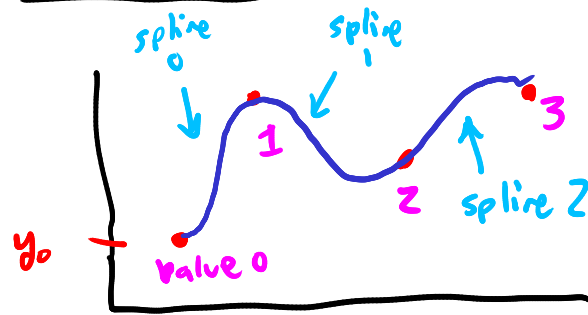


TODAY:

- NUMPY
- CUBIC SPLINES
- CODING!

CUBIC SPLINE PROBLEM



El9- 9/21
 $n+1$ DATA VALUES (SAMPLES AKA CONTROL POINTS)
 $y_0, y_1, y_2, \dots, y_n$

n SPLINE FUNCTIONS - CUBIC POLYNOMIAL

$$y_i(t) = a_i + b_i t + c_i t^2 + d_i t^3$$

$t \in [0, 1]$ $4n$ UNKNOWN COEFFS

CONSTRAINTS - PASS THRU DATA VALUES

$$y_i(0) = y_i \quad y_i(1) = y_{i+1}$$

$$y_0(0) = y_0 \quad y_0(1) = y_1$$

$$y_1(0) = y_1 \quad y_1(1) = y_2$$

$$y_2(0) = y_2 \quad y_2(1) = y_3$$

$2n$ EQUATIONS (LINEAR)

$$y_i(0) = y_i \rightarrow a_i = y_i$$

$$y_i(1) = y_{i+1} \rightarrow a_i + b_i + c_i + d_i = y_{i+1}$$

$$y_i'(t) = b_i + 2c_i t + 3d_i t^2$$

$$y_i''(t) = 2c_i + 6d_i t$$

AGREEMENT IN DERIVATIVE

$$y_i'(1) = y_{i+1}'(0)$$

$$y_i''(1) = y_{i+1}''(0)$$

$2(n-1)$ ADD'L EQN'S

$$b_i + 2c_i + 3d_i = b_{i+1}$$

$$2c_i + 6d_i = 2c_{i+1}$$

$$2n + 2(n-1) = 4n - 2$$

2 ADD'L EQN'S - SET $y'' = 0$ AT START / END

$$y_0''(0) = 0 \quad y_{n-1}''(1) = 0$$

COULD ARRANGE INTO BIG OL' $Ax = b$

$$\begin{bmatrix} \text{"LHS stuff"} \\ 4n \times 4n \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ \vdots \\ a_{n-1} \\ b_{n-1} \\ c_{n-1} \\ d_{n-1} \\ 4n \times 1 \end{bmatrix} = \begin{bmatrix} \text{"RHS stuff"} \\ 4n \times 1 \end{bmatrix}$$

NOT GONNA SOLVE BIG SYSTEM - SOLVE SMALLER TRIDIAGONAL SYSTEM

DEFINE $n+1$ NEW UNKNOWN D_0, D_1, \dots, D_n

$$D_i = y'_i(0) = b_i$$

$$D_i = y'_{i+1}(1)$$

$$D_{i+1} = y_i(1)$$

NEXT SOLVE FOR ALL a_i, b_i, c_i, d_i IN TERMS OF
 y_0, \dots, y_n AND D_0, \dots, D_n

$$a_i = y_i$$

$$b_i = D_i$$

SOLVE THESE
FOR c_i & d_i

$$y_i(1) = a_i + b_i + c_i + d_i = y_{i+1}$$

$$= y_i + D_i + c_i + d_i = y_{i+1}$$

$$y'_i(1) = b_i + 2c_i + 3d_i = D_{i+1}$$

$$= D_i + 2c_i + 3d_i = D_{i+1}$$

AFTER SOME ALGEBRA...

$$c_i = 3(y_{i+1} - y_i) - 2D_i - D_{i+1}$$

$$d_i = 2(y_i - y_{i+1}) + D_i + D_{i+1}$$

AGREEMENT IN y'' :

$$y''_{i-1}(1) = y''_i(0)$$

SUB IN

$$2c_{i-1} + 6d_{i-1} = 2c_i$$

MORE ALGEBRA

$$\cancel{2}D_{i-1} + \cancel{4}D_i + \cancel{2}D_{i+1} = \cancel{6}(y_{i+1} - y_{i-1})$$

$$\begin{matrix} (n+1) \\ \times \\ (n+1) \end{matrix} \begin{bmatrix} ? & & & \\ 1 & 4 & & \\ & 1 & 4 & \\ & & 1 & 4 \\ & & & 1 & 4 & \dots \\ & & & & ? & \end{bmatrix} \begin{bmatrix} D_0 \\ D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} = \begin{bmatrix} ? \\ 3(y_2 - y_0) \\ 3(y_3 - y_1) \\ \vdots \\ 3(y_n - y_{n-2}) \\ ? \end{bmatrix}$$