Model 15: Production Scheduling

Decision Variables:
a_1: number of units of product 1 manufactured in season 1
b_1: number of units of product 2 manufactured in season 1
a_2: number of units of product 1 manufactured in season 2
b_2: number of units of product 2 manufactured in season 2
a_3: number of units of product 1 manufactured in season 3
b_3: number of units of product 2 manufactured in season 3
a_4: number of units of product 1 manufactured in season 4
b_4: number of units of product 2 manufactured in season 4
x_2: number of units of product 1 sold in season 2
y_2: number of units of product 2 sold in season 2
x_3: number of units of product 1 sold in season 3
y_3: number of units of product 2 sold in season 3
x_4: number of units of product 1 sold in season 4
y_4: number of units of product 2 sold in season 4
x_5: number of units of product 1 sold in season 5
y_5: number of units of product 2 sold in season 5
r_1: number of units left unsold at the end of season 2
r_2: number of units left unsold at the end of season 3
r_3: number of units left unsold at the end of season 4
r_4: number of units left unsold at the end of season 5

Let $r_1 = (a_1 + b_1) - (x_2 + y_2)$
$r_2 = (a_2 + b_2 + r_1) - (x_3 + y_3)$
$r_3 = (a_3 + b_3 + r_2) - (x_4 + y_4)$
$r_4 = (a_4 + b_4 + r_3) - (x_5 + y_5)$

Objective: max $70x_2 + 210y_2 + 50x_3 + 40y_3 + 220x_4 + 250y_4 + 12.5x_5 + 50y_5 - 10r_1 - 10r_2 - 10r_3 - 10r_4$

$$
\begin{aligned}
5a_1 + 6b_1 &\leq 12000 \\
5a_2 + 6b_2 &\leq 12000 \\
5a_3 + 6b_3 &\leq 12000 \\
5a_4 + 6b_4 &\leq 12000 \\
3a_1 + b_1 &\leq 15000 \\
3a_2 + b_2 &\leq 15000 \\
3a_3 + b_3 &\leq 15000 \\
3a_4 + b_4 &\leq 15000 \\
2.5x_2 + 4y_2 &\leq 1000 \\
2.5x_3 + 4y_3 &\leq 800 \\
2.5x_4 + 4y_4 &\leq 1000 \\
2.5x_5 + 4y_5 &\leq 900 \\
x_2 + 2y_2 &\leq 200 \\
x_3 + 2y_3 &\leq 400 \\
x_4 + 2y_4 &\leq 200 \\
x_5 + 2y_5 &\leq 300 \\
x_2 + y_2 - a_1 - b_1 &\leq 0 \\
x_3 + y_3 - a_2 - b_2 - r_1 &\leq 0 \\
x_4 + y_4 - a_3 - b_3 - r_2 &\leq 0 \\
x_5 + y_5 - a_4 - b_4 - r_3 &\leq 0 \\
a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, x_2, x_3, x_4, x_5, y_2, y_3, y_4, y_5, r_1, r_2, r_3 &\geq 0
\end{aligned}
$$

Scratch Work

Cost of advertising (premium and non-premium): $20(2.5x_2+4y_2)+40(2.5x_3+4y_3)+10(2.5x_4+4y_4)+25(2.5x_5+4y_5)+80(x_2+2y_2)+100(x_3+2y_3)+55(x_4+2y_4)+75(x_5+2y_5)$

Additional Costs (carry-over): $10r_1+10r_2+10r_3+10r_4$

Profit: $200x_2+450y_2+250x_3+400y_3+300x_4+400y_4+150x_5+300y_5-50x_2-80y_2-100x_3-160y_3-25x_4-40y_4-62.5x_5-100y_5-80x_2-160y_2-100x_3-200y_3-55x_4-110y_4-75x_5-150y_5-10r_1-10r_2-10r_3-10r_4$

Constraints: $5a_i+6b_i \leq 12000$
$3a_i+b_i \leq 15000$
$2.5x_2+4y_2 \leq 1000$
$2.5x_3+4y_3 \leq 800$
$2.5x_4+4y_4 \leq 1000$
$2.5x_5+4y_5 \leq 900$
$x_2+2y_2 \leq 200$
$x_3+2y_3 \leq 400$
$x_4+2y_4 \leq 200$
$x_5+2y_5 \leq 300$
$x_2+y_2 \leq a_1+b_1$
$x_3+y_3 \leq a_2+b_2+r_1$
$x_4+y_4 \leq a_3+b_3++r_2$
$x_5+y_5 \leq a_4+b_4+r_3$