

Transform the following LP's to LP's in standard form:

1. Let $x_1 = z_0 - z_1$ where $z_0, z_1 \geq 0$

Let $x_2 = -z_2$ where $z_2 \geq 0$

Let $x_3 = z_3 + 1$ where $z_3 \geq 0$

$$\begin{aligned} \max \quad & -z_0 + z_1 - 12z_2 + 2z_3 + 2 \\ \text{subject to} \quad & 5z_0 - 5z_1 + z_2 - 2z_3 \leq 12 \\ & -5z_0 + 5z_1 - z_2 + 2z_3 \leq -12 \\ & -2z_0 + 2z_1 + z_2 + 20z_3 \leq 10 \\ & z_3 \leq 3 \\ & z_0, z_1, z_2, z_3 \geq 0 \end{aligned}$$

2. Let $x = x_0 - x_1$ where $x_0, x_1 \geq 0$

Let $y = -y_0$ where $y_0 \geq 0$

Let $z = z_0 + 1$ where $z_0 \geq 0$

$$\begin{aligned} \max \quad & 3x_0 - 3x_1 + 12y_0 + 4z_0 + 4 \\ \text{subject to} \quad & 5x_0 - 5x_1 - 10z_0 \leq 20 \\ & -5x_0 + 5x_1 + 10z_0 \leq -20 \\ & -2x_0 + 2x_1 - y_0 + 17z_0 \leq -7 \\ & x_0 - x_1 - y_0 + z_0 \leq 9 \\ & x_0, x_1, y_0, z_0 \geq 0 \end{aligned}$$

3. Let $x_1 = z_0 - z_1$ where $z_0, z_1 \geq 0$

Let $x_2 = z_2 - 1$ where $z_2 \geq 0$

Let $x_3 = z_3 - 2$ where $z_3 \geq 0$

$$\begin{aligned} \max \quad & -4z_0 + 4z_1 + 2z_2 - z_3 \\ \text{subject to:} \quad & z_0 - z_1 - 3z_2 + z_3 \leq 0 \\ & 5z_2 + 3z_3 \leq 16 \\ & -5z_2 - 3z_3 \leq -16 \\ & z_0 - z_1 + z_2 + z_3 \leq 4 \\ & z_3 \leq 4 \\ & z_0, z_1, z_2, z_3 \geq 0 \end{aligned}$$

4.

$$\begin{aligned} & \text{max: } 3x_1 - 5x_2 \\ \text{subject to: } & -4x_1 - 5x_2 \leq -3 \\ & 6x_1 - 6x_2 \leq 7 \\ & -6x_1 + 6x_2 \leq -7 \\ & x_1 + 8x_2 \leq 20 \\ & x_1, x_2 \geq 0 \end{aligned}$$

5.

$$\begin{aligned} & \text{max: } 8x_1 - 9x_2 - 2x_3 + 6x_4 + 5x_5 \\ \text{subject to: } & -6x_1 - 6x_2 + 10x_3 - 2x_4 + 8x_5 \leq -3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Problem 3: Toll Booth Hours

Decision Variables:

x_i : number of collectors to start working at the i^{th} hour of the day starting at 12am and ending at 11pm for $i = 1$ to $i = 24$. There are a total of 24 variables

Objective: $\max -x_1 - x_2 - x_3 - \dots - x_{23} - x_{24} = -\sum_{i=1}^{24} x_i$

$$\begin{aligned} -x_1 - x_{24} - x_{23} - x_{22} - x_{20} - x_{19} - x_{18} - x_{17} &\leq -2 \text{ (12am to 1am)} \\ -x_2 - x_1 - x_{24} - x_{23} - x_{21} - x_{20} - x_{19} - x_{18} &\leq -2 \text{ (1am to 2am)} \\ -x_3 - x_2 - x_1 - x_{24} - x_{22} - x_{21} - x_{20} - x_{19} &\leq -2 \text{ (2am to 3am)} \\ -x_4 - x_3 - x_2 - x_1 - x_{23} - x_{22} - x_{21} - x_{20} &\leq -2 \text{ (3am to 4am)} \\ -x_5 - x_4 - x_3 - x_2 - x_{24} - x_{23} - x_{22} - x_{21} &\leq -2 \text{ (4am to 5am)} \\ -x_6 - x_5 - x_4 - x_3 - x_1 - x_{24} - x_{23} - x_{22} &\leq -2 \text{ (5am to 6am)} \\ -x_7 - x_6 - x_5 - x_4 - x_2 - x_1 - x_{24} - x_{23} &\leq -8 \text{ (6am to 7am)} \\ -x_8 - x_7 - x_6 - x_5 - x_3 - x_2 - x_1 - x_{24} &\leq -8 \text{ (7am to 8am)} \\ -x_9 - x_8 - x_7 - x_6 - x_4 - x_3 - x_2 - x_1 &\leq -8 \text{ (8am to 9am)} \\ -x_{10} - x_9 - x_8 - x_7 - x_5 - x_4 - x_3 - x_2 &\leq -8 \text{ (9am to 10am)} \\ -x_{11} - x_{10} - x_9 - x_8 - x_6 - x_5 - x_4 - x_3 &\leq -4 \text{ (10am to 11am)} \\ -x_{12} - x_{11} - x_{10} - x_9 - x_7 - x_6 - x_5 - x_4 &\leq -4 \text{ (11am to 12pm)} \\ -x_{13} - x_{12} - x_{11} - x_{10} - x_8 - x_7 - x_6 - x_5 &\leq -3 \text{ (12pm to 1pm)} \\ -x_{14} - x_{13} - x_{12} - x_{11} - x_9 - x_8 - x_7 - x_6 &\leq -3 \text{ (1pm to 2pm)} \\ -x_{15} - x_{14} - x_{13} - x_{12} - x_{10} - x_9 - x_8 - x_7 &\leq -3 \text{ (2pm to 3pm)} \\ -x_{16} - x_{15} - x_{14} - x_{13} - x_{11} - x_{10} - x_9 - x_8 &\leq -3 \text{ (3pm to 4pm)} \\ -x_{17} - x_{16} - x_{15} - x_{14} - x_{12} - x_{11} - x_{10} - x_9 &\leq -6 \text{ (4pm to 5pm)} \\ -x_{18} - x_{17} - x_{16} - x_{15} - x_{13} - x_{12} - x_{11} - x_{10} &\leq -6 \text{ (5pm to 6pm)} \\ -x_{19} - x_{18} - x_{17} - x_{16} - x_{14} - x_{13} - x_{12} - x_{11} &\leq -5 \text{ (6pm to 7pm)} \\ -x_{20} - x_{19} - x_{18} - x_{17} - x_{15} - x_{14} - x_{13} - x_{12} &\leq -5 \text{ (7pm to 8pm)} \\ -x_{21} - x_{20} - x_{19} - x_{18} - x_{16} - x_{15} - x_{14} - x_{13} &\leq -5 \text{ (8pm to 9pm)} \\ -x_{22} - x_{21} - x_{20} - x_{19} - x_{17} - x_{16} - x_{15} - x_{14} &\leq -5 \text{ (9pm to 10pm)} \\ -x_{23} - x_{22} - x_{21} - x_{20} - x_{18} - x_{17} - x_{16} - x_{15} &\leq -3 \text{ (10pm to 11pm)} \\ -x_{24} - x_{23} - x_{22} - x_{21} - x_{19} - x_{18} - x_{17} - x_{16} &\leq -3 \text{ (11pm to 12am)} \\ x_1, x_2, x_3, \dots, x_{22}, x_{23}, x_{24} &\geq 0 \end{aligned}$$

Problem 5: Cereal Blending

Decision Variables:

 x_1 : kg of Food 1 blended in type 1 x_2 : kg of Food 2 blended in type 1 x_3 : kg of Food 3 blended in type 1 x_4 : kg of Food 4 blended in type 1 y_1 : kg of Food 1 blended in type 2 y_2 : kg of Food 2 blended in type 2 y_3 : kg of Food 3 blended in type 2 y_4 : kg of Food 4 blended in type 2**Objective:**

$$\max .895x_1 + 1.305x_2 + 1.265x_3 + 1.125x_4 + .37y_1 + .78y_2 + .74y_3 + .6y_4$$

$$-.2055x_1 + .1631x_2 + .1146x_3 - .0309x_4 \leq 0$$

$$-.201x_1 + .046x_2 - .0775x_3 + .065x_4 \leq 0$$

$$-.015x_1 + .012x_2 + .003x_3 - .006x_4 \leq 0$$

$$.201y_1 - .046y_2 + .0775y_3 - .065y_4 \leq 0$$

$$x_1 + y_1 \leq 1500$$

$$x_2 + y_2 \leq 500$$

$$x_3 + y_3 \leq 1000$$

$$x_4 + y_4 \leq 2000$$

$$x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4 \geq 0$$

Problem 6: Warehousing

Decision Variables:

Let x_{ij} be the amount (in thousands) of square feet leased in month i for a duration of j months.

Objective:

$$\max -650(x_{11} + x_{21} + x_{31} + x_{41} + x_{51}) - 1000(x_{12} + x_{22} + x_{32} + x_{42}) - 1350(x_{13} + x_{23} + x_{33}) - 1600(x_{14} + x_{24}) - 1900x_{15}$$

$$\begin{aligned} -x_{11} - x_{12} - x_{13} - x_{14} - x_{15} &\leq -30 \\ -x_{12} - x_{13} - x_{14} - x_{15} - x_{21} - x_{22} - x_{23} - x_{24} &\leq -20 \\ -x_{13} - x_{14} - x_{15} - x_{22} - x_{23} - x_{24} - x_{31} - x_{32} - x_{33} &\leq -40 \\ -x_{14} - x_{15} - x_{23} - x_{24} - x_{32} - x_{33} - x_{41} - x_{42} &\leq -10 \\ -x_{15} - x_{24} - x_{33} - x_{42} - x_{51} &\leq -50 \\ \forall x_{ij} &\geq 0 \end{aligned}$$

Problem 7: Electronics Company

A_1 : number of employees assembling week 1

A_2 : number of employees assembling week 2

A_3 : number of employees assembling week 3

A_4 : number of employees assembling week 4

I_1 : number of employees instructing week 1

I_2 : number of employees instructing week 2

I_3 : number of employees instructing week 3

Objective: $\max -50A_1 + 650A_2 + 550A_3 + 450A_4 - 2900I_1 - 1500I_2 - 900I_3$

$$50A_1 + 50A_2 + 50A_3 + 50A_4 \leq 21475$$

$$-50A_1 - 50A_2 - 50A_3 - 50A_4 \leq -21475$$

$$A_1 + I_1 \leq 40$$

$$A_2 + I_2 - 3I_1 \leq 40$$

$$A_3 + I_3 - 3I_1 - 3I_2 \leq 40$$

$$A_4 - 3I_1 - 3I_2 - 3I_3 \leq 40$$

$$A_1, A_2, A_3, A_4, I_1, I_2, I_3 \geq 0$$

Problem 8: Summer Job

x_1 : percent of time spent at job 1

x_2 : percent of time spent at job 2

Objective: $\max 4500x_1 + 5000x_2$

$$400x_1 + 500x_2 \leq 600$$

$$5000x_1 + 4000x_2 \leq 6000$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Problem 10: Detergent

x_1 : gallons of NPW sold directly

x_2 : gallons of NPW made into detergent

x_3 : gallons of NPW produced in-house

x_4 : gallons of NPW purchased from outside

Objective: $\max .80x_1 + x_2 - .50x_3 - 1.20x_4$

$$x_1 + x_2 - x_3 - x_4 \leq 0$$

$$-x_1 - x_2 + x_3 + x_4 \leq 0$$

$$x_2 \leq 12000$$

$$x_3 \leq 10000$$

$$x_1, x_2, x_3, x_4 \geq 0$$