AALBORG UNIVERSITY DENMARK

1. Review the following terms.

Functional dependency, Holds, Trivial
Armstrong's axioms
    Reflexive rule
    Augmentation rule
    Transitivity rule
Additional rules
    Union rule
    Decomposition rule
    Pseudotransitivity rule
Closure of Attribute Sets
Decomposition, Lossless decompositions
Third normal form (3NF)
Boyce–Codd normal form (BCNF)

Review the slides or the textbook (Chapter 7).

2. List all nontrivial functional dependencies satisfied by the relation of Figure 1.

A	B	C
a_1	b_1	c_1
a_1	b_1	c_2
a_2	b_1	c_1
a_2	b_1	c_3

Figure 1: Relation of Exercise 2.

Answer

The nontrivial funtional dependencies are: $A \rightarrow B$ and $C \rightarrow B$, and a funtional dependency they logially imply: $AC \rightarrow B$.

3. Given the following relational schema $R = (A, B, C, D, E, F, G)$ and the following set F of functional dependencies (FDs):

$$A \rightarrow BC$$
$$E \rightarrow CGA$$
$$C \rightarrow DA$$
$$CA \rightarrow F$$
$$F \rightarrow CDGB$$
$$CD \rightarrow BF$$

1) Use Armstrong's axioms as well as the additional rules introduced in the lecture to show that the following rule (we name it pseudo II) is correct:

$$\alpha \rightarrow \beta\gamma \wedge \beta \rightarrow \delta \Rightarrow \alpha \rightarrow \beta\delta\gamma$$

2) Please use all Armstrong's axioms, the additional rules introduced in the lecture, and the just derived rule (pseudo II) to derive the following FDs from the FDs given above:

a) $C \rightarrow D$
b) $F \rightarrow A$
c) $E \rightarrow ABCDEFG$

Please state for each step the rule you applied.

3) Compute the attribute closures $(C)^+$ and $(E)^+$ with respect to the above given set F of functional dependencies.

Answer

1)

1: $\alpha \rightarrow \beta\gamma \Rightarrow \alpha \rightarrow \beta \wedge \alpha \rightarrow \gamma$ (decomposition)
2: $\alpha \rightarrow \beta \wedge \beta \rightarrow \delta \Rightarrow \alpha \rightarrow \delta$ (tansitivity)
3: $\alpha \rightarrow \delta \wedge \alpha \rightarrow \gamma \Rightarrow \alpha \rightarrow \delta\gamma$ (union)
4: $\alpha \rightarrow \beta \wedge \alpha \rightarrow \delta\gamma \Rightarrow \alpha \rightarrow \beta\delta\gamma$ (union)

2)

a) $C \rightarrow DA \Rightarrow C \rightarrow D \wedge C \rightarrow A$ (dexomposition)
b) $F \rightarrow CDGB \wedge C \rightarrow DA \Rightarrow F \rightarrow CDAGB$ (pseudo II)
$F \rightarrow CDAGB \Rightarrow F \rightarrow A$ (decomposition)
c) $E \rightarrow CGA \wedge C \rightarrow DA \Rightarrow E \rightarrow ACDG$ (pseudo II)
$E \rightarrow ACDG \wedge A \rightarrow BC \Rightarrow E \rightarrow ABCDG$ (pseudo II)
$E \rightarrow ABCDG \wedge CD \rightarrow BF \Rightarrow E \rightarrow ABCDFG$ (pseudo II)
$E \rightarrow ABCDFG \Rightarrow EE \rightarrow ABCDEFG \Rightarrow E \rightarrow ABCDEFG$ (Augmentation)

3)

$(C)^+ = ABCDFG$
$(E)^+ = ABCDEFG$

4. Given a relational schema $R = (A, B, C, D, E)$ and the following set F of functional dependencies (FDs):

$$A \to BC$$
$$CD \to E$$
$$B \to D$$
$$E \to A$$

1) List the candidate keys for R.
2) Is this relation in BCNF? Why or why not?
3) Suppose that we decompose the schema $R = (A, B, C, D, E)$ into

$$(A, B, C)$$
$$(A, D, E).$$

Is the decomposition lossless? Please explain your answer.

Answer

1) The candidate keys are A, BC, CD, and E.

2) No. $B \to D$ is not trivial and B is not superkey.

3) Yes. A decomposition $\{R_1, R_2\}$ is a lossless decomposition if $R_1 \cap R_2 \to R_1$ or $R_1 \cap R_2 \to R_2$. Let $R_1 = (A, B, C)$, $R_2 = (A, D, E)$, and $R_1 \cap R_2 =$ A. Since A is a candidate key, $R_1 \cap R_2 \to R_1$.