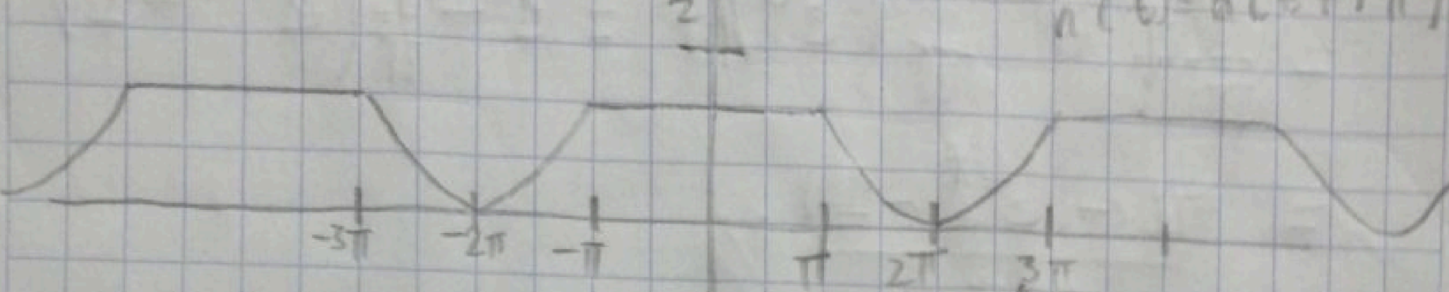
Participación 1.3

$$h(t) = \begin{cases} 2 & -\pi < t < \pi \\ 2e^{-(t-\pi)} & \pi < t < 2\pi \\ 2e^{(t-3\pi)} & 2\pi < t < 3\pi \end{cases}$$

$h(t) = h(t+4\pi)$



$T = 4\pi$

$W_0 = \frac{2\pi}{T} = \frac{1}{2}$

$-2\pi < t < 2\pi$

4π

Par

STF

$$h(t) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos \frac{n}{2} t$$

$$a_n = \frac{4}{T} \int_0^{\frac{T}{2}} s(t) \cos \frac{n}{2} t \, dt$$

$$= \frac{1}{2\pi} \int_0^{\pi} 2 \, dt + \frac{1}{2\pi} \int_{\pi}^{2\pi} 2e^{-(t-\pi)} dt$$

$$= \frac{1}{\pi} \int_0^{\pi} dt + \frac{1}{\pi} \int_{\pi}^{2\pi} e^{-(t-\pi)} dt$$

$u = -(t-\pi)$

$du = -dt$

$$\boxed{a_n = \frac{1}{\pi}(\pi) - \frac{1}{\pi}\int_{\pi}^{2\pi} e^{u}\, du}$$

$$= \frac{1}{\pi}\left(\pi - (e^{u})\Big|_{\pi}^{2\pi}\right) = 1 - \frac{1}{\pi}\left(e^{-(\pi)} - 1\right)$$

$$= \frac{\pi - (e^{-\pi} - 1)}{\pi}$$

$$a_0 \approx 1.3045$$

$$= \int_{\pi}^{2\pi} e^{\pi - t} \cos \frac{n}{2} t \, dt = -e^{\pi - t} \cos\left(\frac{n}{2}t\right) - \int_{\pi}^{2\pi} \frac{n e^{\pi - t} \operatorname{sen}\left(\frac{n}{2}t\right)}{2} dt$$

$$u = -\frac{n \operatorname{sen}\left(\frac{nt}{2}\right)}{2} \qquad dv = -e^{\pi - t}$$

$$du = -\frac{n^2 \cos\left(\frac{nt}{2}\right)}{4} dt \qquad v = e^{\pi - t}$$

$\rightarrow$

$$= -e^{\pi - t}\cos\left(\frac{nt}{2}\right) - \left[-\frac{n e^{\pi - t}\operatorname{sen}\left(\frac{nt}{2}\right)}{2} - \frac{n^2}{4}\int_{\pi}^{2\pi} e^{\pi - t}\cos\left(\frac{nt}{2}\right) dt \right]$$

$\uparrow$ misma ecuación

$\therefore$

$$\rightarrow \int_{\pi}^{2\pi} e^{\pi - t}\cos\left(\frac{nt}{2}\right) dt = \left. \frac{\frac{n e^{\pi - t}\operatorname{sen}\left(\frac{nt}{2}\right)}{2} - e^{\pi - t}\cos\left(\frac{nt}{2}\right)}{\frac{n^2}{4} + 1} \right|_{\pi}^{2\pi}$$

$$= \frac{2n\operatorname{sen}(\pi n) - 4\cos(\pi n)}{e^{\pi} n^2 + 4e^{\pi}} - \frac{2n\operatorname{sen}\left(\frac{\pi n}{2}\right) - 4\cos\left(\frac{\pi n}{2}\right)}{n^2 + 4}$$

$$= \frac{2e^{-\pi}\left(n\operatorname{sen}(n\pi) - 2\cos(n\pi) - e^{\pi} n \operatorname{sen}\left(\frac{\pi n}{2}\right) + 2e^{\pi}\cos\left(\frac{\pi n}{2}\right)\right)}{n^2 + 4}$$

$$\Rightarrow \frac{\operatorname{sen}\left(\frac{n\pi}{2}\right)}{n} - \frac{1}{n^2+4}\left(2e^{-\pi}(-1)^n + n\operatorname{sen}\left(\frac{n\pi}{2}\right) - 2\cos\left(\frac{n\pi}{2}\right)\right)$$

$$h(t) = \frac{\pi - e^{-\pi} + 1}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\left\{\frac{\operatorname{sen}\left(\frac{n\pi}{2}\right)}{n} - \frac{\left(2e^{-\pi}(-1)^n + n\operatorname{sen}\left(\frac{n\pi}{2}\right) - 2\cos\left(\frac{n\pi}{2}\right)\right)}{n^2+4}\right.$$

$\left(\cos\frac{n\pi}{2} t\right)$