Biomedical Engineering Laboratory Report

NHS Medical Engineering in Practice and Stress Analysis

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Abstract

With the help of a thin and thick cylinder, a series of different parameters were measured, proved, and compared to theoretical values to show the behaviour of stress and strain against pressure. The experiment was divided in three stages: thin cylinder open condition, thin cylinder closed condition and thick cylinder. In all three stages, it was proven the linearity of the strain gauges proving that the results were accurate and proving that the position of the strain gauges influences the strain magnitude when pressure is applied. It was also proven that hoop stress magnitude does not change by moving the strain gauges in the axis. Strain and pressure are highly correlated as pressure influences the magnitude of strain. Young Modulus and Poisson's ratio were calculated proving a further relationship respectively between stress and strain and longitudinal and hoop strain. The calculated values for Young Modulus differed by 8% from the theoretical values showing that errors affected the data, it was concluded that the error must be due to the material being used repeatedly. Poisson's ratio calculated value had a difference of 3% from the theoretical value, this could be due to the fact that to calculate the gradient, a large section of the line was taken increasing the percentage of error difference or it could be due to the machine tolerance. The Mohr's circle drawn for both open and closed condition have effectively proven that there is a relationship between direct strain and shear strain and maximum shear strain can be determined by finding the distance between gauge 2 and 6 and dividing it by 2 as it is the radius of the circle. From the Mohr's circle, it was also proven that axial force in the thin cylinder closed condition, influenced the position of the circle and the magnitude, as all the values resulted to be tensile and smaller in magnitude compared to the open condition. When collecting data to analyse the thick cylinder, it was found that strain and stress are directly related as graphs 8 and 9 have very similar curves in comparison with each other. It was also proven that stress and strain diminish at square root intervals, demonstrating the relationship created by Lame's equation. In conclusion, the experiment was carried successfully.

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Introduction

Thin and thick cylinders have been used in many industrial engineering related applications to withstand low and high internal pressure. In this experiment, the difference between thin and thick cylinder will be elaborated and the role of internal pressure inside the cylinders analysed.

The experiment uses two different types of systems: thin and thick cylinder. The thin cylinder will be set in two different conditions: open and closed to examine the role of axial stress and its relationship to strain. The laboratory experiment was carried out with the help of a VDAS-B interface and a computer to create more accurate results.

Aim

The aim of the experiment is to analyse the role of pressure inside thin and thick cylinders in a different range of experiments to understand its relationship with stress and strain.

Objectives

- To find linearity of strain gauges in thin cylinder open condition, closed condition and thick cylinder.
- To find the relationship between stress and strain and deriving the Young Modulus of the material and comparing it to the theoretical value.
- To find the relationship between longitudinal and hoop strain and determining the Poisson's ratio and compare it to the theoretical value.
- To show how Mohr's circles can effectively represent the direct strain values and the relationship between direct strain and shear strain for both open and close thin cylinder conditions.
- To find the relationship between radius and radial and hoop strain in a thick cylinder.
- Show that compared to the other values, in every case for this experiment, longitudinal strain are the smallest.

Background

Background knowledge needed to understand the experiment and the results given.

Thin and thick cylinders

Cylinders have been used within the engineering sector for a series of different results such as pipes, engine cylinders, pressure or storage vessels, boilers or different types of tanks and contain a series of different fluids, gasses, or liquids. This means that the cylinders are subject to fluid pressure. [Slideshare.net. 2021. *Thin and thick cylinders*.]

Figure 1 showing an engine cylinder

[Grabcad.com. 2021.]

Figure 2 showing a storage vessel

[Globustechnomech. 2021. *Storage* Vessels]

Thin and thick cylinders are used in different circumstances because of their characteristics. Thin cylinders are used in low pressure equipment as the cylinder can only resist against internal pressure and therefore has less stress absorption capabilities compared to the thick cylinder, that can resist both internal and external pressure and therefore is used for high pressure applications as it can sustain higher stress. When considering the main difference between thick and thin cylinder, we have to consider if the thickness of the wall is less than 7% of its diameter and if it is less, then the cylinder is considered thin [Mech Eng.- Thin cylinders -online.com. 2021].

In the thick cylinder, stresses are at its maximum at the inner side and at its minimum at the outer side and therefore the distribution of the stress is not uniform throughout the thickness of the thick cylinder, unlike the thin, this causes the thicker cylinder to be a more complex case compared to thin and needs the help of Lame's theory to be solved.

Radial stress magnitude, unlike in thick cylinders, is negligible in thin cylinders compared to hoop and longitudinal stresses that are considered constants over the wall thickness (as shown in figure 3).

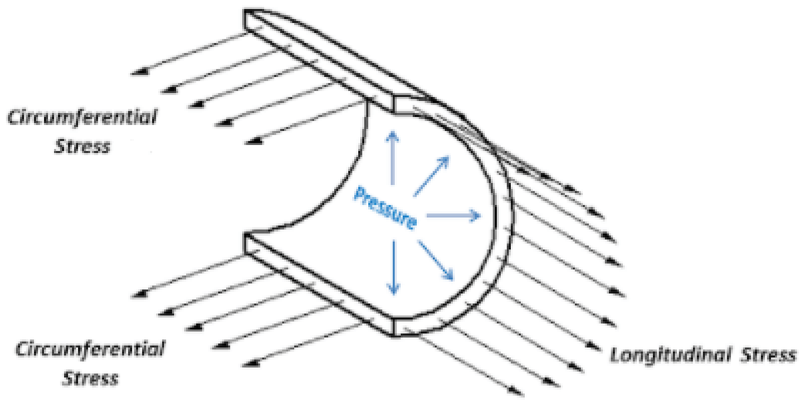


Figure 3 showing circumferential and longitudinal stresses in thin cylinder

[Trenchlesspedia.com. 2021. *What is Hoop Stress*?]

Background theory formulas

Stress

Stress defines the force applied on the area of the same body where the force is applied [Matmatch.com. 2021. *Basic Stress Analysis Calculations*].

$$\sigma = F/A$$

σ = stress obtained (N/m^2).

F= Force applied (N).

A = Area of the object (m^2)

Strain

Strain occurs when a force is applied to the body and causes the body to stretch or deform. [Brilliant.org. 2021. *Terms in Physics: Stress and Strain*].

$$\varepsilon = \Delta L/L$$

ε = strain obtained.

ΔL = change in length due to deformation (m).

L = original length (m).

Young's Modulus

The Young's Modulus measures the stiffness of a material when force is applied to said material. The stiffness might result to be compressive or tensile. [Encyclopedia Britannica. 2021. *Young's modulus*]

$$E = \sigma/\varepsilon$$

E = Young's Modulus (N/m^2)

σ = stress applied (N/m^2).

ε = strain applied.

Poisson's Ratio

When a material is shown to be compressed at one side is because it is been stretched in the other side and vice versa. Poisson's ratio measures the relationship between the transversal strain and axial strain. [BYJUS. 2021. *Poisson's Ratio - Longitudinal Strain and Lateral Strain*]

$$v = -\ \varepsilon\text{-lateral}/\ \varepsilon\text{-axial}$$

V = Poisson's ratio

ε-lateral = strain in direction of the load

ε-axial = strain at the right angle to the load

Background theory formulas relevant to thin cylinder

Direct stresses

Direct stresses in a thin cylinder are divided in three sections: radial stress that can be ignored due to its small magnitude, longitudinal stress acts on the length of the cylinder while the hoop stress acts on the diameter of the cylinder [Codecogs.com. 2021. *Direct Stress and Strain – Materials*].

$$\sigma H = pd/2t$$

σH = hoop stress

p = pressure applied (MPa).

d = diameter of the cylinder (mm).

t = thickness of the cylinder (mm).

$$\sigma L = pd/4t$$

σL = longitudinal stress.

p = pressure applied (MPa).

d = diameter of the cylinder (mm).

t = thickness of the cylinder (mm).

Principal strains

The maximum and minimum strain created at a right angle from each other are the maximum hoop strain and maximum longitudinal strain, these values also represent the diameter of the Mohr's circle [Amesweb.info. 2021. *Plane Strain and Principal Strains*.].

For open condition:

$$\varepsilon H0 = \sigma H/E$$

$$\varepsilon L0 = -v\sigma H/E$$

For closed condition:

$$\varepsilon Hc = (\sigma H - v\sigma l)/E$$

$$\varepsilon Lc = (\sigma l - v\sigma H/E$$

Background theory for thick cylinder

Theoretical strain

$$\varepsilon H = 1/E(\sigma H - v\ \sigma R)$$

$$\varepsilon R = 1/E(\sigma R - v\ \sigma H)$$

$$\varepsilon L = -v/E(\sigma H + \sigma R)$$

Theoretical stress

Minimum $\sigma H = (2p)/(K^2-1)$

Maximum $\sigma H = p(K^2+1)(K^2-1)$

P = applied internal pressure

K = outer radius/ inner radius

Shear stress

Shear stress is the force causing the deformation on the object where a parallel surface of the material itself slips against another one [Study 2021- What is shear stress?].

$$\tau = 1.065p$$

Method

In this section, the procedure to follow will be analysed to allow users to repeat the experiment in the same laboratory settings and the list of equipment has been introduced to understand what is being used to do the experiment.

Procedure

Thin cylinder open and closed condition

Once ready to start the experiment, allow the machine 15 minutes in order to let the oil warm up and start the experiment.

To set up the open-ended conditions, turn anticlockwise the pressure control by screwing the hand wheel.

Then, on VDAS, on Cylinder Parameters, set as experimental set up ‘Open Ends’.

Now, allow the gauge readings to reach zero by turning clockwise the pressure control and hold on ‘Press and zero’ button.

Once the results for strain gauges and pressure reach zero, record the values on the excel table by pressing ‘Add on table’ under options in the VDAS application.

For this experiment, drive anticlockwise the hand pump until it reaches 0.5MPa of pressure, record the values and repeat at intervals of 0.5MPa until it reaches 3.5MPa.

Unlike, the open-ended condition, to start closed end condition, turn anticlockwise the pressure control, and unscrew the hand wheel.

Then, on VDAS, select ‘Closed Ends’ on Cylinder Parameters and close the pressure control by turning it clockwise. Again, to allow the reading to reach zero, press the ‘press and hold to zero’ and insert the results. Unscrew the adjuster at intervals of 0.5Mpa to obtain the readings needed until the pressure reaches 3.5MPa and record the data with the help of VDAS at each interval.

Thick cylinder

This cylinder needs 30 minutes to warm up be ready.

Open a different version of VDAS for the thick cylinder in the computer and connect the VDAS-B Interface to the computer and electronic pressure transducer in the thick cylinder digital output. The Tech equipment VDAS option captures the results at different pressure intervals of 1MPa to a maximum of 7MPa and creates the table on Excel.

List of equipment

A Thin Cylinder SM1007 used for thin open and closed condition experiment is needed.

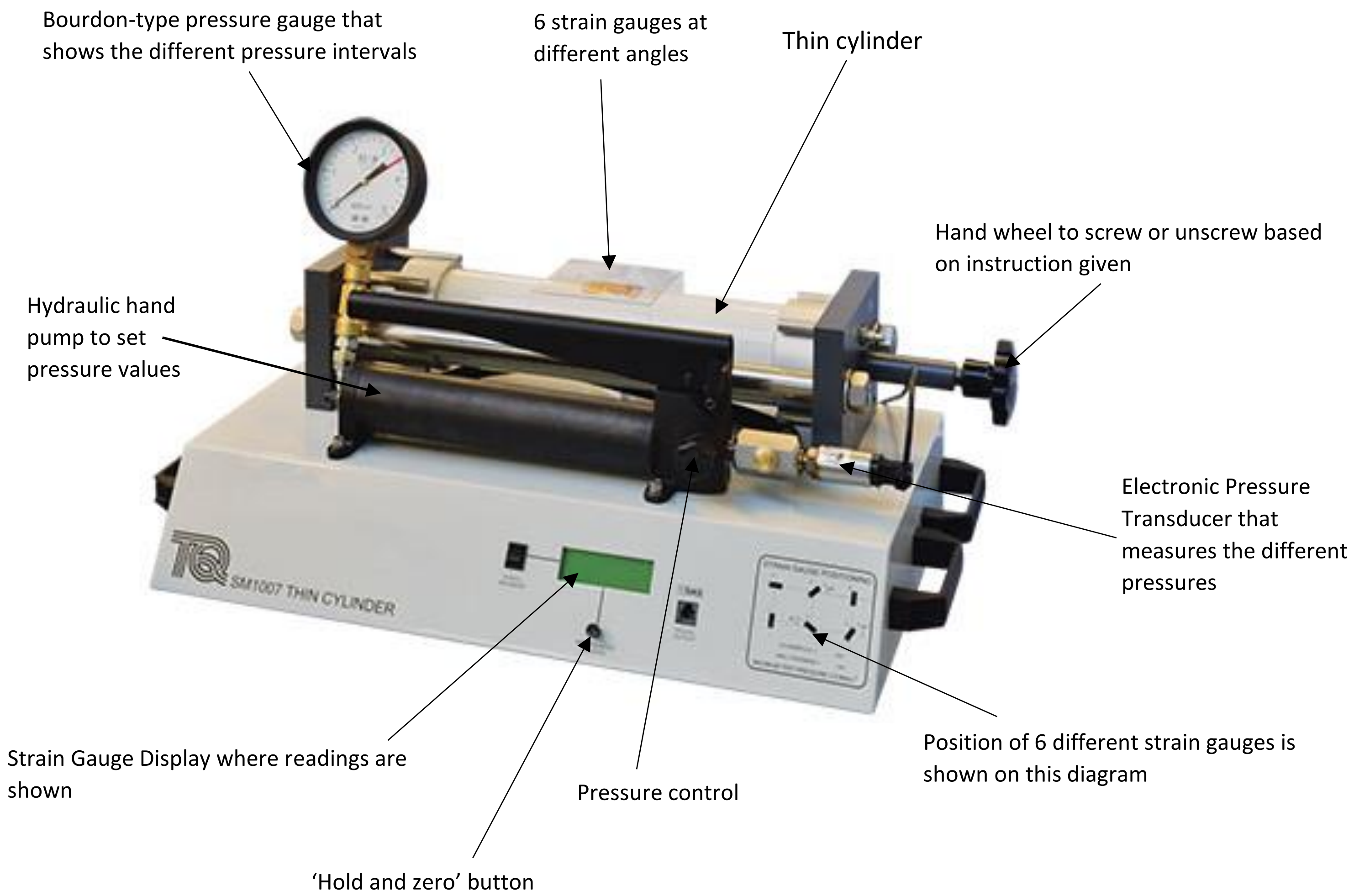


Figure 4 showing the thin cylinder apparatus

[TecQuipment. 2021. *THIN CYLINDER*.]

A Versatile data measurement system with both thin and thick cylinder app downloaded in the computer to collect the data and needed cables.

Figure 5 showing VDAS-B hardware and software [Cste.sut.ac.th. 2021.]

A thick Cylinder SM1011 is used to calculate the data for the thick cylinder experiment in combination with the VDAS Interface and software.

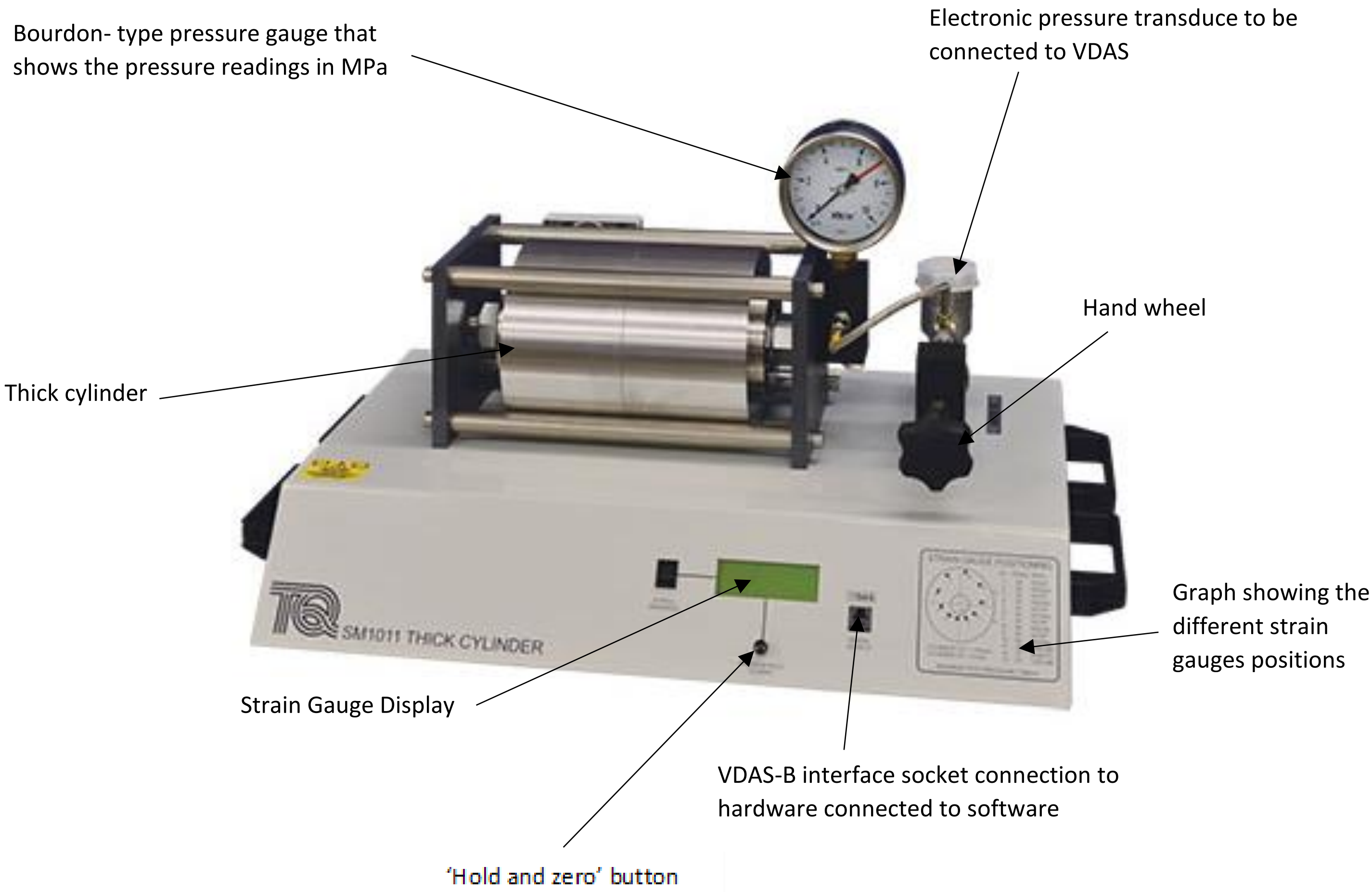


Figure 6 showing thick cylinder SM101 [Ayva.ca. 2021. *TecQuipment Thick Cylinder – SM1011 – AYVA Educational Solutions*.]

Results: data collection and analysis

All the main results have been listed and labelled below.

Experiment 1 open condition thin cylinder

The first experimental section is related to the thin cylinder when set on open condition. In this experimental section, Young's Modulus and Poisson's ratio values will be found and the difference between actual and theoretical Mohr's circle will be analysed.

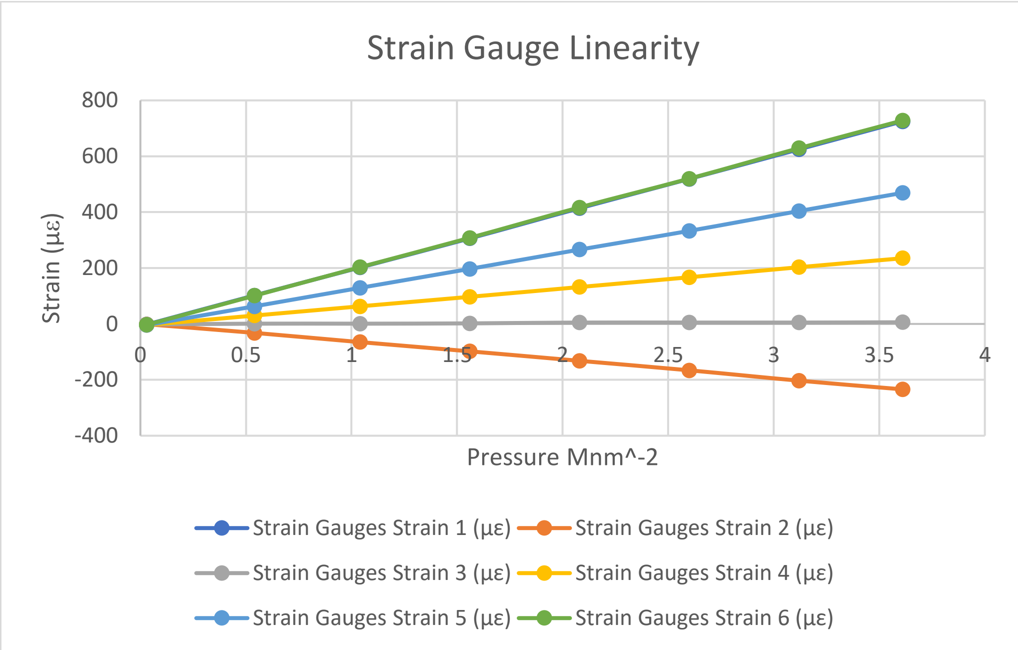
Table 1 shows Strain Gauges 1 to 6 and the corresponded pressure and hoop stress used to calculate them. It also summarises the needed values to draw both actual and theoretical Mohr's circle. It can be noticed that gauges 1 and 6 give us relatively similar values and indicate the hoop strain and gauge 2, as we can notice from the negative sign, is indicating the longitudinal strains. This graph highlights the differences between the actual and theoretical Mohr's circles, and the relationship between the intersection of the actual Mohr's circle with the x axis and the strain gauges from the experiment itself.

Cylinder Condition: OPEN ENDS								
		Direct Hoop Stress ($MN.m^{-2}$)	**Strain (x 10^-6)**					
Reading	**Pressure ($MN.m^{-2}$)**		**Gauge 1**	**Gauge 2**	**Gauge 3**	**Gauge 4**	**Gauge 5**	**Gauge 6**
1	0.03	0.40	-2	-1	-2	-4	-4	-3
2	0.54	7.20	102	-32	1	30	63	101
3	1.04	13.87	202	-65	1	63	129	203
4	1.56	20.80	306	-98	2	97	197	308
5	2.08	27.73	414	-132	5	132	266	417
6	2.60	34.67	519	-166	5	167	333	520
7	3.12	41.60	625	-203	5	203	404	629
8	3.61	48.13	725	-234	6	235	469	728
Values from actual Mohr's Circle (at 3. 5MN.m^2)			726.5	-234.0	6.1	246.0	486.0	726.5
Values from theoretical Mohr's Circle (at 3. 5MN.m^2)			675.3	-222.8	1.7	249.0	450.8	675.3

Table 1 showing conditions for thin cylinder open ends

Strain Gauge linearity

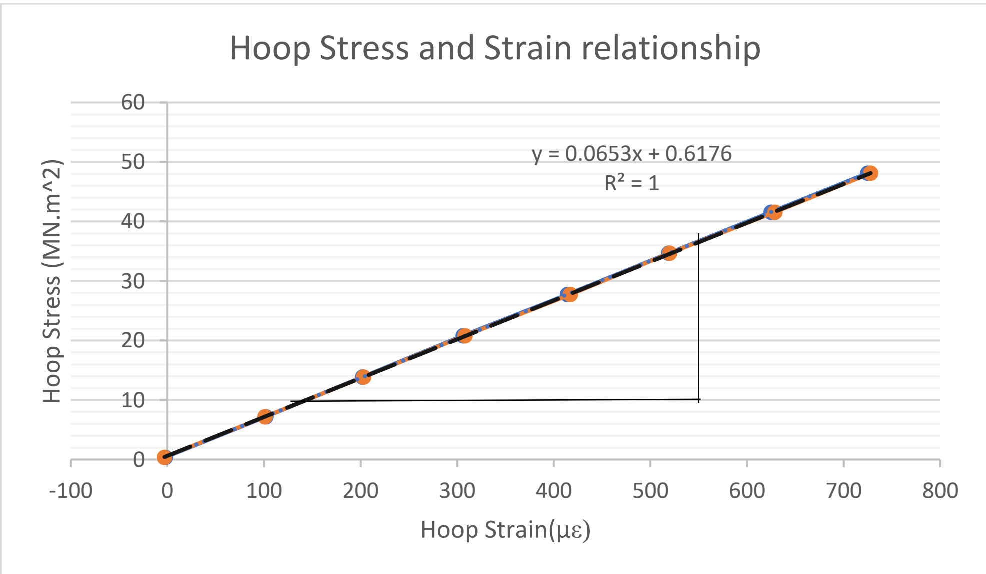
Graph 1 shows that there is proportional correlation between Strain and Pressure in every strain gauge regardless of the degree they are positioned at. The graph shows all 6 strain gauges with strain gauge 1 and 6 that have really similar values at the point where they superimpose.



Graph 1 shows strain gauge linearity for thin cylinder open conditions

The Hoop stress and strain relationship

Graph 2 below shows the relationship between Hoop stress and strain by using gauge 1 and 6 and the hoop stress provided in table 1. The direct hoop stress used is between 0 and 3.61MPa. The gradient of the graph is the young Modulus and set to be of 63.3 x 10^9Pa. The R^2 score shows high correlation between stress and strain.

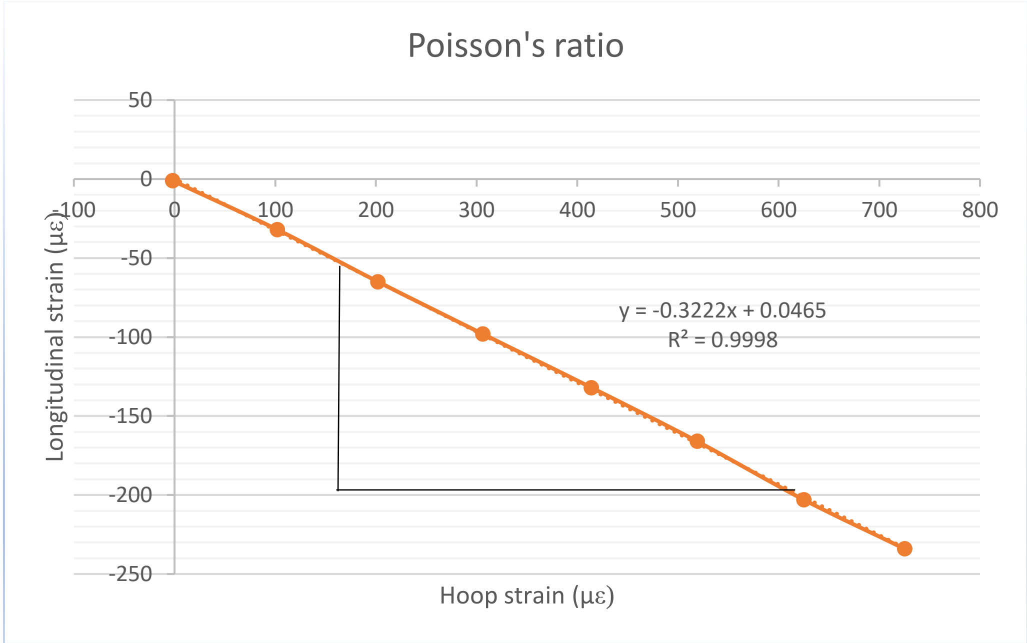


Graph 2 shows Hoop Stress and Strain relationship in a thin cylinder

Young modulus (E) = Hoop Stress/ Hoop Strain = 65.3×10^9

The theoretical Young's Modulus is set to be at 69×10^9MPa creating an error difference of 5.4%.

The relationship between the negative longitudinal strain (strain gauge 2) and strain gauge 1 that indicates the hoop strain, allows us to find the Poisson's ratio that is set to be equal to -0.32.



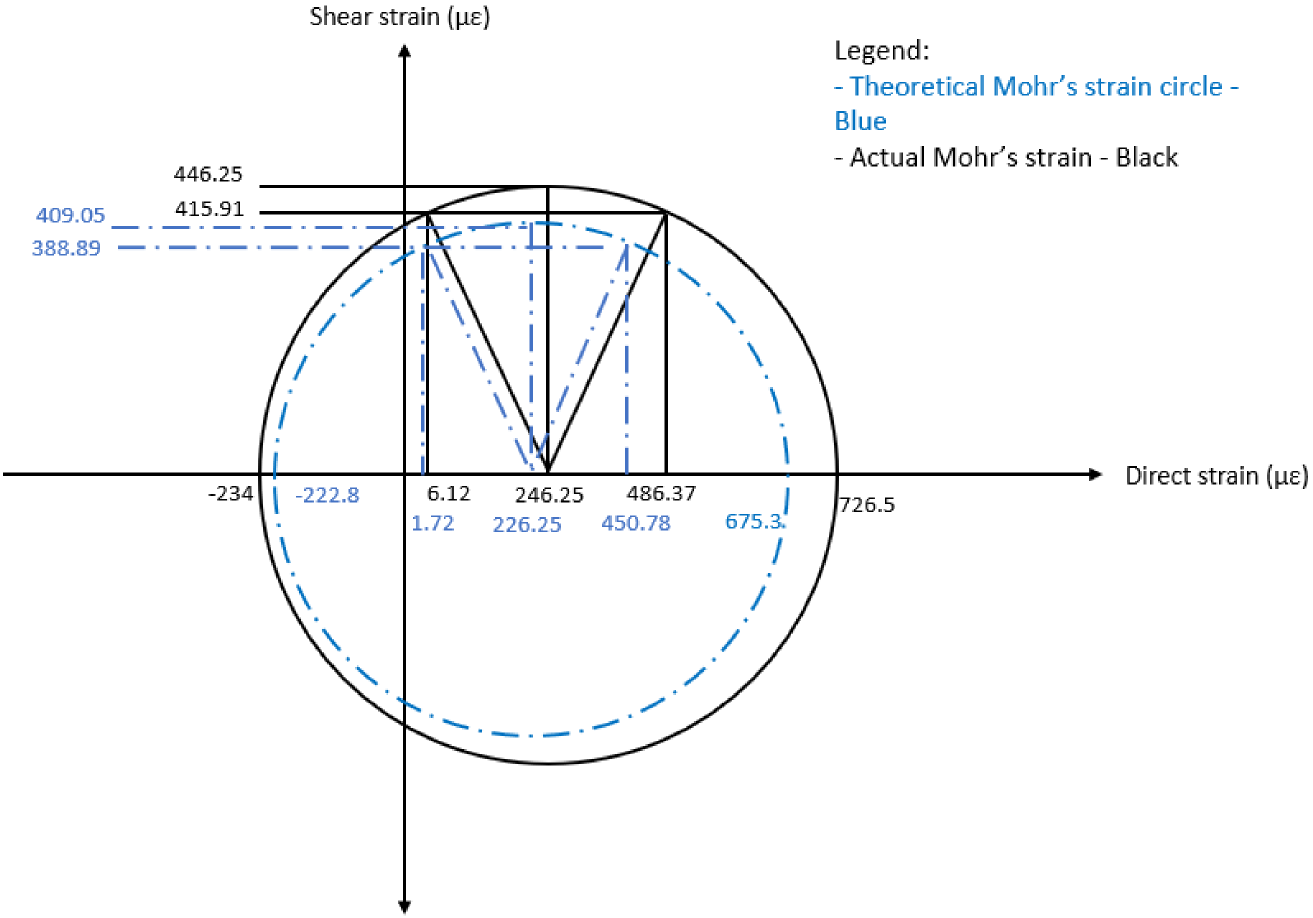
Graph 3 shows Poisson's ratio

Poisson's ratio (v) = - Longitudinal strain/Hoop strain = -0.32

The theoretical Poisson's ratio is set to be -0.33, creating an error difference of 3%.

Principal strain and Mohr's circles

Two Mohr's circles have been drawn above each other to show the difference between theoretical and actual values and find the principal strain values and the direct intermediate strain values at an angle of 60 and 120 degrees to find the corresponded shear strain values at a normal from the horizontal axis and the maximum shear strain applicable.



Graph 4 shows Mohr's circle for open condition

Main values have been listed in table 3 with error differences. On the left, the theoretical values are shown in blue, and on the right of table 3, the actual values followed by the percentage error difference between the two. It is important to notice that error difference between Gauge 3 theoretical and actual is at 255%, showing that the actual value is 255% bigger than the theoretical value, all the other values are under 10% of difference.

Theoretical values (με)		Actual values (με)		% error difference
Gauge 2	-222.8	Gauge 2	-234	5.0%
Gauge 3	1.72	Gauge 3	6.12	255%
Gauge 4	226.25	Gauge 4	246.25	8.8%
Gauge 5	450.78	Gauge 5	486.37	7.8%
Gauge 6	675.3	Gauge 6	726.5	7.6%
Maximum shear strain	409.5	Maximum shear strain	446.25	8.9%
Maximum principal strain	388.89	Maximum principal strain	415.91	6.9%

Table 2 showing main findings from the Mohr's circle for open condition

Experiment 2 close condition thin cylinder

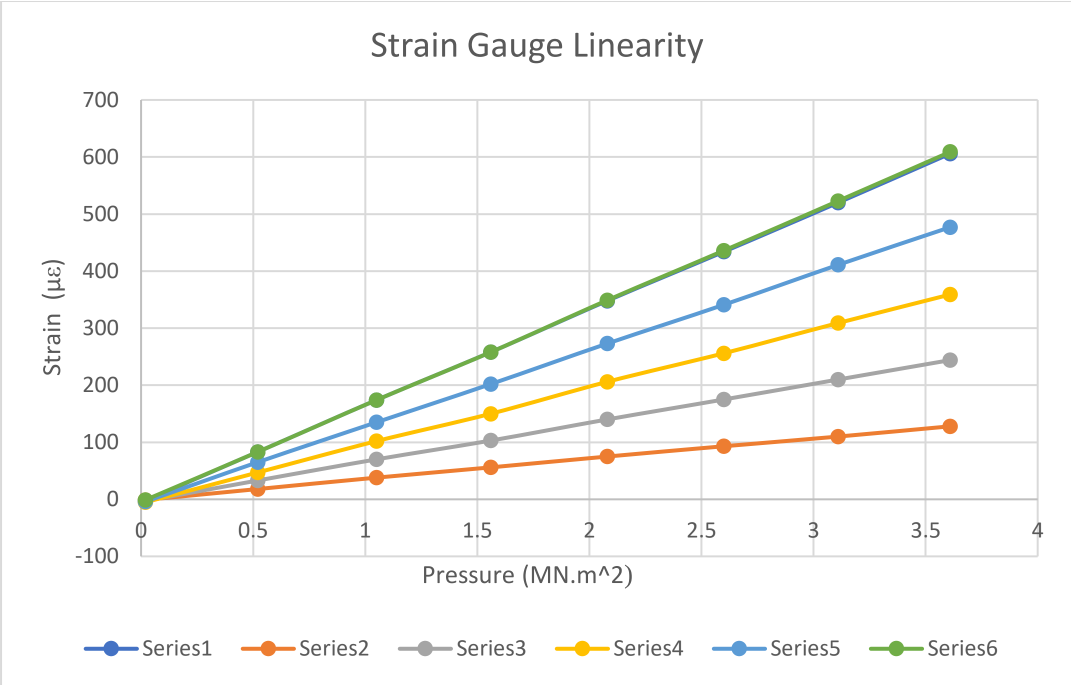
Table 4 shows the strain gauges obtained from the experiment for closed condition while the pressure increased until it reached 3.61MPa. It also shows the direct hoop stress used in the experiment and the values calculated for the theoretical Mohr's circle and the actual values obtained by using the experimental strain gauges at the set pressure of 3.5MPa. It is important to notice that unlike open ends conditions, gauge 2 is positive.

Cylinder Condition: CLOSED ENDS								
Reading	**Pressure ($MN.m^{-2}$)**	**Direct Hoop Stress ($MN.m^{-2}$)**	**Strain**					
			Gauge 1	**Gauge 2**	**Gauge 3**	**Gauge 4**	**Gauge 5**	**Gauge 6**
1	0.02	0.27	-2	-1	-3	-5	-4	-1
2	0.52	6.93	83	18	33	47	65	83
3	1.05	14.00	174	38	70	102	135	174
4	1.56	20.80	258	56	103	150	202	258
5	2.08	27.73	348	75	140	206	273	349
6	2.60	34.67	434	93	175	256	341	436
7	3.11	41.47	520	110	210	309	411	523
8	3.61	48.13	606	128	244	359	477	609
Values from actual Mohr's Circle (at 3. 3.5MN.m^2)			607.5	128.0	247.6	339.8	487.3	607.5
Values from actual Mohr's Circle (at 3. 3.5MN.m^2)			563.9	114.8	229.4	324.6	451.3	563.9

Table 3 shows closed ends condition for a thin cylinder

Strain Gauge Linearity

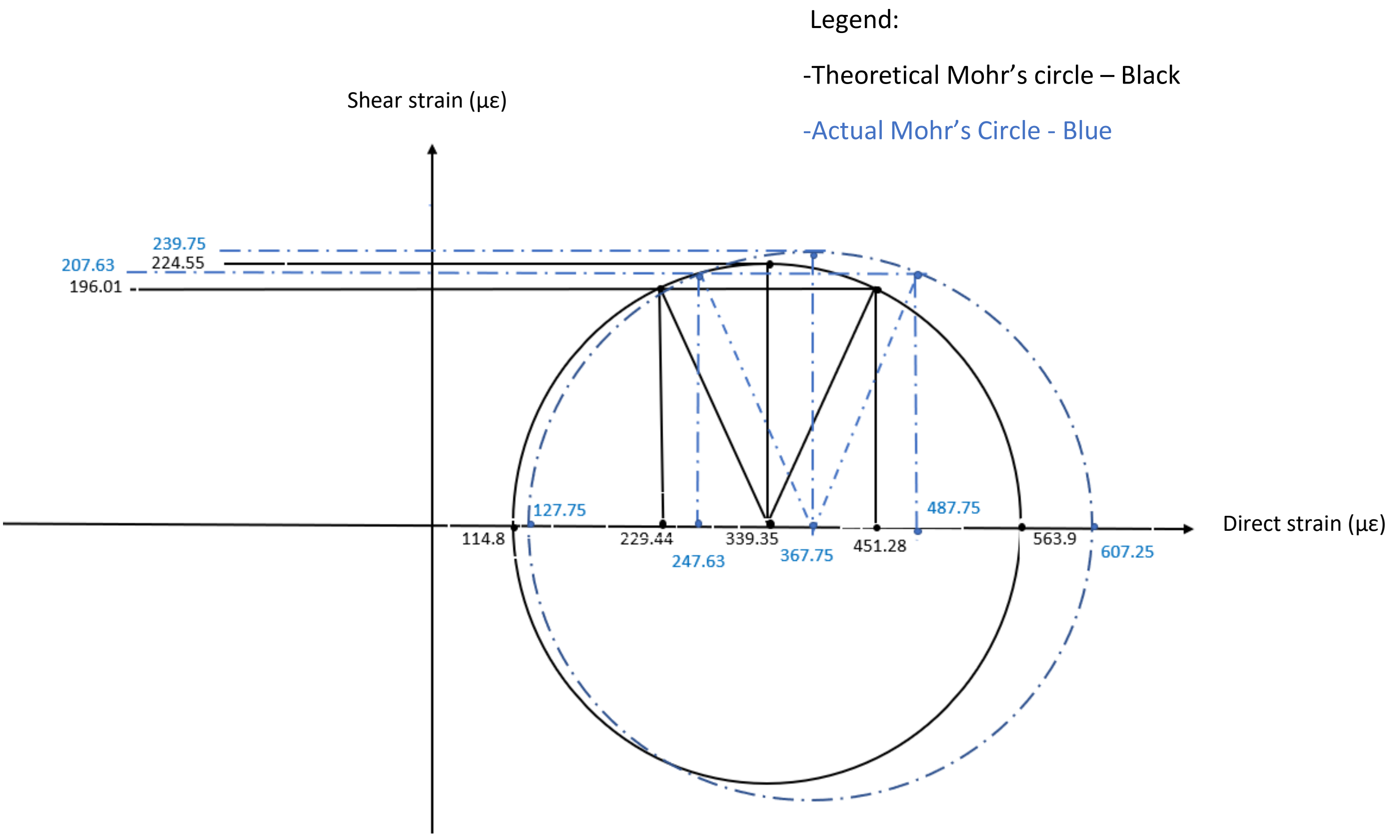
The aim of showing graph 5 is to show the relationship between pressure and strain and the linearity relationship obtained within the closed ends condition in the thin cylinder. Gauges 1 and 6 are superimposing each other again.



Graph 5 shows strain gauge linearity for a closed end thin cylinder

Theoretical principal strain from superposition and Mohr's circle

This graph shows the relationship between theoretical and actual Mohr's circles when closed condition is applied.



Graph 6 shows the Mohr's circle for closed ends condition of a thin cylinder

Table 5 shows the main findings from the Mohr's circles drawn in comparison with each other and the % error difference that is lower than 10% for each point showing high correlation and accuracy between the two circles.

Theoretical values (με)		Actual values (με)		% Error difference
Gauge 2	114.8	Gauge 2	127.75	11%
Gauge 3	229.44	Gauge 3	247.63	7.9%
Gauge 4	339.35	Gauge 4	367.75	8.4%
Gauge 5	451.28	Gauge 5	487.75	8.08%
Gauge 6	563.9	Gauge 6	607.25	7.68%
Maximum shear strain	224.55	Maximum shear strain	239.75	6.77%
Maximum principal strain	196.01	Maximum principal strain	207.63	5.9%

Table 4 showing main findings from the Mohr's circle

Thick cylinder

This section of the experiment shows the main findings from the thick cylinder experiment and highlights points of discussion.

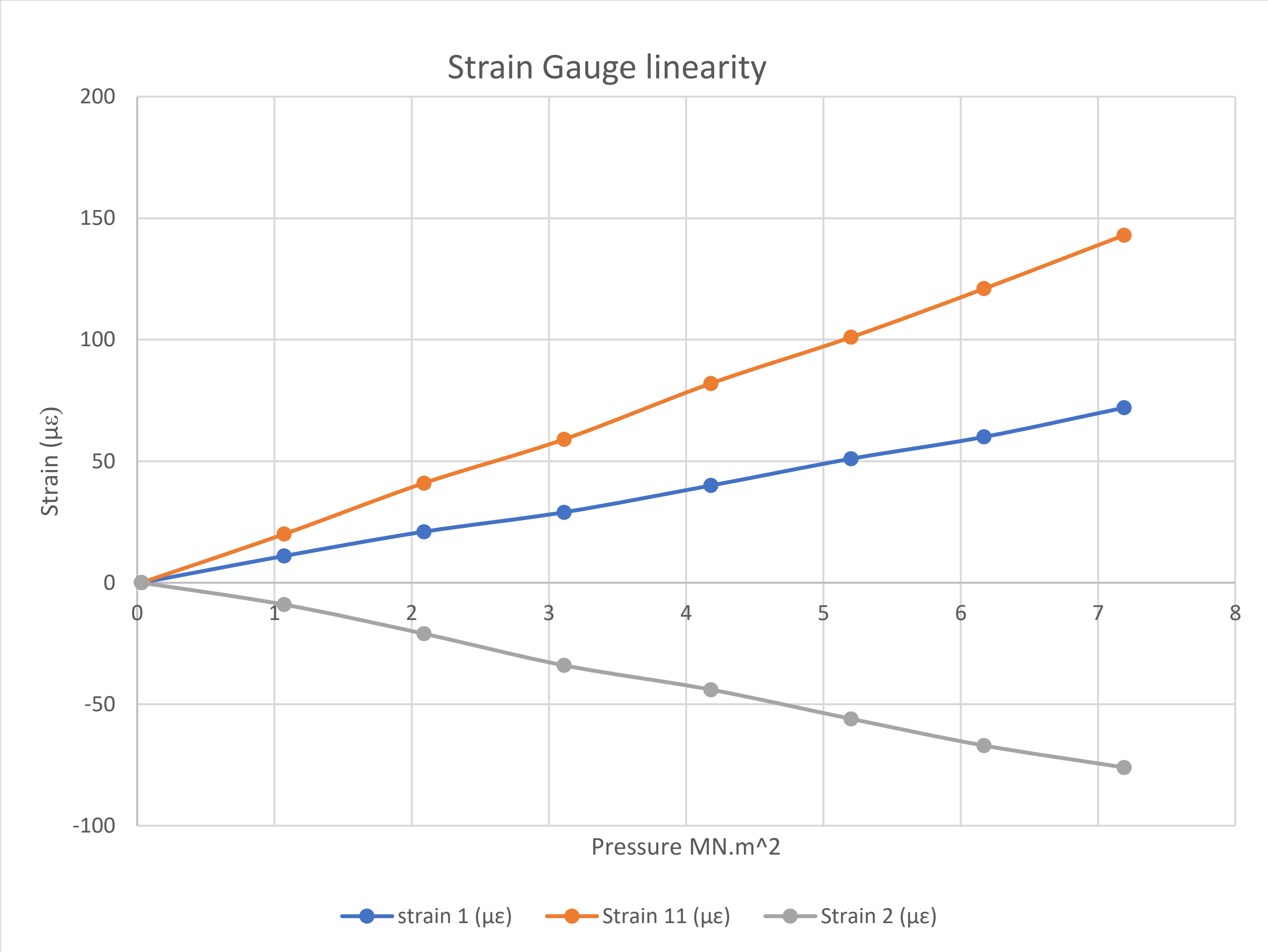
Strain gauge linearity

Table showing strain gauge 1, 2 and 11 used to plot the strain gauge linearity graph against pressure provided in the Excel spreadsheet of the experimental results. It is relevant to notice that strain gauge 2 is shown as positive in the thick cylinder conditions.

Pressure ($MN.m^{-2}$)	**Strain ($x10^{-6}$)**		
	Gauge 1	**Gauge 2**	**Gauge 11**
0.03	0	0	0
1.07	11	20	-9
2.09	21	41	-21
3.11	29	59	-34
4.18	40	82	-44
5.20	51	101	-56
6.17	60	121	-67
7.19	72	143	-76

Table 5 showing gauge 1, 2, 11 and pressure values for a thick cylinder

Graph 7 shows linear relationship between the different strain values for strain 1,2 and 11 that are respectively hoop, longitudinal and radial strains against pressure. These values aim to show us the accuracy of strain gauge results.



Graph 7 showing strain gauge linearity for a thick cylinder

Strain and Stress Distribution

Theoretical hoop strain for the thick cylinder was calculated by combining Lame's equations and obtaining $[P/K^2\text{-}1) \cdot E] \cdot [[1+R^2/r^2]\text{-}v[1\text{-}R^2/r^2]]$.

Theoretical radial strain for the thick cylinder was calculated with $[P/K^2\text{-}1) \cdot E] \cdot [[1\text{-}R^2/r^2]\text{-}v[1+R^2/r^2]]$.

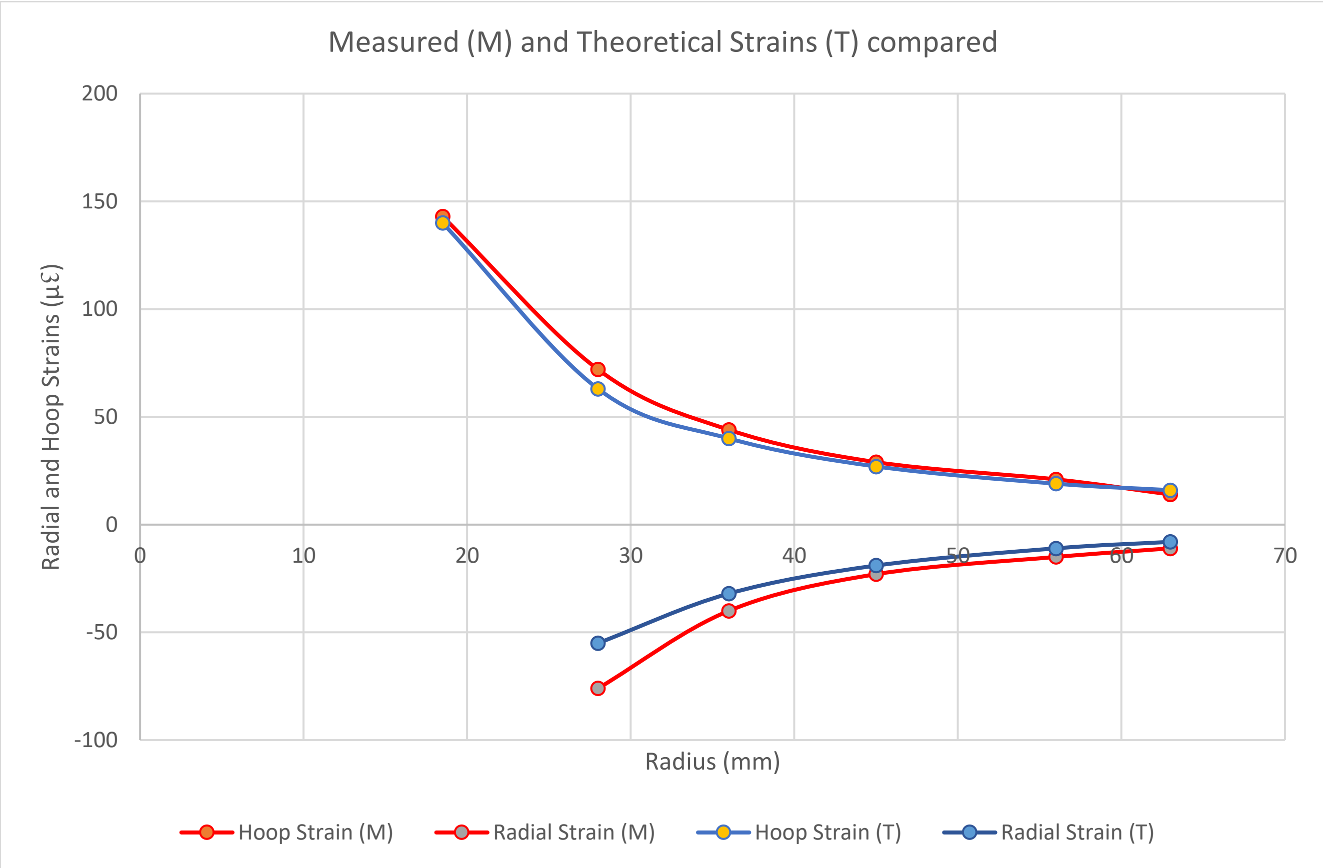
In these two derived equations, P is set to be 7MPa. K is the ratio between inner and outer radius. E is set to be 73GPa and v is -0.33, r is the radius at the different gauges.

Table 7 shows the error difference between theoretical and actual strain values at different radius in the thick cylinder.

Gauge Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Radius at gauge (r)	**28**		**36**		**45**		**56**		**63**		**18.5**	**75**	
Type of Strain	ε_H	ε_R	ε_H	ε_R	ε_H	ε_R	ε_H	ε_R	ε_H	ε_R	ε_H	ε_L	ε_H
Measured Strain (x 10^{-6})	72	-76	44	-40	29	-23	21	-15	14	-11	143	1	10
Calculated Strain (x 10^{-6})	63	-55	40	-32	27	-19	19	-11	16	-8	140	1	12
Error Difference (%)	14	24	10	25	7	21	11	36	13	38	2	0	2
Cylinder Outside Radius: 75 x 10^{-3}						**Cylinder Internal Radius:** 18.5 x 10^{-3}							
Cylinder Pressure: 7MPa													

Table 6 showing strain and stress distribution in a thick cylinder

Graph 8 illustrates the difference between the theoretical and actual values. It can be noticed that the curves superimpose each other on most results showing that the values start to get more and more accurate as the radius increases.



Graph 8 showing comparison between measured and theoretical strains

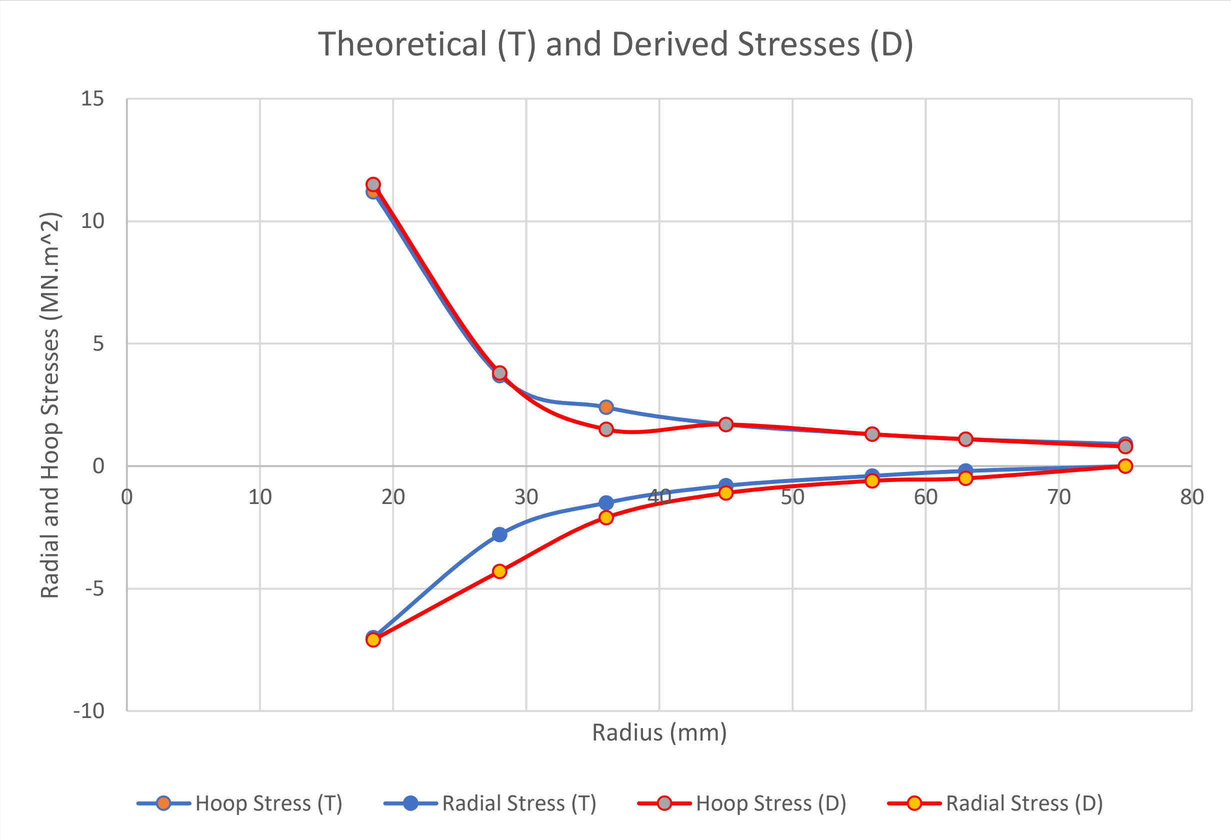
Shear Stress and derived Stress

By using σH= $(E/1-v^2) \cdot (v\varepsilon R+\varepsilon H)$ and σR= $(E/1-v^2) \cdot (v\varepsilon H+\varepsilon R)$, the values in table 8 were calculated using strain data in table 7. The values obtained are the theoretical stress and the ones derived from the actual calculations. It can be noticed that these stress values decrease as radius is increased.

Radius (mm)	**$1/r^2$ (mm) x 1000**	**Theoretical Stress ($MN.m^{-2}$)**		**Derived Stress ($MN.m^{-2}$)**	
		Hoop	**Radial**	**Hoop**	**Radial**
18.5	2.92	11.2	-7	11.5	-7.1
28	1.28	3.7	-2.8	3.8	-4.3
36	0.77	2.4	-1.5	2.5	-2.1
45	0.49	1.7	-0.8	1.7	-1.1
56	0.32	1.3	-0.4	1.3	-0.6
63	0.25	1.1	-0.2	1.1	-0.5
75	0.18	0.9	0	0.8	0
Pressure: 7MPa Shear stress: 7.4MPa					

Table 7 showing shear stress and derived stress in a thick cylinder

With the values calculated and obtained, a comparison graph curves were made to show similarities and differences between the theoretical (T) hoop and radial strain curve and the measured (M) hoop and strain curve.



Graph 9 showing theoretical and derived stresses in comparison

Discussion

Strain gauge Linearity thin and thick cylinder

When measuring strain gauges linearity, we are measuring the strain resistance to the pressure applied. From these graphs (Graphs 3, 5 and 7), we can notice that as pressure increases, the values of strain increase too, and this means that there is a proportional relationship between pressure and strain. We are considering direct proportionality with strain gauges 1, 3, 4, 5 and 6. This means that direct proportionality is seen when the strain gauges are positioned between 30 and 90 degrees. Strain gauge 3 values do change when pressure is applied in graph 3 but the changes in strain are small in magnitude and hard to notice, therefore it seems like the strain magnitude does not increase for strain gauge 3. All these values are tensile strain values while strain gauge 2 is compressive strain as longitudinal strain is compressive and therefore the results, in the strain gauge linearity graph 3, are negative because of this, the relationship between pressure and strain gauge 2 is inverse proportionality. Strain gauge 2 shows compressive results as the current created from the given pressure is flowing on the opposite side. As both values for gauges 1 and 6 are measuring hoop strain and therefore at 90 degrees, in the graph, their values are identical.

In the closed condition, shown in graph 5, the strain gauges values are all positive and therefore it means that the current is flowing in the same direction and each strain gauge is experiencing tensile strain. The strain gauges are increasing at the same rate when pressure is increased creating direct proportionality. The only difference between all these strains, as they are receiving the same pressure, is the angle indicating that the angle at which the strain is located, influences the magnitude of the strain when the same amount of pressure is applied. Again, strain gauge 2 is the lowest value as it is the longitudinal strain and strain gauges 1 and 6 are correlated but also the highest values. In the closed condition, the values are all positive as the force created by the internal pressure at the ends points of the cylinder will be applied and create a longitudinal strain on the same walls, creating the positive values observed [Alkazraji, D., 2021. *Design Approach*.]. Another main difference between open and closed ends conditions is that in open ends, strain gauge 3 tends to be near the x-axis and a significantly small value while in the closed ends, its magnitude increases of 244 με at 3.5MPa against the 6με of the open-ends condition.

From both graphs 1 and 5, we can notice that hoop strain magnitude does not change along the length of the cylinder as gauges 1 and 6 have similar magnitudes even though they are positioned at different distances.

When observing the thick cylinder linearity graph 7, it can be spotted that only three strain gauges have been used: strain gauges 1, 2 and 11 that respectively are hoop strain, longitudinal and radial strain gauges. In the thin cylinder, because radial strain magnitudes tend to be very small and reach approximately zero, they are usually ignored but in thick cylinders, as it can be noticed from the linearity graph, the values tend to be very large and therefore have to be considered and this is the reason Lame's equation is used when observing thick cylinders. In thick cylinders, the magnitude of the longitudinal and hoop strains is significantly smaller compared to the thin cylinder when subject to the same pressure showing that the same amount of pressure creates less deformation in thick cylinders compared to thin. It is also shown that all the strain values in the thick cylinder are tensile from the positive direction of the straight lines, except for strain gauge 2, similar to the open condition for the thin cylinder.

All three graphs (graphs 3,5, and 7) are showing a linear relationship between the strain gauge values as the different strains at set pressure values can be connected with a straight line. This shows that the values tend to be at the right intervals and no anomaly is shown.

Young Modulus and Poisson's Ratio

As shown in the background, the relationship between stress and strain creates what is known as Young Modulus, the measurement of the stiffness of the material based on tensile stress as the material has been stretched. In this part of the experiment, the direct relationship between Hoop stress and strain starts at 0.6176 MPa and the young's modulus is 65.3MPa. These values are set to be accurate as r^2 is 1 and therefore the regression line fits the data created during the experiment and there is a high level of correlation between Hoop strain and stress. This graph was made using gauges 1 and 6 as they are the strain gauges showing hoop strain, against pressure. It also shows that the two strain gauges are strictly related to each other as mentioned in the strain gauge linearity.

If a different material such as steel, with a Young modulus of 210GPa [Steeloncall. 2021], was used and stress maintained at the same rate, the strain would have resulted to be three times smaller than aluminium's strain.

Poisson's ratio was calculated by using the longitudinal strain (strain gauge 2) and the hoop strain (strain gauge 6). The correlation between these values starts at 0.0465 and values tend to be accurate as the straight line created has a r^2 value of 0.9998. The gradient, defining Poisson's ratio, is -0.32. The value of Poisson's ratio is negative as the longitudinal strain is compressive. This value defines that per each millimetre of deformation created in the direction of where stress is applied, there will be a 0.32-millimetre deformation perpendicular to the same direction of stress applied and therefore the material cannot sustain high elastic deformations.

The theoretical values set for young's Modulus are $69x10^9$Pa as the material is aluminium, creating an error difference between measured and theoretical values of 8%. For Poisson's ratio, this error difference is set to be 3%. This difference in values could be related to the calibration of the systems in the laboratory settings as results are based on the stiffness of the machine and the sample itself. As the samples are used often and repeatedly, to do the experiment, the error difference could be due to the sample being damaged as well as altering slightly its properties.

As to find the Young's Modulus, both strain gauges 1 and 6 were used, this could be the cause of such discrepancy between the values for Young Modulus as Poisson's ratio theoretical and actual values tend to be more similar.

The linearity is given in graphs 2 and 3, which further state the linearity of the strain gauges values used.

Mohr's circles

This discussion analyses graphs 4 and 6 and tables 2 and 4.

The principal strains in the Mohr's circles are determined by strain gauge 2 that is in compression and therefore the lowest value. Strain gauges 1 and 6 are the highest values as they are positioned at 90 degrees and indicate the hoop stress. Strain gauge 4 is the centre of the circle. Strain gauge 3 is the low intermediate point of the circle at 60 degrees from the centre and strain gauge 5 is the high intermediate point positioned at 120 degrees from the centre. These two points are a mirror image of each other and therefore have the same y-axis value. The maximum shear strain that indicates the maximum shear stress is indicated by the y axis coordinates of the strain gauge 4 as it is at 45 degrees from the horizontal. Being 45 degrees from the horizontal axis shows that the maximum amount of direct strain is converted into shear strain, determining the radius of the circle. Gauge 3 and 5 show the maximum normal shear strains that can be applied when principal direct strain is applied.

In both circles, the minimum principal strain between theoretical and actual Mohr's circles are near each other while the circles distance themselves as values reach the maximum principal strain values. The radius of the theoretical circle is smaller than the radius of the actual Mohr's circle due to the theoretical one having a smaller value for the average of gauge 1 and 6, this is probably because the theoretical value was obtained at 3.5MPa while the actual value was obtained at 3.61MPa.

Closed condition theoretical Mohr's circle radius is significantly smaller compared to open condition one as the strain values tend to have a smaller interval gap when pressure is increased, this could influence why the shear strain values for closed condition, when examining differences between theoretical and actual Mohr's circle, is within an acceptable range.

It is important to consider that the maximum pressure was found manually and therefore this creates discrepancies in the data as the pressure does not have identical intervals when increasing.

The actual Mohr's circles for both closed and open conditions, predict the direct strain values within an acceptable range of error of less than 10% determining that the values are accurate.

As radius increases, maximum shear strain increases too and with it, the values for principal strain value. This is why the values for shear strain in the open condition are bigger than the values for shear strain in the closed condition. Mohr's circle in the open condition is both in the negative and positive x-axis, further proving that the longitudinal strain gauge 2 is compressive while strain gauge 2 for closed ends is tensile. In the open condition, we can notice from table 3 that there is an interval of around 200 με between each direct strain value while in the closed condition, this interval between values is of 100 με. This difference is because closed ends have direct axial stress caused by the axial load while open condition does not have an axial load. This means that the open condition does not have a force along the x-axis created by internal pressure. The axial load in the closed-end condition stretches the length of the material compressing the cross-sectional area, due to Poisson's effect, this is what causes the Mohr's circle values for the closed condition to be all positive and causes the shear strains to be smaller compared to the open ends condition's shear strains [Yang, K., 2018. *Basic finite element method as applied to injury biomechanics*. pp.231-256.]

Strain and stress, theoretical and derived stress in thick cylinder

When calculating the shear and stress distribution, we can notice that the hoop stress decreases as the radius increases (table 7 and graph 8). This can be found in Lame's equation where radius and stress are inversely proportional and therefore, strains are inversely proportional to the radius as well. And therefore, as stress is inversely proportional to r^2, the stress decreases in each reading of the square root of the reading before making it a square root regression. From graph 8, we can observe that most values match the theoretical values calculated except for the radial strain at 24mm where there is a percentage error of 24% is present. This is probably since it was the first reading and the machine was not warm enough when it started to be able to give accurate values. The strain gauges tend to be more accurate as we differ from the centre of the cylinder.

We can also notice that the last readings in the table are at 18.5mm and 75mm that are the inner and outer radius of the cylinder as well and are the circumferential strains.

As 75mm is the outer circumference and 18.5 the inner circumference, their radial strain is neglected due to the fact that opposite to the internal pressure, atmospheric pressure is applied.

It can be noticed from the theoretical and derived stresses graph 9, that the same issue appears at 28mm radius where theoretical and actual values differ greatly when finding radial stress and then superimpose one above the other again. We can also notice that the stress values diminish very fast as we increase the radius in the first readings but then tend to diminish slowly as the radius reaches 40mm showing that the temperature started stabilising the strain gauges and resistance increases as the radius is increased.

Conclusion

It has been proved that the experiment was carried successfully due to the similarities between theoretical and actual values. It has also been proved that there is a linearity relationship in values between the readings of strain gauges taken in all three experiments: thin cylinder open ends, thin cylinder closed ends and thick cylinder. It was shown that the higher the angle at which the strain gauges were positioned, the higher the strain magnitude. The linearity also shows a direct correlation between pressure and strain, showing that strain is affected directly and proportionally to pressure's magnitude.

For all linearity strain graphs drawn, gauge 2 shows to have the smallest strain values.

The hoop strain in the thin condition is not affected by distance, only by pressure.

The graphs of strain linearity also prove that thick cylinders can endure more pressure compared to thin, as for the same amount of pressure, they create a smaller strain reaction.

Furthermore, it was proven that there is a correlation between stress and strain, defined as Young Modulus, that increases as strain decrease for a set amount of stress.

There is also a high correlation between longitudinal and hoop strain, defined as Poisson's ratio that determines that as longitudinal compressive strain increases, hoop strain increases by the same amount.

Error calculated in the thin cylinder is likely to be due to the calibration of the instruments and the manual error when setting up the intervals of pressure.

The material, as the experiment was repeated repeatedly, may be slightly faulty and this might have affected the results.

It was also proven that in the Mohr's circle for closed ends, there is an axial force acting in the direction of the strain that defines the differences in values between open and closed ends.

The Mohr's circles direct strain points for the actual and theoretical values, match with the strain gauges calculated from the experiment, showing that there is a high correlation between these two too. The highest shear strain is also the radius of the Mohr's circle and gauges 2 determines the minimum principal direct strain, gauge 1 or 6 the maximum principal direct strain and the distance between gauge 2 and 1 or 6 determine, divided by two, the radius of the circle and the maximum shear strain, gauge 3 the intermediate low and gauge 5 the intermediate-high that determine the principal shear strain. The centre of the circle is defined by gauge 4 and also indicates the y coordinates for maximum shear strain. This proves a direct relationship between shear and direct strain.

Axial force in the closed condition for the thin cylinder influences the results by diminishing the radius of the Mohr's circles and creating only tensile direct strains.

Theoretical and actual Mohr's circles, for both open and closed conditions, tend to have an error difference of less than 10%, likely to be due to the excess pressure applied when calculating the strains at each interval and using gauge 1 and 6 average as maximum principal strain.

It has also been proven that there is a relationship between theoretical and manual strain and theoretical and manual stress proving that values collected in the experiment tend to be accurate. As radius increases, strain and stress quickly decrease showing a square root degression.

By comparing graphs 8 and 9 it was also proven that there is a direct correlation between strain and stress and values of strain influence directly and proportionally stress.

Summary

The experiment was carried successfully, and all the data needed was collected. To allow fewer percentage errors between theoretical and actual data, it would be ideal to repeat the experiment several times to improve accuracy. Monitoring the movement of the hand pump carefully when selecting the different pressure intervals would drastically diminish the percentage errors between values. These minor changes can improve data and results.

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