Pattern Recognition Course Assignment 2: Image Segmentation

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1 Introduction

1.1 What is Image Segmentation ?

Image segmentation is the process of partitioning a digital image into multiple segments (sets of pixels, also known as super-pixels). The goal of segmentation is to simplify and/or change the representation of an image into something that is more meaningful and easier to analyze.
Image segmentation is typically used to locate objects and boundaries (lines, curves, etc.) in images. More precisely, image segmentation is the process of assigning a label to every pixel in an image such that pixels with the same label share certain characteristics. The result of image segmentation is a set of segments that collectively cover the entire image, or a set of contours extracted from the image.Each of the pixels in a region are similar with respect to some characteristic or computed property, such as color, intensity, or texture. Adjacent regions are significantly different with respect to the same characteristic(s).

1.2 Applications of Image Segmentation.

Some of the practical applications of image segmentation are:
1. Content-based image retrieval.
2. Machine vision.
3. Medical imaging,including volume rendered images from computed tomography and magnetic resonance imaging.
4. Object detection.(Pedestrian detection,Face detection,Brake light detection, Locate objects in satellite images)
5. Recognition Tasks. (Face recognition, Fingerprint recognition, Iris recognition)
6. Traffic control systems.
7. Video surveillance.
8. Video Object Co-segmentation and action localization.
9. Community detection in social network analytic of social network graphs.

2 Segmentation using K-means algorithms

Several general-purpose algorithms and techniques have been developed for image segmentation. To be useful, these techniques must typically be combined with a domain's specific knowledge in order to effectively solve the domain's segmentation problems.
One of those algorithms is the K-means algorithm is an iterative technique that is used to partition an image into K clusters.
K-means clustering is a method of vector quantization, originally from signal processing, that is popular for cluster analysis in data mining. k-means clustering aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean, serving as a prototype of the cluster. The problem is computationally difficult (NP-hard); however, efficient heuristic algorithms converge quickly to a local optimum.
The term "k-means" was first used by James MacQueen in 1967,[1] though the idea goes back to Hugo Steinhaus in 1957.[2]

2.1 Algorithm

K-means algorithm steps and procedure is provided in Figure 1 which is taken from [3] from Chapter 13.

ALGORITHM 13.1. K-means Algorithm

K-MEANS $(\mathbf{D}, k, \epsilon)$:

$t = 0$
Randomly initialize k centroids: $\boldsymbol{\mu}_1^t, \boldsymbol{\mu}_2^t, \ldots, \boldsymbol{\mu}_k^t \in \mathbb{R}^d$
repeat
  $t \leftarrow t + 1$
  $C_j \leftarrow \emptyset$ for all $j = 1, \cdots, k$
  `// Cluster Assignment Step`
  foreach $\mathbf{x}_j \in \mathbf{D}$ **do**
    $j^* \leftarrow \arg\min_i \left\{ \left\| \mathbf{x}_j - \boldsymbol{\mu}_i^{t-1} \right\|^2 \right\}$ `// Assign` $\mathbf{x}_j$ `to closest centroid`
    $C_{j^*} \leftarrow C_{j^*} \cup \{\mathbf{x}_j\}$
  `// Centroid Update Step`
  foreach $i = 1$ *to* k **do**
    $\boldsymbol{\mu}_i^t \leftarrow \frac{1}{|C_i|} \sum_{\mathbf{x}_j \in C_i} \mathbf{x}_j$
until $\sum_{i=1}^{k} \left\| \boldsymbol{\mu}_i^t - \boldsymbol{\mu}_i^{t-1} \right\|^2 \leq \epsilon$

Figure 1: K-means Algorithm

The steps is as follows, First we Pick K cluster centers, either randomly or based on some heuristic method.Secondly, we assign each pixel in the image to

the cluster that minimizes the distance between the pixel and the cluster center.Then, we re-compute the cluster centers by averaging all of the pixels in the cluster.
Finally, we repeat steps 2 and 3 until convergence is attained (i.e. no pixels change clusters) In this case, distance is the squared or absolute difference between a pixel and a cluster center. The difference is typically based on pixel color, intensity, texture, and location, or a weighted combination of these factors. K can be selected manually, randomly, or by a heuristic. This algorithm is guaranteed to converge, but it may not return the optimal solution. The quality of the solution depends on the initial set of clusters and the value of K. And convergence is either when the centroids have stabilized, meaning that there is no change in their values because the clustering has been successful or the defined number of iterations has been achieved.

2.2 Implementation and Results.

We implemented the above algorithm in Figure 1 using Python and Numpy arrays. The code can be found in our github repo attached with this report in file "clustering.py". The results of our k-means implementation on a selected image is displayed below in figures 3 and 4 under each k value from a range provided (3,5,7,9).

Figure 2: Original Photo

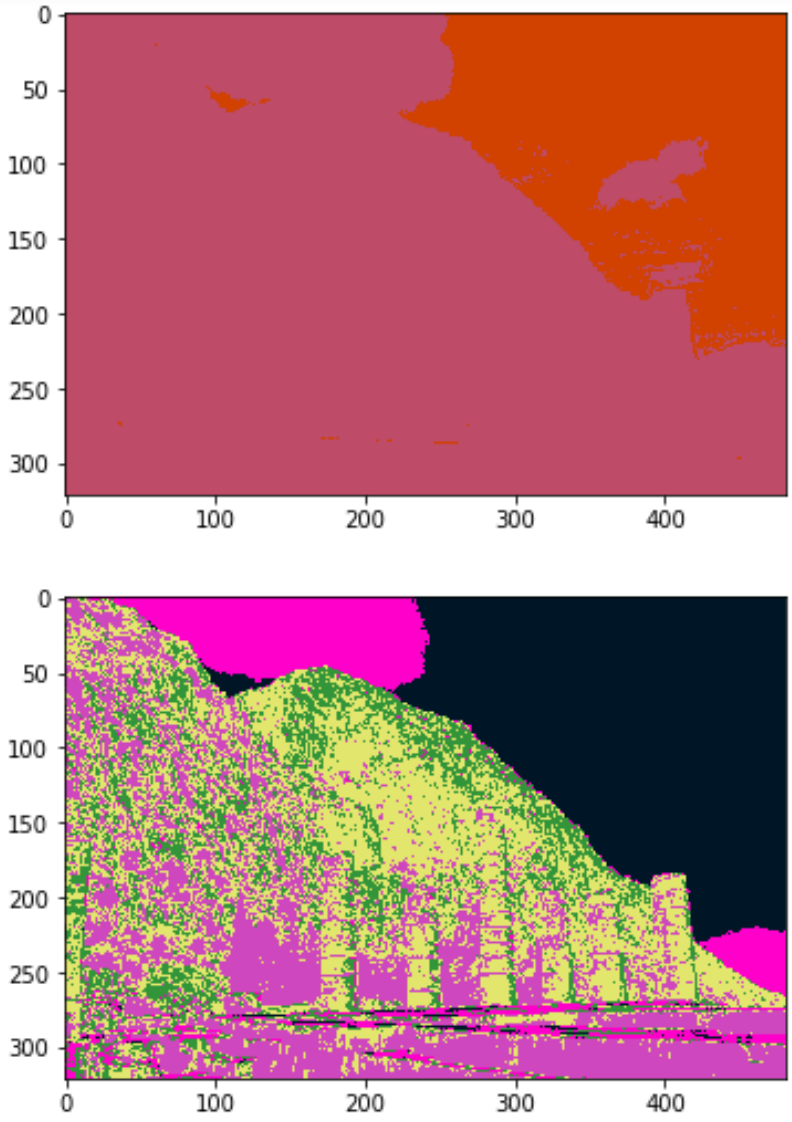


Figure 3: Result 1

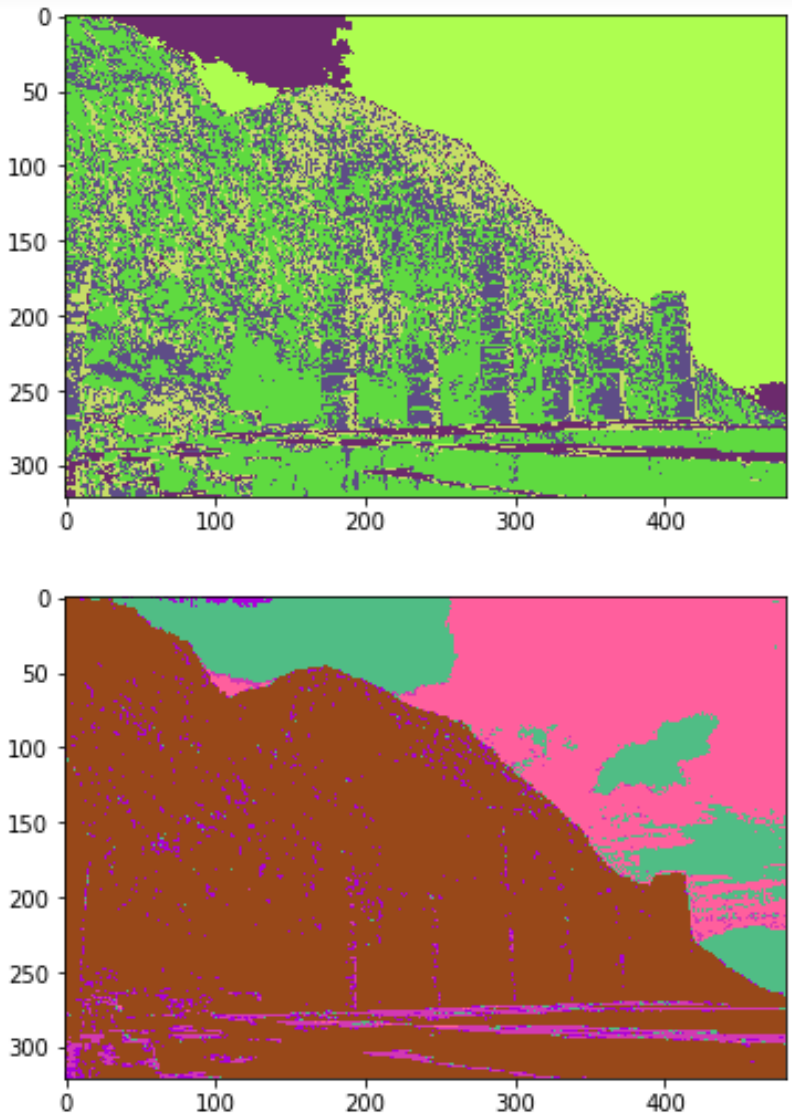


Figure 4: Result 2

The results are imperfect and this is due to the random initialization of the centroids at the start of the algorithm. We then experimented the results using Sklearn's k-means algorithm to obtain better results which are displayed below in figures 5 and 6 under each k value from a range provided (3,5,7,9).

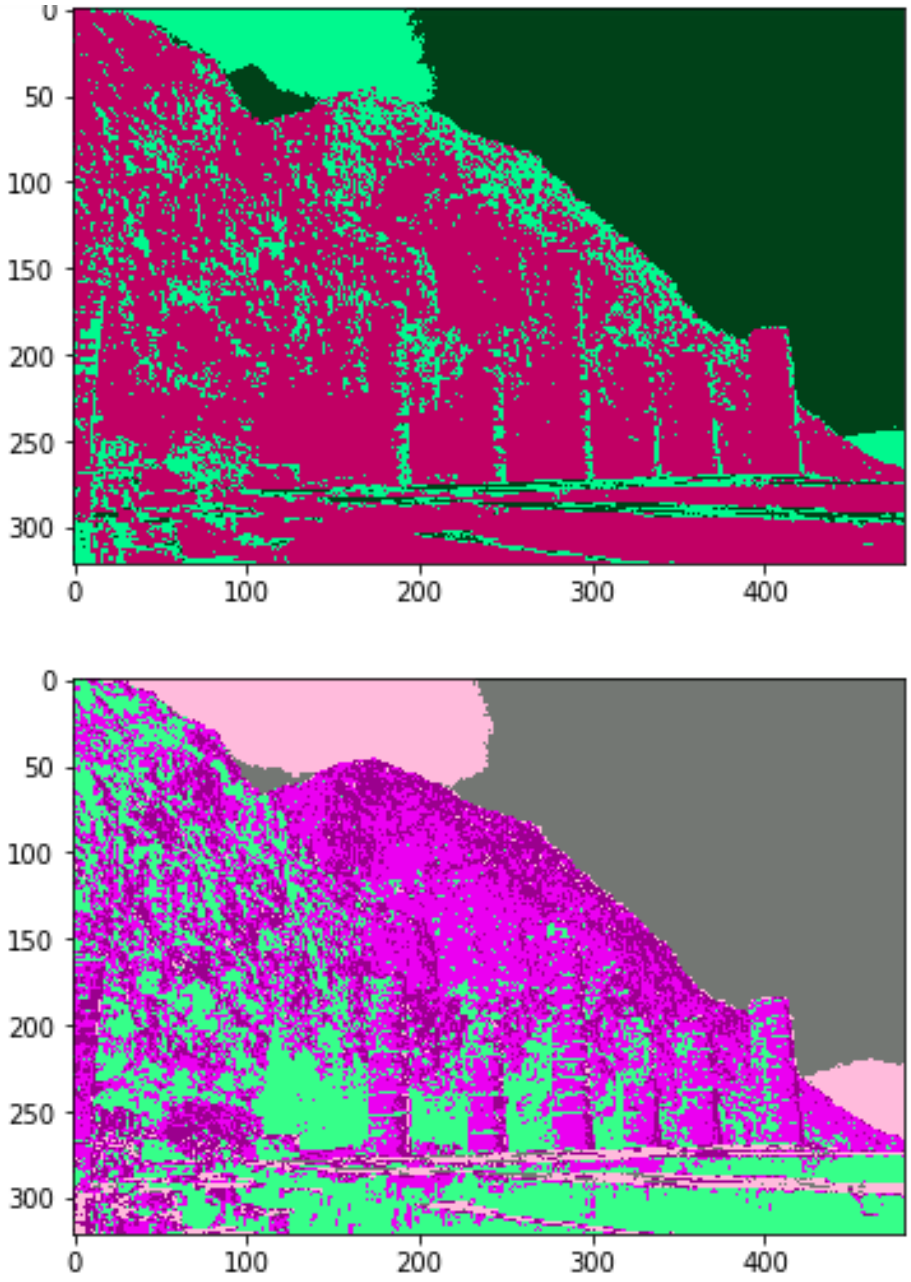


Figure 5: Result 1

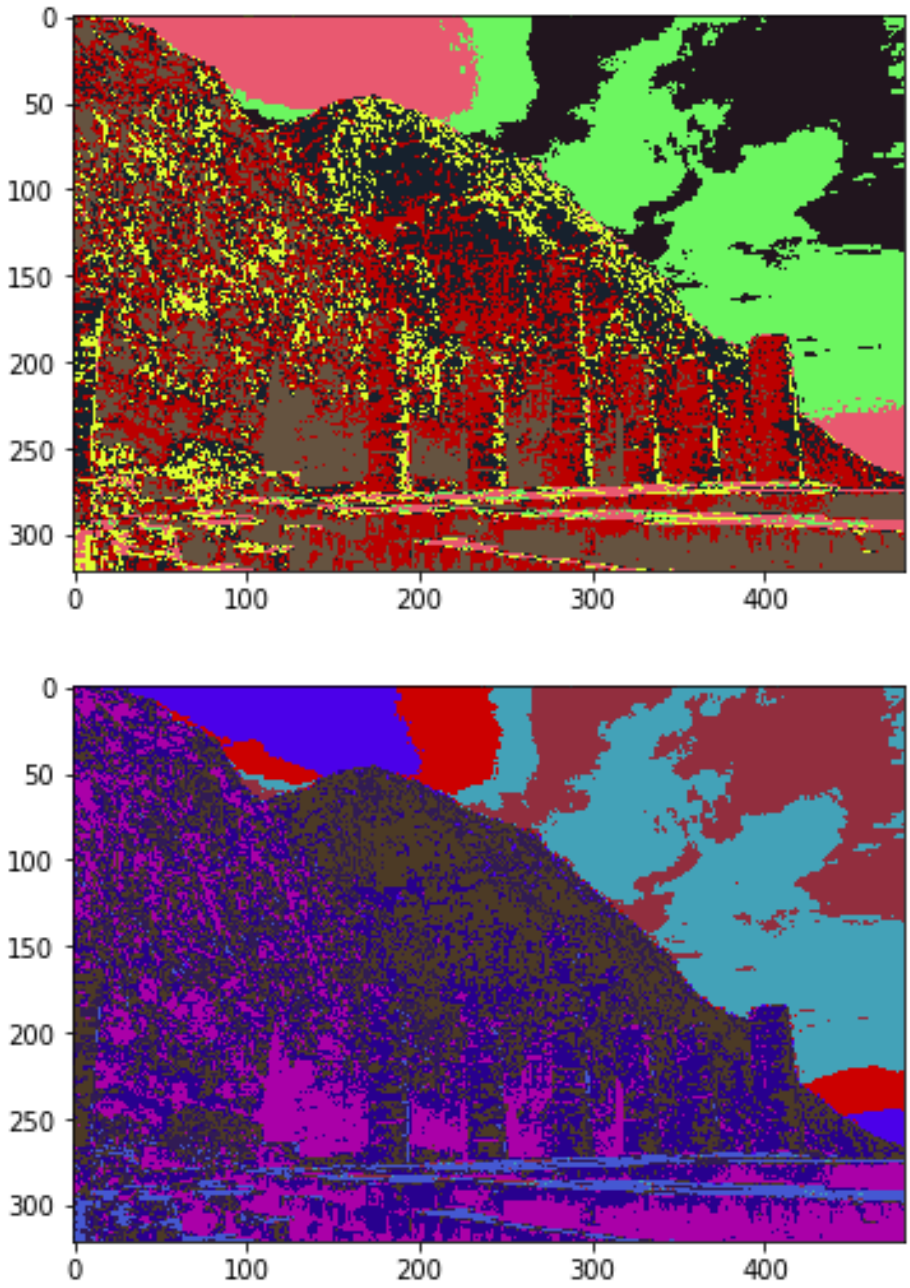


Figure 6: Result 2

As illustrated, the Kmeans algorithm from Sklearn out preforms our implementation due to their usage of kmeans++ algorithm to initialize the cen-

troids instead of a completely random approach and that is why we continue he project then using the Kmeans algorithm from Sklearn to better capture and realize clustering details with future work goals to implement k++ algorithm for centroid initialization.

2.3 Clustering Evaluation

We evaluated the clustering using both F-measure and Conditional Entropy. In statistical analysis of binary classification, the F1 score (also F-score or F-measure) is a measure of a test's accuracy. It considers both the precision p and the recall r of the test to compute the score: p is the number of correct positive results divided by the number of all positive results returned by the classifier, and r is the number of correct positive results divided by the number of all relevant samples (all samples that should have been identified as positive). The F1 score is the harmonic average of the precision and recall, where an F1 score reaches its best value at 1 (perfect precision and recall) and worst at 0. In information theory, the conditional entropy (or equivocation) quantifies the amount of information needed to describe the outcome of a random variable Y given that the value of another random variable X is known. Here, information is measured in shannons, nats, or hartleys.
Conditional entropy equals zero if and only if the value of Y is completely determined by the value of X. The results of the same selected image as above is shown below in figures 7,8 for k values 3,5,7,9,11.

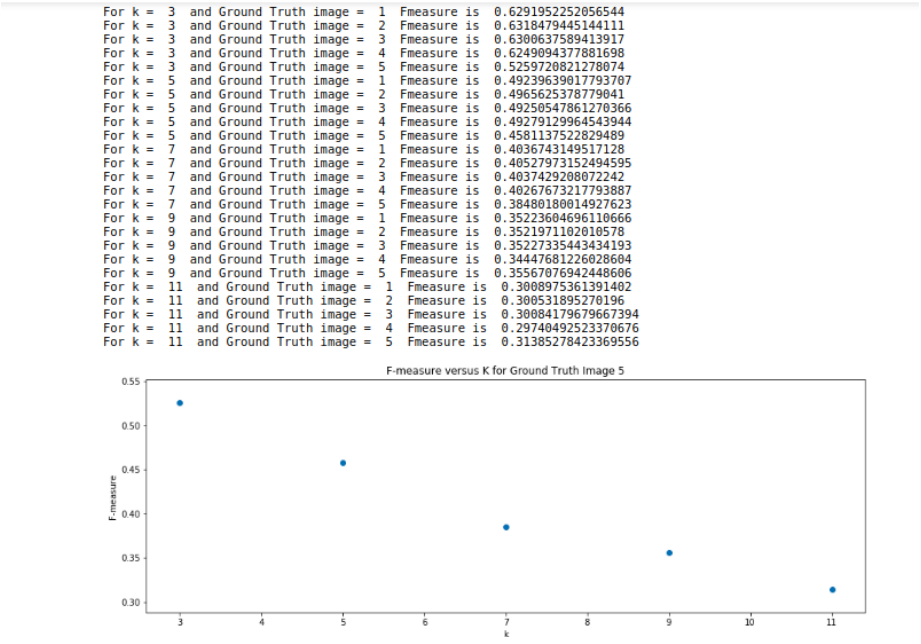


Figure 7: F-measure Results

```
For k =  3  and Ground Truth image =  1  Conditional Entropy is  2.4011382876755896
For k =  3  and Ground Truth image =  2  Conditional Entropy is  2.246060561672166
For k =  3  and Ground Truth image =  3  Conditional Entropy is  2.3766788707342643
For k =  3  and Ground Truth image =  4  Conditional Entropy is  3.0539470636598876
For k =  3  and Ground Truth image =  5  Conditional Entropy is  4.224811956834424
For k =  5  and Ground Truth image =  1  Conditional Entropy is  3.2679623223910164
For k =  5  and Ground Truth image =  2  Conditional Entropy is  3.04915734548019
For k =  5  and Ground Truth image =  3  Conditional Entropy is  3.2380018362564282
For k =  5  and Ground Truth image =  4  Conditional Entropy is  4.10001222729732
For k =  5  and Ground Truth image =  5  Conditional Entropy is  5.643636369477737
For k =  7  and Ground Truth image =  1  Conditional Entropy is  4.566701764231121
For k =  7  and Ground Truth image =  2  Conditional Entropy is  4.3433786726283685
For k =  7  and Ground Truth image =  3  Conditional Entropy is  4.529416178989628
For k =  7  and Ground Truth image =  4  Conditional Entropy is  6.2525345057802255
For k =  7  and Ground Truth image =  5  Conditional Entropy is  8.645300053822002
For k =  9  and Ground Truth image =  1  Conditional Entropy is  4.848517176195985
For k =  9  and Ground Truth image =  2  Conditional Entropy is  4.427846423302367
For k =  9  and Ground Truth image =  3  Conditional Entropy is  4.836510417831613
For k =  9  and Ground Truth image =  4  Conditional Entropy is  7.005370856950953
For k =  9  and Ground Truth image =  5  Conditional Entropy is  9.276306354721O4
For k =  11  and Ground Truth image =  1  Conditional Entropy is  4.970538754430873
For k =  11  and Ground Truth image =  2  Conditional Entropy is  4.333734527768733
For k =  11  and Ground Truth image =  3  Conditional Entropy is  4.953860785111407
For k =  11  and Ground Truth image =  4  Conditional Entropy is  6.286454999866838
For k =  11  and Ground Truth image =  5  Conditional Entropy is  8.89339821027834
```

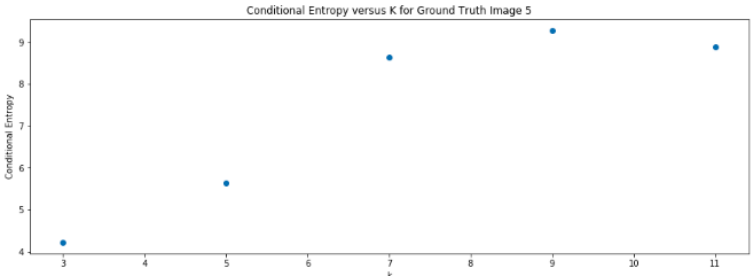


Figure 8: Conditional Entropy Results

It is obvious that with increasing K value against the same ground truth image, the values of the measures computed by our methods increase in case of Entropy and decreases in case of F-measure thus achieving the concept that increasing K degrades performance and doesn't add much in some cases. Take in notice that when experimented on other photos different results and conclusions can be acquired since each image is it's own feature space and different characteristics.

3 Segmentation using Spectral Clustering

So now we open our eyes to see the big picture of the story. There are 2 broad approaches for clustering:
1. Compactness: Points that lie close to each other fall in the same cluster and are compact around the cluster center. The closeness can be measured by the distance between the observations. E.g.: K-Means Clustering
2. Connectivity: Points that are connected or immediately next to each other are put in the same cluster. Even if the distance between 2 points is less, if they are not connected, they are not clustered together. Spectral clustering is a technique that follows this approach.

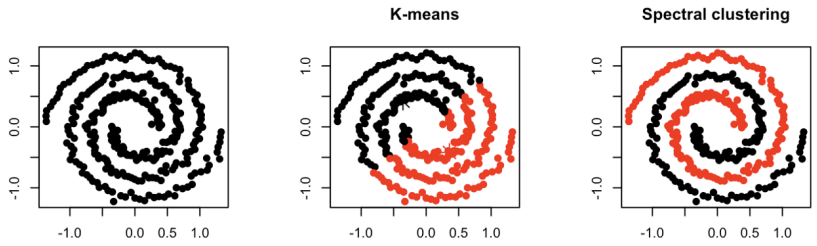


Figure 9: Spectral Clustering vs KMeans

Spectral clustering has become increasingly popular due to its simple implementation and promising performance in many graph-based clustering. It can be solved efficiently by standard linear algebra software, and very often outperforms traditional algorithms such as the k-means algorithm.

3.1 Algorithm

LDA algorithm steps and procedure is provided in Figure 10 which is taken from [3] from Chapter 16.

ALGORITHM 16.1. Spectral Clustering Algorithm

SPECTRAL CLUSTERING $(\mathbf{D}, k)$:

1 Compute the similarity matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$
2 **if** *ratio cut* **then** $\mathbf{B} \leftarrow \mathbf{L}$
3 **else if** *normalized cut* **then** $\mathbf{B} \leftarrow \mathbf{L}^s$ or $\mathbf{L}^a$
4 Solve $\mathbf{B}\mathbf{u}_i = \lambda_i \mathbf{u}_i$ for $i = n, \ldots, n-k+1$, where $\lambda_n \leq \lambda_{n-1} \leq \cdots \leq \lambda_{n-k+1}$
5 $\mathbf{U} \leftarrow \begin{pmatrix} \mathbf{u}_n & \mathbf{u}_{n-1} & \cdots & \mathbf{u}_{n-k+1} \end{pmatrix}$
6 $\mathbf{Y} \leftarrow$ normalize rows of $\mathbf{U}$ using Eq. (16.19)
7 $\mathcal{C} \leftarrow \{C_1, \ldots, C_k\}$ via K-means on $\mathbf{Y}$

Figure 10: Spectral Clustering Algorithm

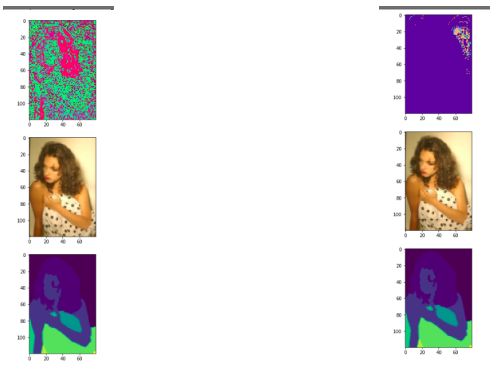
To perform a spectral clustering we need 3 main steps:
1.Create a similarity graph between our N objects to cluster.
2.Compute the first k eigenvectors of its Laplacian matrix to define a feature vector for each object.
3.Run k-means on these features to separate objects into k classes.

3.2 Implementation and results

Again, we implemented the above algorithm in Figure 10 using Python and Numpy arrays. For Step 1, computing the similarity graph we first create an undirected graph G = (V, E) with vertex set V = v1, v2, ..., vn = 1, 2, ..., n observations in the data. This can be represented by an adjacency matrix which has the similarity between each vertex as its elements . To do this, we can either compute The -neighborhood graph, KNN Graph or Fully connected graph. (More details on each of them can be found in [4]) The problem in the implementation of Spectral Clustering is that we have to store the nearest neighbour hood graph in memory which is so huge and we don't actually have enough RAM for such an operation.So we experimented three implementation approaches to find the best one and learn how to deal with such situation.

3.3 Beating the memory limits: Re-sizing the image

An easy approach is to re-size the image enough (we chose 25 percent of the image to keep) such that the NN graph can be constructed and reside in memory resources that we are limited with. So for this approach we tried two different ways one is to pass the re-sized image into our own Spectral clustering algorithm implementation and the other way to pass it into Sklearn Spectral Clustering implementation of the algorithm. We only tried for k=5, and in that case our implementation actually was better for that specific image we tested upon but still both was not good enough. We figure that choosing another picture can be actually obtain better results but leave it as a future work on the project. The results of this approach is shown in figure 11 below.



(a) Re-sizing, running our implementation

(b) Re-sizing, running SKlearn implementation

Figure 11: Results of Re-sizing on a specific photo

3.4 Beating the memory limits: Annoy and Sparsity

The second we approach is more complex and less intuitive, though it also it didn't give best or even better results but we intend to study more about it and make it work better as our main goal in the future work.
Annoy [6](Approximate Nearest Neighbors Oh Yeah) is a C++ library with Python bindings to search for points in space that are close to a given query point. It also creates large read-only file-based data structures that are mapped into memory so that many processes may share the same data.
Annoy has the ability to use static files as indexes. In particular, this means you can share index across processes. Annoy also decouples creating indexes from loading them, so you can pass around indexes as files and map them into memory quickly. Another nice thing of Annoy is that it tries to minimize memory footprint so the indexes are quite small. Why is this useful? If you want to find nearest neighbors and you have many CPU's, you only need to build the index once. You can also pass around and distribute static files to use in production environment, in Hadoop jobs, etc. Any process will be able to load (mmap) the index into memory and will be able to do lookups immediately. More details on how it works can be found in the documentation and source code in [6]. We used annoy to store our NN graph efficiently without re-sizing the image and then we used Sparse matrices to code up our way through the Spectral clustering algorithm where we used everything as sparse matrices as we noticed the obvious sparsity of our data (only 5 non-zero values in each row of the image) and then we run our algorithm accordingly. The results of this approach is found below in figure 12 below.

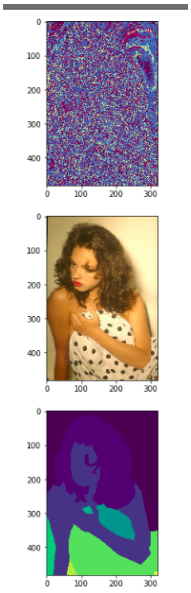


Figure 12: Results of approach 2

4 Adding spices to the mix: Spatial Feature encoding in KMeans

In the previous parts, we used the color features RGB. We didn't encode the layout of the pixels. We want to modify that for K-means clustering to encode the spatial layout of the pixels.
The spatial layout of a pixel means it's location or coordinate in the data matrix you are passing to the KMeans algorithm. Our Implementation to this added feature is that we concatenated to our original data matrix with three features (R,G,B), two more features one for the x-coordinate and one for the y-coordinate of the location of the pixel in the matrix where each row represents a pixel.
The conventional K-means approaches only involves the segmentation in feature space. Without consideration of spatial constrains, and often generate over segmented results. In [5], they proposed a novel K-means based approach that combines spatial constrains. Much more details, mathematical background and results on the process can be found there.
Our results of the implementation to such additive feature is shown below in figure 13.

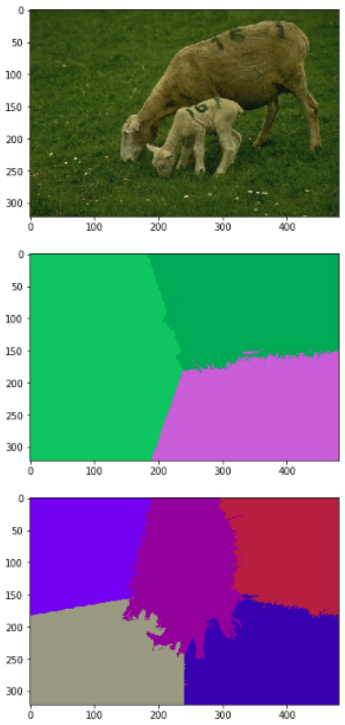


Figure 13: Sample Result of adding spatial features.

5 Future Improvements and Goals

In this section we would like to point out some final notes that we would like to do futuristic improvements in our source code and implementation.
1- Adding kmeans++ algorithm for centroid initialization in our KMeans algorithm implementation.
2- Further improve our spectral clustering algorithm implementation by better understanding Annoy and the usage of sparse matrices efficiently.
3-Answering of the following questions in mind and developing a conclusion section to this document upon.
- Is KMeans training algorithm on a train set ? Does it differ to input the whole training set or one image only ?
- Does Spectral clustering work good only in case of bad results in KMeans and otherwise since data is already in good dimension representation so it doesn't add anything maybe turns it worse ?
- What did the spatial layout encoding add exactly to our clustering ? How is the output obtained beneficial ?
- How can i (if at all) use my test and validation data split ? Don't i just need my train set ?

6 References

[**1**] MacQueen, J. B. (1967). Some Methods for classification and Analysis of Multivariate Observations. Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability. 1. University of California Press. pp. 281–297. MR 0214227. Zbl 0214.46201. Retrieved 2009-04-07.

[**2**] Steinhaus, H. (1957). "Sur la division des corps matériels en parties". Bull. Acad. Polon. Sci. (in French). 4 (12): 801–804. MR 0090073. Zbl 0079.16403.

[**3**] DATA MINING AND ANALYSIS,Fundamental Concepts and Algorithms MOHAMMED J. ZAKI Rensselaer Polytechnic Institute, Troy, New York WAGNER MEIRA JR. Universidade Federal de Minas Gerais, Brazil

[**4**] Spectral clustering: The intuition and math behind how it works! -Neerja Doshi

[**5**] A Spatial Constrained K-Means Approach to Image Segmentation- Ming Luo, Yu-Fei Ma, Hong-Jiang Zhang

[**6**] Annoy open source library repository

No further Information,End of Report