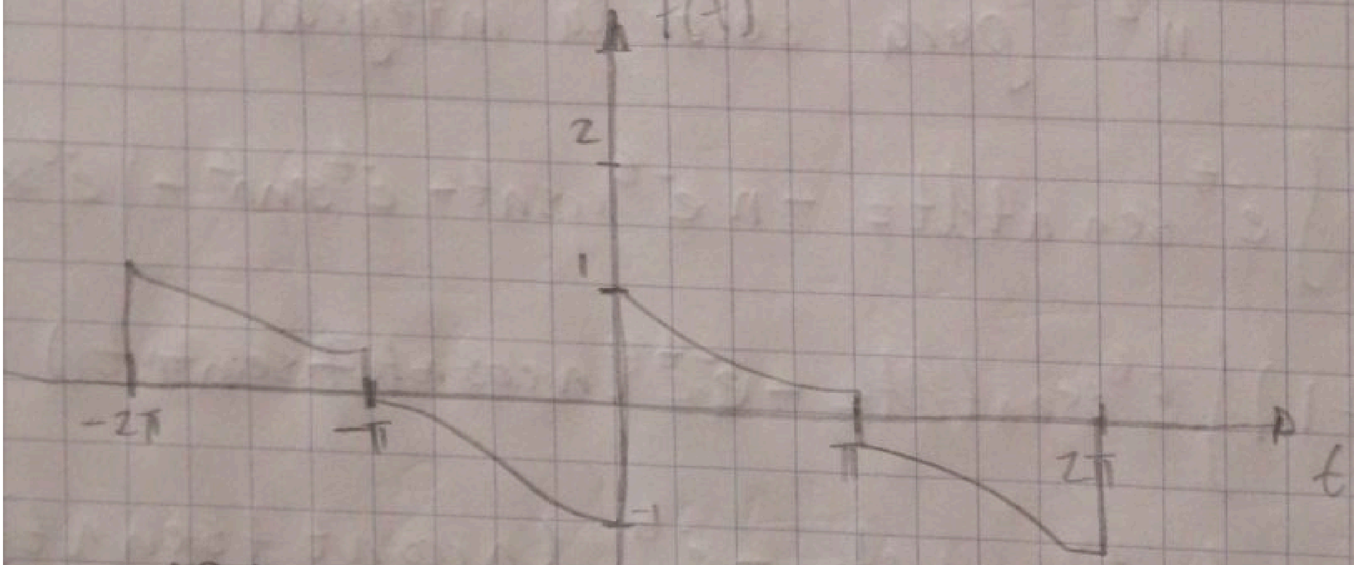
Evidencia 1.4 Encuentre y grafique la STF de $f(t)$

$$f(t)=\begin{cases} e^{-t} & ; \; 0<t<\pi \\ -e^{t} & ; \; -\pi<t<0 \end{cases}$$



IMPAR

$$T=2\pi \qquad W_0=\frac{2\pi}{T}=1$$

↓

$$b_n=\frac{4}{T}\int_0^{\frac{T}{2}} f(t)\,\text{sen}\,n w_0 t\,dt$$

$$=\frac{4}{2\pi}\int^{\frac{2\pi}{2}} e^{-t}\,\text{sen}\,nt\,dt=\frac{2}{\pi}\int_0^{\pi} e^{-t}\,\text{sen}\,nt\,dt$$

$u=e^{-t}$ $\qquad dv=\text{sen}\,nt\,dt$

$du=-e^{-t}\,dt$ $\qquad v=-\frac{\cos nt}{n}$

$$b_n=\frac{2}{\pi}\left[-\frac{e^{-t}\cos nt}{n}-\frac{1}{n}\int_0^{\pi} e^{-t}\cos nt\,dt\right]$$

$u=e^{-t}$ $\qquad dv=\cos nt\,dt$

$du=-e^{-t}\,dt$ $\qquad v=\frac{1}{n}\text{sen}\,nt$

$$\int_0^{\pi} e^{-t} \operatorname{sen} nt\,dt = -\frac{e^{-t}\cos nt}{n} - \frac{1}{n}\left(\frac{e^{-t}\operatorname{sen} nt}{n} + \frac{1}{n}\int e^{-t}\operatorname{sen} nt\,dt \right)$$

multiplicamos toda la expresión por n^2 para sacar la integral

$$n^2 \int e^{-t} \operatorname{sen} nt\,dt = -n e^{-t}\cos nt - e^{-t}\operatorname{sen} nt - \int e^{-t}\operatorname{sen} nt\,dt$$

$$(n^2+1)\int e^{-t}\operatorname{sen} nt\,dt = -e^{-t}(n\cos nt + \operatorname{sen} nt)$$

$$\int e^{-t}\operatorname{sen} nt\,dt = \frac{-e^{-t}}{n^2+1}(n\cos nt + \operatorname{sen} nt)$$

$$b_n = \frac{\pi}{2}\left[\frac{-e^{-t}}{n^2+1}(n\cos nt + \operatorname{sen} nt)\right]\Bigg|_0^{\pi}$$

$$= \frac{\pi}{2}\left[-\frac{e^{-\pi}\,n(-1)^n}{n^2+1} + \frac{n}{n^2+1}\right]$$

$$= \frac{2n}{\pi(n^2+1)}\left(1 - e^{-\pi}(-1)^n\right)$$

$$f(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen} nt$$

$$= \frac{2}{\pi} \sum_{n=1}^{\infty} \left[\frac{n\left(1 - e^{-\pi}(-1)^n\right)}{n^2+1} \operatorname{sen} nt \right]$$