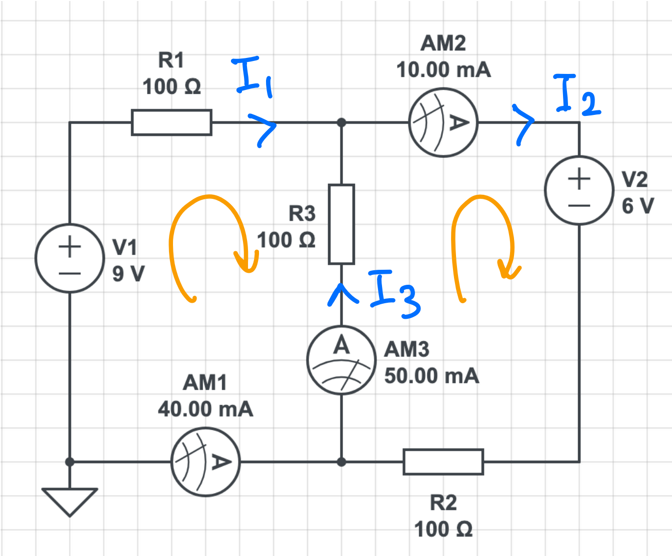
Write down a matrix-vector equation where the unknown is a vector of currents, I_1, I_2 and I_3. Verify that the ammeter values shown in this diagram are correct.



$$+9 - 100 I_1 + 100 I_3 = 0$$
$$-100 I_3 - 6 - 100 I_2 = 0$$
$$I_1 + I_3 = I_2$$

$$\begin{bmatrix} -100 & 0 & +100 \\ 0 & -100 & -100 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ 0 \end{bmatrix}$$

Solving the above with Doolittle's method Write down the system matrix, A, using Kirchhoff's Laws and find its LU Decomposition using Doolittle's method. What is the right-hand side?

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

In the following decompsition, determine all the entries such as L_{21} and U_{12}.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Use L and U to calculate the value of the currents in the circuit.

For Doolittle's method, use Gaussian Elimination

$$\begin{bmatrix} -100 & 0 & +100 \\ 0 & -100 & -100 \\ 1 & -1 & 1 \end{bmatrix}$$

row 2 = row 2 − 0 × row 1

row 3 = row 3 − $\left(\frac{-1}{100}\right)$ × row 1

$\cancel{1 - \frac{-1}{100} \times -100} \rightarrow 0$, $-1 - (\cdots) \times 0$, $1 - \left(\frac{-1}{100}\right) \times 100$

0 , −1 , 2

Now matrix is

$$\begin{bmatrix} -100 & 0 & +100 \\ 0 & -100 & -100 \\ 0 & -1 & 2 \end{bmatrix}$$

row 3 = row 3 − $\frac{1}{100}$ × row 2

$0 - \frac{1}{100} \times 0$, $\cancel{-1 - \frac{1}{100} \times -100} \rightarrow 0$, $2 - \frac{1}{100} \times -100$

0 , 0 , 2 + 1

$$\begin{bmatrix} -100 & 0 & 100 \\ 0 & -100 & -100 \\ 0 & 0 & 3 \end{bmatrix} = U.$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{100} & \frac{1}{100} & 1 \end{bmatrix}$$

$$\underbrace{LUx}_{y} = b \qquad \text{or} \qquad Ax = b$$

$Ly = b$ by forward elimination.

$$b = \begin{bmatrix} -9 \\ 6 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{100} & \frac{1}{100} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ 0 \end{bmatrix}$$

$$y_1 = -9$$

$$0 \times -9 + y_2 = 6 \Rightarrow y_2 = 6$$

$$+\frac{9}{100} + \frac{6}{100} + y_3 = 0 \Rightarrow y_3 = \frac{-15}{100} = -3/20$$

now, $Ux = y$ by backward elimination

$$\begin{bmatrix} -100 & 0 & 100 \\ 0 & -100 & -100 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -9 \\ 6 \\ -0.15 \end{bmatrix}$$

1) $3x_3 = -0.15 \Rightarrow \boxed{x_3 = -0.05}$

2) $-100x_2 - 100.x_3 = 6$

$-100x_2 + 5 = 6 \Rightarrow \boxed{x_2 = -0.01}$

3) $-100x_1 + 100x_3 = -9$

$-100x_1 - 5 = -9 \Rightarrow \boxed{x_1 = 0.04}$

$I_1 = 40$ mA

$I_2 = -10$ mA

$I_3 = -5$ mA

Calculate the currents when (a) $V_1 = 5$ and $V_2 = 4$ Volts, and (b) $V_1 = 100$ and $V_2 = 1$ Volts. Use the LU decomposition each time.

The same LU decomposition can be used: $b_1 = -V_1$, $b_2 = +V_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{100} & \frac{1}{100} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad y_1 = -5, \quad y_2 = 4, \quad y_3 = ?$$

$$-\frac{-5}{100} + \frac{4}{100} + y_3 = 0 \quad \Rightarrow \quad y_3 = -0.09$$

$$\begin{bmatrix} -100 & 0 & 100 \\ 0 & -100 & -100 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -9/100 \end{bmatrix} \Rightarrow \boxed{\begin{aligned} x_3 &= -0.03 \\ x_2 &= -0.01 \\ x_1 &= +0.02 \end{aligned}}$$

$$-100x_2 - 100 \times \frac{-3}{100} = 4, \quad -100x_2 = 4 - 3$$

$$-100x_1 + 100 \times \frac{-3}{100} = -5, \quad -100x_1 = -5 + 3$$