Instructions 'Sketch' means you should use a writing implement. 'Plot' means you should use a program of some kind. 'Draw' means you may either sketch or plot. In all cases, you must draw and label axes and properly annotate your diagrams.

1. For the system
$$\dot{x} = f(x; r) = r - \cosh x, \tag{1}$$
   (a) Draw all the qualitatively different phase portraits on separate axes for different values of r. In each phase portrait, draw arrows on the x axis to show the direction of 'flow', and mark the fixed points. Note the stability or instability of each fixed point. **Note:** you should only need one, two or three different values of r to cover all qualitatively different cases. Ignore any phase portraits with semi-stable fixed points; these are 'edge cases' that are not very important for now.
   (b) Draw a bifurcation diagram, i.e., a plot with r on the horizontal axis and x on the vertical axis, with a curve (or curves) showing the fixed points x^* that exist for each value of r. Recall that fixed points are those values of x for which $f(x) = 0$. Use dashed lines for the unstable fixed points and solid lines for the stable fixed points.
   (c) Determine the critical value of r at which a bifurcation occurs.

2. For the system
$$\dot{x} = f(x; r) = rx^3 + x^4, \tag{2}$$
   (a) Draw all the qualitatively different phase portraits on separate axes for different values of r. In each phase portrait, draw arrows on the x axis to show the direction of 'flow', and mark the fixed points. Note the stability or instability of each fixed point. **Note:** This time, you should show semi-stable fixed points as well as stable and unstable ones.
   (b) Sketch a bifurcation diagram, i.e., a plot with r on the horizontal axis and x on the vertical axis, with a curve (or curves) showing the fixed points x^* that exist for each value of r. Recall that fixed points are those values of x for which $f(x) = 0$. Use dashed lines for the unstable fixed points and solid lines for the stable fixed points.
   (c) Determine the critical value of r at which a bifurcation occurs.

3. Consider two systems governed by the differential equations
$$\dot{x} = f(x; r) = rx \pm x^3, \tag{3}$$
   where one of the systems has a '+' sign and the other has a '−' sign. These systems undergo pitchfork bifurcations as the parameter r is varied from below $r = 0$ to above $r = 0$. The bifurcation diagrams for supercritical and subcritical pitchfork bifurcations are shown in fig. 1 for your reference. If both systems are initialized with $x(0) = 0.3$ — indicated with ● — use the bifurcation diagrams to explain, with words and sketches as necessary, what will happen to $x(t)$ as time passes and r is slowly increased from $r = -1$ to $r = +1$. You must write two different explanations, one for the system with a supercritical pitchfork bifurcation and one for the system with a subcritical pitchfork bifurcation. You should not need more than a few sentences for each system.

4. Draw the bifurcation diagram for
$$\dot{x} = rx + x^3 - x^5, \tag{4}$$
   with r in the range $-0.5 < r < 0.5$. Your diagram should have dashed lines for unstable fixed points, solid lines for stable fixed points, and should be qualitatively correct with respect to the direction and curvature of the lines that you draw.

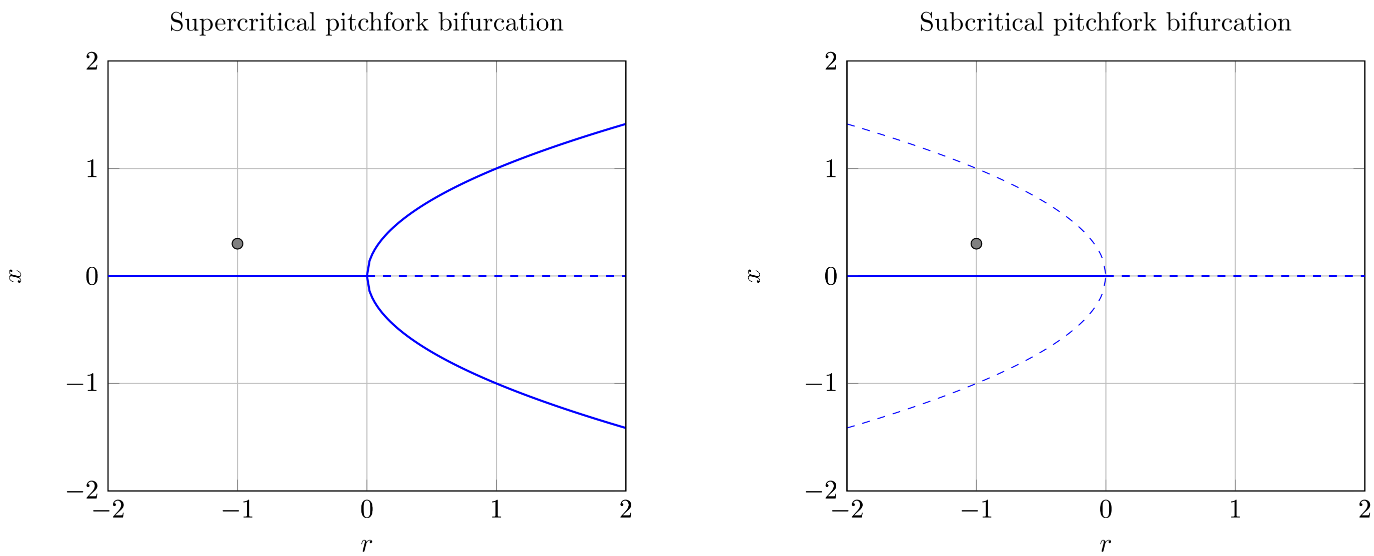

Figure 1: Bifurcation diagrams for the pitchfork bifurcation