Emma Deckers
Math 301 Spring 22
Homework 4

Page 54 Problem 4 Find $a \equiv 97(mod 7)$ for $0 \leq a \leq 6$.
Notice $97 = 13(7) + 6$, so $6 \equiv 97(mod 7)$.

Page 64 Problem 6 Let k be an integer such that $k^2 = n$ for a positive integer in $n \in \mathbb{Z}$. Then k has 8 possible values mod 8: $k = \{0, 1, 2, 3, 4, 5, 6, 7\}$. Then $k^2(mod 8)$ has the following values: $k^2 = \{0, 1, 4, 1, 0, 1, 4, 1\}$. Thus if $n = k^2$ is a perfect square, n is congruent mod 8 to either 0,1, or 4.

Page 64 Problem 15 Show that for $m \geq 0$, $17|(3 \cdot 5^{2m+1} + 2^{3m+1})$.
Let m be a positive integer. First, notice that $5^2 \equiv 2^3 mod 17$. Let $m = 0$, then it is true that 17 divides $3 \cdot 5^1 + 2^1 = 17$. Suppose that for some $n > m$ that $17|(3 \cdot 5^{2n+1} + 2^{3n+1})$. By definition, this means that $3 \cdot 5^{2n+1} \equiv -2^{3n+1} mod 17$. Using theorem 3.2 from the textbook, $3 \cdot 5^{2n+1}(5^2) \equiv -2^{3n+1}(2^3) mod 17$. Observe that

$$\begin{gathered} 3 \cdot 5^{2n+1} \cdot 5^2 = 3 \cdot 5^{2n+3} \\ = 3 \cdot 5^{2(n+1)+1} \\ \text{And,} \\ -2^{3n+1} \cdot 2^3 = -2^{3n+4} \\ = -2^{3(n+1)+1} \end{gathered}$$

Therefore, $3 \cdot 5^{2(n+1)+1} \equiv -2^{3(n+1)+1} mod 17$ which by definition means $17|(3 \cdot 5^{2(n+1)+1} + 2^{3(n+1)+1})$.By induction we have proven the theorem.

Page 76 Problem 6 Solve $9x \equiv 21(mod 12)$.
The associated diophantine equation is given by $9x + 12y = 21$. $(9, 12) = 3|21$ so there will be a solution. Using the Euclidean Algorithm,

$$\begin{gathered} 12 = 9 + 3 \\ 3 = 12(1) + 9(-1) \\ 21 = 12(7) + 9(-7) \end{gathered}$$

The general solution is given by $X = -7 + 4t$ for $t \in \mathbb{Z}$.