

Q1 Solve $f(x) = x^7 - \cos(x) = 0$ using the **inverse quadratic interpolation method** (IQIM) and output the solution "x". Given three initial guesses x_1, x_2, x_3 , IQIM calculates x_4, x_5, \dots , by

$$x_{j+1} = x_j - c_1 y_j + c_2 y_j^2, \quad j = 3, 4, \dots$$

where $y_j = f(x_j)$, c_1 and c_2 satisfy

$$c_1 + (y_{j-1} - y_j)c_2 = \frac{x_{j-1} - x_j}{y_{j-1} - y_j}$$

$$c_1 + (y_{j-2} - y_j)c_2 = \frac{x_{j-2} - x_j}{y_{j-2} - y_j}$$

The method is derived as follows. We find a quadratic polynomial of y , say $x = P(y)$, such that $x_j = P(y_j)$, $x_{j-1} = P(y_{j-1})$ and $x_{j-2} = P(y_{j-2})$, then let $x_{j+1} = P(0)$. We let the polynomial be given as

$$P(y) = x_j + c_1(y - y_j) + c_2(y - y_j)^2,$$

then $x_j = P(y_j)$ is already satisfied. The conditions $x_{j-1} = P(y_{j-1})$ and $x_{j-2} = P(y_{j-2})$ give the two equations for c_1 and c_2 . Like the secant method, IQIM does not require the derivative of f . But it requires three initial guesses. It can be proved that the method is faster than secant method.