

Q1 : For $j = 1, 3, 5, \dots, 197$ and $n = 1, 2, \dots, 99$, we define $a_n = j/(1+j)$, $b_n = \sqrt{j}$ and the 100×100 matrix

$$A = \begin{bmatrix} 0 & 0 & a_{99} & & & & b_{99} & 0 \\ 0 & 0 & 0 & a_{98} & & & b_{98} & \vdots \\ a_{99} & 0 & \ddots & \ddots & \ddots & & \vdots & \vdots \\ & a_{98} & \ddots & \ddots & \ddots & a_4 & b_4 & 0 \\ & & \ddots & 0 & \ddots & 0 & b_3 & 0 \\ & & & a_4 & 0 & 0 & 0 & 0 \\ b_{99} & b_{98} & \dots & b_4 & b_3 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find the three smallest eigenvalues of A , and plot the eigenvectors corresponding to the 2nd and 3rd smallest eigenvalues.

ps: $a_1 = 1/(1+1), a_2 = 3/(1+3), \dots$ and $b_1 = \sqrt{1}, b_2 = \sqrt{3}, \dots$