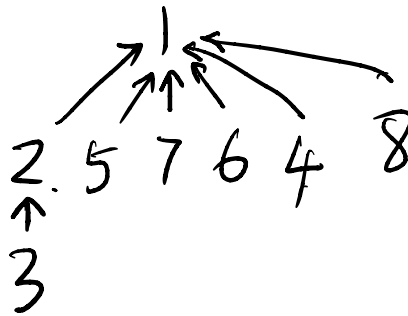


## 1 WQU and Path Compression

Assume we have eight sets, represented by integers 1 through 8, that start off as completely disjoint sets. Draw the WQU Tree after the series of `union()` and `find()` operations with path compression. Write down the result of `find()` operations. Break ties by choosing the smaller integer to be the root.

```
union(2, 3);  
union(1, 6);  
union(5, 7);  
union(8, 4);  
union(7, 2);  
find(3); 2  
union(6, 4);  
union(6, 3);  
find(7); 1  
find(8); 1
```



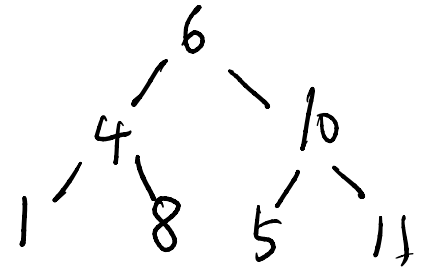
## 2 Is This a BST?

The following method `isBSTBad` is supposed to check if a given binary tree is a BST, though for some binary trees, it is returning the wrong answer. Think about an example of a binary tree for which `isBSTBad` fails. Then, write `isBSTGood` so that it returns the correct answer for any binary tree. The `TreeNode` class is defined as follows:

```
class TreeNode {
    int val;
    TreeNode left;
    TreeNode right;
}
```

**Hint:** You will find `Integer.MIN_VALUE` and `Integer.MAX_VALUE` helpful when writing `isBSTGood`.

```
public static boolean isBSTBad(TreeNode T) {
    if (T == null) {
        return true;
    } else if (T.left != null && T.left.val > T.val) {
        return false;
    } else if (T.right != null && T.right.val < T.val) {
        return false;
    } else {
        return isBSTBad(T.left) && isBSTBad(T.right);
    }
}
```



```
public static boolean isBSTGood(TreeNode T) {
    return isBSTHelper(T, Integer.MAX_VALUE, Integer.MIN_VALUE)
}
```

```
public static boolean isBSTHelper(TreeNode T, int MaxVal, int MinVal)
```

```
    if (T == null) {
        return true;
    }
```

```
    return T.val > MinVal && T.val < MaxVal &&
```

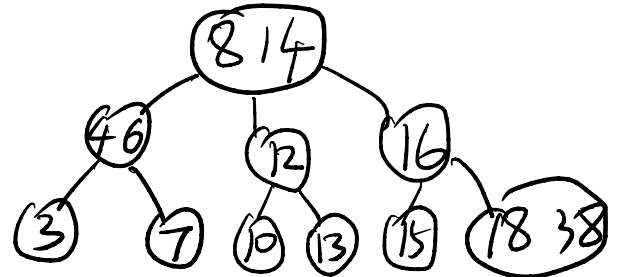
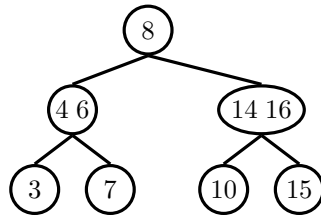
```
        isBSTHelper(T.left, T.val, MinVal)
```

```
        && isBSTHelper(T.right, MaxVal, T.val);
```

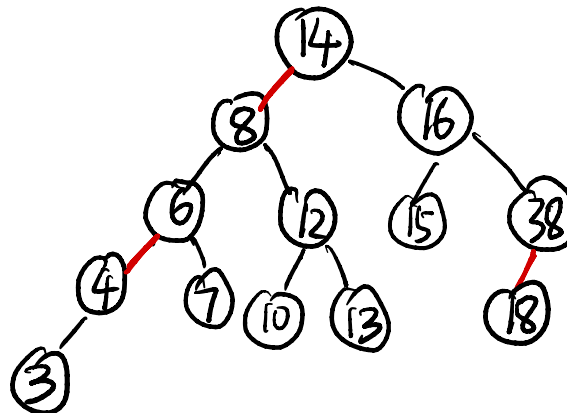
```
}
```

## 3 2-3 Trees and LLRB's

- 3.1 Draw what the following 2-3 tree would look like after inserting 18, 38, 12, and 13.



- 3.2 Now, convert the resulting 2-3 tree to a left-leaning red-black tree.



- 3.3 *Extra:* If a 2-3 tree has depth  $H$  (that is, the leaves are at distance  $H$  from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

$$2H+2$$

## 4 Hashing

- 4.1 Here are three potential implementations of the `Integer`'s `hashCode()` function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw or disadvantage.

```
1 public int hashCode() {
2     return -1;
3 }
```



```
1 public int hashCode() {
2     return intValue() * intValue();
3 }
```



```
1 public int hashCode() {
2     return super.hashCode();
3 }
```



- 4.2 *Extra, but highly recommended:* For each of the following questions, answer **Always**, **Sometimes**, or **Never**.

- (a) When you modify a key that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

*Sometimes*

- (b) When you modify a value that has been inserted into a `HashMap` will you be able to retrieve that entry again? Explain.

*Always*