Simulations of an Exponential Distribution

Simulation exercises

Task: Simulation of the exponential distribution (rexp) with lambda=0.2. The distribution of the averages of 40 exponentials is investigated (n=40). The simulation is run 1000 times (nosim=1000). The mean and variance are stored as columns in the rexpDistF data frame. Running values mean and variance are also stored in the data frame.

```
lambda <- 0.2; nosim <- 1000; n <- 40
rexpDistF <- data.frame(id=numeric(nosim), mean=numeric(nosim), var=numeric(nosim),
run_mean=numeric(nosim), run_variance=numeric(nosim))
for(i in 1:nosim) {
rdm <- rexp(n, rate=lambda)
rexpDistF$id[i] <- i; rexpDistF$mean[i] <- mean(rdm); rexpDistF$var[i] <- var(rdm)
if (i > 1) {
rexpDistF$run_mean[i] <- (rexpDistF$run_mean[i-1] * (i-1) + mean(rdm)) / i
rexpDistF$run_variance[i] <- (rexpDistF$run_variance[i-1] * (i-1) + var(rdm)) / i
} else {
rexpDistF$run_mean[i] <- mean(rdm); rexpDistF$run_variance[i] <- var(rdm)
}
}
```

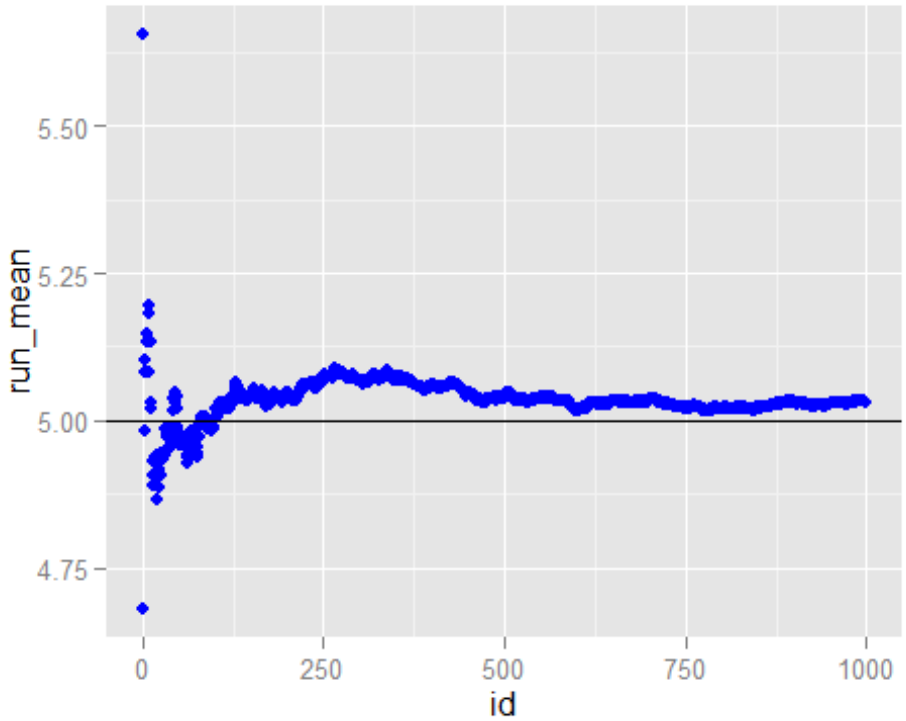
The center of the distribution

With increasing number of simulations the "running mean" (5.0301) settles close to the value of 1/lambda (5).

```
library(ggplot2)

## Warning: package 'ggplot2' was built under R version 3.0.3

ggplot(rexpDistF, aes(x=id, y=run_mean)) + geom_point(colour="blue") +
geom_hline(yintercept=1/lambda)
```



The variance of the distribution

The variance converges towards (1/lambda)^2 (25). This can be compared with the actual variance from simulation (25.2873). By taking the square root of the variance (standard deviation) it can be seen that it tends towards the value of 1/lambda (5.0286).

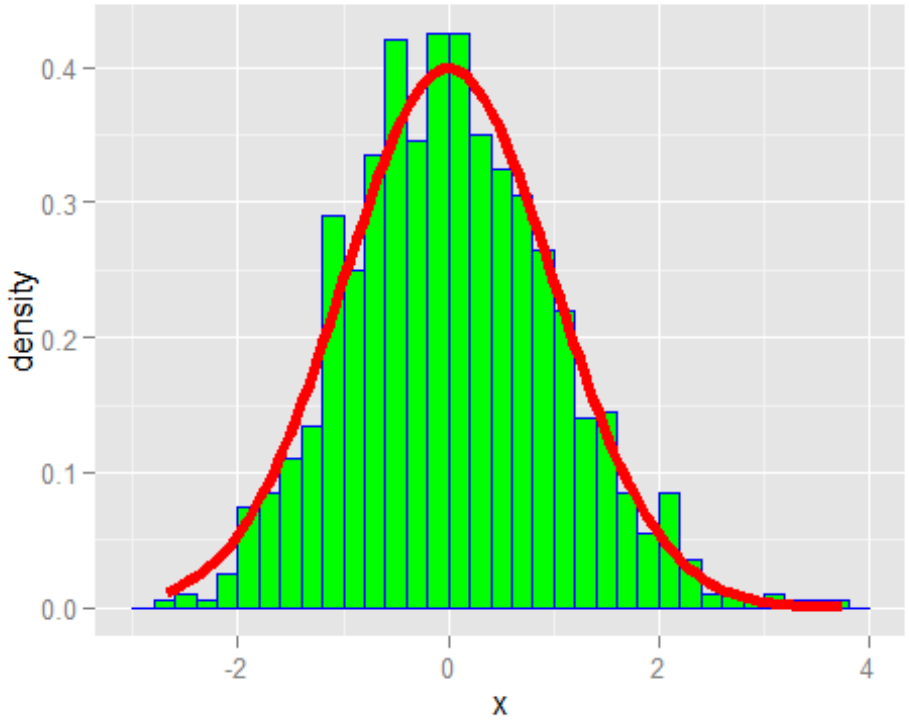
The shape of the distribution is approximately normal

By plotting the distribution as a histogram we can see that it appears as a normal distribution. First, it is normalized by centering around 0 and dividing to get a variance of 1.

```
normalMeans <- data.frame(x=numeric(nosim))
normalMeans$x <- (rexpDistF$mean - (1/lambda)) / ((1/lambda)/sqrt(n))
```

Below, the histogram is compared with a normal distribution (red bell-shaped line). It can be seen that the histogram approximates the normal distribution plot.

```
g3 <- ggplot(normalMeans, aes(x=x))
g3 <- g3 + geom_histogram(binwidth=.2, color="blue", fill="green", aes(y=..density..))
g3 + stat_function(fun=dnorm, color="red", size=2)
```



Evaluation of the coverage of the confidence interval for 1/lambda

The coverage of the confidence interval of 95% is calculated as:

```
m <- mean(rexpDistF$mean); sd <- sqrt(mean(rexpDistF$var))
m + c(-1, 1) * 1.96 * sd / sqrt(n)

## [1] 3.472 6.588
```