multiplication by the small matrices **P**, **R** and **T**. Numerical results depicting the composition of personnel within the hierarchical levels

TABLE 2

The Population Characteristics for a Simulated Hierarchy for α = .10.

Level	A_i	B_i	C_i	Efficiency
0	10.000%	70.000%	20.000%	78.000%
1	31.053	51.754	17.193	82.772
2	39.321	42.893	17.785	84.307
3	41.534	42.159	16.307	85.045
4	42.007	42.091	15.902	85.221
5	42.080	42.076	15.844	85.247
6	42.055	42.059	15.886	85.234
7	41.995	42.035	15.970	85.205
8	41.917	42.003	16.080	85.167
9	41.827	41.965	16.208	85.124
10	41.732	41.921	16.347	85.077
11	41.636	41.874	16.490	85.029
12	41.541	41.824	16.635	84.981
13	41.450	41.772	16.778	84.934
14	41.363	41.719	16.918	84.889
15	41.281	41.665	17.054	84.845
16	41.204	41.611	17.185	84.804
17	41.132	41.558	17.310	84.764
18	41.066	41.504	17.430	84.727
19	41.004	41.451	17.545	84.692
20	40.948	41.398	17.654	84.650
21	40.896	41.345	17.759	84.627
22	40.848	41.294	17.858	84.598
23	40.804	41.242	17.953	84.570
24	40.764	41.192	18.044	84.544
25	40.728	41.142	18.130	84.519
26	40.694	41.093	18.213	84.496
27	40.664	41.044	18.291	84.475
28	40.637	40.997	18.366	84.454
29	40.613	40.950	18.438	84.435
30	40.590	40.903	18.506	84.417
31	40.571	40.858	18.572	84.400
32	40.553	40.813	18.634	84.384
33	40.538	40.768	18.694	84.369
34	40.524	40.725	18.751	84.355
35	40.512	40.682	18.806	84.341
36	40.502	40.639	18.859	84.329
37	40.494	40.598	18.909	84.317
38	40.486	40.556	18.957	84.306
39	40.481	40.516	19.003	84.295
40	40.476	40.476	19.048	84.286

are shown in Table 2 for α = .10. These are the steady state figures found after repeated iterations of an arbitrary, initial, hierarchical distribution. The calculations were performed on an IBM 360/91, and take about a half minute to compute the limit distribution for any value of α (some 200 iterations).

From Table 2 it will be seen that the fraction of a_i's rapidly climbs from its initial distribution of 10%, reaching a maximum of 42.080% at level 5; after that, there is a slow but steady *decline*. The b_i's drop steadily from 70% to an asymptotic value of 40.476%. Most surprising, however, is the variation of c_i's. From a rather large initial value of 20%, they are rapidly whittled down until their ranks are depleted to 15.844%

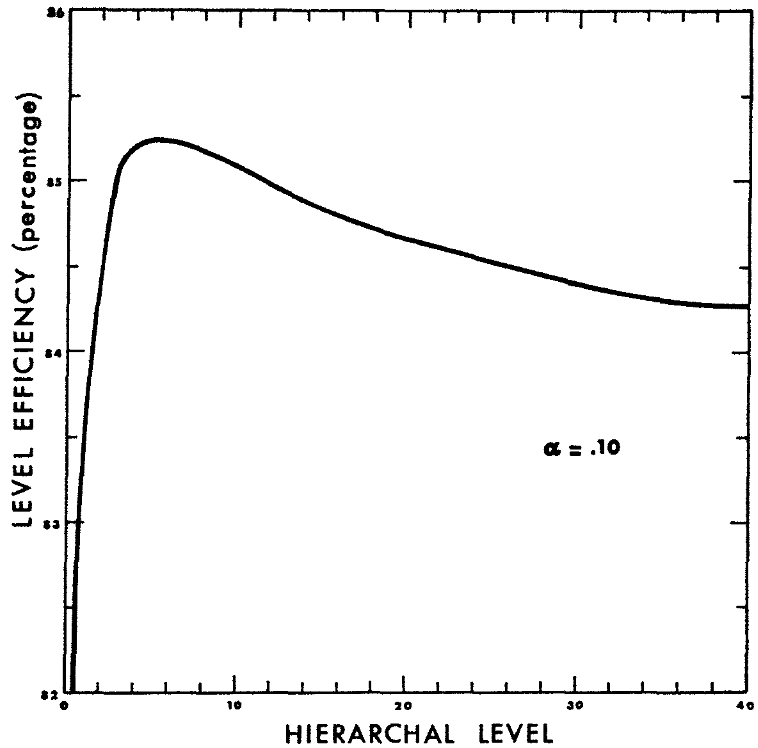

FIGURE 1. The overall efficiency E_i for each level of a 40 level hierarchy. In this illustration α, the growth parameter has the value .10.

of the total at level 5. After reaching this minimum, their proportion *begins to climb*, approaching a limiting value of 19.048%. In other words, *the same mechanism that is effective in weeding out incompetents at lower levels, creates them at higher levels.* The overall effect on the firm is illustrated by Figure 1. This curve, labeled E_i for efficiency, is a weighted average, giving the efficiency at each level. Each a_i is weighted at 100%, b_i—80% and c_i—60%. As can be seen, the personnel within the hierarchy grow rapidly more efficient until level five. After that, a slow but steady deterioration sets in. Is this a mathematical fiction or a fact of life?—theoretical confirmation of the heuristically observed Peter Principle. What is going on?

Discussion

The phenomena just encountered is not fictitious and has a simple explanation. As we have emphasized by our notation, the annual review is really a combination of *two* filtering processes, promotion and recycling. Let us consider each separately. A man promoted is unlikely to perform as well at his new level as his old level. He has new tasks and responsibility. Much of his past experience is of little help. On the other hand, a man passed over for promotion will have the benefits of his experience intact as well as another year's advantage on the learning curve. These considerations are implicit in the parameters selected for the transition coefficients. In some sense, the recycling matrix R is doing a better job of filtering, or enhancing efficiency, than the promotion matrix P. How can this be made precise and exactly what is its significance?

Consider first those people who manage to reach the top, level 40. With mandatory retirement after 40 years service, this population is characterized by never having been passed over. In other words, they have never felt the effects of the R filter but rather have been processed 40 times through the P filter, i.e. P^{40}. Let v be the eigenvector corresponding to the largest eigenvalue of P.

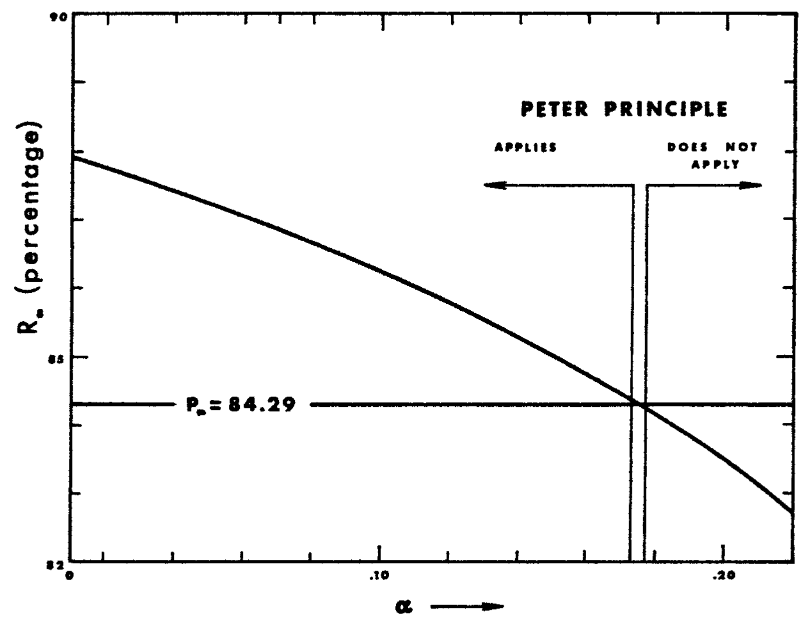

FIGURE 2. The variation of $R_\infty(\alpha)$. For $\alpha < .175$, $R_\infty > P_\infty$ and the Peter Principle holds

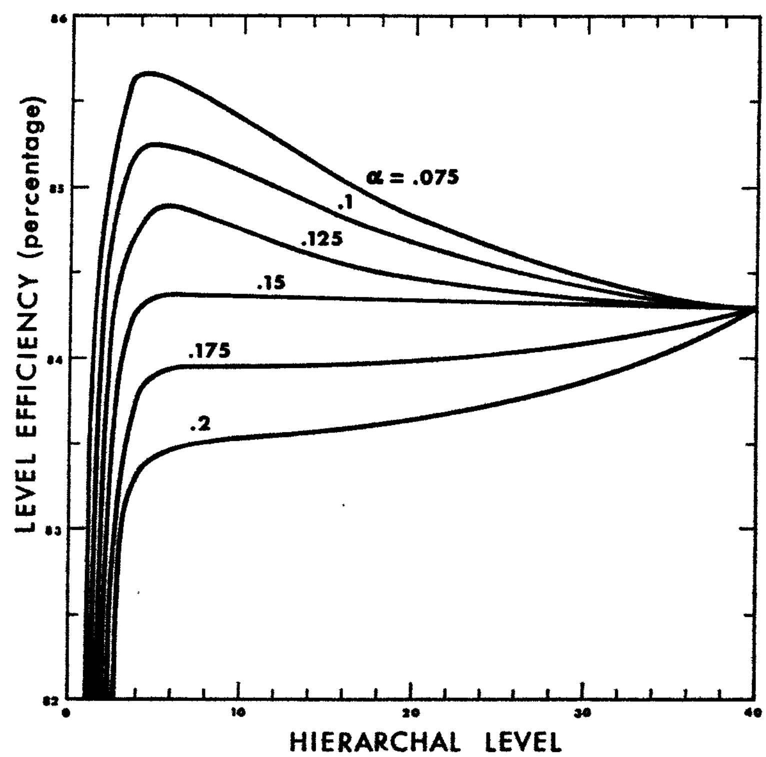

FIGURE 3. Hierarchal efficiencies for firms with varying rates of internal mobility

$$\mathbf{v} = \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix}.$$

Then the population at level 40 will have an efficiency P_∞ comparable to

$$P_\infty = \frac{100. (v_A + .8v_B + .6v_C)}{(v_A + v_B + v_C)}.$$

TABLE 3

The Population Characteristics for a Simulated Hierarchy for α = .15.

Level	A_i	B_i	C_i	Efficiency
0	10.000%	70.000%	20.000%	78.000%
1	27.916	53.424	18.659	81.851
2	36.224	43.448	20.328	83.179
3	38.885	42.510	18.605	84.056
4	39.584	42.424	17.992	84.319
5	39.752	42.413	17.835	84.383
6	39.787	42.403	17.810	84.395
7	39.789	42.386	17.825	84.393
8	39.781	42.360	17.859	84.384
9	39.769	42.324	17.907	84.372
10	39.757	42.279	17.964	84.359
11	39.747	42.226	18.027	84.344
12	39.739	42.168	18.093	84.329
13	39.734	42.106	18.160	84.315
14	39.732	42.041	18.226	84.301
15	39.735	41.975	18.291	84.289
16	39.740	41.907	18.352	84.278
17	39.750	41.840	18.411	84.268
18	39.762	41.772	18.466	84.259
19	39.778	41.704	18.518	84.252
20	39.796	41.637	18.567	84.246
21	39.817	41.570	18.613	84.241
22	39.840	41.505	18.655	84.237
23	39.866	41.440	18.695	84.234
24	39.893	41.376	18.731	84.232
25	39.922	41.313	18.765	84.231
26	39.953	41.250	18.797	84.231
27	39.985	41.189	18.826	84.232
28	40.018	41.129	18.853	84.233
29	40.052	41.070	18.878	84.235
30	40.088	41.011	18.901	84.237
31	40.124	40.954	18.922	84.240
32	40.161	40.897	18.942	84.244
33	40.199	40.841	18.960	84.248
34	40.237	40.787	18.976	84.252
35	40.276	40.733	18.991	84.257
36	40.316	40.680	19.005	84.262
37	40.355	40.628	19.017	84.268
38	40.395	40.576	19.028	84.273
39	40.436	40.526	19.038	84.279
40	40.476	40.476	19.048	84.286

We shall call P_∞ the asymptotic efficiency corresponding to the promotion matrix. On the other hand, the lower levels are populated by people who have often been passed over for promotion. They have been processed much less by P than by repeated applications of R. In similar fashion we can define R_∞ as the asymptotic efficiency of the recycling matrix. Three cases can now arise according as $R_\infty > P_\infty$, $R_\infty = P_\infty$ and $R_\infty < P_\infty$.

If $R_\infty > P_\infty$ then we can expect the Peter Principle to hold. In such instances, low or middle management is more effectively filtered than top management. Accordingly, we can expect a fast rise to some maximum in efficiency and then a subsequent decline.

TABLE 4

The Population Characteristics for a Simulated Hierarchy for α = .2.

Level	A_i	B_i	C_i	Efficiency
0	10.000%	70.000%	20.000%	78.000%
1	25.222	54.839	19.939	81.056
2	33.334	43.888	22.778	82.111
3	36.318	42.751	20.931	83.077
4	37.228	42.647	20.125	83.421
5	37.484	42.640	19.876	83.522
6	37.556	42.638	19.806	83.550
7	37.582	42.630	19.788	83.559
8	37.602	42.615	19.783	83.564
9	37.627	42.591	19.782	83.569
10	37.663	42.556	19.781	83.576
11	37.710	42.511	19.778	83.586
12	37.769	42.457	19.774	83.599
13	37.837	42.396	19.767	83.614
14	37.913	42.329	19.758	83.631
15	37.996	42.258	19.746	83.650
16	38.084	42.184	19.731	83.671
17	38.176	42.109	19.715	83.692
18	38.272	42.032	19.696	83.715
19	38.370	41.954	19.676	83.739
20	38.470	41.876	19.653	83.763
21	38.571	41.799	19.630	83.788
22	38.674	41.722	19.605	83.814
23	38.777	41.645	19.578	83.840
24	38.880	41.569	19.551	83.866
25	38.983	41.494	19.523	83.892
26	39.087	41.420	19.494	83.919
27	39.190	41.346	19.464	83.945
28	39.293	41.274	19.433	83.972
29	39.395	41.202	19.402	83.999
30	39.498	41.131	19.371	84.025
31	39.599	41.062	19.339	84.052
32	39.700	40.993	19.308	84.078
33	39.800	40.925	19.275	84.105
34	39.899	40.858	19.243	84.131
35	39.998	40.792	19.210	84.157
36	40.095	40.727	19.178	84.183
37	40.192	40.663	19.145	84.209
38	40.288	40.600	19.113	84.235
39	40.382	40.538	19.080	84.260
40	40.476	40.476	19.048	84.286

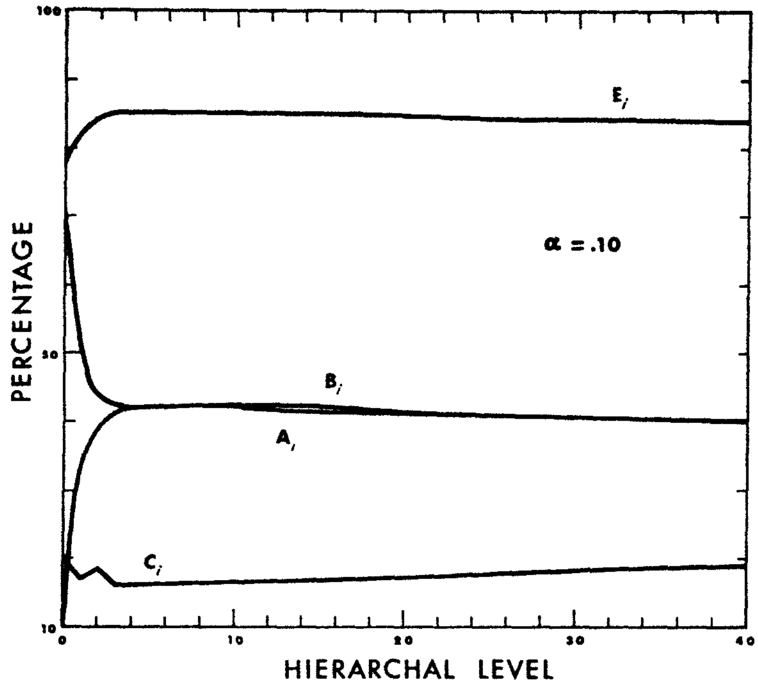

FIGURE 4. The variation of the population characteristics A_i, B_i, C_i, and E_i as a function of hierarchal level for $\alpha = .1$.

If $R_\infty = P_\infty$ or $R_\infty < P_\infty$ then the selectivity of **R** is the same as, or more than, **P** insofar as efficiency is concerned. In such an event we can expect a continuing rise in efficiency, approaching an asymptotic value without evidence of a relative maximum. For such hypothetical firms the Peter Principle would not apply. But it is hard to visualize a screening mechanism that could achieve this. After all, promotion is a much more ambiguous undertaking than recycling, and realistic transition probabilities would have to allow for this.

These ideas can be illustrated by considering the variations of $R_\infty(\alpha)$, a nontrivial function of α. Its variation is indicated in Figure 2.

For small promotion rates, $\alpha < .175$, and $R_\infty = P$ so that the Peter Principle holds. This is understandable because with slow promotion, people tend to master their tasks, become quite proficient before moving on to a new position.

At about $\alpha = .175$, $R_\infty = P_\infty$ and the Peter Principle no longer holds. In Figure 3 we compare the hierarchical efficiency of firms operating with promotion parameters $\alpha = .075, .100, .125, .150, .175$, and $.200$. As anticipated, the Peter Principle is quite evident for $\alpha = .10$, which is substantially less than the critical parameter. For $\alpha = .15$, it is much less apparent, and by $\alpha = .175$ the Peter Principle is no longer evident.

It is important however to note that at least in our model, the Peter Principle is a *relative* phenomenon, not an absolute one. That is, while lower values of α tend to imply decreasing competence at higher levels within a firm, it should be noted—and emphasized—that the *absolute* competence of such firms is uniformly higher than those having higher values of α. Thus firms which promote rapidly will *seem internally* to have increasing proficiency at higher levels. On the other hand, slowly growing firms

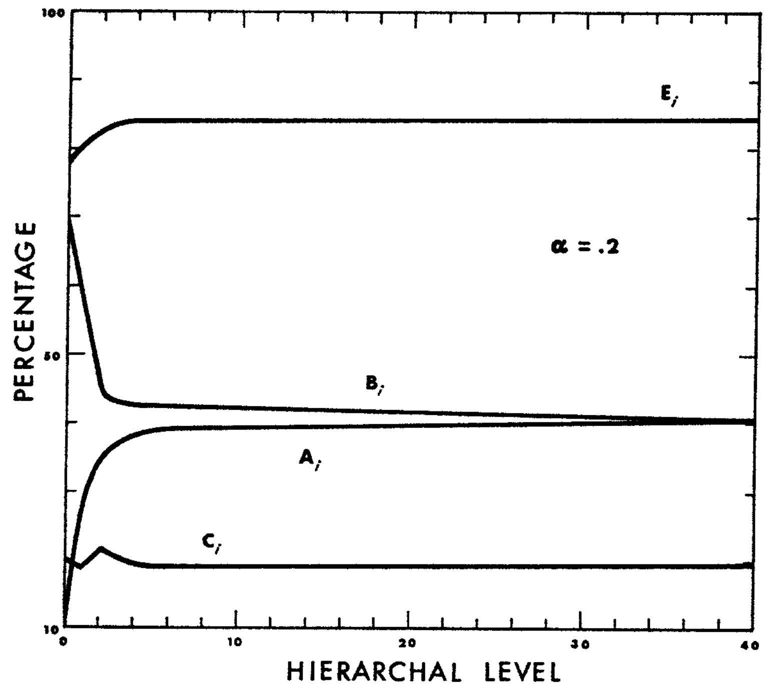

FIGURE 5. The variation of the population characteristics A_i , B_i , C_i , and E_i as a function of hierarchal level for $\alpha = .2$.

which are actually doing quite well in absolute terms might *seem internally* to present an anomalous situation.

For reference, Tables 3 and 4 give the full range of hierarchal characteristics for $\alpha = .15, .20$. Graphs illustrating the variation for $\alpha = .1$ and $.2$ are displayed in Figures 4 and 5.

Intermediate Hiring

In our model we have not explicitly considered entry into the hierarchy except at level 1. This is, of course, somewhat unrealistic. However, *a posteriori*, we can obtain a fair inkling into the effects of intermediate hiring. Whatever the level of entry, personnel are subjected to repeated applications of the **R**, **P**, and **T** filters. To get to the top from level 1 a man would be subjected to $\mathbf{P}^{39}$. However, if he entered the firm at say, level six, he would reach the top after being processed by $\mathbf{P}^{33}$. In other words, intermediate entry at level j merely delays the appearance of the stationary state, but come it must. As can be seen by the rapid rise of the figures from the entry level at 1: the stationary state is approached rather rapidly. Accordingly intermediate hiring would roughly perturb our results as follows:

If the intermediate personnel have competence statistics with efficiency higher than that of the stationary state, then they would exert upward "noise" upon the displayed curves but would leave the asymptotic values unchanged. On the other hand, if their performance abilities were less than the average of the entry level, they would skew the figures by contributing "downward noise." In any event, recovery would be rapid and the asymptotic values, unchanged.

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