

Q1 : For  $0 \leq a \leq 5$ , calculate the eigenvalues of

$$A = \begin{bmatrix} 0 & 1 & a & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ -a & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -a & 2 & 1 \end{bmatrix}.$$

Plot the real and imaginary parts of the eigenvalues as functions of  $a$ .

hints: you can first discretize  $a$ , then store the eigenvalues of  $A$ . Please use subplot to plot the real parts and imaginary parts. The  $x$ -coordinate is the discretized points of  $a$ .

Q2 We define the following function  $u$

$$u = \frac{\cos(\kappa r)}{\kappa} - \frac{\cos(\kappa) + \sin(\kappa)i}{\kappa(J_0(\kappa r) + J_1(\kappa)i)} J_0(\kappa r)$$

where  $J_\nu(z)$  is the Bessel function of the first kind (in matlab you can use `besselj(nu,z)`, and more details can be found by typing `doc besselj` in the command window). Here  $i$  is the imaginary unit.

Let  $\kappa = 10$  and  $0 \leq r \leq 2\pi$ , plot the real part and imaginary part of  $u$  in one figure (hints:  $u$  is a function of  $r$ , so you can use  $r$  as the  $x$ -coordinate).