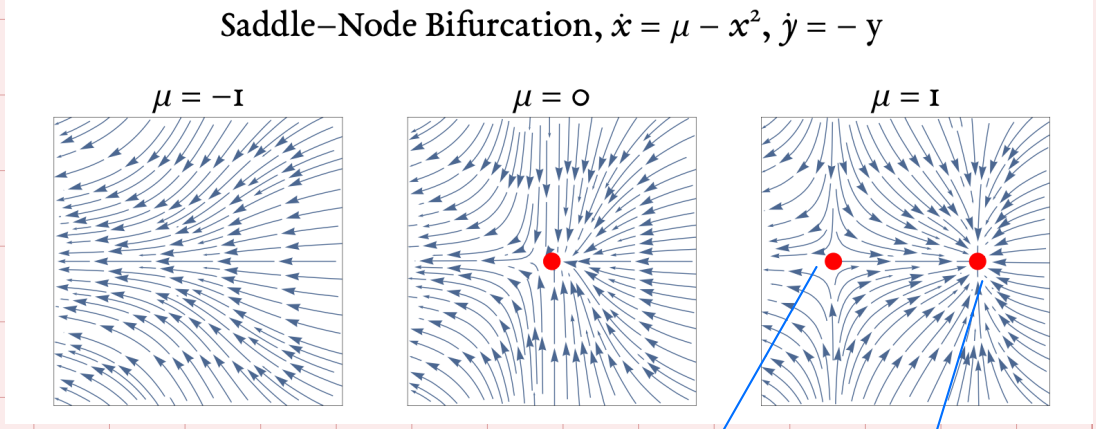
Mon, Mar 24 Lecture 15

Bifurcations

A bifurcation occurs when the topology of phase space changes (qualitatively)

$$\dot{x} = \mu - x^2$$
$$\dot{y} = -y$$

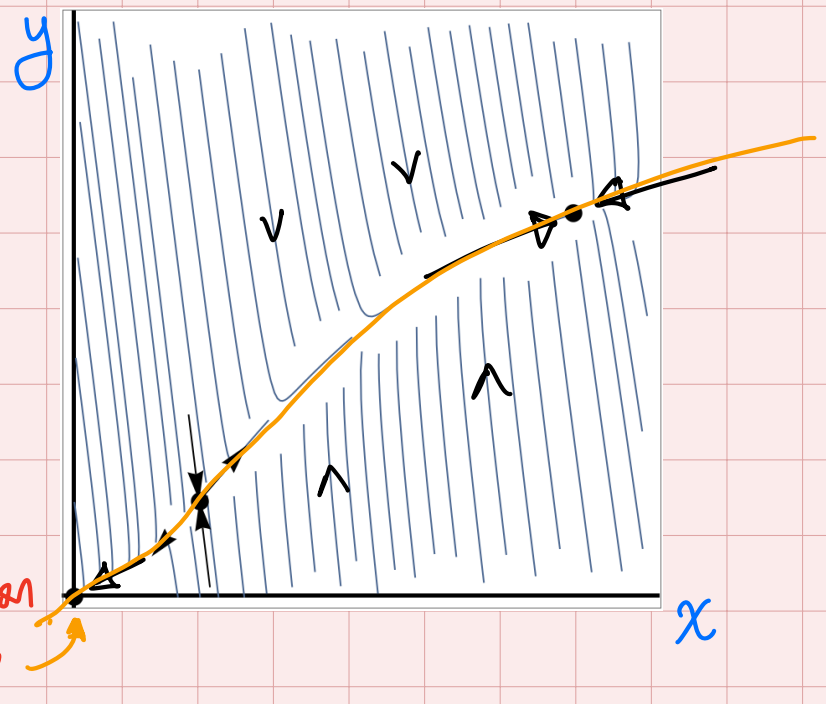


Saddle-node Bifurcation in 2-d occurs in one dimension

$$\dot{x} = -ax + y$$
$$\dot{y} = \frac{x^2}{1+x^2} - by$$

$a, b > 0$

$x, y > 0$

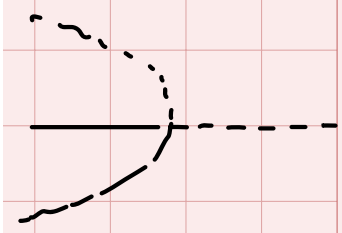


in this system, bifurcation occurs on this curve

tinyurl.com/E91bifurcations2d

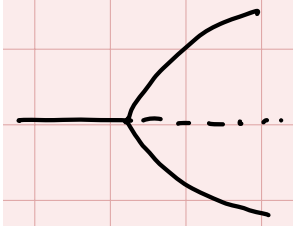
Subcritical Pitchfork bifurcation, $\dot{x} = \mu\, x + x^3$, $\dot{y} = -\, y$

$\mu = -1$ $\mu = 0$ $\mu = 1$



Supercritical Pitchfork bifurcation, $\dot{x} = \mu\, x - x^3$, $\dot{y} = -\, y$

$\mu = -1$ $\mu = 0$ $\mu = 1$

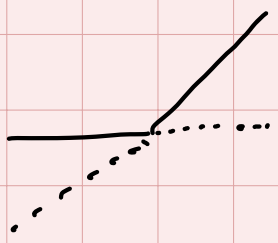


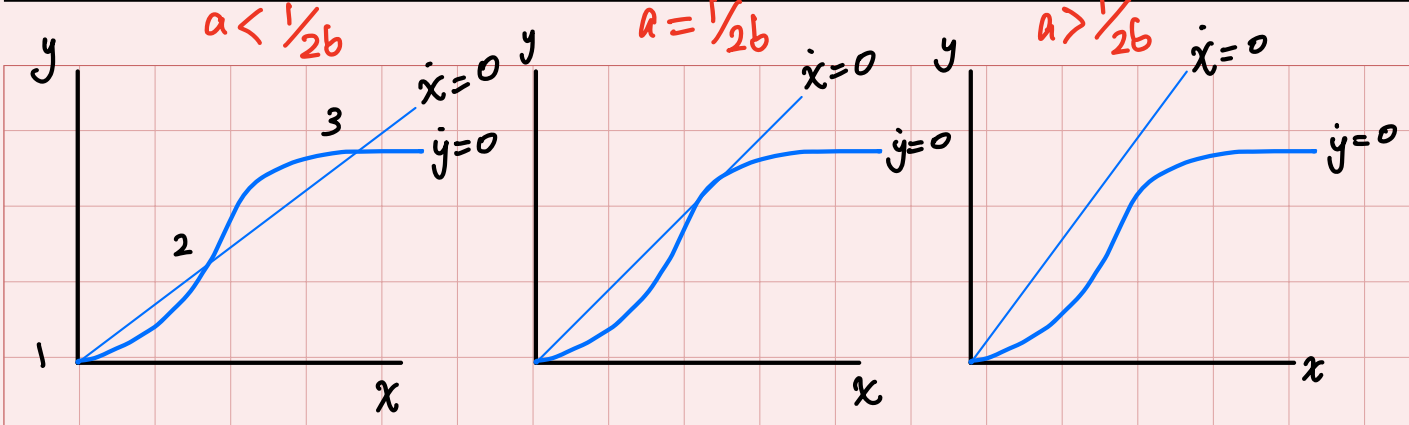
Saddle–Node Bifurcation, $\dot{x} = \mu - x^2$, $\dot{y} = -\, y$

$\mu = -1$ $\mu = 0$ $\mu = 1$

Transcritical Bifurcation, $\dot{x} = \mu\, x - x^2$, $\dot{y} = -\, y$

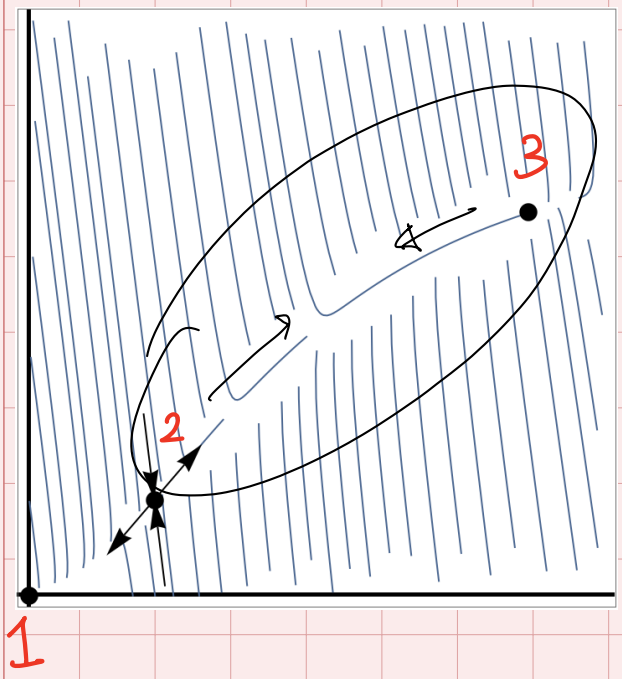
$\mu = -1$ $\mu = 0$ $\mu = 1$





Fixed pts are at $ax\,b(1+x^2) = x^2$: (0, 0)

and at $x = \dfrac{1 \pm \sqrt{1-(2ab)^2}}{2ab}$



Two of the fixed points meet and annihilate each other. (2 and 3)

Example of a saddle-node bifurcation

< in-class exercise >