Page 86 Problem 1 Find $\phi(n)$ for $n = 20, 60, 63, 341, 561$.

By theorem 3.15, $\phi(n)$ is multiplicative. So,

$$\phi(20) = \phi(2^2)\phi(5) = (2^2 - 2)(4) = 8$$
$$\phi(60) = \phi(3)\phi(20) = 2 * 8 = 16$$
$$\phi(63) = \phi(3^2)\phi(7) = (3^2 - 3) * 6 = 36$$
$$\phi(341) = \phi(11)\phi(31) = 10 * 30 = 300$$
$$\phi(561) = \phi(3)\phi(11)\phi(17) = 2 * 10 * 16 = 320$$

Page 86 Problem 3 Show that if n is even, $\phi(2n) = 2\phi(n)$.

Let n be even. Then we can write $n = 2^k m$ for some integer $m \in \mathbb{Z}$, such that $(m, 2) = 1$. Note that m can be equal to 1 here. Then

$$\phi(2n) = \phi(2^{k+1}m) = \phi(2^{k+1})\phi(m)$$
$$(2^{k+1} - 2^k)\phi(m) = 2^k\phi(m)$$
$$2 \cdot 2^{k-1}\phi(m) = 2\phi(2^k)\phi(m)$$
$$2\phi(2^k m) = 2\phi(n).$$