CS 61B Spring 2018 — More Asymptotic Analysis

Discussion 8: March 6, 2018

Here is a review of some formulas that you will find useful when doing asymptotic analysis.

- $\sum_{i=1}^{N} i = 1 + 2 + 3 + 4 + \cdots + N = \frac{N(N+1)}{2} = \mathbf{\frac{N^2+N}{2}}$
- $\sum_{i=0}^{N-1} 2^i = 1 + 2 + 4 + 8 + \cdots + 2^{N-1} = 2 \cdot 2^{N-1} - 1 = \mathbf{2^N - 1}$

Intuition

For the following recursive functions, give the worst case and best case running time in the appropriate $O(\cdot)$, $\Omega(\cdot)$, or $\Theta(\cdot)$ notation.

1.1 Give the running time in terms of N.

```java
public void andslam(int N) {
    if (N > 0) {
        for (int i = 0; i < N; i += 1) {
            System.out.println("datboi.jpg");
        }
        andslam(N / 2);
    }
}
```

best case: $\Theta(N)$

worst case: $\Theta(N)$

$$N! \quad \sum_{i=0}^{n} \frac{n!}{(n-i-1)!}$$

$$\sum_{i=0}^{n} \frac{1}{(n-i-1)!}$$

1.2 Give the running time for `andwelcome(arr, 0, N)` where N is the length of the input array `arr`.

```java
public static void andwelcome(int[] arr, int low, int high) {
    System.out.print("[ ");
    for (int i = low; i < high; i += 1) {
         System.out.print("loyal  ");
    }
    System.out.println("]");
    if (high - low > 0) {
        double coin = Math.random();
        if (coin > 0.5) {
            andwelcome(arr, low, low + (high - low) / 2);
        } else {
            andwelcome(arr, low, low + (high - low) / 2);
            andwelcome(arr, low + (high - low) / 2, high);
        }
    }
}
```

$$N + \frac{N}{2} + \frac{N}{4} + \dots + 1 = 2N - 1$$

$$N + N + N + \dots + N$$

best: $\Theta(N)$ worst: $\Theta(N \log N)$

1.3 Give the running time in terms of N.

```java
public int tothe(int N) {
    if (N <= 1) {
        return N;
    }
    return tothe(N - 1) + tothe(N - 1);
}
```

best & worst: $\Theta(2^N)$

1.4 Give the running time in terms of N.

```java
public static void spacejam(int N) {
    if (N <= 1) {
        return;
    }
    for (int i = 0; i < N; i += 1) {
        spacejam(N - 1);
    }
}
```

$$R(N) = N \cdot \left[R(N-1) + 1\right] = N\,R(N-1) + N$$

$$O(N \cdot N!)$$

$$\sum_{i=0}^{n} \frac{n!}{(n-i)!}(n-i) = \sum_{i=0}^{n} \frac{n!}{(n-i-1)!} \le \sum_{i=0}^{n} n! = n \cdot n!$$

$$\in O(N \cdot N!)$$

Hey you watchu gon do

2.1 For each example below, there are two algorithms solving the same problem. Given the asymptotic runtimes for each, is one of the algorithms **guaranteed** to be faster? If so, which? And if neither is always faster, explain why.

(a) Algorithm 1: $\Theta(N)$, Algorithm 2: $\Theta(N^2)$

(b) Algorithm 1: $\Omega(N)$, Algorithm 2: $\Omega(N^2)$

(c) Algorithm 1: $O(N)$, Algorithm 2: $O(N^2)$

(d) Algorithm 1: $\Theta(N^2)$, Algorithm 2: $O(\log N)$

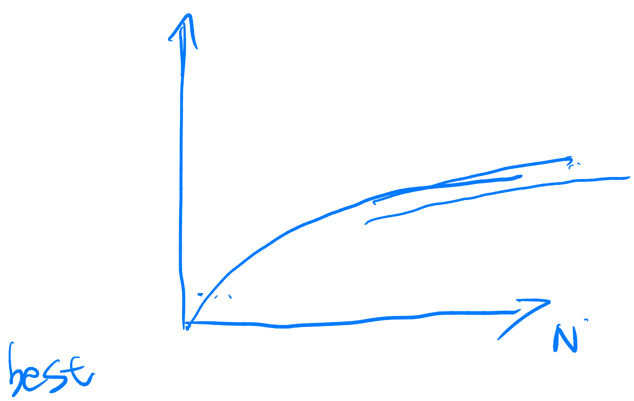
(e) Algorithm 1: $O(N \log N)$, Algorithm 2: $\Omega(N \log N)$

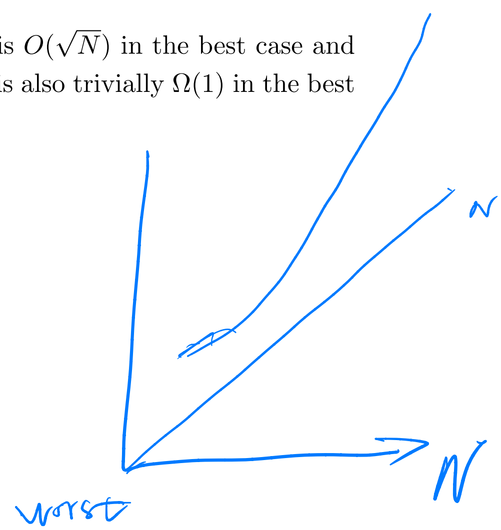
Would your answers above change if we did not assume that N was very large (for example, if there was a maximum value for N, or if N was constant)?

if so, all can't

Asymptotic Notation

3.1 Draw the running time graph of an algorithm that is $O(\sqrt{N})$ in the best case and $\Omega(N)$ in the worst case. Assume that the algorithm is also trivially $\Omega(1)$ in the best case and $O(\infty)$ in the worst case.





Extra: Following is a question from last week, now that you have properly learned about $O(\cdot)$, $\Omega(\cdot)$, or $\Theta(\cdot)$.

3.2 Are the statements in the right column true or false? If false, correct the asymptotic notation ($\Omega(\cdot)$, $\Theta(\cdot)$, $O(\cdot)$). Be sure to give the tightest bound. $\Omega(\cdot)$ is the opposite of $O(\cdot)$, i.e. $f(n) \in \Omega(g(n)) \iff g(n) \in O(f(n))$.

$f(n) = 20501$	$g(n) = 1$	$f(n) \in O(g(n))$ ✓	
$f(n) = n^2 + n$	$g(n) = 0.000001n^3$	$f(n) \in \Omega(g(n))$ ✗	$f(n) \in O(g(n)$
$f(n) = 2^{2n} + 1000$	$g(n) = 4^n + n^{100}$	$f(n) \in O(g(n))$ ✓	
$f(n) = \log(n^{100})$	$g(n) = n \log n$	$f(n) \in \Theta(g(n))$ ✗	$f(n) \in O(g(n)$
$f(n) = n \log n + 3^n + n$	$g(n) = n^2 + n + \log n$	$f(n) \in \Omega(g(n))$ ✓	
$f(n) = n \log n + n^2$	$g(n) = \log n + n^2$	$f(n) \in \Theta(g(n))$ ✓	
$f(n) = n \log n$	$g(n) = (\log n)^2$	$f(n) \in O(g(n))$ ✗	$f(n) \in \Omega(g(n))$

Fall 2015 *Extra*

4.1 If you have time, try to answer this challenge question. For each answer true or false. If true, explain why and if false provide a counterexample.

(a) If $f(n) \in O(n^2)$ and $g(n) \in O(n)$ are positive-valued functions (that is for all n, $f(n), g(n) > 0$), then $\frac{f(n)}{g(n)} \in O(n)$. ✗

$f(n) = n^2$

$g(n) = 1$

(b) If $f(n) \in \Theta(n^2)$ and $g(n) \in \Theta(n)$ are positive-valued functions, then $\frac{f(n)}{g(n)} \in \Theta(n)$. ✓

$k_1 n^2 \leq f(n) \leq k_2 n^2$

$k_3 n \leq g(n) \leq k_4 n$

$$\frac{k_1}{k_4} n \leq \frac{f(n)}{g(n)} \leq \frac{k_2}{k_3} n$$