Q1 *For $1 \le a \le 4$, find a solution of*

$$f(x) = \tan(x) + a\sin(x)/x = 0$$

using Newton's method. Plot the solution as a function of a.

Q2 Given n points (x_j, y_j), $1 \le j \le n$, we can try to fit them by a quadratic polynomial $P(x) = c_1 + c_2 x + c_3 x^2$ using least squares. That means, we solve the minimization problem $\min \sum_{j=1}^{n} [c_1 + c_2 x_j + c_3 x_j^2 - y_j]^2$. This leads to $\min_{\mathbf{c}} ||A\mathbf{c} - \mathbf{y}||$, where

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}.$$

Now for $n = 10$, $x_j = 1 + (j-1)/(n-1)$ and $y_j = e^{x_j}$, find c_1, c_2 and c_3.