Q1 Solve the following differential equations

$$\frac{du_1}{dt} = tu_1,$$
$$\frac{d^2u_2}{dt^2} = u_2(\frac{du_1}{dt} - au_2 + b)^2 - cu_1$$

$a = 1/4, b = 4/5$ and $c = 8/3$. The initial conditions are given as $u_1(0) = 1, u_2(0) = 3.3, \frac{du_2}{dt}(0) = 2$. The time intervals are defined as $[0, 1]$. Plot each component of the solution with respect to time using subplot.

Q2 Define the following polynomial

$$l_i(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_{i-1})(x - x_{i+1})\cdots(x - x_n)}{(x_i - x_0)(x_i - x_1)\cdots(x_i - x_{i-1})(x_i - x_{i+1})\cdots(x_i - x_n)}$$
$$= \prod_{i=0, j\neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

where x_i is the collection of all the x-coordinates of the data points (refer to the table for x_i, i.e., $x_0 = 1, x_1 = 3, x_2 = 6, ...$) and $l_i(x)$ is polynomial of degree n and satisfy

$$l_i(x_j) = \begin{cases} 0 & j \neq i \\ 1 & j = i. \end{cases}$$

We can define the following polynomial interpolation using the data points (x_i, y_i)

$$L_n(x) = \sum_{i=0}^{n} y_i l_i(x)$$

where y_i is the the collection of the data points as given in the following table, see y_0.

Use the following data points and based on the above definition for $L_n(x)$ to write a code for the polynomial approximation $L_n(x)$ and use it to approximate y for given $x = 1, 2, 3, 4, \cdots 16$.

x_i	1	3	6	9	11	15
y_i	1	9	35	79	120	210

Plot the data points (x_0, y_0) and the approximated values (x, y) in one graph, where the data points (x_0, y_0) are denoted as dots.