

Compiler

Lecture 6

Syntax analysis Part 3

BOTTOM UP Parsing

Parsing Techniques

- *Top-down parsers* (*LL(1), recursive descent*)
 - ❑ Start at the root of the parse tree from the start symbol and grow toward leaves.
 - ❑ Pick a production and try to match the input
 - ❑ Bad “pick” \Rightarrow may need to backtrack
 - ❑ Some grammars are backtrack-free (*predictive parsing*)
- *Bottom-up parsers* (*LR(1), operator precedence*)
 - ❑ Start at the leaves and grow toward root
 - ❑ We can think of the process as reducing the input string to the start symbol
 - ❑ At each reduction step a particular substring matching the right-side of a production is replaced by the symbol on the left-side of the production
 - ❑ Bottom-up parsers handle a large class of grammars

Bottom-Up Parsing

- Bottom-up parsing is also known as *shift-reduce parsing* because its two main actions are **shift and reduce.**
 - ❑ At each shift action, the current symbol in the input string is pushed to a stack.
 - ❑ At each reduction step, the symbols at the top of the stack (this symbol sequence is the right side of a production) will be replaced by the non-terminal at the left side of that production.
- If the substring is chosen correctly, **the right most derivation** of that string is created in the **reverse order.**

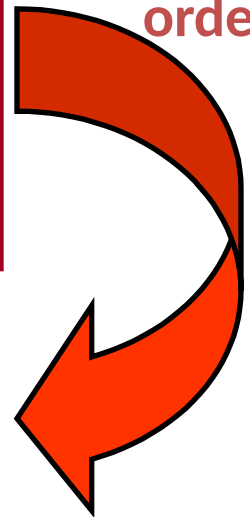
Shift-Reduce Parsing

- “Shift-Reduce” Parsing
- Reduce a string to the start symbol of the grammar.
- At every step a particular sub-string is matched (in left-to-right fashion) to the right side of some production and then it is substituted by the **non-terminal** in the left hand side of the production.

$$\begin{aligned} S &\rightarrow aABe \\ A &\rightarrow Abc \mid b \\ B &\rightarrow d \end{aligned}$$

abbcde
aAbcde
aAde
aABe
S

Reverse
order



Rightmost Derivation:

$S \Rightarrow aABe \Rightarrow aAde \Rightarrow aAbcde \Rightarrow abbcde$

Handles

- **Handle of a string:** Substring that matches the RHS of some production AND whose reduction to the non-terminal on the LHS is a step along the reverse of some rightmost derivation.

Example: Handle

Grammar

$S \rightarrow a A B e$

$A \rightarrow A b c \mid b$

$B \rightarrow d$

a b b c d e

a A b c d e

a A d e

a A B e

S

Handle

a b b c d e

a A b c d e

a A A e

... ?

NOT a handle, because
further reductions will fail
(result is not a sentential form)

A Stack Implementation of A Shift-Reduce Parser

- A stack is used to hold grammar symbols
- Handle always appear on top of the stack
- Initial configuration:

Stack Input

\$ w\$

- Acceptance configuration

Stack Input

\$S \$

- Basic operations:
 - Shift
 - Reduce
 - Accept
 - Error

A Stack Implementation of A Shift-Reduce Parser

$$E \rightarrow E + T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow (E) \mid \mathbf{id}$$

Stack	Input	Action
\$	id₁ * id₂ \$	shift
\$ id₁	* id₂ \$	reduce by $F \rightarrow \mathbf{id}$
\$F	* id₂ \$	reduce by $T \rightarrow F$
\$T	* id₂ \$	shift
\$T *	id₂ \$	shift
\$T * id₂	\$	reduce by $F \rightarrow \mathbf{id}$
\$T * F	\$	reduce by $T \rightarrow T * F$
\$T	\$	reduce by $E \rightarrow T$
\$E	\$	accept

Shift-Reduce Parsers

- There are two main categories of shift-reduce parsers

1. Operator-Precedence Parser

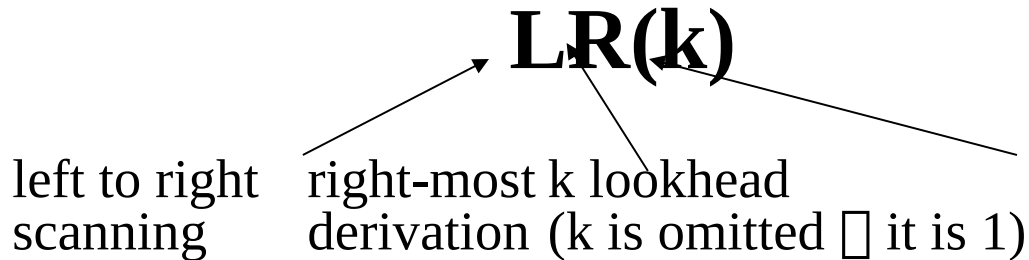
- simple, but only a small class of grammars.

2. LR-Parsers

- covers wide range of grammars.
 - SLR – simple LR parser
 - LR – most general LR parser
 - LALR – intermediate LR parser (lookahead LR parser)
- SLR, LR and LALR work same, only their parsing tables are different.

LR Parsers

- The most powerful shift-reduce parsing is:



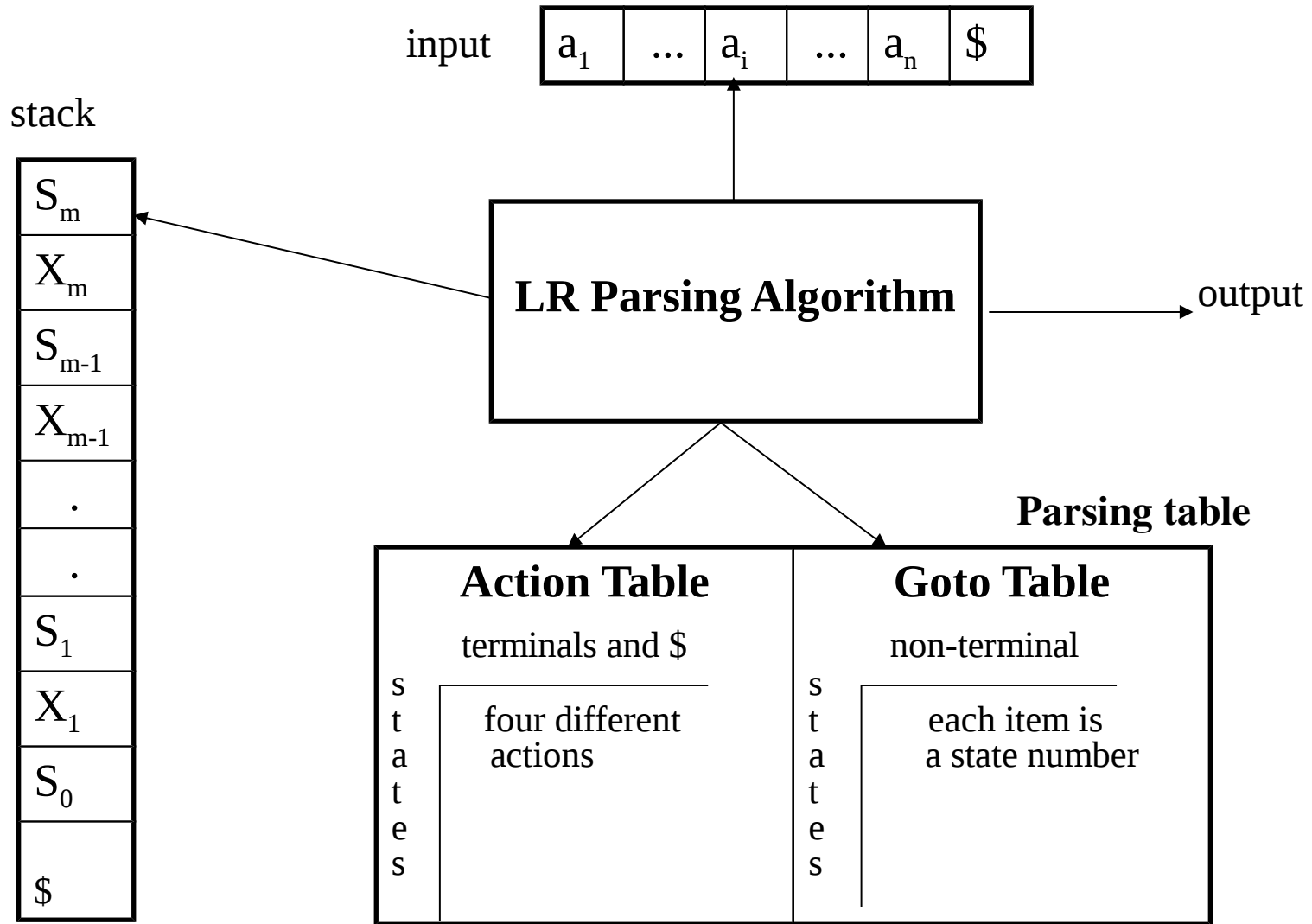
parsing.

- **LR parsing is attractive because:**
 - LR parsing is most general non-backtracking shift-reduce parsing, yet it is still efficient.
 - The class of grammars that can be parsed using LR methods is a proper superset of the class of grammars that can be parsed with predictive parsers.
 $LL(1)\text{-Grammars} \subset LR(1)\text{-Grammars}$
 - An LR-parser can detect a syntactic error as soon as it is possible to do so a left-to-right scan of the input.

Potential Problems

- How do we know which action to take: whether to shift or reduce, and which production to apply
- Issues
 - Sometimes can reduce but should not
 - Sometimes can reduce in different ways

LR Parsing Algorithm



Constructing SLR Parsing Tables – LR(0) Items

- An LR parser makes shift-reduce decisions by maintaining states to keep track of where we are in a parse.
- States represent sets of items.
- An item of a grammar G is a production of G with a dot at some position of the body.

– Example

$$A \rightarrow X Y Z$$

$$\text{items} \left\{ \begin{array}{l} A \rightarrow \cdot X Y Z \\ A \rightarrow X \cdot Y Z \\ A \rightarrow X Y \cdot Z \\ A \rightarrow X Y Z \cdot \end{array} \right.$$

$$A \rightarrow \underbrace{X}_{\downarrow} \cdot \underbrace{Y Z}_{\downarrow}$$

stack

next derivations

with input strings

Note that production $A \rightarrow \epsilon$ has one item $[A \rightarrow \bullet]$

LR(0) Items

- Augmented the grammar
 - If G is a grammar with start symbol S , then G' is the augmented grammar for G with new start symbol S' and new production $S' \rightarrow S\$$.
 - The purpose of this new starting production is to indicate to the parser when it should stop parsing and announce acceptance of the input.
- Closure function
- Goto function

Closure function

- To construct states, we begin with a particular LR(0) item and construct its closure
 - the closure adds more items to a set when the “.” appears to the left of a non-terminal
 - if the state includes $X \rightarrow s . Y s'$ and $Y \rightarrow t$ is a rule then the state also includes $Y \rightarrow . t$

Augmented Grammar:

0. $S' \rightarrow S \$$

- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L , S$

$S \rightarrow (L) \mid x$
 $L \rightarrow S \mid L , S$

First state

$S' \rightarrow . S \$$
 $S \rightarrow . (L)$
 $S \rightarrow . x$

} Full
Closure

Goto Function

- Function $Goto(I, X)$
 - I is a set of items
 - X is a grammar symbol
 - $Goto(I, X)$ is defined to be the closure of the set of all items $[A \rightarrow \alpha X \cdot \beta]$ such that $[A \rightarrow \alpha \cdot X \beta]$ is in I .
 - Goto function is used to define the transitions in the LR(0) automation for a grammar.

Augmented Grammar: G'

0. $S' \rightarrow S \$$

- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L , S$

$S' \rightarrow .S \$$

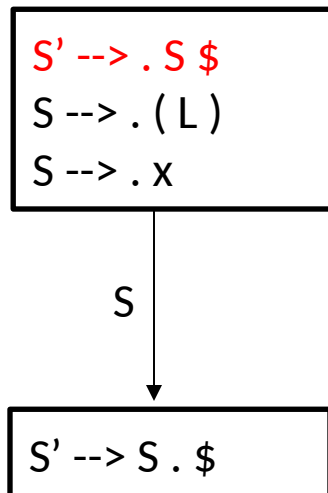
$S \rightarrow . (L)$

$S \rightarrow . x$

Grammar: G'

0. $S' \rightarrow S \$$

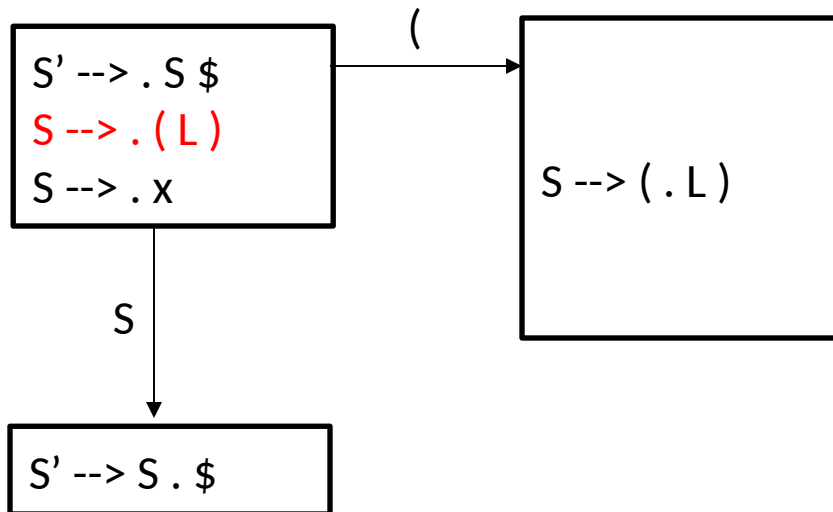
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

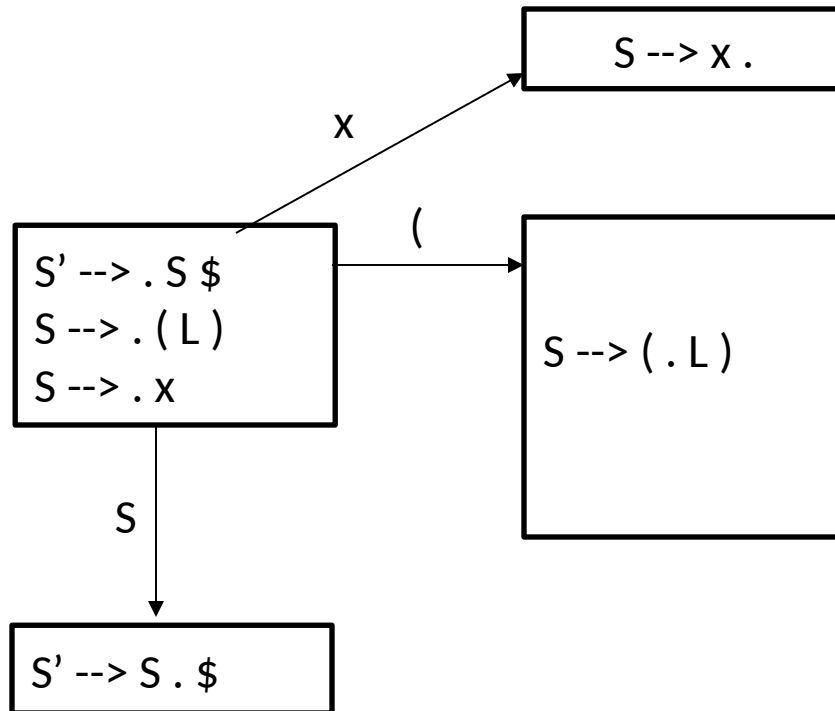
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L , S$



Grammar: G'

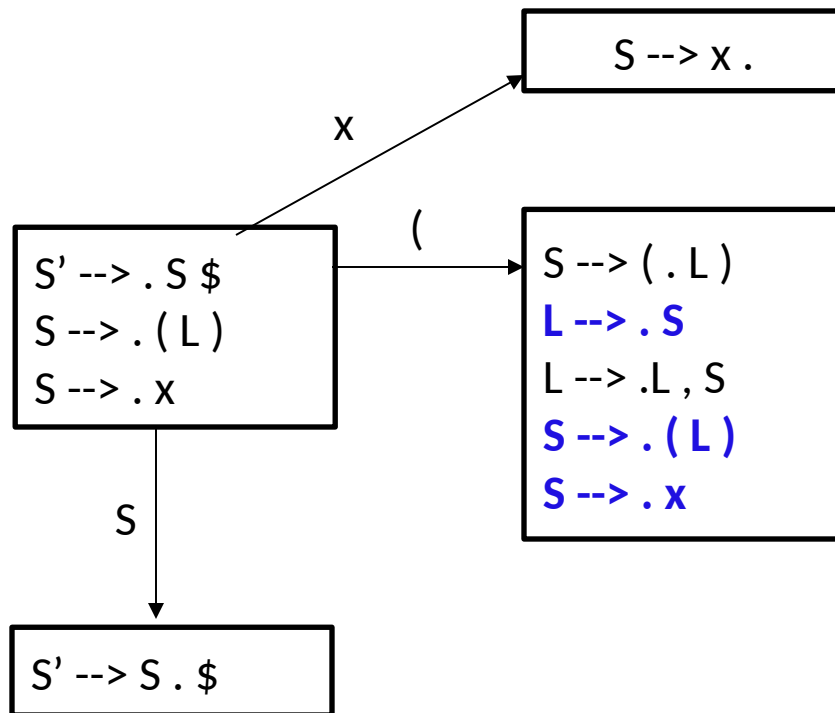
0. $S' \rightarrow S \$$

- $S \rightarrow (L)$
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- $L \rightarrow L , S$



Grammar: G'

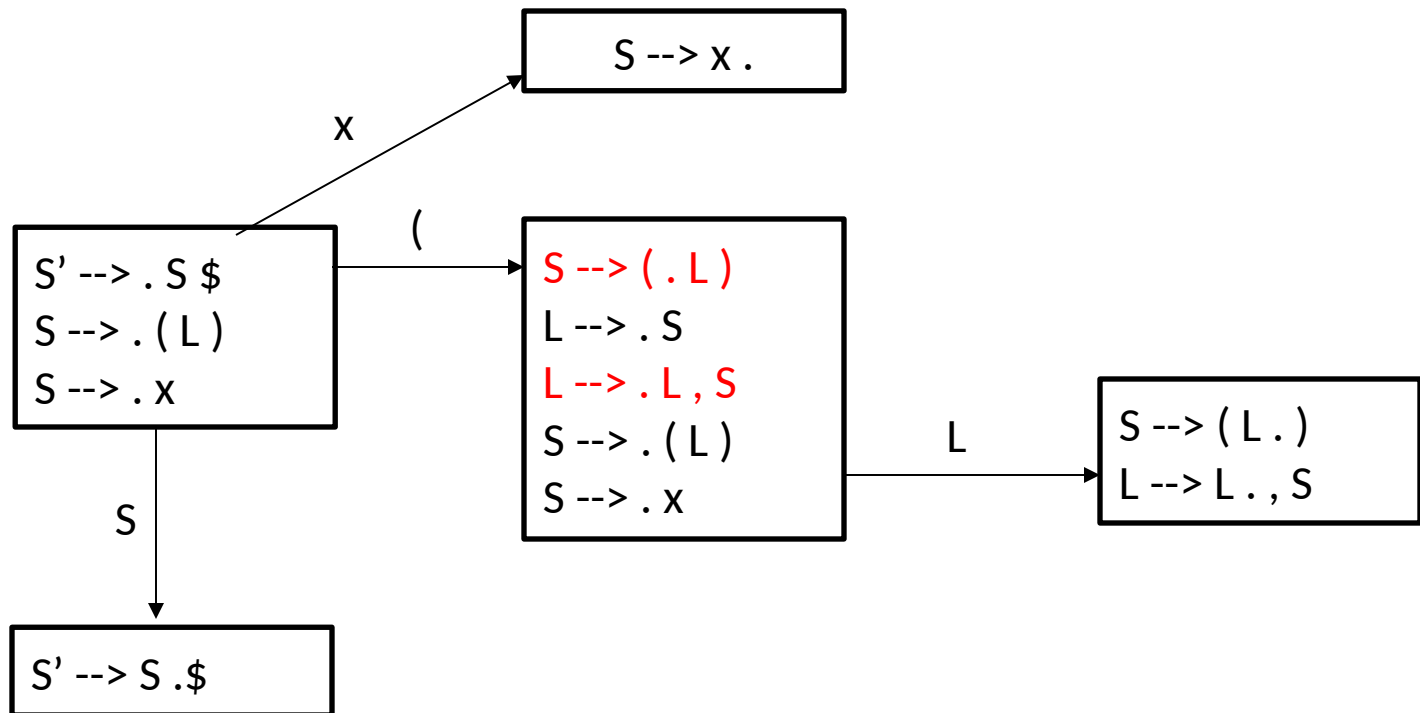
0. $S' \rightarrow S \$$
- $S \rightarrow (L)$
 - $S \rightarrow x$
 - $L \rightarrow S$
 - $L \rightarrow L , S$



Grammar: G'

0. $S' \rightarrow S \$$

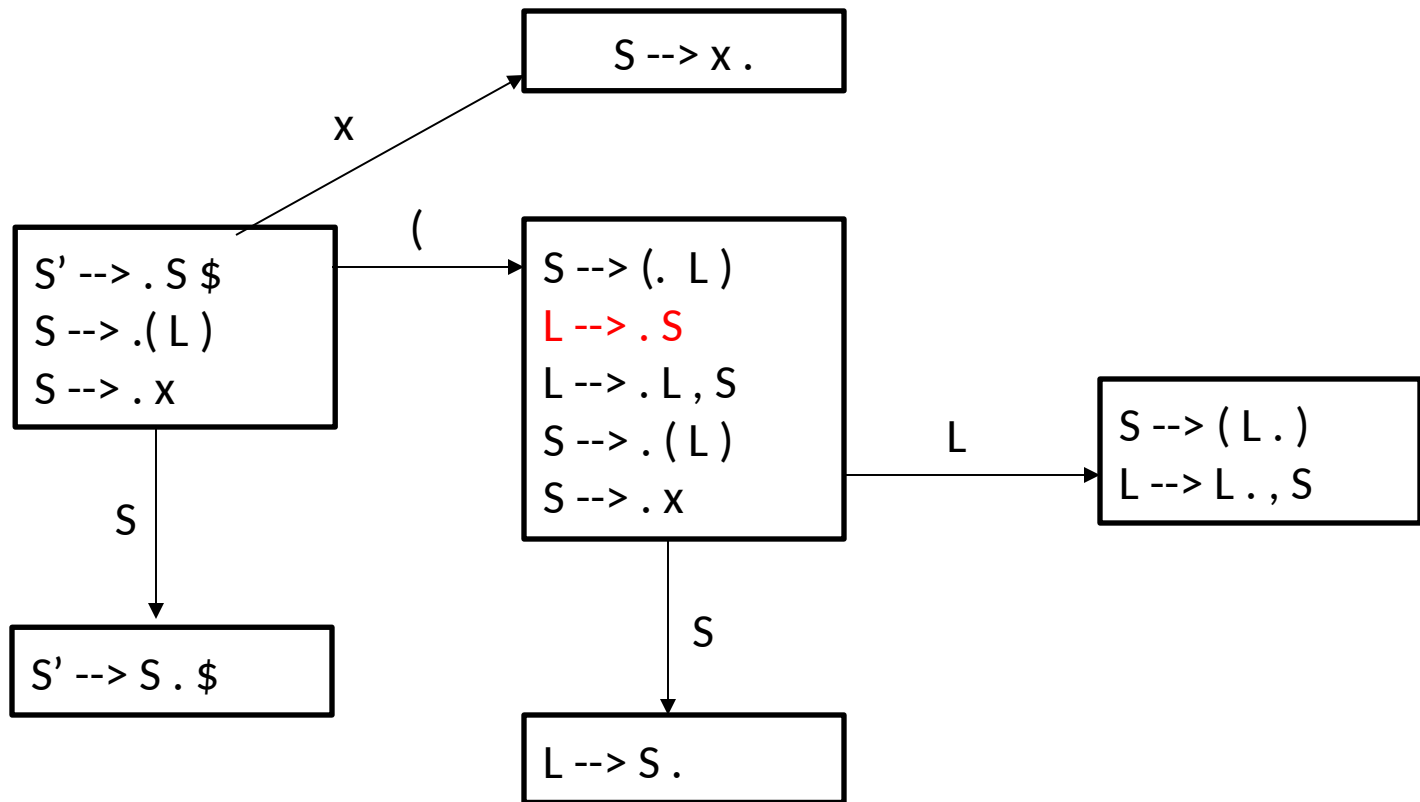
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

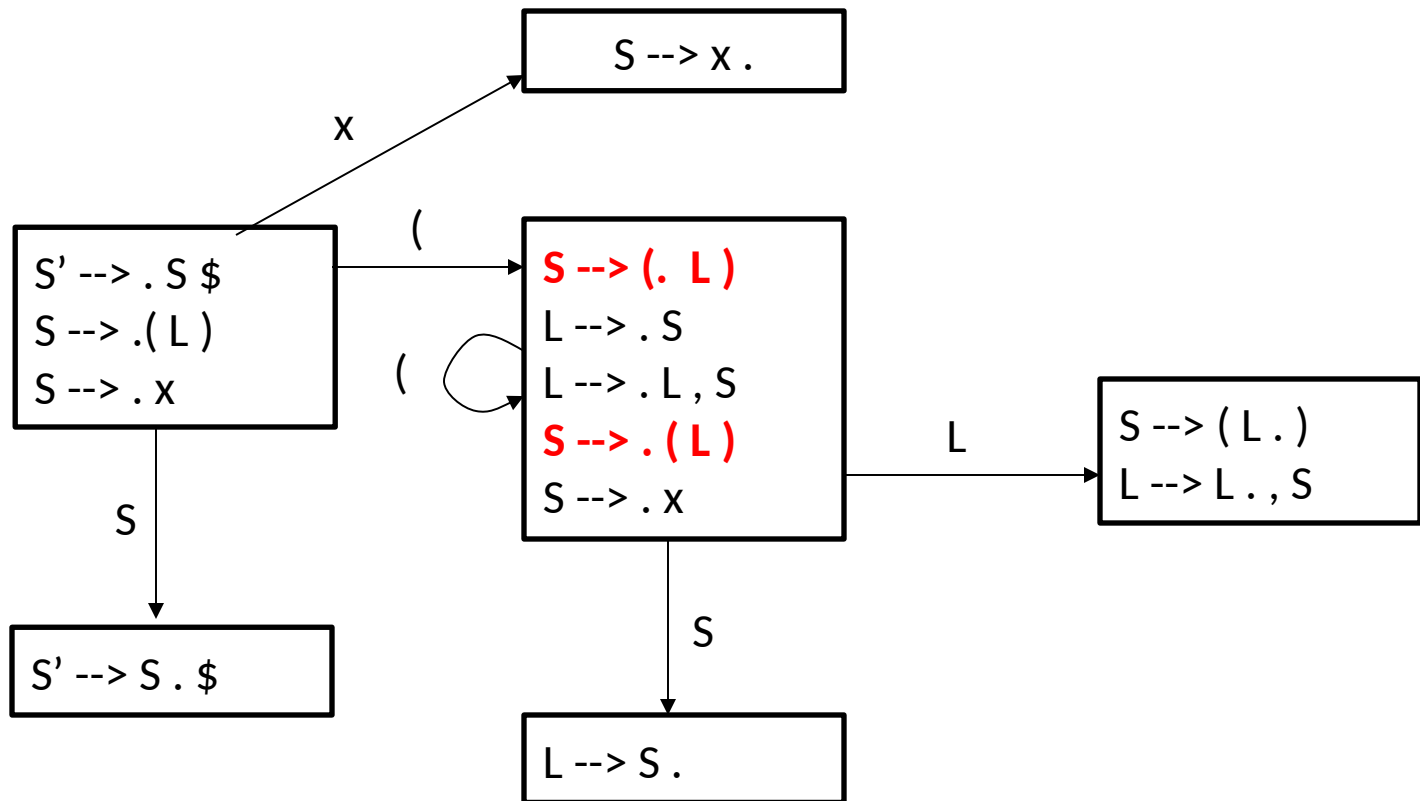
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

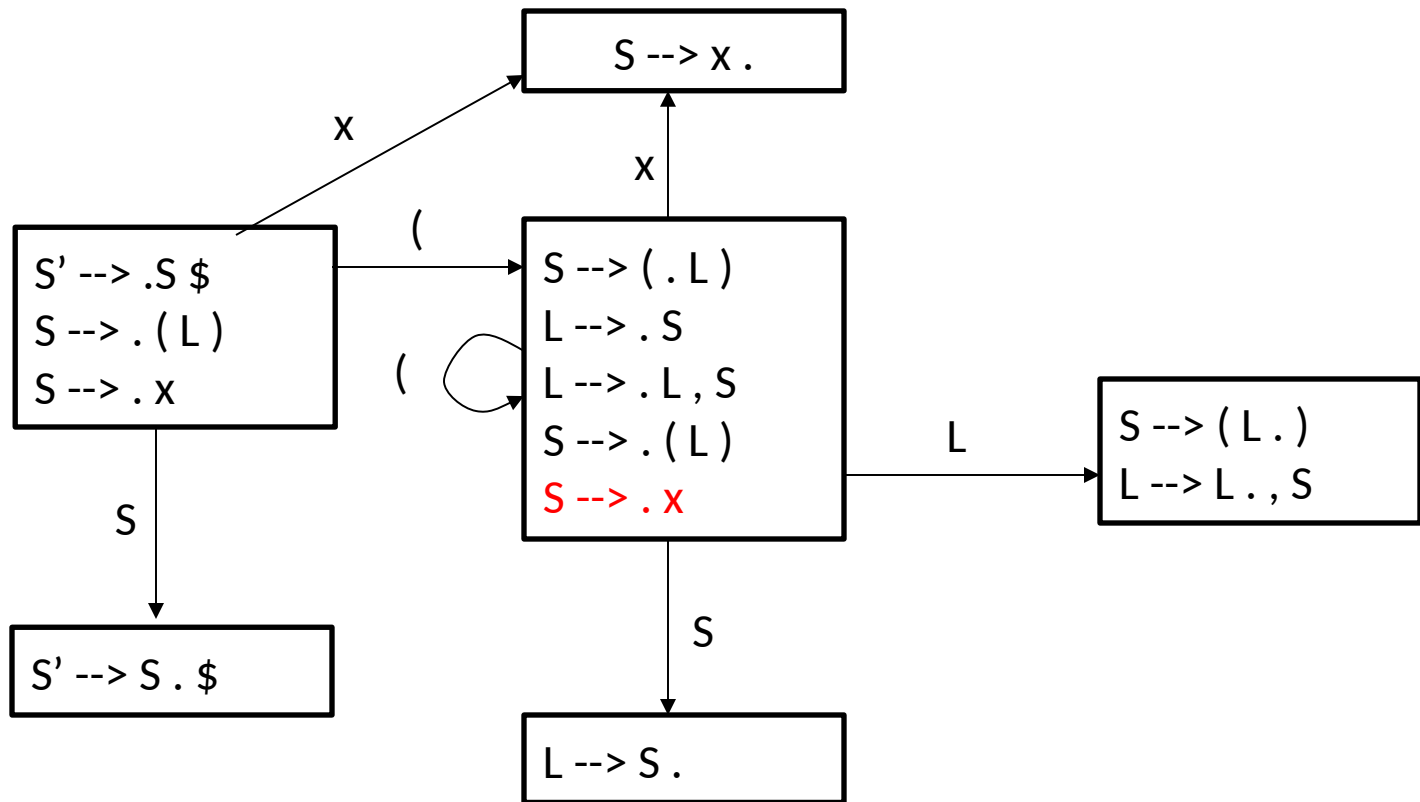
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

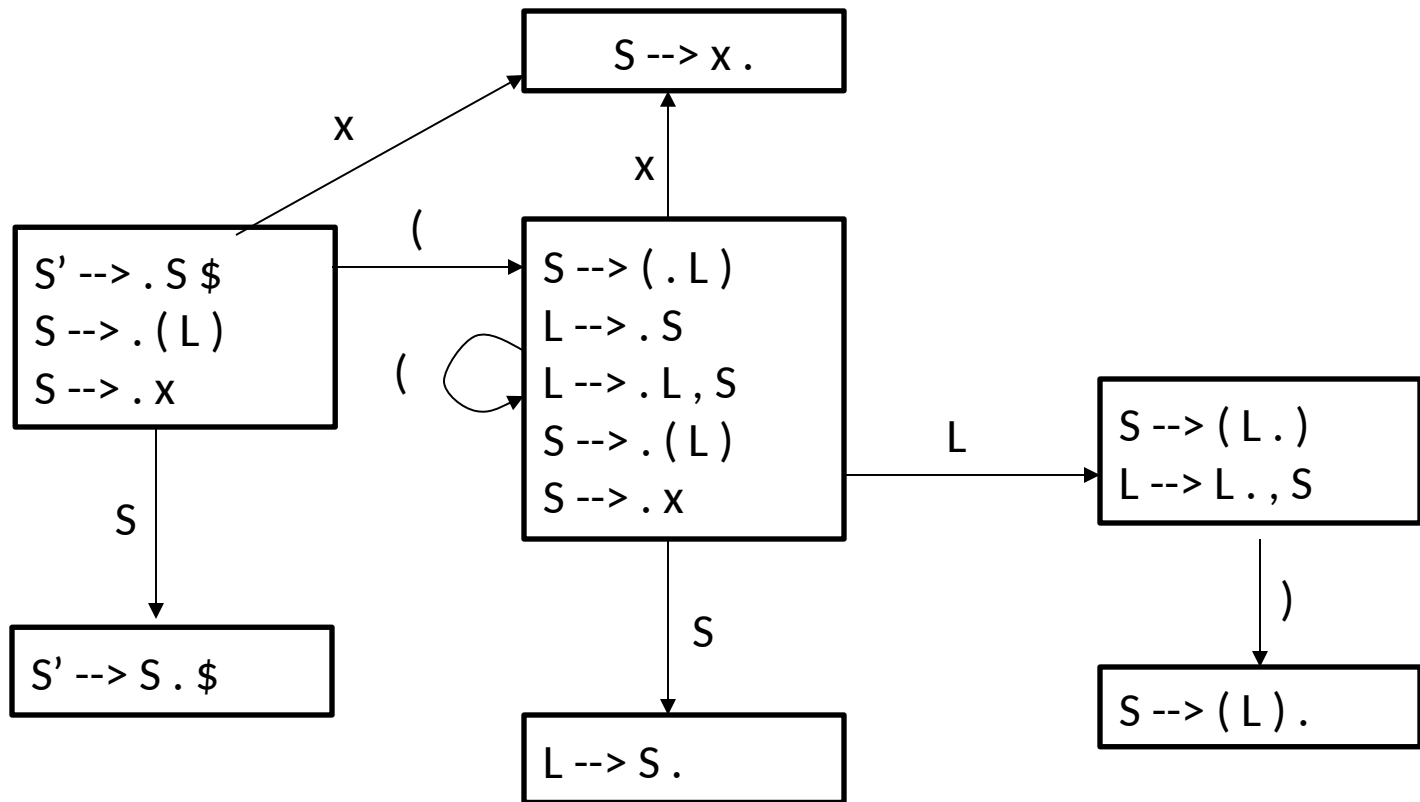
0. $S' \rightarrow S \$$

- $S \rightarrow (L)$

- $S \rightarrow x$

- $L \rightarrow S$

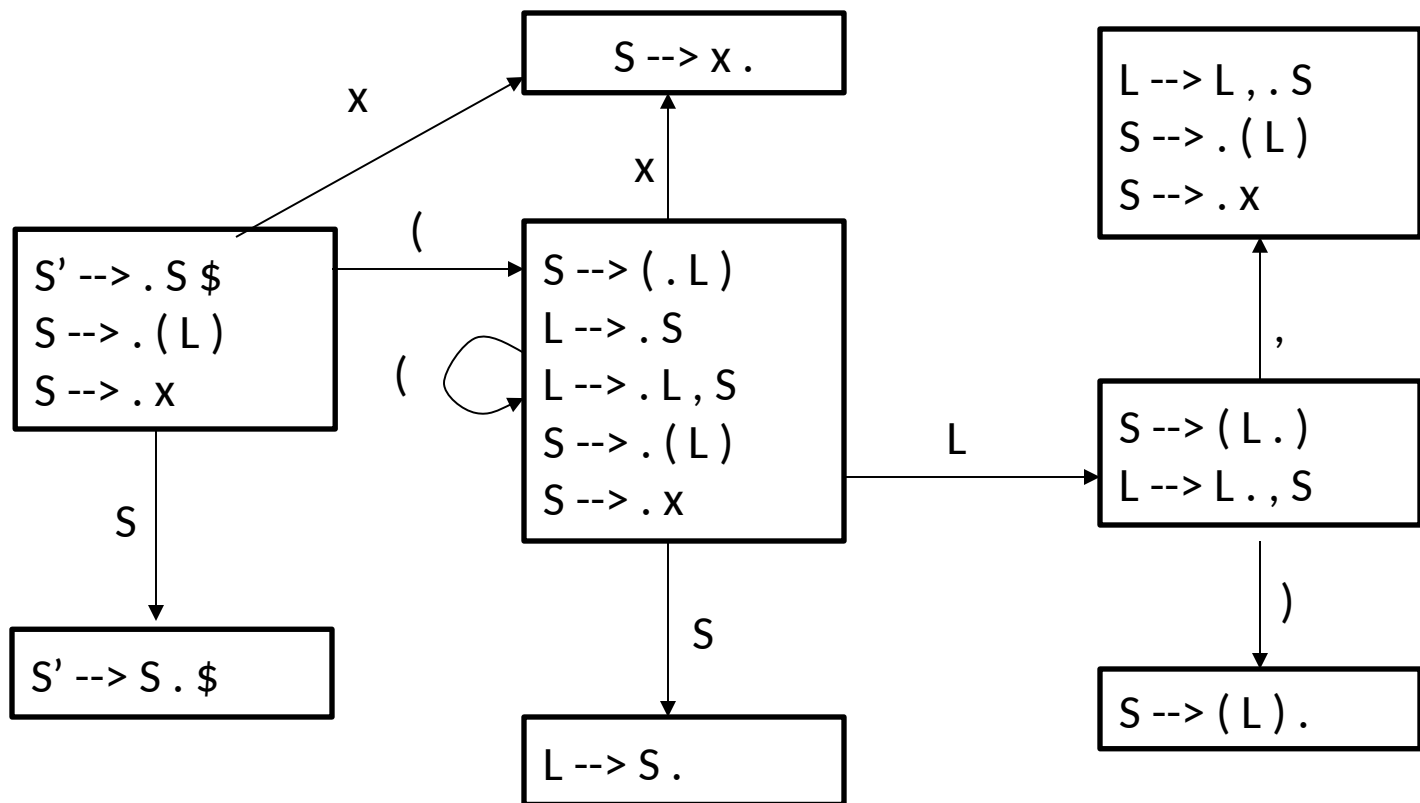
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

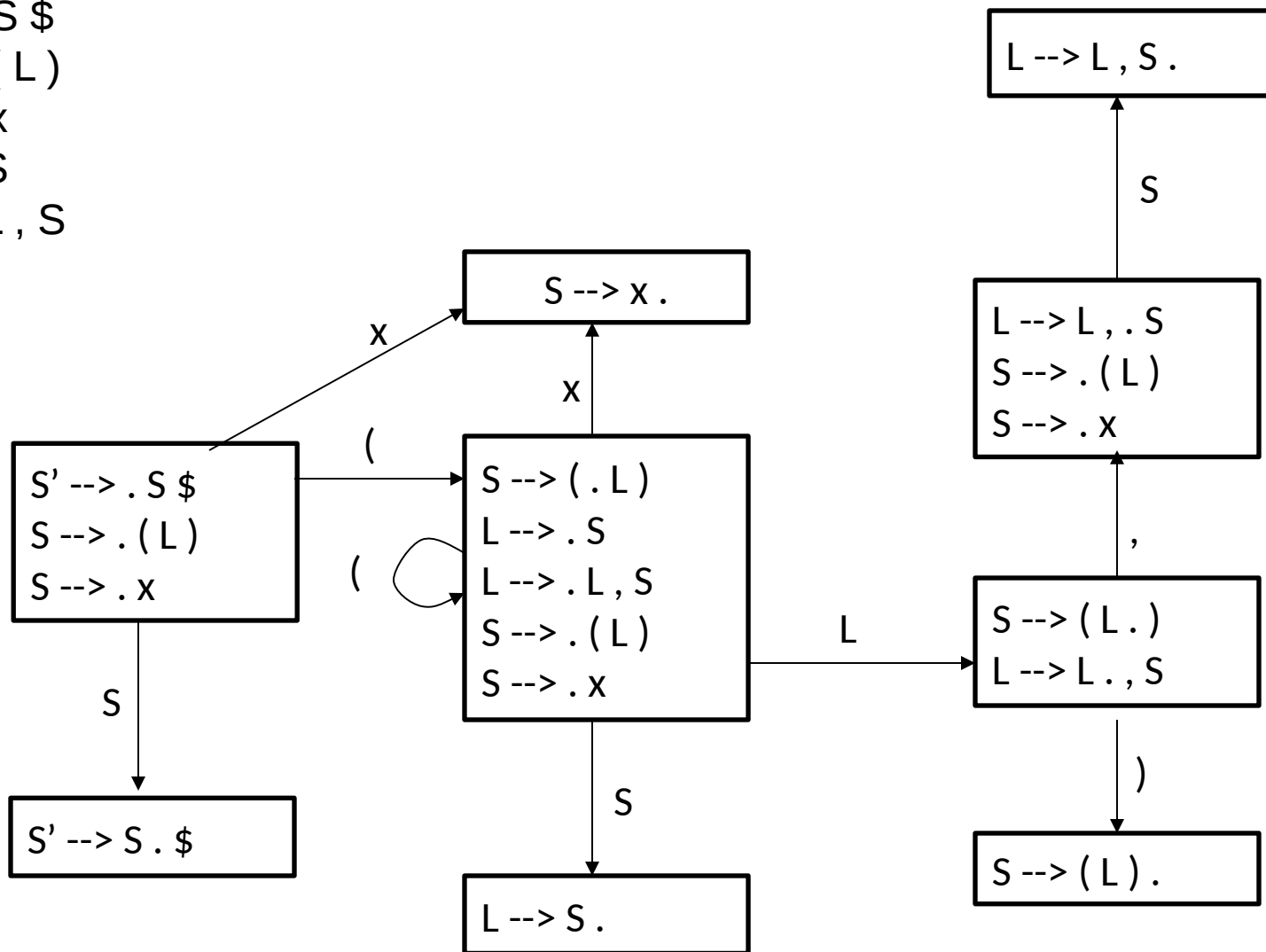
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

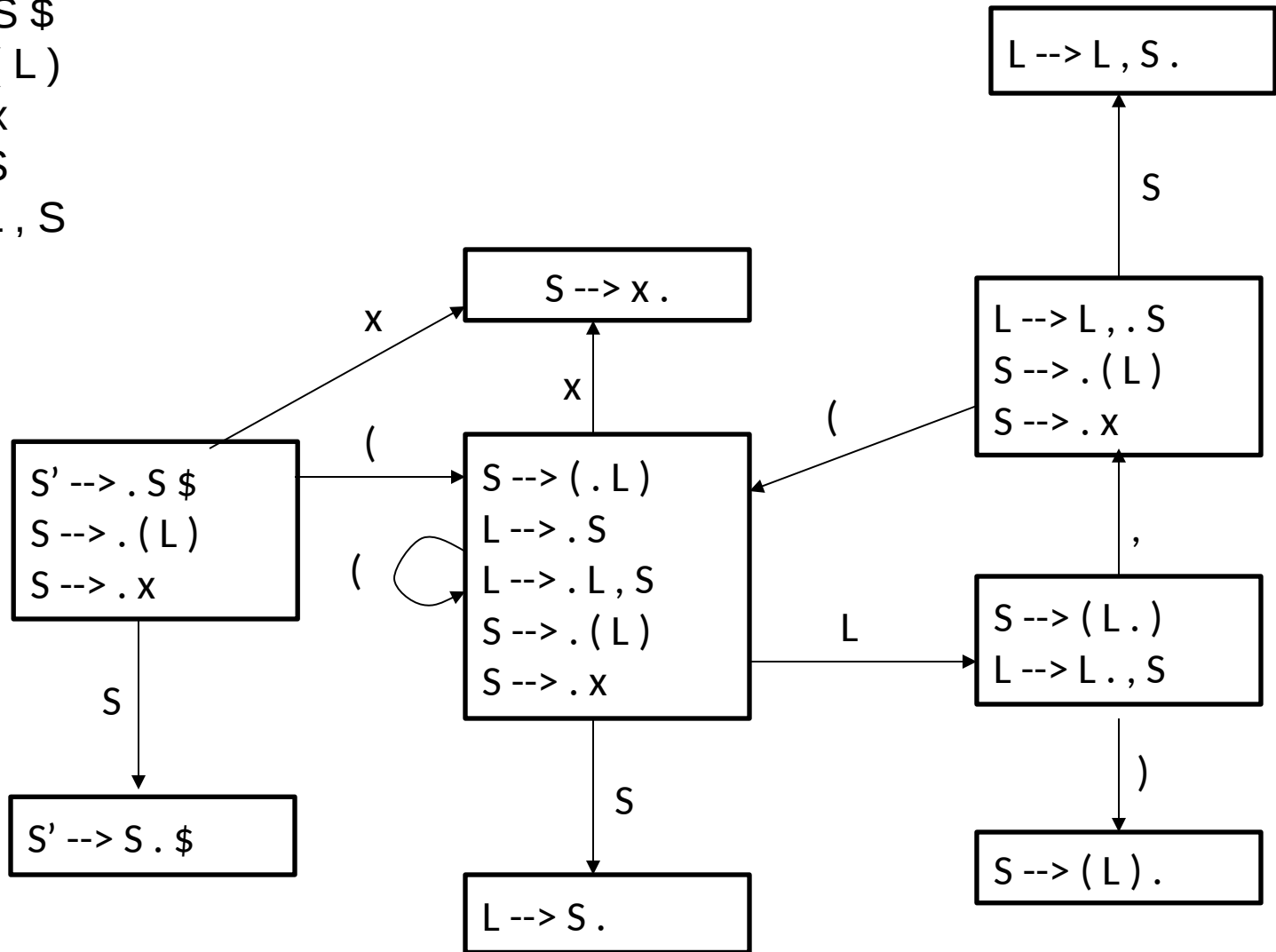
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

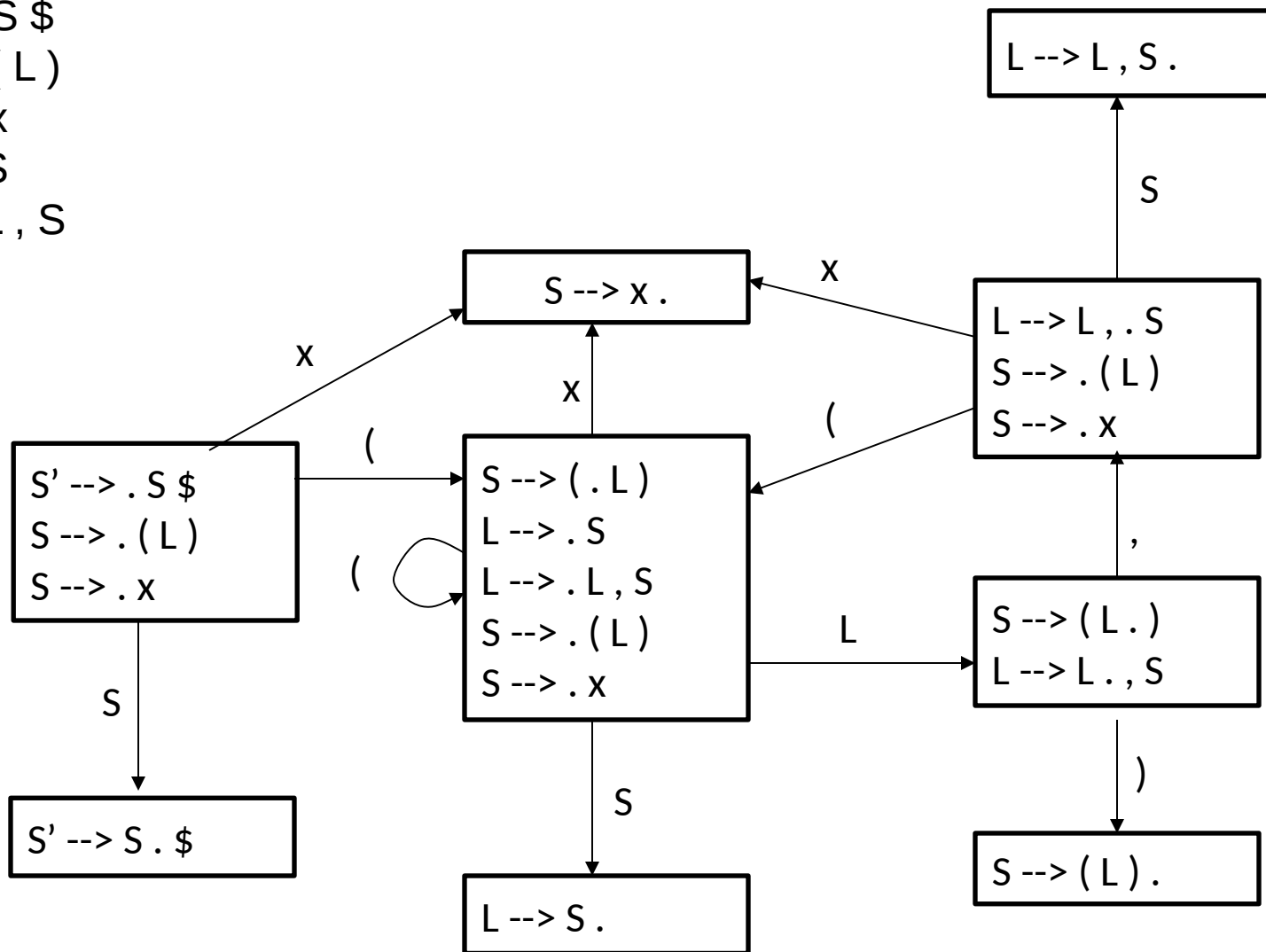
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

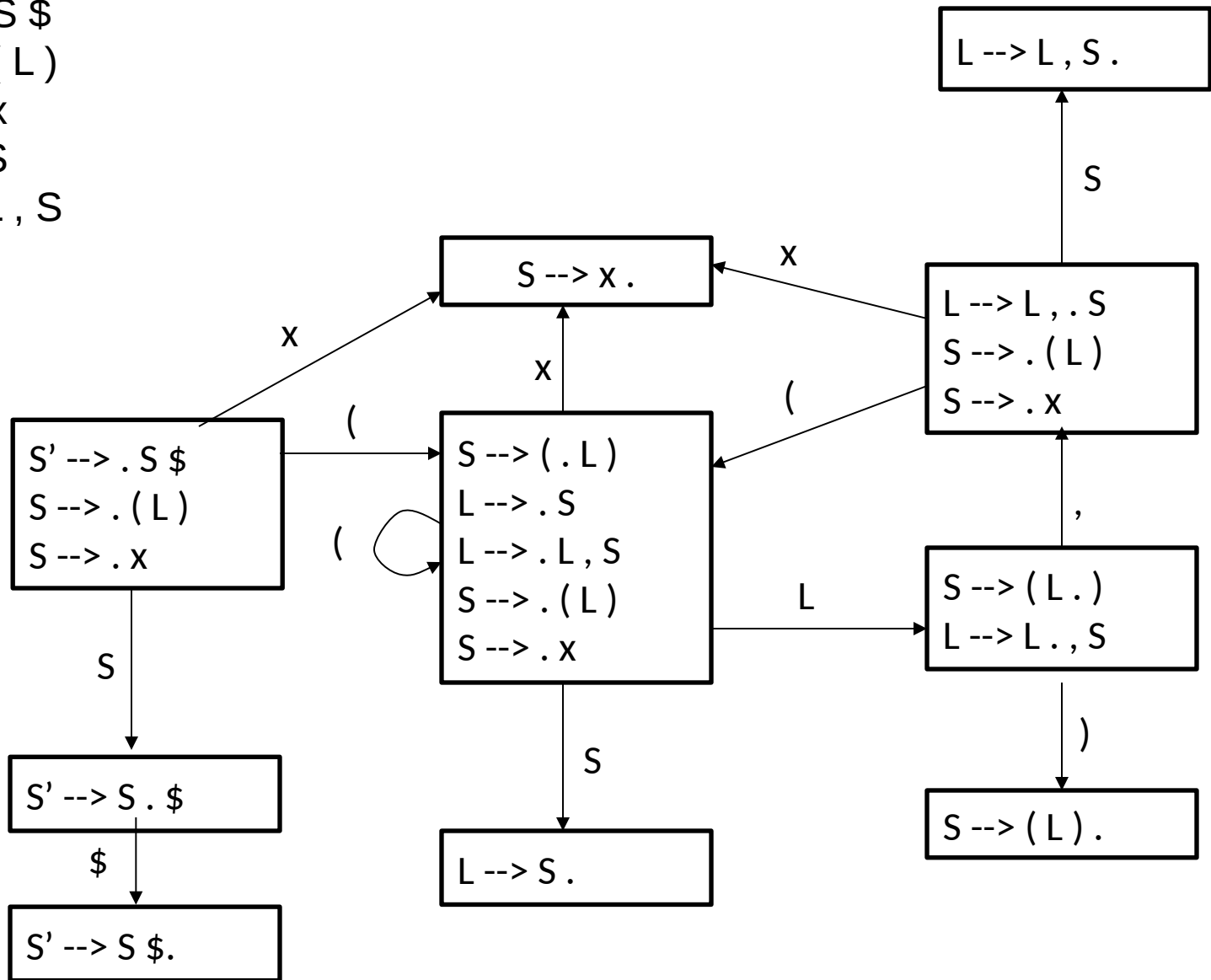
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

0. $S' \rightarrow S \$$

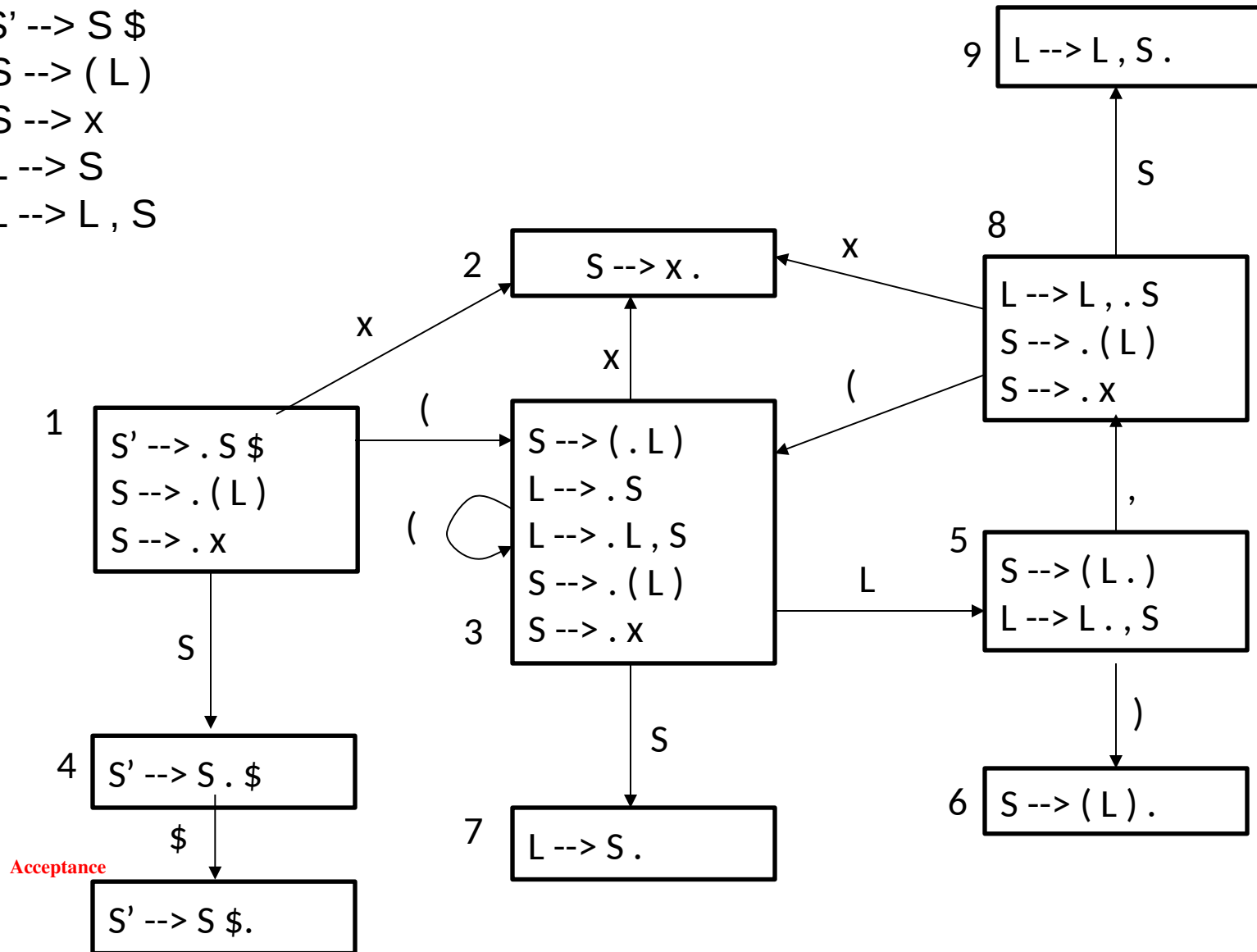
- $S \rightarrow (L)$
- $S \rightarrow x$
- $L \rightarrow S$
- $L \rightarrow L, S$



Grammar: G'

Complete LR(0) states for G'

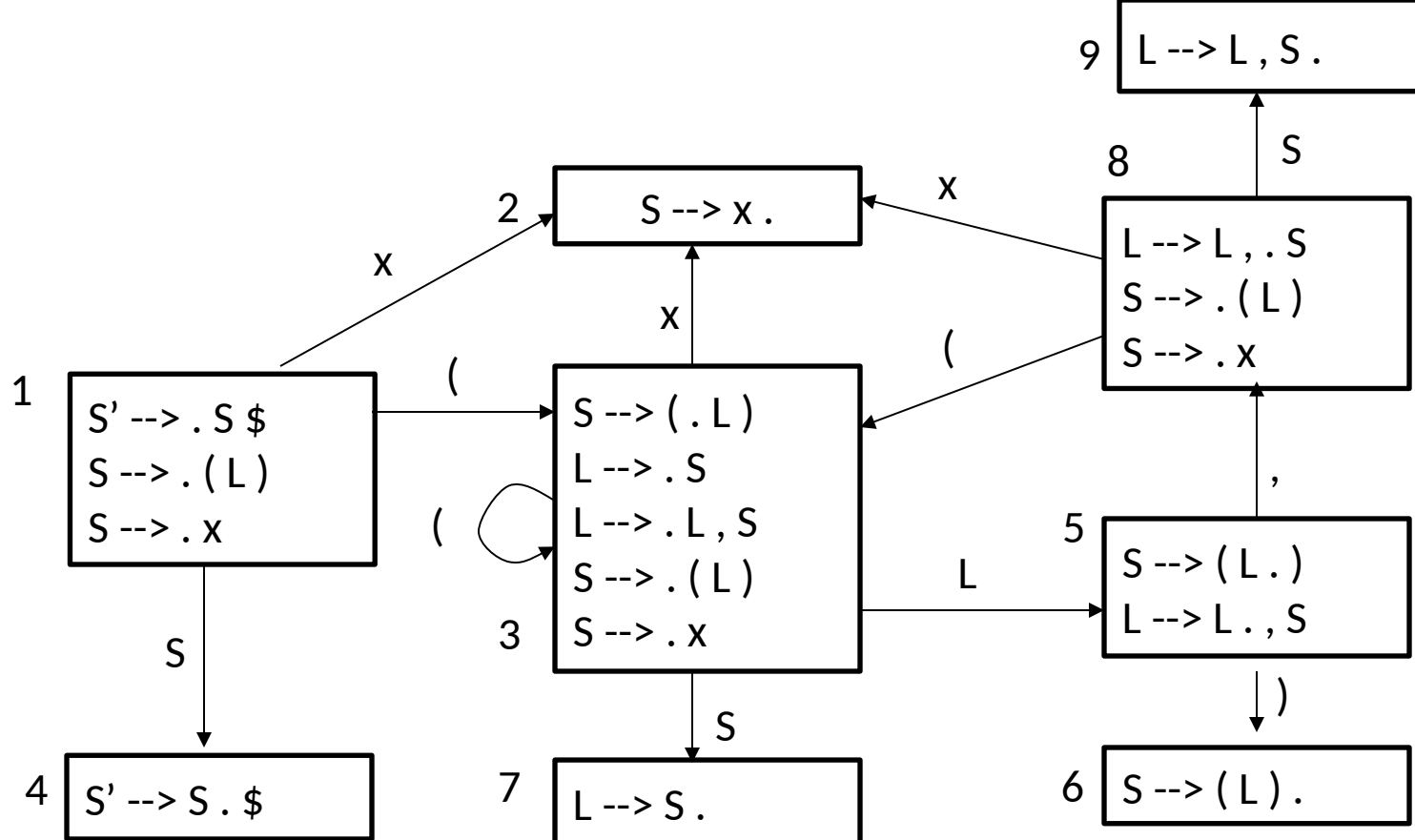
0. $S' \rightarrow S \$$
- $S \rightarrow (L)$
 - $S \rightarrow x$
 - $L \rightarrow S$
 - $L \rightarrow L, S$



Constructing SLR Parsing Tables

- Input : Augment the grammar with $S' \rightarrow S$
 - *Output the SLR parsing table*
 - *Method :*
1. Construct the canonical collection of sets of LR(0) items for G' .
 $C \leftarrow \{I_0, \dots, I_n\}$
 2. State i is constructed from I_i
 - If $[A \rightarrow \alpha \bullet a \beta] \in I_i$ and $\text{goto}(I_i, a) = I_j$ then set $\text{action}[i, a] = \text{shift } j$. Here a must be a terminal
 - If $[A \rightarrow \alpha \bullet] \in I_i$ then set $\text{action}[i, a] = \text{reduce } A \rightarrow \alpha$ for all $a \in \text{FOLLOW}(A)$ (apply only if $A \neq S'$)
 - If $[S' \rightarrow S \bullet]$ is in I_i then set $\text{action}[i, \$] = \text{accept}$
 3. If $\text{goto}(I_i, A) = I_j$ then set $\text{goto}[i, A] = j$ set **goto table**
 4. All entries not defined by rules (2) and (3) are made “error”.
 5. The initial state i is the I_i holding item $[S' \rightarrow \bullet S]$

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$



ACTION table

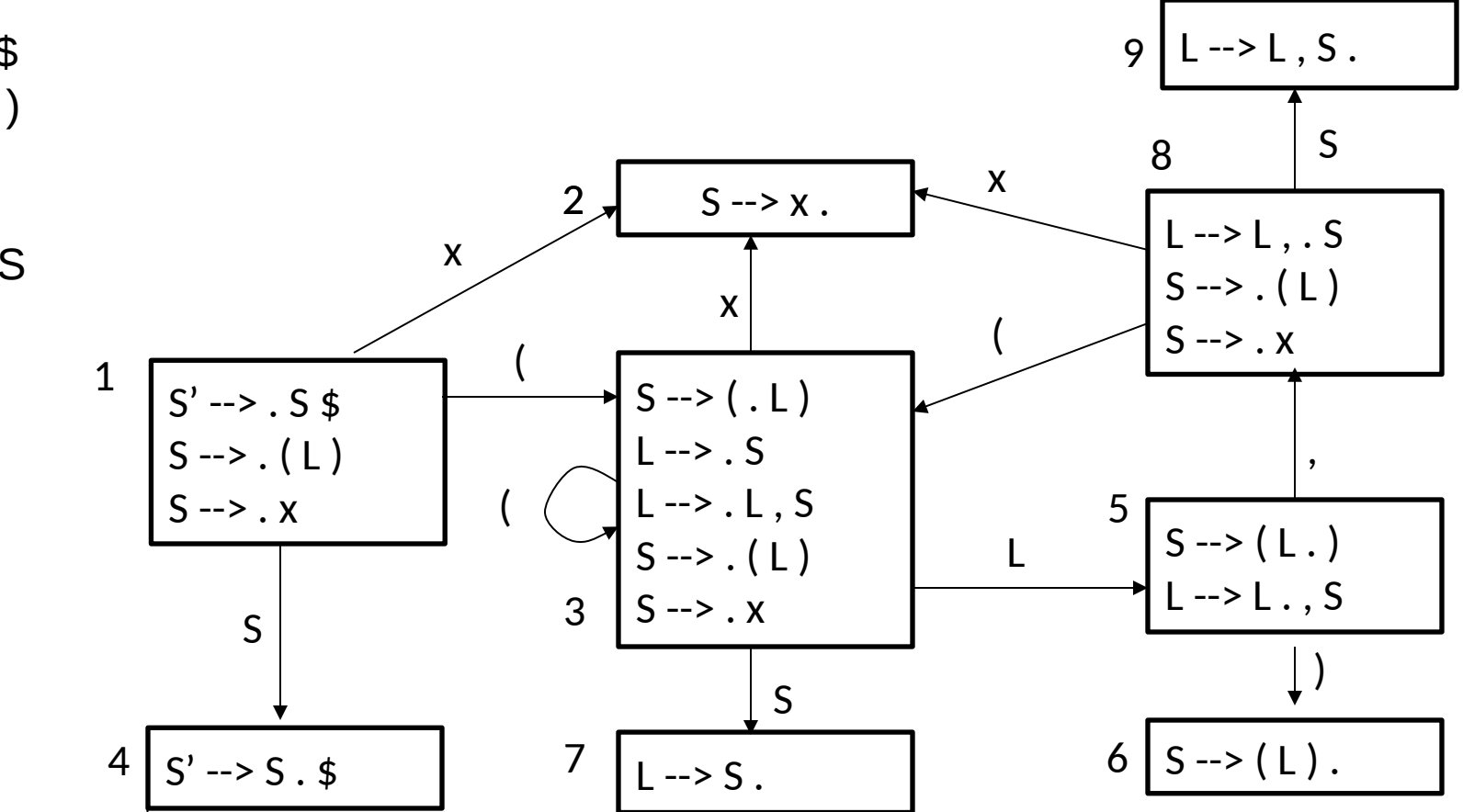
GOTO table

states	()	x	,	\$	S	L
1							
2							
3							
4							

Acceptance

$S' \rightarrow S \$$

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$



ACTION table

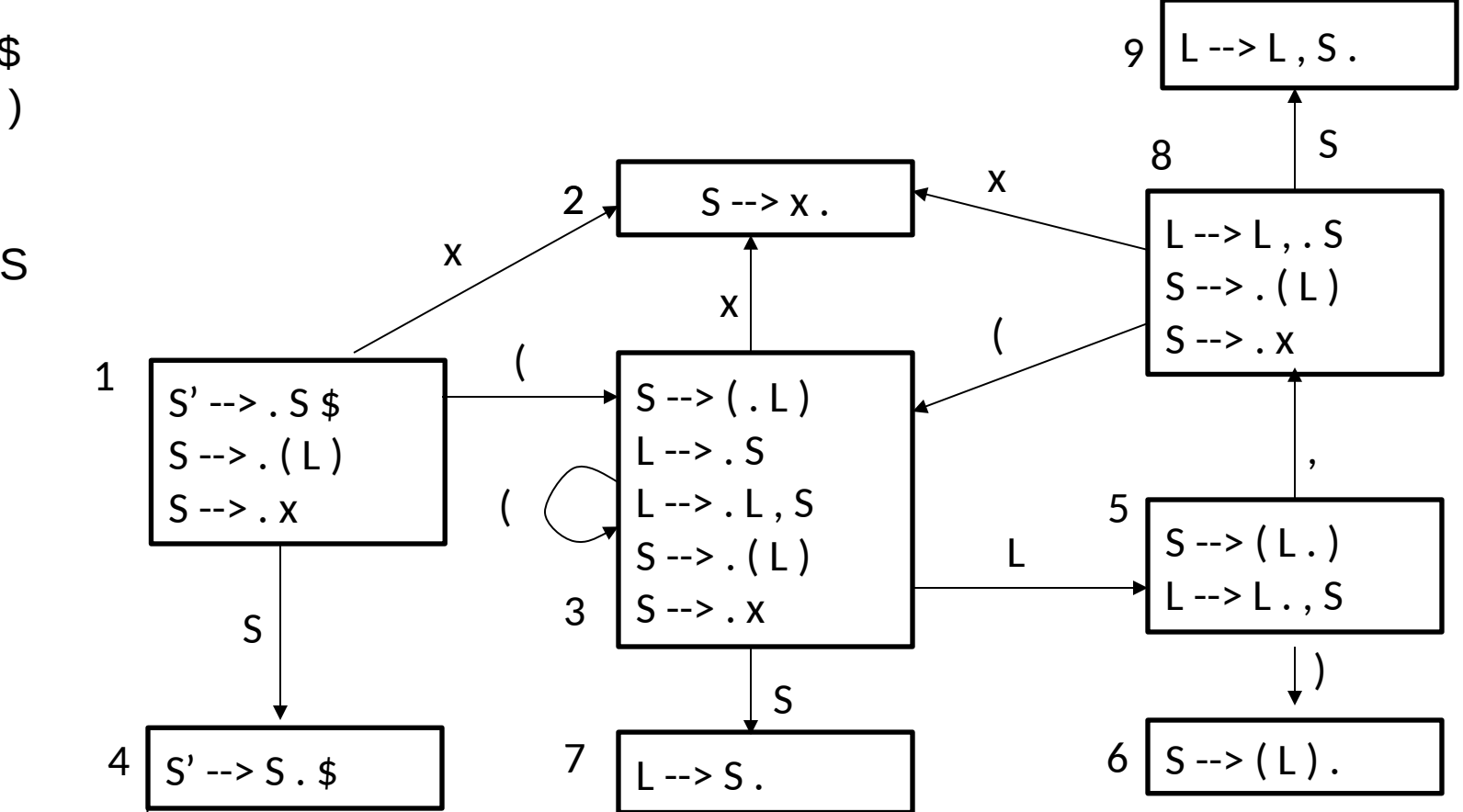
GOTO table

Acceptance

$S' \rightarrow S \$.$

states	()	x	,	\$	S	L
1	s3						
2							
3							
4							

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$



ACTION table

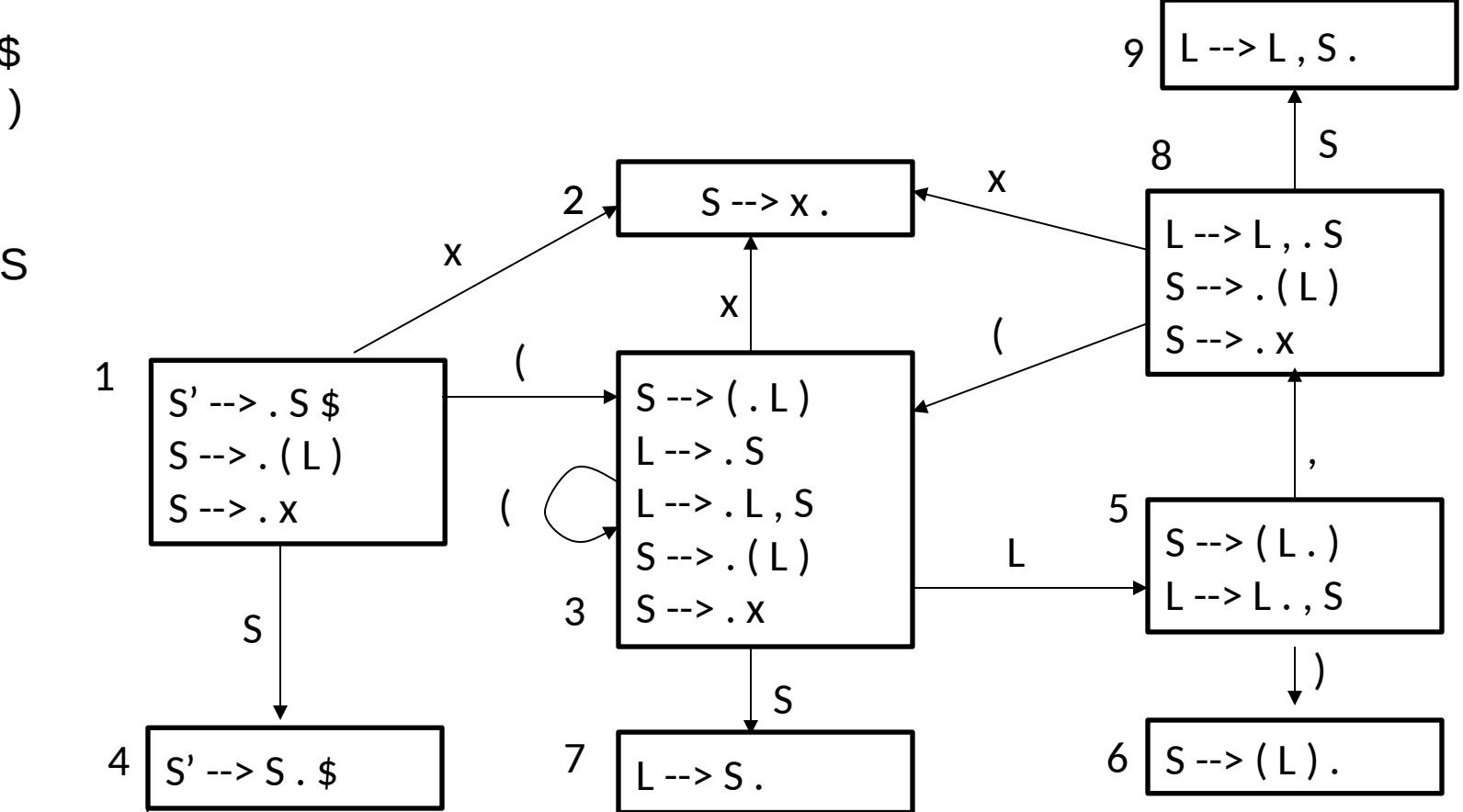
GOTO table

Acceptance

$S' \rightarrow S \$ \cdot$

states	()	x	,	\$	S	L
1	s3		s2				
2							
3							
4							

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$

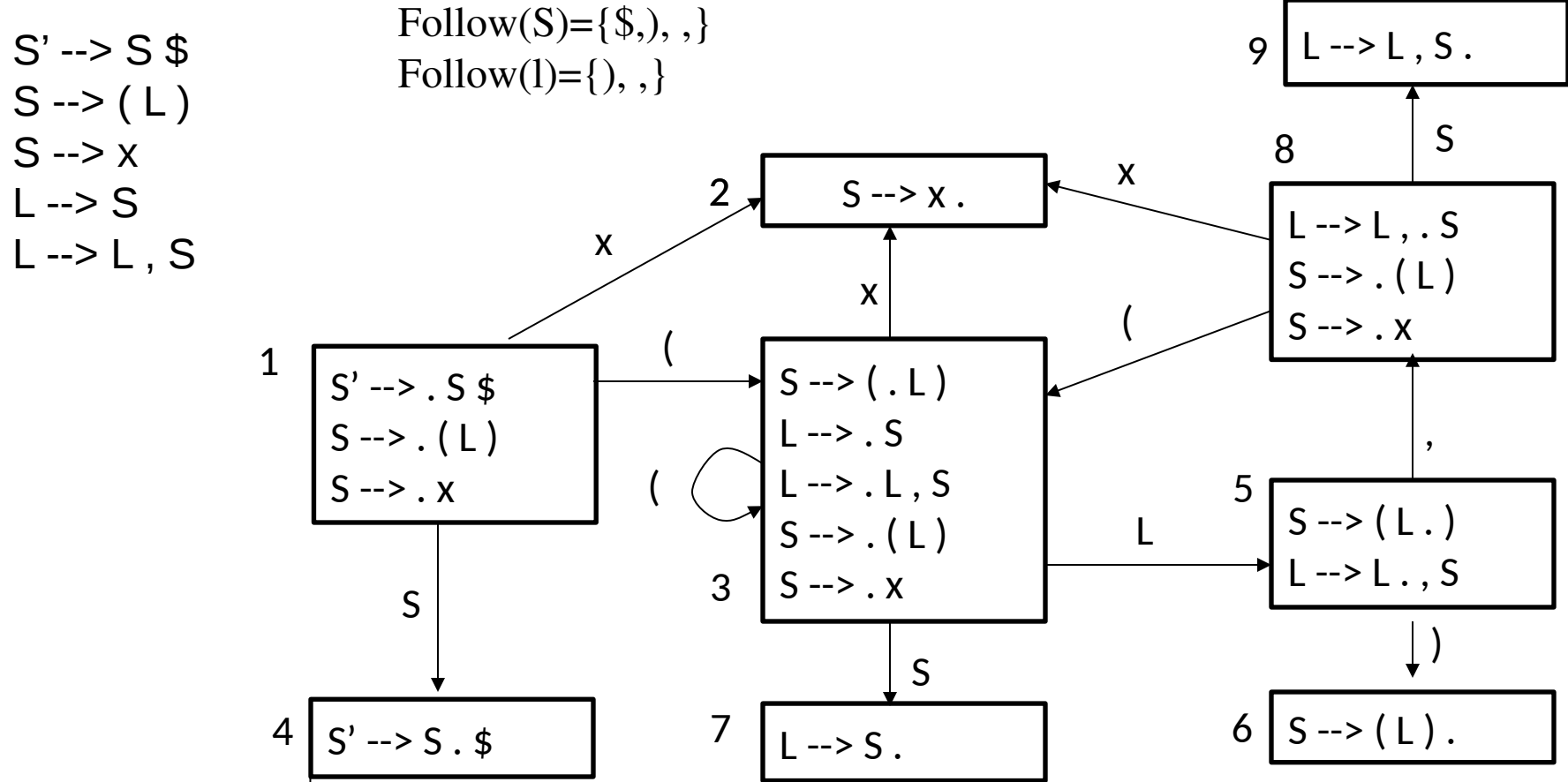


ACTION table

GOTO table

states	()	x	,	\$	S	L
1	s3		s2			4	
2							
3							
4							

Acceptance



ACTION table

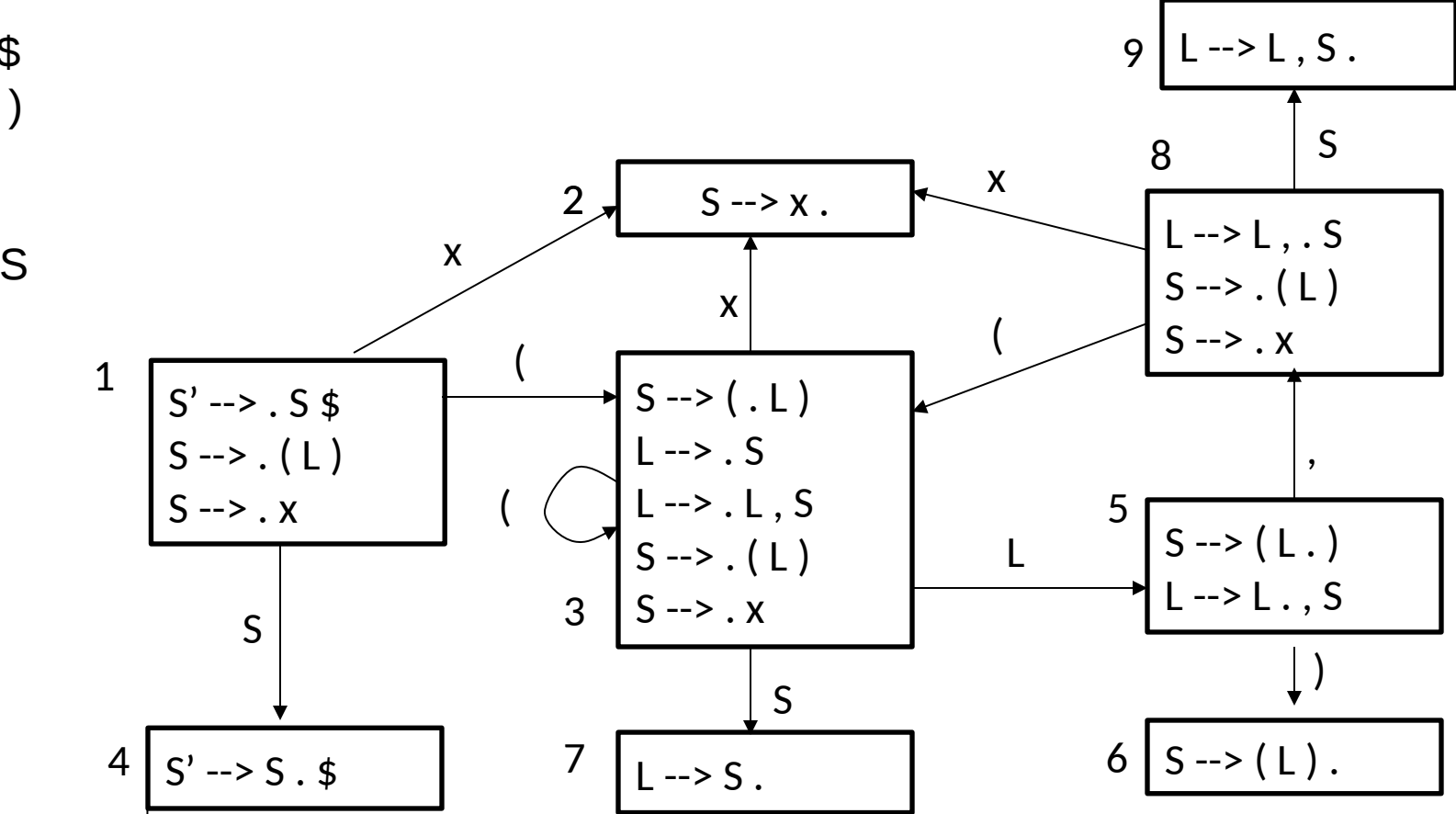
GOTO table

states	()	x	,	\$	S	L
1	s3		s2			4	
2		r2		r2	r2		
3	s3		s2				
4							

Acceptance

$S' \rightarrow S \$.$

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$



ACTION table

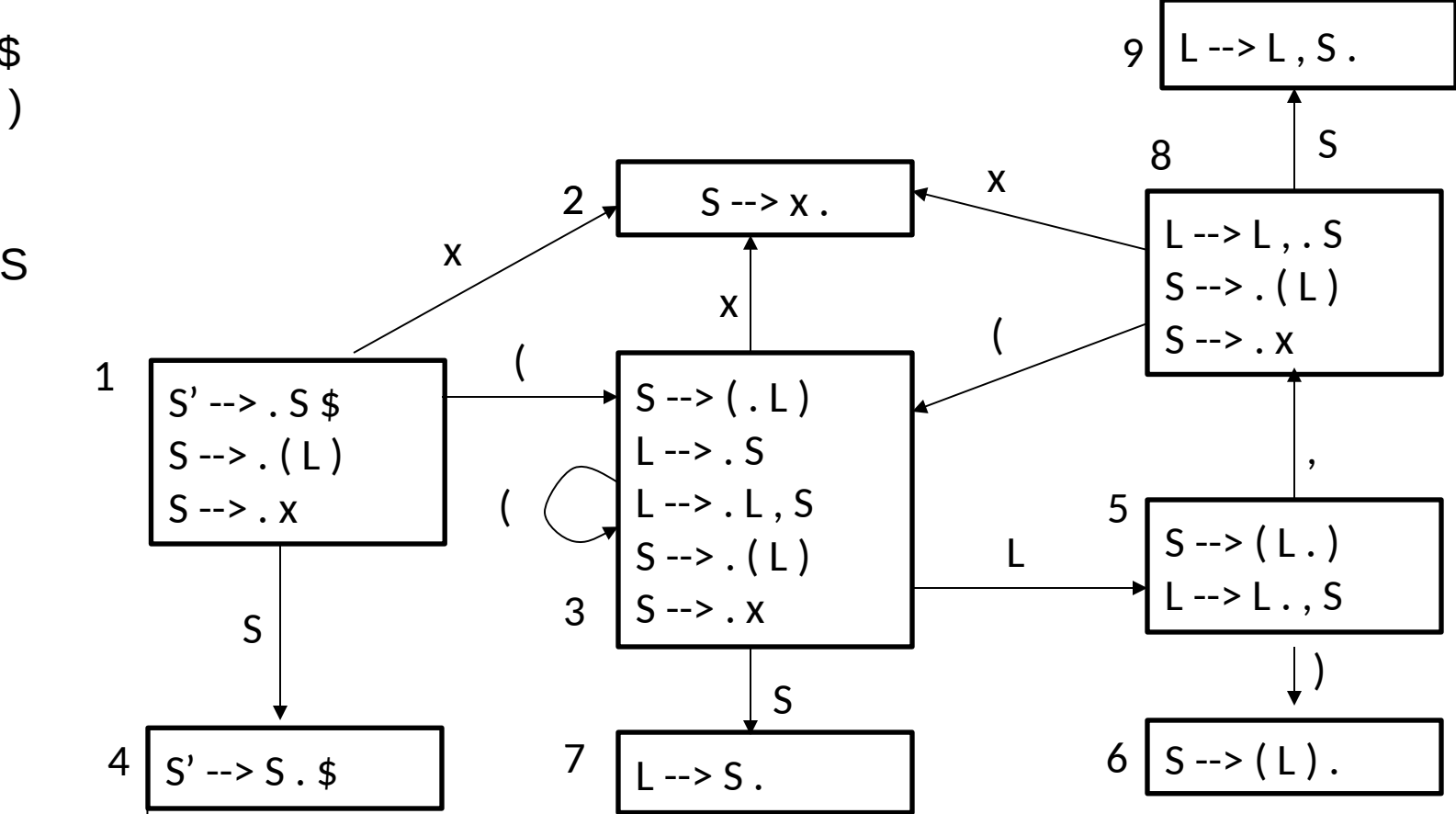
GOTO table

states	()	x	,	\$	S	L
1	s3		s2			4	
2		r2		r2	r2		
3	s3		s2			7	5
4							

Acceptance

$S' \rightarrow S \$$

$S' \rightarrow S \$$
 $S \rightarrow (L)$
 $S \rightarrow x$
 $L \rightarrow S$
 $L \rightarrow L, S$



ACTION table

GOTO table

states	()	x	,	\$	S	L
1	s3		s2			4	
2		r2		r2	r2		
3	s3		s2			7	5
4					accept		

Acceptance

$S' \rightarrow S \$.$

states	()	x	,	\$	S	L
1	s3		s2			4	
2		r2		r2	r2		
3	s3		s2			7	5
4					accept		
5		s6		s8			
6		r1		r1	r1		
7		r3		r3			
8	s3		s2			9	
SLR parsing table for G							

r0 S' --> S \$

r1 S --> (L)

r2 S --> x

r3 L --> S

r4 L --> L , S

Follow(S)={\$,), , }

Follow(L)={), , }

Example: Moves of LR parser on (x,x) input.

Stack	Input	Action
1	(x,x)\$	shift
1(3	x,x)\$	shift
1(3x2	,x)\$	Reduce $S \rightarrow x$
1(3S7	,x)\$	Reduce $L \rightarrow S$
1(3L5	,x)\$	shift
1(3L5,8	x)\$	shift
1(3L5,8x2)\$	Reduce $S \rightarrow x$
1(3L5,8S9)\$	Reduce $L \rightarrow L, S$
1(3L5)\$	shift
1(3L)6	\$	Reduce $S \rightarrow (L)$
1S4	\$	accept

Task

- Task Given the following CFG grammar G with P:

$$\begin{aligned} S &\rightarrow aABe \\ A &\rightarrow Abc \mid b \\ B &\rightarrow d \end{aligned}$$

- Construct the corresponding parsing table using SLR parsing algorithm.
- Show the stack contents, the input and the action used during parsing for the input string $w = abbcde$

Exercise

- Task Given the following CFG grammars G1 & G2 with P:

G1:

$E \rightarrow E+T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

G2:

$S \rightarrow L=R$

$S \rightarrow R$

$L \rightarrow *R$

$L \rightarrow id$

$R \rightarrow L$

- Construct the corresponding parsing tables for G1 & G2 using SLR parsing algorithm.
- Show the stack contents, the input and the action used during parsing for the input string $w = id+id*id$ using parsing table of G1 .