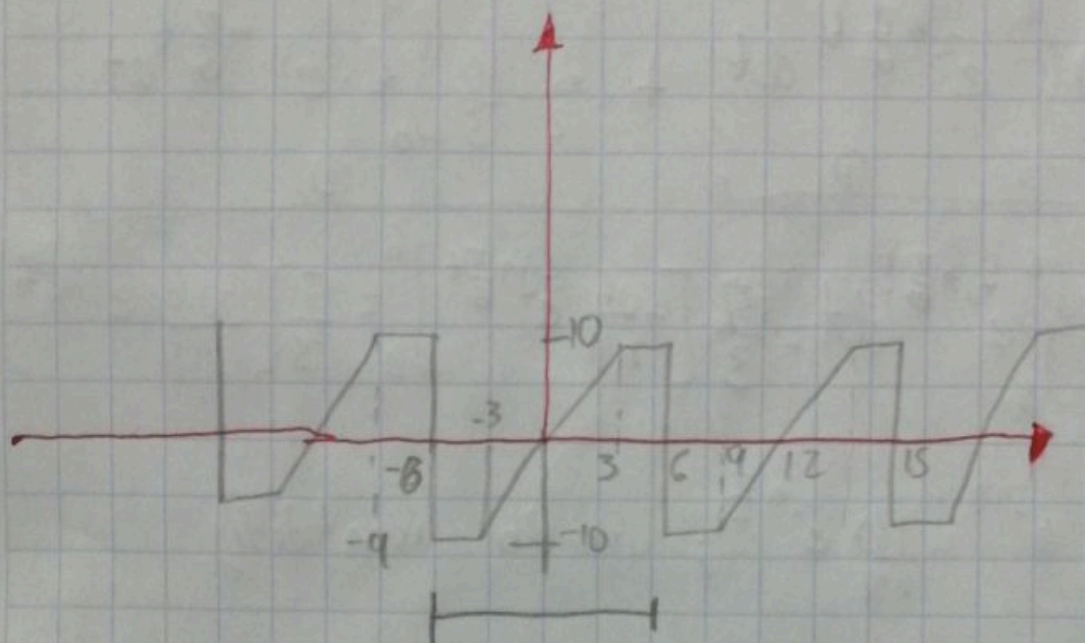


10/09/21

Ejercicio: Desarrollar la S.E.F de $g(t)$



$$T = 12 \quad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{6}$$

$$g(t) = \begin{cases} -10 & ; -6 < t < -3 \\ \frac{10}{3} t & ; -3 < t < 3 \\ 10 & ; 3 < t < 6 \\ g(t+12) & ; \text{otro caso} \end{cases}$$

$$C_n = \frac{1}{T} \int_{t_0}^{t_0+T} g(t) e^{-jn\omega_0 t} dt$$

$$C_n = \frac{1}{12} \int_{-6}^6 g(t) e^{-jn\frac{\pi}{6}t} dt$$

EXERCISE 10

$$= \underbrace{-\frac{20}{12} \int_3^6 e^{-jn\frac{\pi}{6}t} dt}_{(1)} + \underbrace{\frac{2}{12} \cdot \frac{16}{3} \int_0^3 t \cdot e^{-jn\frac{\pi}{6}t} dt}_{(2)}$$

$$(1) = \frac{10}{6} \int_3^6 e^{-jn\frac{\pi}{6}t} dt = \frac{5}{3} \left[\frac{-j6}{n\pi} e^{-jn\frac{\pi}{6}t} \right]_3^6 = \frac{5}{3} \left[\frac{-j6}{n\pi} e^{-jn\frac{\pi}{2}} \right]_3^6$$

$$= \frac{10j}{n\pi} e^{-jn\pi} - \frac{10}{n\pi} e^{-jn\frac{\pi}{2}}; \text{ usando identidades}$$

$$= \frac{10j}{n\pi} (\cos(n\pi) - j\sin(n\pi)) - \frac{10}{n\pi} (\cos(n\frac{\pi}{2}) - j\sin(n\frac{\pi}{2}))$$

$$= \frac{10j}{n\pi} \cos(n\pi) - \frac{10j}{n\pi} \cos(n\frac{\pi}{2}) - \frac{10}{n\pi} \sin(n\frac{\pi}{2})$$

$$(2) = \frac{5}{9} \int_0^3 t e^{-jn\frac{\pi}{6}t} dt = \frac{5}{9} \left[\frac{-j6t}{jn\pi} e^{-jn\frac{\pi}{6}t} \right]_0^3 + \frac{6}{jn\pi} \int_0^3 e^{-jn\frac{\pi}{6}t} dt$$

$$= \frac{5}{9} \left[\frac{6tj}{n\pi} e^{-jn\frac{\pi}{6}t} \right]_0^3 - \frac{j6}{n\pi} \left[\frac{6}{n\pi} e^{-jn\frac{\pi}{6}t} \right]_0^3$$

$$= \frac{5}{9} \left[\frac{18j}{n\pi} e^{-jn\frac{\pi}{2}} + \frac{36}{n^2\pi^2} e^{-jn\frac{\pi}{2}} - \frac{36}{n^2\pi^2} e^{-jn\frac{\pi}{2} \cdot 0} \right]$$

$$= \frac{5}{9} \left(\frac{18j}{n\pi} e^{-jn\frac{\pi}{2}} + \frac{36}{n^2\pi^2} e^{-jn\frac{\pi}{2}} - \frac{36}{n^2\pi^2} \right)$$

$$= \frac{10j}{n\pi} e^{-jn\frac{\pi}{2}} + \frac{20}{n^2\pi^2} e^{-jn\frac{\pi}{2}} - \frac{20}{n^2\pi^2} \quad \text{usando identidad}$$

$$= \frac{10j}{n\pi} \left(\cos\left(n\frac{\pi}{2}\right) - j\sin\left(n\frac{\pi}{2}\right) \right) + \frac{20}{n^2\pi^2} \left(\cos\left(n\frac{\pi}{2}\right) - j\sin\left(n\frac{\pi}{2}\right) \right) - \frac{20}{n^2\pi^2}$$

$$= \frac{10}{n\pi} \sin\left(n\frac{\pi}{2}\right) + \frac{20}{n^2\pi^2} \cos\left(n\frac{\pi}{2}\right) - \frac{20}{n^2\pi^2} + j \left(\frac{10}{n\pi} \cos\left(n\frac{\pi}{2}\right) - \frac{20}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right) \right)$$

$$b_n = -2 \{ \text{Im}[c_n] \} = -2 \left(\frac{10}{n\pi} \cos\left(n\frac{\pi}{2}\right) - \frac{20}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right) \right)$$

$$\Rightarrow G(t) = \sum_{n=1}^{\infty} -2 \left(\frac{10}{n\pi} \cos\left(n\frac{\pi}{2}\right) - \frac{20}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right) \right)$$