ENGR91 Nonlinear Dynamics & Chaos

Spring 2025
CRN 29577
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Instructor	Emad Masroor	emasroo1@swarthmore.edu
Lectures	MW 11:45 — 1:00	221 Singer Hall
Office hours	W 2:00 — 4:00 PM & by appt.	112 Singer Hall
Grader	Brandon Mickelson	
Textbook	Nonlinear Dynamics and Chaos Steven Strogatz (2nd or 3rd edition)	
Prerequisites	MATH 15 & 25, ENGR 12, & (MATH 43/44 or CHEM 42 or PHYS 7)	
Lab	Self-scheduled; Details TBA	
Website	emadmasroor.github.io/classes/E91_S25/ and moodle.swarthmore.edu	
Final Exam	TBD	

Course description This course will introduce you to the exciting field of nonlinear dynamics and chaos, with an ephasis on its applications in engineering, science, and the social sciences. The course will emphasize how the *nonlinearity* of the differential equations that govern the world around us adds significant complexity compared with linear systems — such as those you learned about in E12 — and will illustrate this with 1-D and 2-D systems of the form $\dot{x} = f(x), x \in \mathbb{R}^d$ before moving on to 3-D systems after spring break, starting with the Lorenz equations as a chaotic model of atmospheric dynamics. Chaos will then be treated rigorously for both differential equations and maps, such as the logistic map $x_{n+1} = rx_n(1-x_n)$. The course will end with a technical introduction to the study of fractals, motivated by their appearance in chaotic systems. Throughout the course, there will be a complementary emphasis on analytical techniques and numerical simulations in MATLAB or Python.

Course website Course material will be uploaded to https://emadmasroor.github.io/classes/E91_S25/. This primarily includes lecture notes and homework assignments. You can expect each lecture's notes to be uploaded to this website within a few hours of the lecture, and you can expect homework assignments due in week $n+1$ to be uploaded here on week n. Moodle will be used to submit assignments and communicate grades.

Learning Objectives After completing this course successfully, you should be able to:

- Understand how the mathematics of nonlinear ordinary differential equations describe processes of change in the world.
- Quantitatively and qualitatively interpret nonlinear ordinary differential equations using analytical, numerical, and graphical techniques. This includes the use of software and/or programming languages, depending on your interest and motivation.
- Identify when a process of change is or can be linear vs. nonlinear, stable vs. unstable, and integrable vs. chaotic, and apply this information in a scientific or engineering context.
- Understand and explain, to a lay audience, the concept of chaos, and how it differs from what is usually called 'randomness'.

Policies

0.1 Class meetings

This course will be conducted in the form of (in-person) lectures during the regularly scheduled meeting times (i.e., Mondays and Wednesdays at 11:45 AM for 75 minutes per class meeting). Students are expected to attend every class meeting and take notes. The instructor will make available the lecture notes developed in class, but these do not replace the need for students to actively engage with the material, ideally by taking their own notes. Please be respectful of everyone else's time by being punctual. You must not use cell phones or laptop computers during class, but you are permitted to take notes on a tablet or tablet-like device. Note that you need explicit permission of the instructor to take notes on your laptop, and such permission will be granted only in exceptional circumstances. If you must use your cell phone during class, you may excuse yourself from the classroom.

0.2 Exams

There will be one midterm exam and one final exam for this course. You should expect the midterm to be held around spring break, and you should expect the final to be a **take-home exam** which you can complete over a 24-hour period of your choosing during finals week. The final exam will be cumulative. If you miss an exam without prior notice, it will not be rescheduled. If you need to take an exam at an alternate time for any reason, you must request this in writing at least one week prior to the scheduled exam. Such a request will be granted on a case-by-case basis at the discretion of the instructor.

0.3 Homework

Homework assignments are an integral part of this course. Typically, they will be **due on Wednesdays at 11:59 PM**. You should expect to turn in approximately 12 homework assignments over the course of the semester. These assignments will collectively serve as a written document testifying to your ability to solve problems in nonlinear dynamics. As such, you should strive to turn in your best work.

- Your submissions should be in PDF format, either typeset on your computer or handwritten and neatly scanned.
- Although you will need to write code to solve some problems, your code will typically not be graded. You may, however, be asked to submit your code as an appendix.
- Diagrams should be 'publication-quality' whenever feasible. At a minimum, they should include all axis labels and legends where appropriate, and should use a font size that is large enough to be legible. You should pay attention to the thickness of lines, the shape of markers, and the contrast between colors.

Due dates are strict on the timescale of days (e.g., you can't normally submit a homework assignment a day or two late) but flexible on the time scale of an hour (i.e., you can typically submit a homework assignment up to an hour-ish after the deadline and still 'have it count' as if it was submitted on time. A penalty of 50% will be applied for late homework assignments, and they will not be accepted more than seven days after the deadline. The purpose of the 50%

penalty is two-fold: it eliminates the need for negotiations about exactly what happens when you miss a deadline, and it encourages you to stick to the deadlines to stay on track. You get one 'free' one-week extension to use at any time in the semester by invoking this policy in an email to the instructor any time before the deadline.

0.4 Collaboration and Attribution

Working together with others on homework assignments is acceptable and even encouraged. However, what you submit must either be entirely your own work, or clearly name your collaborators. If you worked closely enough with a fellow student that you suspect your solutions will be substantially similar, you should attribute each other on your homework on the front page with a statement like "I worked together with X on problem 2". You are also encouraged to attribute online resources if you use them.

0.5 Artificial Intelligence

With the widespread availability of Large Language Models and other general-purpose artificial intelligence tools, you now have access to a powerful piece of technology that has the potential to drastically change the way we obtain information. At the same time, these tools threaten to reshape our relationship to *knowledge* and *understanding*, and some of these changes are inimical to the values of a college. With this in mind, note that

- You are not prohibited from using these tools — the same way you would use a search engine or an online forum — to help you answer questions when you are studying for this class.
- You are encouraged to use these tools creatively to aid in your understanding. If you have a question about the topics you are learning about in this course and you 'ask an AI', you may get a good answer. But to get a *really* good answer, you'll need to know how to write a good question, which takes some practice.
- Large Language Models are essentially very sophisticated plagiarism machines. After ingesting everything that humans have ever written that's on the internet, these tools can regurgitate this information by sequentially predicting the 'best' words in a given 'conversation'.
- AI is (still) often wrong. There are many reasons for this, but it's important to note that there are basically no guarantees against against finding something wrong on a Google search or a ChatGPT query.
- Everything you turn in for this class must be your own work. Asking an AI tool how to solve a homework or exam problem, and then turning in that solution in your own words is a violation of academic integrity and will be treated as such. In this, AI tools are no different than using Chegg or StackExchange to find out how to solve a problem.

0.6 Assessment and Grades

Grade components and tentative thresholds are shown below.

Homework	25%	A−	80+
Midterm	25%	B−	70+
Final	35%	C−	60+
Lab	15%	D−	50+

Tentative Schedule

Below the horizontal line, the schedule becomes tentative although the topics are correct. This class is being taught for the first time, and we'll see what we cover.

Class	**Date**	**Topic**	**Readings**	**Due**
1.1	Wed, Jan 22	Introduction & Review		
2.1	Mon, Jan 27	Three approaches to dynamics	2.0, 2.1, 4.0, 4.1	
2.2	Wed, Jan 29	Potentials & Linear Stability	2.4, 2.7	HW 1
3.1	Mon, Feb 3	Bifurcations in 1D	Ch. 3	
3.2	Wed, Feb 5	Bifurcations in 1D	Ch. 3	HW 2
4.1	Mon, Feb 10	Linear systems	Ch. 5	
4.2	Wed, Feb 12	Linear systems	Ch. 5	HW 3
5.1	Mon, Feb 17	Nonlinear phase plane	Ch. 6	
5.2	Wed, Feb 19	Nonlinear phase plane	Ch. 6	HW 4
6.1	Mon, Feb 24	Limit cycles	Ch. 7	
6.2	Wed, Feb 26	Limit cycles	Ch. 7	HW 5
7.1	Mon, Mar 3	Bifurcations in 2D	Ch. 8	
7.2	Wed, Mar 5	Bifurcations in 2D	Ch. 8	HW 6
8.1	Mon, Mar 17	The Lorenz System	Ch. 9	
8.2	Wed, Mar 19	Introduction to chaos	Ch. 9	Midterm
9.1	Mon, Mar 24	Lorenz Equations	Ch. 9	
9.2	Wed, Mar 26	'Strange Attractors'	Ch. 9; 12	HW 7
10.1	Mon, Mar 31	One-dimensional maps	Ch. 10	
10.2	Wed, Apr 2	Universality; Feigenbaum const.	Ch. 10	HW 8
11.1	Mon, Apr 7	Cantor set	Ch. 11	
11.2	Wed, Apr 9	Fractals	Ch. 11	HW 9
12.1	Mon, Apr 14	...		
12.2	Wed, Apr 16			HW 10
13.1	Mon, Apr 21			
13.2	Wed, Apr 23			HW 11
14.1	Mon, Apr 28			
14.2	Wed, Apr 30			HW 12
13.2	Wed, Apr 23			
14.1	Mon, Apr 28			
14.2	Wed, Apr 30			