Evidencia 1.6

Determina TF de la función $g(t)$

$A \operatorname{sen} \omega t \rightarrow 1 \operatorname{sen} 1t$ $\qquad g(t)\begin{cases} \operatorname{sen} t ; & -\pi < t < \pi \\ 0 ; & \text{otro caso} \end{cases}$

$\omega = \frac{2\pi}{T} = 1$

$\operatorname{sen} t = \frac{e^{it} - e^{-it}}{2i} ; \quad 2i \operatorname{sen} t = e^{it} - e^{-it}$

$$\mathcal{F}\{g(t)\} = \int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt = \int_{-\pi}^{\pi} \operatorname{sen} t\, e^{-i\omega t} dt$$

$$= \int_{-\pi}^{\pi} \frac{e^{it} - e^{-it}}{2i} e^{-i\omega t} = \frac{1}{2i}\int_{-\pi}^{\pi} \left[e^{it(1-\omega)} - e^{-it(1+\omega)} \right] dt$$

$$= \frac{1}{2i}\left[\frac{1}{i(1-\omega)} e^{it(1-\omega)} \Big|_{-\pi}^{\pi} + \frac{1}{i(1+\omega)} e^{-it(1+\omega)} \Big|_{-\pi}^{\pi} \right]$$

$$= \frac{1}{2i^2}\left[\frac{1}{1-\omega}\left(e^{i\pi(1-\omega)} - e^{-i\pi(1-\omega)} \right) + \frac{1}{1+\omega}\left(e^{-i\pi(1+\omega)} - e^{i\pi(1+\omega)} \right) \right]$$

$$= -\frac{1}{2}\left[\frac{1}{1-\omega}\left(e^{i\pi} \cdot e^{-i\pi\omega} - e^{-i\pi} e^{i\pi\omega} \right) + \frac{1}{1+\omega}\left(e^{-i\pi} e^{i\pi\omega} - e^{i\pi} \cdot e^{-i\pi\omega} \right) \right]$$

$$= -\frac{1}{2}\left[\frac{1}{1-\omega}\left((-1) e^{-i\pi\omega} - (-1) e^{i\pi\omega} \right) + \frac{1}{1+\omega}\left((-1) e^{i\pi\omega} - (-1) e^{-i\pi\omega} \right) \right]$$

$$= -\frac{1}{2}\left[\frac{1}{1-\omega}\left(-e^{-i\pi\omega} + e^{i\pi\omega} \right) + \frac{1}{1+\omega}\left(-e^{i\pi\omega} + e^{-i\pi\omega} \right) \right]$$

$$= -\frac{1}{2}\left[\frac{2i}{1-\omega}(\operatorname{sen} \pi\omega) + \frac{2i}{1+\omega} \operatorname{sen} \pi\omega \right] = -i \operatorname{sen} \pi\omega \left(\frac{1}{1-\omega} + \frac{1}{1+\omega} \right)$$

$$= -i\,\text{sen}\,\pi\omega\left(\frac{1+\omega+1-\omega}{(1-\omega)(1+\omega)}\right) = -i\,\text{sen}\,\pi\omega\left[\frac{2}{1-\omega^2}\right]$$

$$= i\left[\frac{2\,\text{sen}\,\pi\omega}{\omega^2-1}\right]$$

$$F(\omega) = 0 + i\left(\frac{2\,\text{sen}\,\pi\omega}{\omega^2-1}\right)$$

$$|F(\omega)| = \left|\frac{2\,\text{sen}\,\pi\omega}{\omega^2-1}\right|$$

$$\theta(\omega) = \tan^{-1}\left(\frac{\frac{2\,\text{sen}\,\pi\omega}{\omega^2-1}}{0}\right) = \pm 90^\circ$$