Problema 1

$$x(t)=\begin{cases}\frac{1}{2}t & ;0<t\leq 2\\ 1 & ;2<t\leq 4\\ -\frac{1}{2}(t-6) & ;4<t\leq 6\end{cases}$$

$T=6 \therefore \omega_0=\frac{2\pi}{T}=\frac{\pi}{3}$

ya que $x(t)$ es par $\Rightarrow b_n=0$

$$a_n=\frac{4}{T}\int_0^{\frac{T}{2}} x(t)\cos n\omega_0 t\,dt \quad ; \quad a_0=\frac{2}{T}\int_0^{\frac{T}{2}} x(t)\,dt$$

$$=\frac{4}{6}\int_0^3 x(t)\cos\frac{n\pi}{3}t\,dt$$

$$=\frac{2}{3}\left[\int_0^2 \frac{1}{2}t\cos\frac{n\pi}{3}t\,dt+\int_2^3 \cos\frac{n\pi}{3}t\,dt\right]$$

$u=t \quad dv=\cos\frac{n\pi}{3}t\,dt$

$du=dt \quad v=\frac{3}{n\pi}\operatorname{sen}\frac{n\pi}{3}t$

$$a_n=\frac{1}{3}\left[\frac{3t}{n\pi}\operatorname{sen}\frac{n\pi}{3}t\Big|_0^2-\frac{3}{n\pi}\int_0^2\operatorname{sen}\frac{n\pi}{3}t\,dt\right]\dots$$

$$\dots+\frac{2}{3}\left(\frac{3}{n\pi}\right)\operatorname{sen}\frac{n\pi}{3}t\Big|_2^3$$

$$=\frac{1}{3}\left[\frac{6}{n\pi}\operatorname{sen}\frac{2}{3}\pi n+\frac{9}{n^2\pi^2}\cos\frac{n\pi}{3}t\Big|_0^2\right.\dots$$

$$\dots+\frac{2}{n\pi}\left(\underset{0}{\operatorname{sen} n\pi}-\operatorname{sen}\frac{2}{3}n\pi\right)$$

$$=\frac{2}{n\pi}\operatorname{sen}\frac{2}{3}n\pi+\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]\dots$$

$$\dots-\frac{2}{n\pi}\operatorname{sen}\frac{2}{3}n\pi \quad \forall n\neq 0$$

$$=\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]$$

$$=\frac{2}{6}\int_0^3 x(t)\,dt$$

$$=\frac{1}{3}\left[\int_0^2\frac{1}{2}t\,dt+\int_2^3 dt\right]$$

$$=\frac{1}{6}\left[\frac{t^2}{2}\right]_0^2+\frac{1}{3}t\Big|_2^3$$

$$=\frac{1}{12}[4-0]+\frac{1}{3}[3-2]$$

$$=\frac{1}{3}+\frac{1}{3}$$

$$a_0=\frac{2}{3}$$

Entonces la STF es:

$$x(t)=\frac{2}{3}+\sum_{n=1}^{\infty}\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]\cdot\dots$$

$$\dots\cdot\cos\frac{n\pi}{3}t$$

$$x(t)=\frac{2}{3}+\sum_{n=1}^{\infty}\frac{3}{n^2\pi^2}\left[\cos\frac{2}{3}n\pi-1\right]\cdot\cos\frac{n\pi}{3}t$$

(2) Si $x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2}\left(\cos\frac{2}{3}n\pi - 1\right) \cdot \cos\frac{n\pi}{3}t$

la SEF se define como

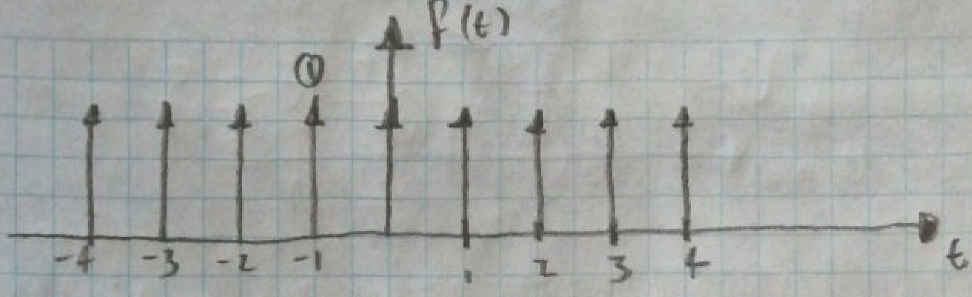
$$C_n = \frac{1}{2}(a_n - ib_n); \quad \text{si } b_n = 0$$

$$\Rightarrow C_n = \frac{1}{2}\left[\frac{3}{n^2\pi^2}\left(\cos\left(\frac{2}{3}n\pi\right) - 1\right)\right] \quad \forall n \neq 0$$

$$C_0 = a_0 = \frac{2}{3}$$

$$\Rightarrow x(t) = \frac{2}{3} + \sum_{\substack{n \neq 0 \\ y\, n = -\infty}}^{\infty} \frac{3}{2n^2\pi^2}\left(\cos\frac{2}{3}n\pi - 1\right) \cdot e^{in\frac{\pi}{3}t}$$

3. Encuentre la transformada de $f(t)$



como $f(t)$ es periódica, entonces $T=1$

$$\Rightarrow \mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(w - n w_0)$$

$$w_0 = \frac{2\pi}{T} = 2\pi \, ; \quad C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) \, e^{-jnw_0 t} \, dt$$

$$C_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} f(t) \, e^{-jn2\pi t} \, dt = \int_{0^-}^{0^+} \delta(t) \, e^{-jn2\pi t} \, dt \Rightarrow C_n = e^{0} = \underline{1}$$

$$\Rightarrow F(\omega) = \mathcal{F}\{f(t)\} = 2\pi \sum_{n=-\infty}^{\infty} \delta(w - 2\pi n)$$

5. Usando las propiedades de la transformada de Fourier, complete la pareja de transformadas siguiente:

$$\underbrace{\frac{t^2}{3-jt}}_{①} + \underbrace{e^{j4t}(t-1)}_{②} + \underbrace{5\,Sa(t-1)}_{③} \longleftrightarrow$$

① Si $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$$\frac{1}{a+jt} \longleftrightarrow 2\pi\left[e^{a\omega}u(-\omega)\right]; \text{ con } a=3$$

$$\frac{1}{3+jt} \longleftrightarrow 2\pi\left[e^{3\omega}u(-\omega)\right]$$

$$\frac{1}{3-jt} \longleftrightarrow 2\pi\left[e^{-3\omega}u(\omega)\right]$$

$$(-jt)^2\cdot\frac{1}{3-jt} \longleftrightarrow \frac{d^2}{d^2\omega}\left[2\pi\left(e^{-3\omega}u(\omega)\right)\right]$$

$$-\frac{t^2}{3-jt} \longleftrightarrow \frac{d^2}{d^2\omega}\left[2\pi\left(e^{-3\omega}\cdot u(\omega)\right)\right]$$

① $$\frac{t^2}{3-jt} \longleftrightarrow -\frac{d^2}{d^2\omega}\left[2\pi\left(e^{-3\omega}\cdot u(\omega)\right)\right]$$

③ Si $A\,C_d(t) \longleftrightarrow Ad\,Sa\frac{\omega d}{2}$

$$Ad\,Sa\frac{td}{2} \longleftrightarrow A\,C_d(\omega)\cdot 2\pi$$

con $d=2$ y $A=\frac{5}{2}$

$$\frac{5}{2}\cdot 2\,Sa\,t\frac{2}{2} \longleftrightarrow 2\pi\left[\frac{5}{2}C_2(-\omega)\right]$$

$$5\,Sa\,t \longleftrightarrow \pi\left[5C_2(-\omega)\right]$$

$$5\,Sa(t-1) \longleftrightarrow \pi(5C_2(-\omega))\cdot e^{-j\omega}$$

② Si $\delta(t) \longleftrightarrow 1$; $1 \longleftrightarrow 2\pi\delta(\omega)$

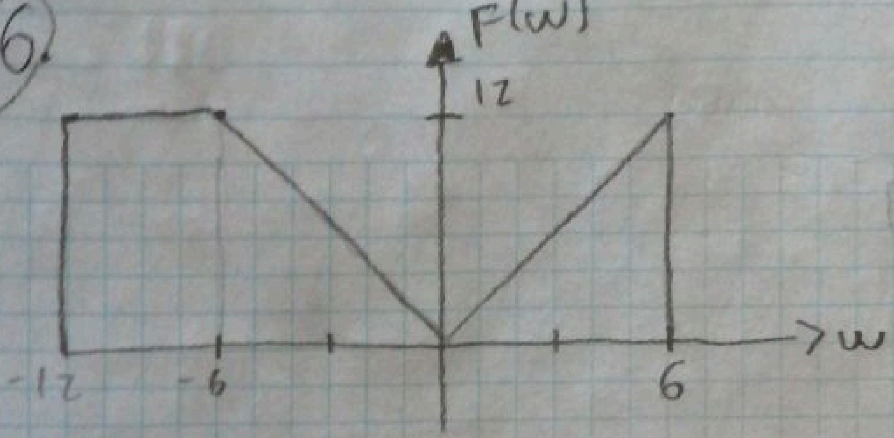
$$-jt \longleftrightarrow \frac{d}{d\omega}\left[2\pi\delta(\omega)\right]; \quad t \longleftrightarrow +j\frac{d}{d\omega}\left[2\pi\delta(\omega)\right]$$

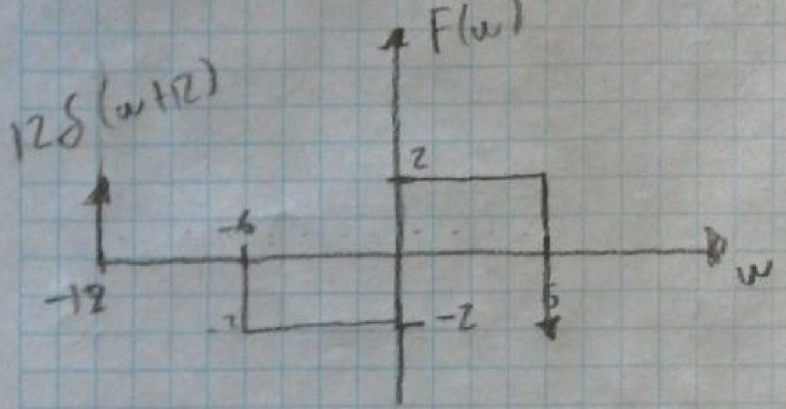
$$(t-1) \longleftrightarrow +j\frac{d}{d\omega}\left[2\pi\delta(\omega)\right]\cdot e^{-j\omega}$$

② $$e^{j4t}(t-1) \longleftrightarrow 2\pi j\frac{d}{d\omega}\left[\delta(\omega-4)\right]\cdot e^{-j(\omega-4)}$$

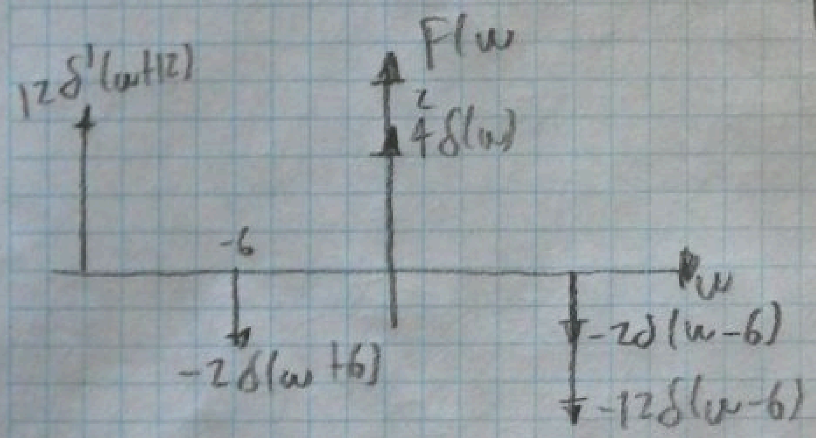
$$f(t) \longleftrightarrow -\frac{d^2}{d^2\omega}\left[2\pi\left(e^{-3\omega}\cdot u(\omega)\right)\right] + 2\pi j\frac{d}{d\omega}\left[\delta(\omega-4)\right]\cdot e^{-j(\omega-4)} + (5\pi C_2(-\omega))\cdot e^{-j\omega}$$

6.





$F'(\omega) = 12\delta(\omega+12) - 2 + 2 - 12\delta(\omega$



$F''(\omega) = 12\delta'(\omega+12) - 2\delta(\omega+6) + 4\delta(\omega) - 2\delta(\omega-6) - 12\delta'(\omega-6)$

② $\frac{d^2}{dt^2} F(\omega) = 12\delta'(\omega+12) - 2\delta(\omega+6) + 4\delta(\omega) - 2\delta(\omega-6) - 12\delta'(\omega-6)$

$\mathcal{F}^{-1}\left\{ \frac{d^2}{dt^2} F(\omega) \right\} = 12\mathcal{F}^{-1}\{\delta'(\omega+12)\} - 2\,\mathcal{F}^{-1}\{\delta(\omega+6)$