1 Connectivity

2 Traffic model and throughput

In this part of the assignement we compare the throughput bound of the Fat-tree topology with that of a random (i.e. Jellyfish) topology having the same equipment. Let N denote the number of servers, S the number of n-ports switches and L the number of biderectional links of the network.

1. Once n is fixed, the values of N, S and L are can be immediately derived from the properties of the Fat-tree topology. Specifically, they can be written as functions of n as follows:

$$N = \frac{n^3}{4} \qquad\qquad S = \frac{5}{4}n^2 \qquad\qquad L = 3N = \frac{3}{4}n^3$$

In the Jellyfish topology the number r of switch ports to be connected to other switches is involved in the inequalitiy $N \leq S(n-r)$. Therefore, assuming the network to be equipped with the maximum possible number of servers, we derive the following for r:

$$r = n - \frac{N}{S} = n - \frac{n^3}{4}\frac{4}{5n^2} = n - \frac{n}{5} = \frac{4}{5}n$$

. Notice that this choice of r also leads to same number of links of Fat-tree:

$$L_J = \frac{Sr}{2} = \frac{1}{2} \cdot \frac{5}{4}n^2 \cdot \frac{4}{5}n = \frac{3}{4}n^3 = L_{FT} = L$$

. We also check that this value of r satisfies the requirement of a general r-regular graph:

- $r < S$: $\frac{4}{5}n < \frac{5}{4}n^2 \Leftrightarrow n > \lceil\frac{16}{25}\rceil = 1$
- the product rS is even, i.e. $rS = 2k$ for some $k \in \mathbb{N}$

2. We can now write the expression for the application-oblivious throughput bound TH for an all-to-all traffic matrix as a function of n and of the average shortest path length $\overline{h}$. In particular, we have that:

- $\ell = L = 3N = \frac{3}{4}n^3$
- $\nu_f = \binom{N}{2} = \frac{N(N-1)}{2} = \frac{n^3}{8}(\frac{n^3}{4} - 1) = \frac{n^3(n^3-4)}{32}$

Therefore, substituting in the general formula we get the following:

$$TH = TH(n, \overline{h}) \leq \frac{\ell}{\overline{h}\mu_f} = \frac{6}{\overline{h}(N-1)} = \frac{24}{\overline{h}(n^3-4)}.$$

3. Finally, we compare numerically the bound on TH in both topologies for $n = 5\ell,\ \ell = 1, \ldots, 10$. In the case of the Fat-tree topology, we can use the exact value of $\overline{h}$, which is given by:

$$\overline{h} = \frac{1}{\nu_f}\sum_{1=1}^{\nu_f} h_i = \left(2\left[\frac{n^3}{8}\left(\frac{n}{2}-1\right)\right] + 4\left[\frac{n^4}{16}\left(\frac{n}{2}-1\right)\right] + 6\left[\frac{n^5}{32}(n-1)\right] = \right)\frac{1}{\nu_f} =$$
$$= 2 \cdot \frac{3n^3 - n^2 - 2n - 4}{n^3 - 4}.$$

Instead, in order to estimate the lower bound on $\overline{h}$ in the case of the r-regular random graph, we first have to calculate these quantities:

$$K = K(n) = 1 + \left\lfloor \frac{\log(N - \frac{2(N-1)}{r})}{\log(r-1)} \right\rfloor = 1 + \left\lfloor \frac{\log(\frac{10n^3 - 25n^2 - 20}{8n})}{\log(\frac{4n-5}{5})} \right\rfloor$$

$$R = R(n,K) = N - 1 - \sum_{j=1}^{K-1} r(r-1)^{j-1} = \frac{5}{4}n^2 - 1 - \sum_{j=1}^{K-1} \frac{4n}{5}\left(\frac{4n}{5} - 1\right)^{j-1}$$

Thus, from a known result we get the lower bound: $\overline{h} \geq \frac{\sum_{j=1}^{K-1} j \frac{4n}{5}(\frac{4n}{5}-1)^{j-1} + Kr}{\frac{5}{4}n^2 - 1}$

Our results are summarized in the following table, where both TH_{FT} and TH_J are intended to be upper bound on the traffic flow:

n	N	S	L	TH_{FT}	TH_J
5	31.25	31.25	93.75	0.03571	0.05621
10	250	125	750	0.00417	0.00638
15	843.75	281.25	2531.25	0.00122	0.00185
20	2000	500	6000	0.00051	0.00077
25	3906.25	781.25	11718.75	0.00026	0.00039
30	6750	1125	20250	0.00015	0.00023
35	10718	1531.25	32156.35	9e-05	0.00014
40	16000	2000	48000	6e-05	9e-05
45	22781.25	2531.25	68343.75	4e-05	7e-05
50	31250	3125	93750	3e-05	5e-05

What we learn from the table above is that the Jellyfish topology supports more flows at high throughput thanks to its average shortest path length.