1. The Lorenz equations. Consider the system of ordinary differential equations studied by Edward Lorenz as a model for atmospheric convection.

$$\dot{x} = \sigma(y - x) \tag{1a}$$
$$\dot{y} = rx - y - xz \tag{1b}$$
$$\dot{x} = xy - bz, \tag{1c}$$

where $\sigma, r, b > 0$. For each of the following parts, you should produce two plots:

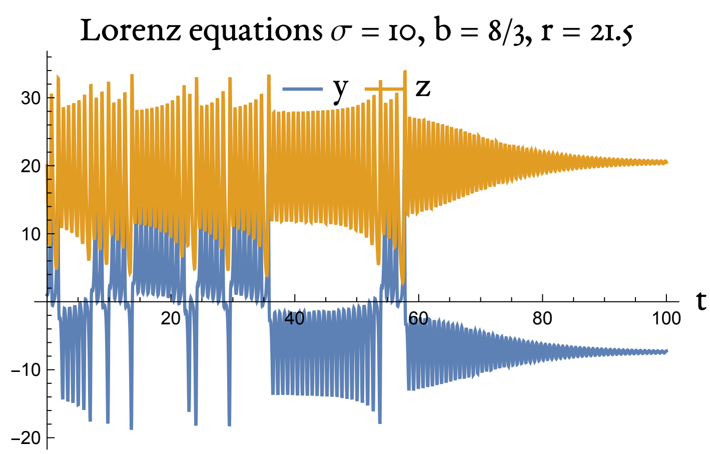
1. (Time series) Plot $y(t)$ and $z(t)$ on the same set of axes, with t on the horizontal axis.
2. (Parametric) Plot $x(t)$ against $z(t)$, i.e., place x on the vertical axis and z on the horizontal axis. The parametric plot should have the **same** axis limits across all plots.

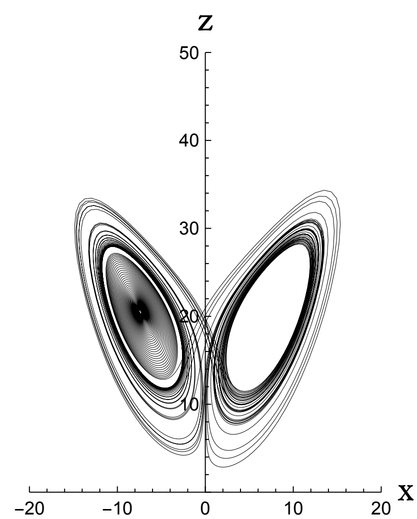
(a) Numerically solve (1) using a programming language of your choice for the following parameters

- $\sigma = 10$, $b = 8/3$, $r = 21.5$
- $x(0) = 0, y(0) = 1, z(0) = 20$.
- integrate from $t = 0$ to $t = 100$,

and plot the results. Describe this behaviour in words. Do you see any fixed points, limit cycles, or chaos?

Ans. The numerical solution is shown below.





(b) Numerically solve (1) using a programming language of your choice for the following parameters

- $\sigma = 10$, $b = 8/3$, $r = 14$
- $x(0) = 0, y(0) = 1, z(0) = 0$.
- integrate from $t = 0$ to $t = 20$,

and plot the results. Describe this behaviour in words. Do you see any fixed points, limit cycles, or chaos?

Ans. The numerical solution is shown below.