ALGUNAS SUMATORIAS

1) $$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

2) $$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$$

3) $$\sum_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2} \right)^2 = \left(\sum_{j=1}^{n} j \right)^2$$

4) $$\sum_{j=1}^{n} j^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

5) $$\sum_{j=0}^{n} 2^j = 2^{n+1} - 1$$

6) $$\sum_{j=0}^{n} x^j = \frac{x^{n+1}-1}{x-1}$$

7) $$\sum_{j=1}^{n} j2^j = (n-1)2^{n+1} + 2$$

8) $\sum_{j=1}^{n} jx^j = \dfrac{x + x\left(\dfrac{1-x^n}{1-x}\right) - x^{n+1}n}{(1-x)}$

$$S = \sum_{j=1}^{n} jx^j$$

$$S = x + \sum_{j=2}^{n} jx^j$$

$$S = x + x\sum_{j=2}^{n} jx^{j-1}$$

$$S = x + x\sum_{j=1}^{n-1} (j+1)x^j$$

$$S = x + x\sum_{j=1}^{n-1} jx^j + x\sum_{j=1}^{n-1} x^j$$

$$S = x + x\sum_{j=1}^{n-1} jx^j + x\sum_{j=1}^{n-1} x^j + x\ nx^n - x\ nx^n$$

$$S = x + x\ \sum_{j=1}^{n} jx^j + x\ \sum_{j=1}^{n-1} x^j - x^{n+1}n$$

$$S = x + x\ \ S\ \ + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S - xS = x + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S(1-x) = x + x\left(\frac{1-x^n}{1-x}\right) - x^{n+1}n$$

$$S = \frac{x + x\left(\dfrac{1-x^n}{1-x}\right) - x^{n+1}n}{(1-x)}$$

9) $\sum_{i=2}^{n} \lfloor \log_2 i \rfloor = \sum_{j=1}^{\lfloor \log_2 n \rfloor - 1} j * 2^j + \left(n - 2^{\lfloor \log_2 n \rfloor} + 1\right) * \lfloor \log_2 n \rfloor$

i	$\lfloor \log_2 i \rfloor$	**Nro. de veces que se suma** $\lfloor \log_2 \rfloor$
2	1	$2 * \mathbf{1}$
3	1	
4	2	$2^2 * \mathbf{2}$
5	2	
6	2	
7	2	
8	3	$2^3 * \mathbf{3}$
...	...	
15	3	
16	4	$(20 - 16 + 1) * \mathbf{4} = 20$
17	4	
...	...	
n = 20	4	

$\sum_{j=1}^{\lfloor \log_2 n \rfloor - 1} j * 2^j$

$+$

$\left(n - 2^{\lfloor \log_2 n \rfloor} + 1\right) * \lfloor \log_2 n \rfloor$

Bibliografía

Cormen, Leiserson, Rivest y Stein (2009). *Apéndice A: Summations* en Introduction to Algorithms, 3er. Edición.