Leading University, Sylhet
Department of Computer Science & Engineering
Final In-Course Evaluation, Spring-2021

Course Title: Differential Equation and Fourier Analysis   **Course Code: MAT-1315**
Full Marks: 30   **Time: 2 Hours**

[Answer any three (3) questions including question-1]

1. **(a)** Define Gamma function.   **1**

    (b) Prove that $(i) \lceil (n+1) = n \lceil n \quad ; n > 1$ **3**
            $(ii) \lceil (n+1) = n! \quad ; n = 1,2,3, ...$

    (c) Prove that   **3**
        (i) $\mathcal{L}\{\sin at\} = \dfrac{a}{s^2+a^2}$
        (ii) $\mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}$

    (d) Define Bessel function. Also find the Laplace transform of the Bessel function of order one and two.   **3**

2. **(a)** Solve the Ordinary Differential equation by using Laplace transforms,   **5**

$$Y'' - 5Y' + 6Y = e^{3t}\sin 6t, \quad Y(0) = -3, Y'(0) = 5$$

    (b) Solve $\begin{cases} \dfrac{dX}{dt} = X - 3Y \\ \dfrac{dY}{dt} = 3Y - 2X \end{cases}$ subject to X(0)=8 , Y(0)=3   **5**

3. **(a)** Find the Fourier series of the function $f(x) = x \ on - \pi < x < \pi$ **4**

    (b) Find the Fourier series of the function   **6**
        $F(x) = \begin{cases} x^2 & ; -L < x < 0 \\ 2x & ; \ 0 < x < L \end{cases}$ on $\ -L < x < L$

4. Find $(i) \mathcal{L}^{-1}\left\{\dfrac{5s^2-15s-11}{(s+1)(s-2)^3}\right\}$ **10**
        $(ii) \mathcal{L}^{-1}\left\{\dfrac{3s+1}{(s-1)(s^2+3)}\right\}$