How many 16-bit floats between 2^4 and 2^5?

$2^4 = 16$:

$$\underbrace{010011}_{19}\underbrace{0000000000}$$

bias: 15

$$1.000\ldots0_2 \times 2^{19-15}$$

The next number will increment significand.

```
0|10011|0000000001   *
0|10011|0000000010   **
0|10011|0000000011
.|.....|
0|10011|1111111111
```

$$1.111\ldots1_2 \times 2^4$$

and so on. There are 10 binary digits in significand.

$\Rightarrow$ 2^{10} numbers b/w 2^4 and 2^5.

Gap Size

Difference b/w * and ** is :

$$\overbrace{\left(1 + \frac{1}{2^9}\right) \times 2^4}^{*} - \overbrace{\left(1 + \frac{1}{2^{10}}\right) \times 2^4}^{**}$$

can cancel 1's

$$= 2^4\left[\frac{1}{2^9} - \frac{1}{2^{10}}\right]$$

$$= 2^4\left[\frac{2}{2^{10}} - \frac{1}{2^{10}}\right] = 2^4 \times \frac{1}{2^{10}} = \boxed{2^{-6}} \approx 0.0156$$

How many 16-bit floats between 2^7 and 2^8?

$2^7 = 128$:

$$\underbrace{01011}_{22}\underbrace{00000000000}$$

bias: 15

$$1.000\ldots0_2 \times 2^{22-15}$$

The next number will increment significand.

```
0|10110|0000000001   *
0|10110|0000000010   **
0|10110|0000000000
...
0|10110|1111111111
```

$$1.111\ldots1_2 \times 2^4$$

and so on. There are 10 binary digits in significand.

$\Rightarrow$ 2^{10} numbers b/w 2^7 and 2^8.

Gap Size

Difference b/w * and ** is:

$$\overbrace{\left(1 + \frac{1}{2^9}\right) \times 2^7}^{*} - \overbrace{\left(1 + \frac{1}{2^{10}}\right) \times 2^7}^{**}$$

can cancel 1's

$$= 2^7 \left[\frac{1}{2^9} - \frac{1}{2^{10}}\right]$$

$$= 2^7 \left[\frac{2}{2^{10}} - \frac{1}{2^{10}}\right] = 2^7 \times \frac{1}{2^{10}} = \boxed{2^{-3}} = 0.125$$

How many 16-bit floats between 2^{-5} and 2^{-4}?

$2^{-5} = 1/32$

$0|\underbrace{01010}|0000000000$ $\quad 1.000\ldots00_2 \times 2^{-5}$

10 . minus bias = -5

The next number will increment significand.

$0|01010|0000000001$ ★

$0|01010|0000000010$ ★★

$0|01010|0000000011$

...

$0|01010|1111111111$ → biggest number with exponent -5

$\Rightarrow$ 2^{10} 16-bit floats between 2^{-5} and 2^{-4}.

Gap Size

Difference b/w ★ and ★★ is :

$$\underbrace{\left(1 + \frac{1}{2^9}\right) \times 2^{-5}}_{★} - \underbrace{\left(1 + \frac{1}{2^{10}}\right) \times 2^{-5}}_{★★}$$ can cancel 1's

$$= 2^{-5}\left[\frac{1}{2^9} - \frac{1}{2^{10}}\right]$$

$$= 2^{-5}\left[\frac{2}{2^{10}} - \frac{1}{2^{10}}\right] = 2^{-5} \times \frac{1}{2^{10}} = \boxed{2^{-15}} \approx 0.0000305$$

How many 16-bit floats between 2^{-8} and 2^{-7} ?

$$2^{-8} = 1/256$$

$$\underbrace{0|00111|}_{7.\ \text{minus bias} = -8}0000000000 \qquad 1.000\ldots00_2 \times 2^{-8}$$

The next number will increment significand.

0|00111|0000000001 ☆

0|00111|0000000010 ☆☆

0|00111|0000000011

...

0|00111|1111111111 → biggest number with exponent −8

$\Rightarrow$ 2^{10} 16-bit floats between 2^{-8} and 2^{-7}.

Gap Size

Difference b/w ☆ and ☆☆ is :

$$\underbrace{\left(1 + \frac{1}{2^9}\right) \times 2^{-8}}_{☆} - \underbrace{\left(1 + \frac{1}{2^{10}}\right) \times 2^{-8}}_{☆☆}$$

can cancel 1's

$$= 2^{-8}\left[\frac{1}{2^9} - \frac{1}{2^{10}}\right]$$

$$= 2^{-8}\left[\frac{2}{2^{10}} - \frac{1}{2^{10}}\right] = 2^{-8} \times \frac{1}{2^{10}} = \boxed{2^{-18}} \approx 0.00000381$$