(1) $\underbrace{0}_{\text{sign}}|\underbrace{10011}_{\text{exponent}}|\underbrace{0000001010}_{\text{significand}}$

~~0100110000001011~~

Exponent: $2^0 \times 1 + 2^1 \times 1 + 2^2 \times 0 + 2^3 \times 0 + 2^4 \times 1$

$= 1 + 2 + 16$

$= 19.$

subtract bias

Power of 2: $19 - 15 = 4$

Significand

$$1.0000001010 \times 2^{19-15}$$

$2^{-1} \times 0 \quad 2^{-2} \times 0 \quad 2^{-7} \times 1 \quad 2^{-9} \times 1$

Fraction

$$\left(1 + \frac{1}{2^7} + \frac{1}{2^9}\right) \times 2^4$$

$$= \left(1 + \frac{1}{128} + \frac{1}{512}\right) \times 2^4$$

$$= \left(\frac{512}{512} + \frac{4}{512} + \frac{1}{512}\right) \times 2^4$$

$$= \frac{517}{512} \times 16 = \boxed{\frac{517}{32}} \approx \boxed{16.16}$$

(2) The next biggest number should increment the significand, leaving the same exponent.

a) $0|10011|000000\underline{\underline{1011}}$

Here, we used the fact that 1011 is the next biggest binary number after 1010

(b) and (c) using a similar procedure:

$$1.0000001011 \times 2^{19-15}$$

$$2^{-1}\times 0 \quad 2^{-2}\times 0 \quad 2^{-7}\times 1 \quad 2^{-9}\times 1 \quad 2^{-10}\times 1$$

$$= \left(1 + \frac{1}{2^7} + \frac{1}{2^9} \times \frac{1}{2^{10}}\right) \times 2^4$$

$$= \left(\frac{1024}{1024} + \frac{8}{1024} + \frac{2}{1024} + \frac{1}{1024}\right) \times 2^4$$

$$= \frac{1035}{1024} \times 2^4 = \boxed{\frac{1035}{64}} \approx \boxed{16.17}$$

Machine Epsilon : difference between 1 and next bigge number.

0011110000000000
0011110000000001 $\longrightarrow (1+2^{-10}) - 1 = 1/2^{10} \approx 9.7\times 10^{-4}$

00111111100...0
$\underbrace{00111111100...1}_{\text{23 bits}}$ $\longrightarrow (1+2^{-23}) - 1 = 1/2^{23} \approx 1.2\times 10^{-7}$

001111111111100...0
$\underbrace{001111111111100...1}_{\text{52 bits}}$ $\longrightarrow (1+2^{-52}) - 1 = 1/2^{52} \approx 2.2\times 10^{-16}$

Largest number

We set both exponent and significand to be the highest they can be without showing up as "infinity" or "NaN".

0111101111111111

$$= 1.111....1 \times 2^{15}$$

$$= \left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{1024}\right)\times 2^{15}$$

Exponent = 30
minus bias = 15

$$= \frac{2047}{1024} \times 2^{15}$$

$$= 65{,}504$$

$\underbrace{011111110}\underbrace{11...1}_{\text{23 bits}}$

$$= 1.111...1 \times 2^{127}$$

$$= \left(1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{23}}\right)\times 2^{127}$$

Exponent = 254
minus bias = 127

$$= 1.999.... \times 2^{15}$$

$$\approx 2\times 2^{127} = 2^{128}$$

$$= 3.4\times 10^{38}$$

$\underbrace{011111111110}\underbrace{11..1}_{\text{52 bits}}$

$$= 1.11...1 \times 2^{5}$$

$$= \left(1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^{52}}\right)\times 2^{1023}$$

Exponent = 2046
minus bias = 1023

$$= 1.99... \times 2^{1023}$$

$$\approx 2\times 2^{1023} = 2^{1024}$$

$$= 1.8\times 10^{308}$$

Smallest Number (greater than zero)

$000\ldots1$

significand : $00\cdots1$
exponent : $00\cdots0$

General form

$$\boxed{0.00\ldots1 \times 2^{0-\text{bias}+1}}$$

16-bit $0.\overbrace{00\ldots1}^{10} \times 2^{-15+1}$

$$= \left(0+\frac{1}{2^{10}}\right) \times 2^{-14} = 2^{-24} \approx 5.97 \times 10^{-8}$$

32-bit $0.\overbrace{00\ldots.1}^{23} \times 2^{-127+1}$

$$= \left(0+\frac{1}{2^{23}}\right) \times 2^{-126} = 2^{-149} \approx 1.4 \times 10^{-45}$$

64-bit $0.\overbrace{00\ldots1}^{52} \times 2^{-1023+1}$

$$= \left(0+\frac{1}{2^{52}}\right) \times 2^{-1022} = 2^{-1074} \approx 5 \times 10^{-324}$$

Number of Numbers

including zero but excluding ∞ and NaN.

16-bit: possible numbers: 2^{16}

When exponent is all 1's, we don't have numbers

$$\underbrace{0}_{1\text{ bit}}11111\underbrace{0000000000}_{10\text{ bits}}$$

Whether these are zero or one, no admissable numbers.

2^{11} possible numbers here.

$$\Rightarrow 2^{16} - 2^{11} = 63{,}488$$

32-bit possible numbers: 2^{32}

When exponent is all 1's, we don't have numbers

$$\underbrace{0}_{1\text{ bit}}11111\underbrace{000000\cdots000}_{23\text{ bits}}$$

Whether these are zero or one, no admissable numbers.

2^{24} possible numbers here.

$$\Rightarrow 2^{32} - 2^{24} \approx 4.27\times10^{9}$$

64-bit possible numbers: 2^{64}

When exponent is all 1's, we don't have numbers

$$\underbrace{0}_{1\text{ bit}}11111\underbrace{000000\cdots000}_{52\text{ bits}}$$

Whether these are zero or one, no admissable numbers.

2^{53} possible numbers here.

$$\Rightarrow 2^{64} - 2^{53} \approx 1.84\times10^{19}$$