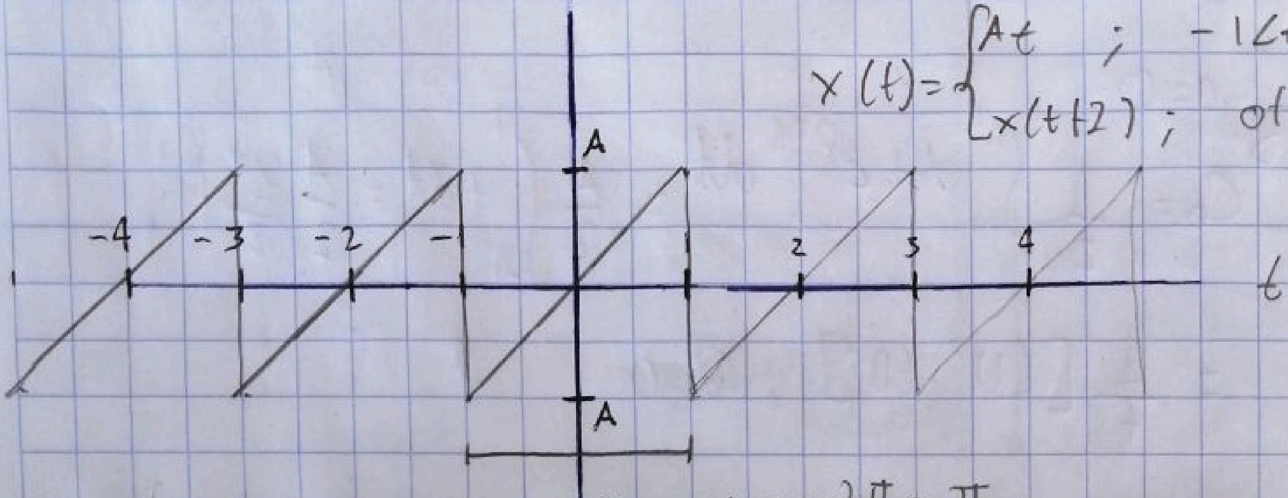
Evidencia 1.7

Determinar la transformada de Fourier de la función $x(t)$ y grafica su espectro de frecuencia

$$x(t) = \begin{cases} At & ; \quad -1 < t < 1 \\ x(t+2) & ; \quad \text{otro caso} \end{cases}$$



$T = 2 \qquad \omega_0 = \frac{2\pi}{T} = \pi$

$$\mathcal{F}\{x(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(\omega - n\pi)$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-jn\pi t}\, dt$$

$$= \frac{A}{2}\int_{-1}^{1} t \cdot e^{-jn\pi t}\, dt \quad ; \quad \begin{array}{ll} u = t & dv = e^{-jn\pi t}\, dt \\ du = dt & v = -\frac{1}{jn\pi}\, e^{-jn\pi t} \end{array}$$

$$= \frac{A}{2}\left[-\frac{t}{jn\pi} e^{-jn\pi t} \Big|_{-1}^{1} + \frac{1}{jn\pi}\int e^{-jn\pi t}\, dt \right]$$

$$= \frac{A}{2}\left[-\frac{1}{jn\pi} e^{-jn\pi} - \frac{1}{jn\pi} e^{jn\pi} \left(-\frac{1}{j^2 n^2 \pi^2}\, e^{-jn\pi t} \right) \Big|_{-1}^{1} \right]$$

$$= \frac{A}{2}\left\{ -\frac{1}{jn\pi}\left[e^{-jn\pi} + e^{jn\pi} \right] + \frac{1}{n^2\pi}\left[e^{-jn\pi} - e^{jn\pi} \right] \right\}$$

$$= \frac{A}{2}\left\{ \frac{-2}{jn\pi}(-1)^n - \frac{2i}{n^2\pi^2} \cancel{\operatorname{sen} n\pi} \right\}$$

$$\rightarrow 0$$

$$= -\frac{A(-1)^n}{jn\pi} \quad \forall n \neq 0 \quad \& \quad \frac{jA}{n\pi}(-1)^n \quad \forall n \neq 0$$

Si $n=0$

$$\Rightarrow C_0 = \frac{1}{2}\int_{-1}^{1} At\, e^{j0\pi t}\, dt = \frac{A}{2}\int_{-1}^{1} t\, dt = \frac{A}{2}\frac{t^2}{2}\Big|_{-1}^{1}$$

$$= \frac{A}{4}\left[(1)^2 - (-1)^2\right] \; ; \; C_0 = 0$$

$$\mathcal{F}\{x(t)\} = X(\omega) = 2\pi \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{jA}{n\pi}(-1)^n \cdot \delta(\omega - n\pi)$$

$$= j2A \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(-1)^n}{n} \cdot \delta(\omega - n\pi)$$

imaginaria

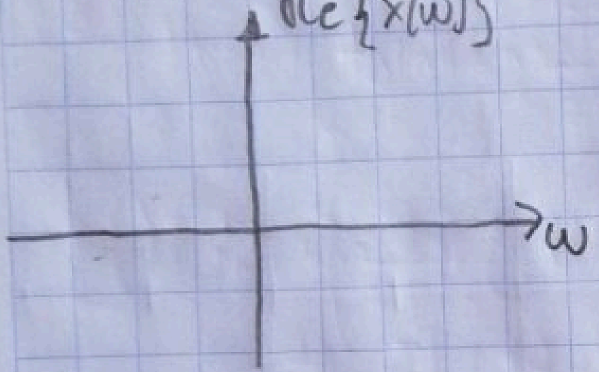
$$|c| = |y|$$

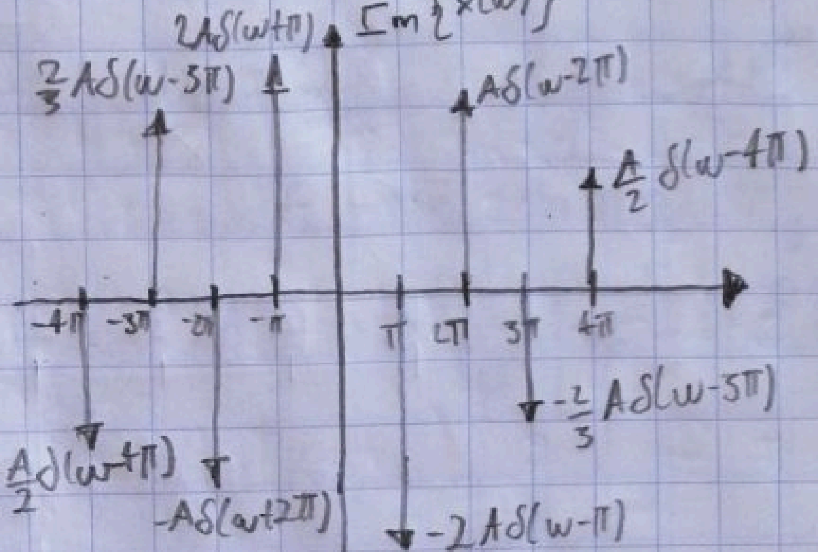
$$\theta = \pm 90^\circ, \quad +90^\circ \text{ si } y > 0$$

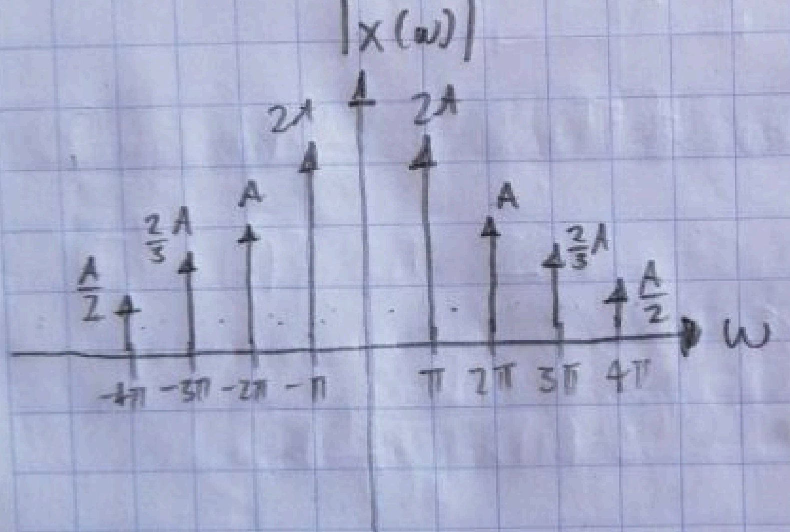
$$-90^\circ \text{ si } y < 0$$

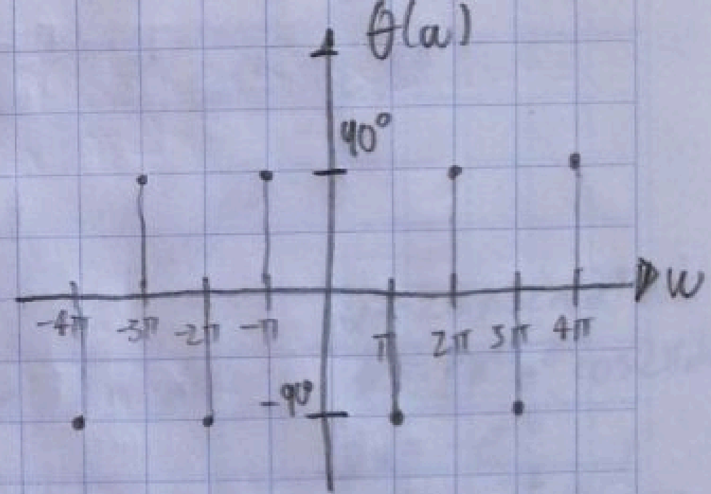
$$|X(\omega)| = \left| 2A \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(-1)^n}{n} \delta(\omega - n\pi) \right|$$

$$\theta(\omega) = \begin{cases} 90^\circ ; & \mathrm{Im}\{X(\omega)\} > 0 \\ -90^\circ ; & \mathrm{Im}\{X(\omega)\} < 0 \end{cases}$$









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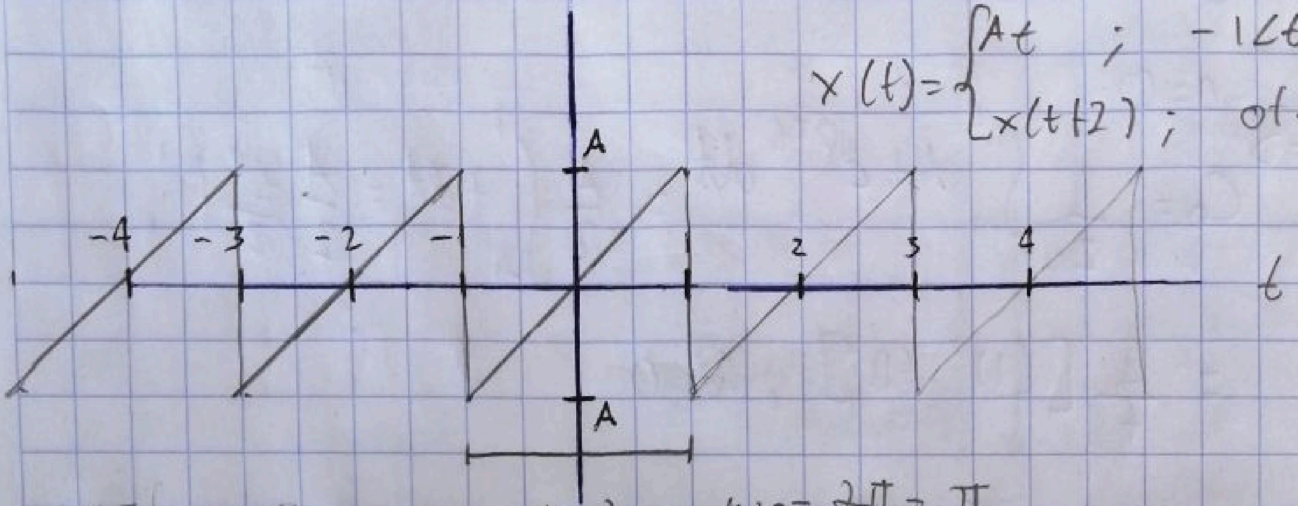

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$$= \frac{A}{2}\left\{ -\frac{1}{jn\pi}\left[e^{-jn\pi} + e^{jn\pi} \right] + \frac{1}{n^2\pi}\left[e^{-jn\pi} - e^{jn\pi} \right] \right\}$$

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