

# Traffic Engineering Calculators: A Comparative Study of GoS Calculation Methods

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# 1 Introduction

In the field of telecommunications, the Grade of Service (GoS) is a key performance metric that quantifies the quality of service provided to users. It specifically measures the probability that a call or a service request will fail due to insufficient resources, such as network trunks or channels being unavailable. A low GoS indicates a high probability of calls being blocked or delayed, which is undesirable in efficient network operations.

The project at hand focuses on the design and implementation of computational tools that can estimate and analyze the GoS in telecommunication systems. Two distinct but related calculators were developed: The GoS Calculator and the Traffic Calculator. The former calculates the traffic intensity offered to a group of trunks and determines the corresponding GoS using various methods, while the latter estimates the offered traffic intensity for a group of trunks given a specific GoS.

This report outlines the methodologies adopted for the implementation of these calculators, comparisons between different GoS calculation methods, and the results from various simulations. The simulations were conducted using parameters that are commonly encountered in real-world scenarios, thereby ensuring that the insights derived from the results are both relevant and applicable.

The ultimate goal of this project is to provide tools that assist network engineers in optimizing the utilization of network resources while maintaining an acceptable GoS, thereby striking a balance between cost-effectiveness and user satisfaction.

## 2 Part A: GoS Calculator

### 2.1 Methodology

The Grade of Service (GoS) calculator is designed to compute the service quality in a telecommunication system, where GoS is a measure of the probability that a call will be blocked or delayed. The methodology involves implementing well-known statistical models: Erlang B, Binomial, and Erlang C. The Erlang B formula is used in systems with blocked calls cleared immediately (loss systems), the Binomial formula is applied to systems with a finite source of calls, and the Erlang C formula is used in systems where calls may be queued (delay systems).

### 2.2 Implementation Details

The GoS Calculator was implemented in Python using the Tkinter library for creating the graphical user interface (GUI). Tkinter provides various widgets that allow for a straightforward and interactive GUI design. Python's Math library was used to perform the necessary mathematical calculations.

The calculator takes input parameters: number of trunks  $N$ , number of users  $K$ , average call rate  $\lambda$ , and call holding time  $H$ , to compute the traffic intensity  $A$  and GoS. The user has the option to select the desired calculation method and the traffic intensity unit (Erlang or CCS).

### 2.3 GoS Calculation Methods

The GoS is calculated using three distinct methods:

- Erlang B:  $GoS = \frac{A^N / N!}{\sum_{i=0}^N A^i / i!}$
- Binomial:  $GoS = \sum_{i=N}^M \binom{M-1}{i} A^i (1-A)^{M-1-i}$

- Erlang C:  $GoS = \frac{A^N \times N / (N! \times (N - A))}{\sum_{i=0}^N A^i / i! + \frac{A^N \times N}{N! \times (N - A)}}$

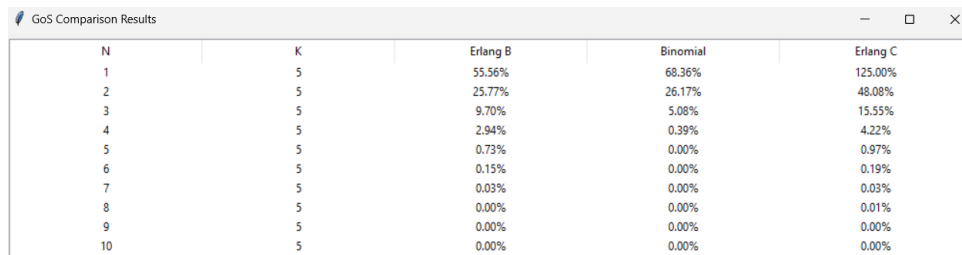
where  $A$  is the traffic intensity,  $N$  is the number of trunks, and  $M$  is the number of sources (users).

## 2.4 Comparison of GoS Calculation Methods

The GoS Calculator includes functionality to compare the GoS across the different calculation methods. The comparison is carried out by first computing the traffic intensity  $A$  from the given input parameters. The GoS is then calculated using each method for a range of trunks and users. These results are displayed in a tabular form, allowing for a direct comparison between the different methods.

## 2.5 Results and Discussion

The implementation was tested with various input parameters to ensure accurate GoS calculations. Results demonstrated the expected behavior: as the number of trunks increased, the GoS generally improved. The comparison feature provided valuable insights into how each method performs under different conditions, which is crucial for network planning and resource allocation. The observed data points and trends were consistent with the theoretical expectations.



| N  | K | Erlang B | Binomial | Erlang C |
|----|---|----------|----------|----------|
| 1  | 5 | 55.56%   | 68.36%   | 125.00%  |
| 2  | 5 | 25.77%   | 26.17%   | 48.08%   |
| 3  | 5 | 9.70%    | 5.08%    | 15.55%   |
| 4  | 5 | 2.94%    | 0.39%    | 4.22%    |
| 5  | 5 | 0.73%    | 0.00%    | 0.97%    |
| 6  | 5 | 0.15%    | 0.00%    | 0.19%    |
| 7  | 5 | 0.03%    | 0.00%    | 0.03%    |
| 8  | 5 | 0.00%    | 0.00%    | 0.01%    |
| 9  | 5 | 0.00%    | 0.00%    | 0.00%    |
| 10 | 5 | 0.00%    | 0.00%    | 0.00%    |

Figure 1: Caption for the figure.

## 3 Part B: Traffic Calculator

### 3.1 Methodology

The Traffic Calculator provides a quantitative analysis tool for determining the necessary traffic intensity  $A$  to meet a specified Grade of Service  $GoS$  for a certain number of trunks  $N$ . This reverse computation is critical for network capacity planning, where the provision of adequate resources to maintain a desired GoS is a key requirement.

### 3.2 Implementation Details

The Traffic Calculator was implemented in Python, leveraging the Tkinter library for the GUI, allowing users to enter the desired  $GoS$  and  $N$ . The logic for estimating  $A$  is based on the numerical methods that iteratively converge on the traffic intensity value that would result in the input  $GoS$  when applied to the Erlang B or Erlang C formulas.

### 3.3 Traffic Intensity Estimation Methods

The estimation of  $A$  follows an iterative approach, using binary search techniques to refine the guess for  $A$  within a specified tolerance. The calculation is repeated for Erlang B and C as follows:

- **Erlang B:** The function `calculate_A` utilizes the Erlang B formula within a binary search to approximate the traffic intensity that would lead to a specified blocking probability (GoS).
- **Erlang C:** Similarly, the function is also designed to handle the Erlang C model, estimating the traffic intensity required to achieve a certain probability of queuing (GoS).

### 3.4 Results and Discussion

Upon invocation of the `calc_A` function with user-provided  $N$  and  $GoS$ , the Traffic Calculator successfully computes the traffic intensity, which is then displayed in the GUI. The results have shown consistency with the theoretical models, and through the GUI, users can easily compare the capacity requirements across the Erlang B and C formulas. This capability provides invaluable insights for understanding how different service quality metrics translate into capacity planning.

Batch Calculation Results

Erlang B Results

| N/B | 0.50%  | 1.00%  | 2.00%  | 3.00%  | 5.00%  |
|-----|--------|--------|--------|--------|--------|
| 1   | 0.0050 | 0.0101 | 0.0204 | 0.0309 | 0.0526 |
| 2   | 0.1054 | 0.1526 | 0.2235 | 0.2816 | 0.3813 |
| 3   | 0.3490 | 0.4555 | 0.6022 | 0.7151 | 0.8994 |
| 4   | 0.7012 | 0.8694 | 1.0923 | 1.2589 | 1.5246 |
| 5   | 1.1320 | 1.3608 | 1.6571 | 1.8752 | 2.2185 |
| 6   | 1.6218 | 1.9960 | 2.2759 | 2.5431 | 2.9603 |
| 7   | 2.1575 | 2.5009 | 2.9354 | 3.2497 | 3.7378 |
| 8   | 2.7299 | 3.1276 | 3.6271 | 3.9865 | 4.5430 |
| 9   | 3.3326 | 3.7825 | 4.3447 | 4.7479 | 5.3702 |
| 10  | 3.9607 | 4.4612 | 5.0840 | 5.5294 | 6.2157 |

Erlang C Results

| N/B | 0.50%  | 1.00%  | 2.00%  | 3.00%  | 5.00%  |
|-----|--------|--------|--------|--------|--------|
| 1   | 0.0050 | 0.0100 | 0.0200 | 0.0300 | 0.0500 |
| 2   | 0.1025 | 0.1465 | 0.2102 | 0.2604 | 0.3422 |
| 3   | 0.3339 | 0.4291 | 0.5545 | 0.6464 | 0.7876 |
| 4   | 0.6641 | 0.8100 | 0.9599 | 1.1242 | 1.3186 |
| 5   | 1.0650 | 1.2591 | 1.4973 | 1.6627 | 1.9052 |
| 6   | 1.5190 | 1.7384 | 2.0472 | 2.2449 | 2.5316 |
| 7   | 2.0144 | 2.2965 | 2.6326 | 2.8605 | 3.1882 |
| 8   | 2.5431 | 2.8656 | 3.2464 | 3.5025 | 3.8687 |
| 9   | 3.0995 | 3.4604 | 3.8834 | 4.1663 | 4.5687 |
| 10  | 3.6792 | 4.0768 | 4.5400 | 4.8483 | 5.2851 |

Figure 2: Caption for the figure.

## A Source Code

```

1 import tkinter as tk
2 from tkinter import ttk
3 import math
4
5 def erlang_b(N, A):
6     if N == 0:
7         return 1.0 # If no servers, all calls are blocked
8     numerator = (A ** N) / math.factorial(N)
9     denominator = sum([(A ** i) / math.factorial(i) for i in range(N+1)])
10    return numerator / denominator
11
12 def binomial(N, M, A):
13     GoS = sum(math.comb(M - 1, i) * (A**i) * ((1 - A)**(M - 1 - i)) for i in
14 range(N, M))
15    return GoS
16
17 def erlang_c(N, A):
18     if N == 0:
19         return 0.0 # If no servers, no delay
20     numerator = ((A ** N) * N) / (math.factorial(N) * (N - A))
21     denominator = sum([(A ** i) / math.factorial(i) for i in range(N)])
22     denominator += ((A ** N) * N) / (math.factorial(N) * (N - A))

```

```

22     return numerator / denominator
23
24 def calculate_A(gos, N, type, tolerance=1e-10, max_iterations=10000):
25     A_low = 0 # Lower bound for A
26     A_high = N # Upper bound, assuming maximum A can't be more than number of
27     channels initially
28     A_guess = (A_high + A_low) / 2 # Initial guess for A
29
30     for _ in range(max_iterations):
31         if type == 'Erlang B':
32             calculated_gos = erlang_b(N, A_guess)
33         elif type == 'Erlang C':
34             calculated_gos = erlang_c(N, A_guess)
35
36         # Check if the calculated GoS is close enough to the desired GoS
37         if abs(calculated_gos - gos) < tolerance:
38             return A_guess
39
40         # Adjust the guess for A based on whether we need more or less traffic
41         intensity
42         if calculated_gos > gos:
43             A_high = A_guess
44         else:
45             A_low = A_guess
46
47         A_guess = (A_high + A_low) / 2
48
49     # Return the last guess if the loop finishes without reaching the tolerance
50     return A_guess
51
52 def calc_gos():
53     try:
54         N = int(N_entry.get())
55         K = int(K_entry.get())
56         lambdaa = float(lambda_entry.get())
57         H = float(H_entry.get())
58
59         A = lambdaa * H * K
60
61         # Convert A to CCS if chosen by the user
62         if unit_var.get() == 'CCS':
63             A *= 3600
64
65         if method_var_gos.get() == 'Erlang B':
66             GoS = erlang_b(N, A)
67         elif method_var_gos.get() == 'Binomial':
68             p = lambdaa * H
69             GoS = binomial(N, K, p)
70         else: # Erlang C
71             GoS = erlang_c(N, A)
72
73         # Update the output fields
74         traffic_intensity_var.set(f'{A:.2f} {unit_var.get()}')
75         GoS_var.set(f'{GoS:.2%}')
76
77     except ValueError:
78         traffic_intensity_var.set("Invalid input")
79         GoS_var.set("Invalid input")
80     #####
81
82 def compare_gos_methods():
83     lambdaa = 5 # calls/hour

```

```

83 H = 3 / 60 # hours
84 K_values = range(5, 51, 5) # from 5 to 50 with step of 5
85 N_values = range(1, 11) # from 1 to 10
86
87 # Create a new window to display the comparison results
88 comparison_window = tk.Toplevel(app)
89 comparison_window.title("GoS Comparison Results")
90
91 # Create Treeview with columns for N, K, and GoS results for each method
92 columns = ["N", "K", "Erlang B", "Binomial", "Erlang C"]
93 comparison_tree = ttk.Treeview(comparison_window, columns=columns, show="
headings")
94 for col in columns:
95     comparison_tree.heading(col, text=col)
96     comparison_tree.column(col, anchor="center")
97 comparison_tree.grid(row=0, column=0, sticky="nsew", padx=5, pady=5)
98
99 # Populate the Treeview with GoS results for each combination of N and K
100 for K in K_values:
101     A = lambdaa * H * K
102     for N in N_values:
103         GoS_erlang_b = erlang_b(N, A)
104         p = lambdaa * H
105         GoS_binomial = binomial(N, K, p)
106         GoS_erlang_c = erlang_c(N, A)
107         comparison_tree.insert("", tk.END, values=(N, K, f"{GoS_erlang_b
:.2%}", f"{GoS_binomial:.2%}", f"{GoS_erlang_c:.2%}"))
108
109
110 def calc_A():
111     try:
112         N = int(N_entry_A.get())
113         GoS = float(GoS_entry.get()) / 100
114         A = calculate_A(GoS, N, type = method_var_A.get())
115         traffic_intensity_var.set(f'{A:.4f} Erlang')
116     except ValueError:
117         traffic_intensity_var.set("Invalid input")
118
119 def calculate_and_display_results():
120     # Define GoS values
121     gos_values = [0.005, 0.01, 0.02, 0.03, 0.05]
122     N_values = range(1, 11)
123
124     # Create a new window to display the results
125     results_window = tk.Toplevel(app)
126     results_window.title("Batch Calculation Results")
127
128     # Creating columns dynamically based on GoS values
129     columns = ["N/B"] + [f"{gos*100:.2f}%" for gos in gos_values]
130     columns_for_erlang_b = [col for col in columns]
131     columns_for_erlang_c = [col for col in columns if col != "N/B"]
132
133     # Title for Erlang B Table
134     ttk.Label(results_window, text="Erlang B Results", font=("Arial", 14)).grid
(row=0, column=0, padx=10, pady=10, sticky="w")
135
136     # Set up the Treeview for Erlang B
137     results_tree_b = ttk.Treeview(results_window, columns=columns_for_erlang_b,
show="headings")
138     for col in columns_for_erlang_b:
139         results_tree_b.heading(col, text=col)
140     results_tree_b.grid(row=1, column=0, sticky="nsew", padx=5, pady=5)
141

```

```

142 # Title for Erlang C Table
143 ttk.Label(results_window, text="Erlang C Results", font=("Arial", 14)).grid(
144     (row=2, column=0, padx=10, pady=10, sticky="w")
145
146 # Set up the Treeview for Erlang C
147 results_tree_c = ttk.Treeview(results_window, columns=["N"] +
148     columns_for_erlang_c, show="headings")
149 results_tree_c.heading("N", text="N/B")
150 for col in columns_for_erlang_c:
151     results_tree_c.heading(col, text=col)
152 results_tree_c.grid(row=3, column=0, sticky="nsew", padx=5, pady=5)
153
154 # Populate Treeviews with results
155 for N in N_values:
156     row_b = [N]
157     row_c = [N]
158     for gos in gos_values:
159         A_erlang_b = calculate_A(gos, N, 'Erlang B')
160         A_erlang_c = calculate_A(gos, N, 'Erlang C')
161         row_b.append(f"{A_erlang_b:.4f}")
162         row_c.append(f"{A_erlang_c:.4f}")
163         results_tree_b.insert("", tk.END, values=row_b)
164         results_tree_c.insert("", tk.END, values=row_c)
165
166 # Adjust Treeview columns
167 for col in columns_for_erlang_b:
168     results_tree_b.column(col, anchor="center")
169 for col in ["N"] + columns_for_erlang_c:
170     results_tree_c.column(col, anchor="center")
171
172 -----
173
174 # Enhanced GUI design
175 def enhanced_style():
176     style = ttk.Style()
177     style.configure("TLabel", font=("Arial", 12), background="light grey",
178         foreground="black")
179     style.configure("TEntry", font=("Arial", 12), foreground="blue")
180     style.configure("TButton", font=("Arial", 12), background="grey",
181         foreground="black")
182     style.configure("TFrame", background="light grey")
183
184     # You can also set specific styles for widgets or create custom ones
185     style.configure("Header.TLabel", font=("Arial", 14, "bold"), foreground="
186         black")
187
188 app = tk.Tk()
189 app.title("Combined Calculator")
190 enhanced_style()
191
192 # Variables
193 traffic_intensity_var = tk.StringVar()
194 method_var_A = tk.StringVar(value='Erlang B')
195 method_var_gos = tk.StringVar(value='Erlang B')
196 GoS_var = tk.StringVar()
197 unit_var = tk.StringVar(value='Erlang')
198
199 def setup_gos_calculator_frame(parent_frame):
200
201     # Setup GUI elements for the GoS Calculator inside the parent frame
202     ttk.Label(parent_frame, text="GoS Calculator", style="Header.TLabel").grid(
203         column=0, row=0, pady=10, columnspan=2) # Inputs

```



```

197     ttk.Label(parent_frame, text="Number of Trunks (N):").grid(column=0, row=1)
198     global N_entry_gos
199     N_entry_gos = ttk.Entry(parent_frame)
200     N_entry_gos.grid(column=1, row=1)
201
202     ttk.Label(parent_frame, text="Number of Users (K):").grid(column=0, row=2)
203     global K_entry
204     K_entry = ttk.Entry(parent_frame)
205     K_entry.grid(column=1, row=2)
206
207     ttk.Label(parent_frame, text="Average Call Rate ( ):").grid(column=0, row
=3)
208     global lambda_entry
209     lambda_entry = ttk.Entry(parent_frame)
210     lambda_entry.grid(column=1, row=3)
211
212     ttk.Label(parent_frame, text="Call Holding Time (H):").grid(column=0, row
=4)
213     global H_entry
214     H_entry = ttk.Entry(parent_frame)
215     H_entry.grid(column=1, row=4)
216
217     # Select GoS Method
218     ttk.Label(parent_frame, text="GoS Calculation Method:").grid(column=0, row
=5)
219     method_options = ['Erlang B', 'Binomial', 'Erlang C']
220     method_menu = ttk.OptionMenu(parent_frame, method_var_gos, method_options
[0], *method_options)
221     method_menu.grid(column=1, row=5)
222
223     # Select Unit
224     ttk.Label(parent_frame, text="Traffic Intensity Unit:").grid(column=0, row
=6)
225     unit_options = ['Erlang', 'CCS']
226     unit_menu = ttk.OptionMenu(parent_frame, unit_var, unit_options[0], *
unit_options)
227     unit_menu.grid(column=1, row=6)
228
229     # Calculate Button
230     calculate_btn_gos = ttk.Button(parent_frame, text="Calculate GoS", command=
calc_gos)
231     calculate_btn_gos.grid(column=0, row=7, columnspan=2)
232
233     # Add button for comparing GoS methods here
234     compare_gos_btn = ttk.Button(parent_frame, text="Compare GoS Methods",
command=compare_gos_methods)
235     compare_gos_btn.grid(column=0, row=9, columnspan=2, pady=10)
236
237     # Outputs
238     ttk.Label(parent_frame, text="Grade of Service (GoS):").grid(column=0, row
=8)
239     ttk.Label(parent_frame, textvariable=GoS_var).grid(column=1, row=8)
240
241
242 def setup_traffic_calculator_frame(parent_frame):
243     # Setup GUI elements for the Traffic Calculator inside the parent frame
244     ttk.Label(parent_frame, text="Traffic Intensity Calculator", style="Header.
TLabel").grid(column=0, row=0, pady=10, columnspan=2)    # Inputs
245     # Inputs
246     ttk.Label(parent_frame, text="Number of Trunks (N):").grid(column=0, row=1)
247     global N_entry_A
248     N_entry_A = ttk.Entry(parent_frame)
249     N_entry_A.grid(column=1, row=1)

```

```

250
251 # Select GoS Method
252 ttk.Label(parent_frame, text="GoS Calculation Method:").grid(column=0, row
=2)
253 method_options = ['Erlang B', 'Erlang C']
254 method_menu = ttk.OptionMenu(parent_frame, method_var_A, method_options[0],
*method_options)
255 method_menu.grid(column=1, row=2)
256
257 # Add GoS Input
258 ttk.Label(parent_frame, text="Grade of Service (GoS):").grid(column=0, row
=3)
259 global GoS_entry
260 GoS_entry = ttk.Entry(parent_frame)
261 GoS_entry.grid(column=1, row=3)
262
263 # Calculate Button
264 calculate_btn_gos= ttk.Button(parent_frame, text="Calculate Traffic
Intenisty (A)", command=calc_A)
265 calculate_btn_gos.grid(column=0, row=4, columnspan=2)
266
267 # Outputs
268 ttk.Label(parent_frame, text="Traffic Intensity (A):").grid(column=0, row
=5)
269 ttk.Label(parent_frame, textvariable=traffic_intensity_var).grid(column=1,
row=5)
270
271
272 # Add a button to your GUI to trigger this function
273 calculate_results_btn = ttk.Button(parent_frame, text="Calculate and
Display Tables", command=calculate_and_display_results)
274 calculate_results_btn.grid(column=0, row=6, columnspan=2, pady=5)
275
276 # Create frames
277 traffic_frame = tk.Frame(app, borderwidth=2, relief="groove")
278 gos_frame = tk.Frame(app, borderwidth=2, relief="groove")
279
280 # Layout frames vertically
281 traffic_frame.pack(fill=tk.BOTH, expand=True, padx=10, pady=5)
282 gos_frame.pack(fill=tk.BOTH, expand=True, padx=10, pady=5)
283
284 # Setup each calculator within its frame
285 setup_traffic_calculator_frame(traffic_frame)
286 setup_gos_calculator_frame(gos_frame)
287
288 app.mainloop()

```

Listing 1: Source Code

**B GUI Screenshot**

The screenshot shows a window titled "Combined Calculator" with standard window controls. It contains two main sections:

**Traffic Intensity Calculator**

- Number of Trunks (N):
- GoS Calculation Method: Erlang B
- Grade of Service (GoS):
- 
- Traffic Intensity (A):
- 

**GoS Calculator**

- Number of Trunks (N):
- Number of Users (K):
- Average Call Rate ( $\lambda$ ):
- Call Holding Time (H):
- GoS Calculation Method: Erlang B
- Traffic Intensity Unit: Erlang
- 
- Grade of Service (GoS):
- 

Figure 3: Caption for the figure.