Consider the differential equation

$$m\ddot{x} + c\dot{x} + kx = \sin(\omega t). \tag{1}$$

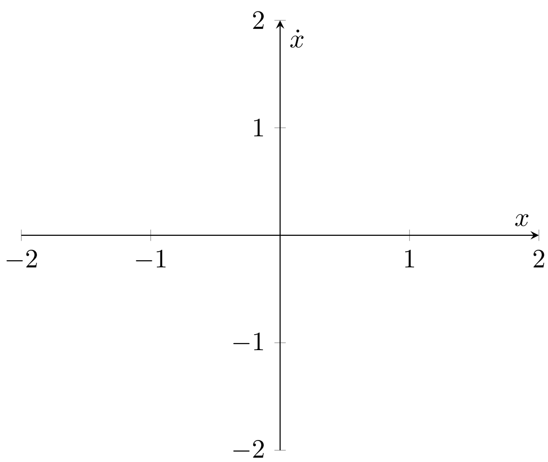
✍ What is the **order** of this differential equation?

✍ Is this an autonomous differential equation or a non-autonomous differential equation?

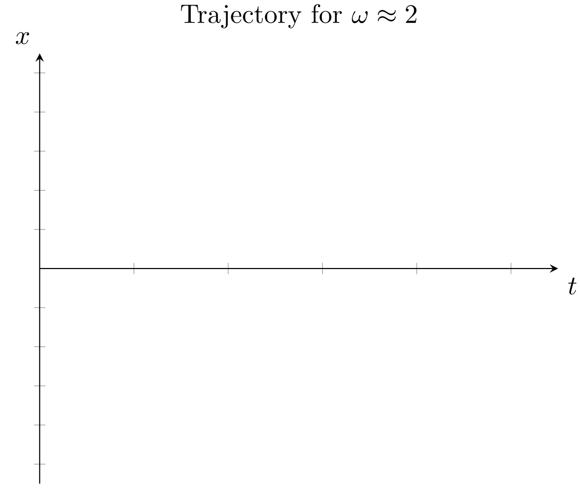
✍ The term on the left represents a spring-mass-dashpot system as usual. What is the physical meaning of the term on the right side?

✍ Visit `https://tinyurl.com/E91limitcycle1` and observe the dynamics at $\omega \approx 2$ and $\omega \approx 1/4$. Alternatively, you can visit `https://emadmasroor.github.io/classes/E91_S25/Resources/ForcedHarmonicOscillator.nb` to download the Mathematica notebook directly. Sketch x against time and $\dot{x}$ against x for long times below. Let the initial condition for your plots be $x(0) = 1, \dot{x}(0) = 0$.

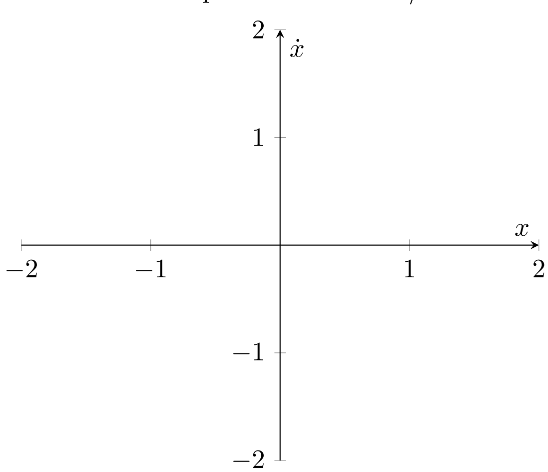
Phase portrait for $\omega \approx 2$



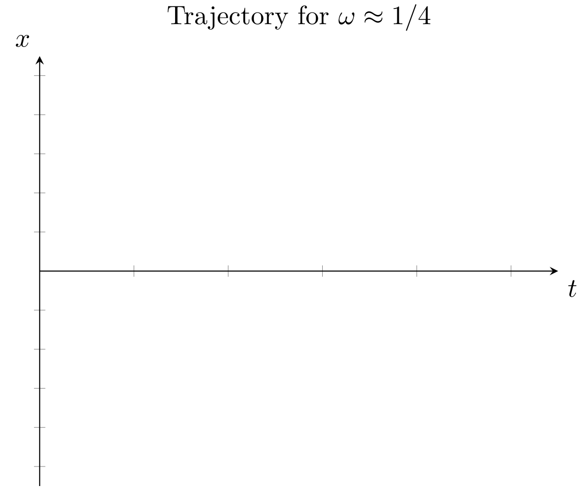
Trajectory for $\omega \approx 2$



Phase portrait for $\omega \approx 1/4$



Trajectory for $\omega \approx 1/4$



Consider the differential equation

$$\ddot{x} + \mu \left(x^2 - 1\right) \dot{x} + x = 0. \tag{2}$$

✍ What is the **order** of this differential equation?

✍ Is this an autonomous differential equation or a non-autonomous differential equation?

✍ Interpret the terms in this equation using the usual language of oscillators. What do they each mean?

✍ For two values of $\mu = 0.1, 4$, and using the initial condition $x(0) = 1, \dot{x}(0) = 0$, numerically integrate these equations using a computer program of your choice, and sketch the resulting trajectories $x(t)$. Use the accompanying graph paper to sketch what your computer program tells you.
For $\mu = 0.1$, plot $t = 0$ to $t = 100$. For $\mu = 4$, plot $t = 0$ to $t = 50$.