Rodríguez Martínez Heber Emanuel

Problemario 1^{er} departamental

Problema 1. Expresar las señales e^{-t}, t^2, y $2t$ como serie trigonométrica de Fourier en el intervalo $(0, 1)$

1.1 $f(t) = e^{-t}$ $\quad T = 1 \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1} = 2\pi$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{1}\int_0^1 e^{-t}\,dt = -e^{-t}\Big|_0^1 = \left[-e^{-1} - (-e^{0})\right]$$

$$= -e^{-1} - (-1); \quad a_0 = 1 - e^{-1} = 1 - \frac{1}{e}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+t} f(t)\cos n\omega_0 t\,dt = \frac{2}{1}\int_0^1 e^{-t}\cos 2n\pi t\,dt$$

$u = \cos 2\pi n t \qquad dv = e^{-t}\,dt$

$du = -2\pi n \operatorname{sen} 2\pi n t\,dt \qquad v = -e^{-t}$

$$a_n = -e^{-t}\cos 2\pi n t - \int_0^1 2\pi n e^{-t}\operatorname{sen} 2\pi n t\,dt;$$

$u = -2\pi n \operatorname{sen} 2\pi n t$

$du = 4\pi^2 n^2 \cos 2\pi n t\,dt$

$dv = -e^{-t}$

$v = e^{-t}$

$$= -e^{-t}\cos 2\pi n t - \left[-2\pi n e^{-t}\operatorname{sen} 2\pi n t + 4\pi^2 n^2 \cdot \int_0^1 e^{-t}\cdot\cos 2\pi n t\,dt\right]$$

$$= 2\left[\frac{2\pi n e^{-t}\operatorname{sen} 2\pi n t - e^{-t}\cos 2\pi n t}{4\pi^2 n^2 + 1}\right]\Bigg|_0^1 = 2\left[\frac{e^{-t}(2\pi n \operatorname{sen} 2\pi n t - \cos 2\pi n t)}{4\pi n^2 + 1}\right.$$

$$= 2\left[\frac{e^{-1}(2\pi n \overset{0}{\cancel{\operatorname{sen} 2\pi n}} - \overset{-1}{\cancel{\cos 2\pi n}})}{4\pi^2 n^2 + 1}\right] - 2\left[\frac{\overset{1}{\cancel{e^{0}}}(2\pi n \overset{0}{\operatorname{sen} 0} - \overset{-1}{\cos 0})}{4\pi^2 n^2 + 1}\right]$$

$$a_n = 2\left[\frac{1}{4\pi^2 n^2 + 1}\left\{(e^{-1}(-1)) + 1\right\}\right] = 2\left[\frac{1 - e^{-1}}{4\pi^2 n^2 + 1}\right] = \frac{2 - 2e^{-1}}{4\pi^2 n^2 + 1}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\,\text{sen}\,\omega_0 t\,dt = 2\int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt \quad \begin{cases} u = \text{sen}\,2\pi n t \\ du = 2\pi n\cos 2\pi n t\,dt \\ dv = e^{-t}\,dt \\ v = -e^{-t} \end{cases}$$

$$= 2\left[-e^{-t}\,\text{sen}\,2\pi n t + \int_0^1 2\pi n e^{-t}\cos 2\pi n t\,dt\right. \quad \begin{cases} u = 2\pi n\cos 2\pi n t \\ du = -4\pi^2 n^2\,\text{sen}\,2\pi n t \\ dv = -e^{-t}\,dt \\ v = e^{-t} \end{cases}$$

$$= 2\left[-e^{-t}\,\text{sen}\,2\pi n t - \left(2\pi n e^{-t} + \int_0^1 4\pi^2 n^2 e^{-t}\,\text{sen}\,2\pi n t\,dt\right)\right]$$

$$= 2\left[-e^{-t}\,\text{sen}\,2\pi n t - \left(2\pi n e^{-t}\cos 2\pi n t + 4\pi^2 n^2 \cdot \int_0^1 e^{-t}\,\text{sen}\,2\pi n t\,dt\right)\right]$$

$$= 2\left[-\frac{e^{-t}(\text{sen}\,2\pi n t + 2\pi n\cos 2\pi n t)}{4\pi^2 n^2 + 1}\right]\Bigg|_0^1$$

$$= 2\left\{\left[-e^{-1}\frac{(\text{sen}\,\overset{0}{\cancel{2\pi n}} + 2\pi n\overset{1}{\cancel{\cos 2\pi n}})}{4\pi^2 n^2 + 1}\right] - \left[-\frac{e^{0}(\text{sen}\,\overset{0}{0} + 2\pi n\overset{1}{\cancel{\cos 0}})}{4\pi^2 n^2 + 1}\right]\right\}$$

$$= 2\left\{\frac{-e^{-1}(2\pi n) + (2\pi n)}{4\pi^2 n^2 + 1}\right\} = 2\left[\frac{2\pi n - 2e^{-1}\pi n}{4\pi^2 n^2 + 1}\right]$$

$$b_n = \frac{4\pi n(1 - e^{-1})}{4\pi^2 n^2 + 1}$$

$$f(t) = 1 - e^{-1} + \sum_{n=1}^{\infty} \frac{2 - 2e^{-1}}{4\pi^2 n^2 + 1}\cos 2n\pi t + \frac{4\pi n(1 - e^{-1})}{4\pi^2 n^2 + 1}\,\text{sen}\,2n\pi t$$

$$0 < t < 1$$

1.2 $f(t)=t^2 \quad T=1 \quad w_0=2\pi$

$$f(t)=a_0+\sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n \operatorname{sen} n w_0 t$$

$$a_0=\int_0^1 t^2 dt=\left[\frac{t^3}{3}\right]\Big|_0^1=\frac{1}{3}$$

$u=t^2$ $\quad dv=\cos 2\pi n t\, dt$

$du=2t\,dt$ $\quad v=\frac{\operatorname{sen} 2\pi n t}{2\pi n}$

$$a_n=2\int_0^1 t^2 \cos 2\pi n t\, dt=$$

$u=t$ $\quad dv=\operatorname{sen} 2\pi n t$

$du=dt$ $\quad v=-\frac{\cos 2\pi n t}{2\pi n}$

$$=2\left[\frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n}-\frac{1}{\pi n}\int_0^1 t \operatorname{sen} 2\pi n t\, dt\right]$$

$u=2\pi n t$

$du=2\pi n\, dt$

$dt=\frac{1}{2\pi n}$

$$=2\left[\frac{t^2 \operatorname{sen} 2\pi n t}{2\pi n}-\frac{t\cos 2\pi n t}{2\pi n}+\int_0^1 \frac{\cos 2\pi n t}{2\pi n} dt\right]$$

$$=2\left[\frac{t^2 \operatorname{sen}^2 2\pi n t}{2\pi n}-\frac{t\cos 2\pi n t}{2\pi n}+\frac{1}{4\pi^2 n^2}\int_0^1 \cos u\, du\right]$$

sustituyendo:

$$a_n=2\left[\frac{t^2 \operatorname{sen}^2 2\pi n t}{2\pi n}-\frac{\operatorname{sen} 2\pi n t}{4\pi^3 n^3}+\frac{t\cos 2\pi n t}{2\pi^2 n^2}\right]\Big|_0^1$$

$$=2\left\{\left[\frac{\overset{0}{\operatorname{sen} 2\pi n}}{2\pi n}-\frac{\overset{0}{\operatorname{sen} 2\pi n}}{4\pi^3 n^3}+\frac{\overset{1}{\cos 2\pi n}}{2\pi^2 n^2}\right]-\left[\frac{\operatorname{sen} 0}{2\pi n}-\frac{\operatorname{sen} 0}{4\pi^3 n^3}\,\frac{t\cos 0}{2\pi^2 n^2}\right]\right.$$

$$=\cancel{2}\left[\frac{1}{\cancel{2}\pi^2 n^2}\right]=\frac{1}{\pi^2 n^2}$$

$$b_n=2\int_0^1 t^2 \operatorname{sen} 2\pi n t\, dt$$

$u=t^2$

$du=2t\,dt$

$dv=\operatorname{sen} 2\pi n t\, dt$

$v=-\frac{\cos 2\pi n t}{\pi n}$

$u = t \quad dv = \cos 2\pi n t\,dt$
$du = dt \quad v = \frac{\sin 2\pi n t}{2\pi n}$

$$b_n = 2\left[-\frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{1}{\pi n}\int_0^1 t\cos 2\pi n t\,dt\right]$$

$$= 2\left[\frac{-t^2 \cos 2\pi n t}{2\pi n} + \frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \int_0^1 \frac{\operatorname{sen} 2\pi n t}{2\pi n}\,dt\right]$$

$u = 2\pi n t$
$du = 2\pi n\,dt$
$dt = \frac{1}{2\pi n}$

$$= 2\left[-\frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{4\pi^2 n^2}\int_0^1 \operatorname{sen} 2\pi n t\,dt\right]$$

$$= 2\left[-\frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{t \operatorname{sen} 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{4 n^2 \pi^2}\right]$$

sostituyendo

$$b_n = 2\left[\frac{t \operatorname{sen} 2\pi n t}{2\pi^2 n^2} + \frac{t^2 \cos 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{4\pi^3 n^3}\right]\Bigg|_0^1$$

$$= 2\left\{\left[\frac{\operatorname{sen} 2\pi n}{2\pi^2 n^2} + \frac{\cos 2\pi n}{2\pi n} + \frac{\cos 2\pi n}{4\pi^3 n^3}\right] - \left[\frac{0 \operatorname{sen} 0}{2\pi^2 n^2} + \frac{0 \cos 0}{2\pi n} + \frac{\cos 0}{4\pi^3 n^3}\right]\right.$$

$$= 2\left[\frac{1}{2\pi n}\right] = \frac{1}{\pi n}$$ //

$$f(t) = \frac{1}{3} + \sum_{n=1}^{\infty}\left(\frac{1}{\pi^2 n^2}\cos 2\pi n t + \frac{1}{\pi n}\operatorname{sen} 2 n \pi t\right)$$

$$0 < t < 1$$

$1.3 \quad f(t) = 2t \qquad T = 1 \quad W_0 = 2\pi$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n \operatorname{sen} n w_0 t$$

$$a_0 = \int_0^1 2\,dt = t^2 \Big|_0^1 = \underline{1} \,//$$

$u = t \quad dv = \cos 2n\pi t\,dt$

$du = dt \quad v = \frac{\operatorname{sen} 2n\pi t}{2\pi n}$

$$a_n = 2\int_0^1 2t \cos 2n\pi t\,dt = 4\int_0^1 t\cos 2n\pi t\,dt$$

$u = 2\pi n t \quad du = 2\pi n\,dt$

$dt = \frac{1}{2\pi n}$

$$= 4\left[\frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \int_0^1 \frac{\operatorname{sen} 2\pi n t\,dt}{2\pi n}\right]$$

$$= 4\left[\frac{t \operatorname{sen} 2\pi n t}{2\pi n} - \frac{1}{4\pi^2 n^2}\int_0^1 \operatorname{sen} u\,du\right] = 4\left[\frac{t \operatorname{sen} 2\pi n t}{2\pi n} + \frac{\cos 2\pi n t}{4\pi^2 n^2}\right]\Bigg|_0^1$$

$$= \left[\frac{2 \operatorname{sen} 2\pi n}{\pi n} + \frac{\overset{1}{\cos 2\pi n}}{\pi^2 n^2}\right] - \left[\frac{0 \operatorname{sen} 0}{\pi n} + \frac{\overset{1}{\cos 0}}{\pi^2 n^2}\right]$$

$$= \left[\frac{1}{\pi^2 n^2} - \frac{1}{\pi^2 n^2}\right] = \underline{0} \,//$$

$u = t \quad dv = \operatorname{sen} 2n\pi t\,dt$

$du = dt \quad v = -\frac{\cos 2\pi n t}{2\pi n}$

$u = 2\pi n t$

$du = 2\pi n\,dt$

$dt = \frac{1}{2\pi n}$

$$b_n = 2\int_0^1 2t \operatorname{sen} 2n\pi t\,dt = 4\int_0^1 t \operatorname{sen} 2n\pi t\,dt$$

$$= 4\left[-\frac{t\cos 2n\pi t}{2\pi n} + \int_0^1 \frac{\cos 2\pi n t\,dt}{2\pi n}\right] = 4\left[\frac{-t\cos 2n\pi t}{2\pi n} + \frac{1}{4\pi^2 n^2}\int \cos u\,du\right]$$

$$= 4\left[\frac{-t\cos 2n\pi t}{2\pi n} + \frac{\operatorname{sen} 2\pi n t}{4\pi^2 n^2}\right] = \left[\frac{\operatorname{sen} 2n\pi t}{\pi^2 n^2} - \frac{2t\cos 2n\pi t}{\pi n}\right]\Bigg|_0^1$$

$$= \left[\frac{\overset{0}{\operatorname{sen} 2n\pi}}{\pi^2 n^2} - \frac{2\overset{1}{\cos 2n\pi}}{\pi n}\right] - \left[\frac{\operatorname{sen} 0}{\pi^2 n^2} - \frac{0 \cos 0}{\pi n}\right] = \frac{-2}{\pi n}$$

$$f(t) = \frac{1}{2} - \sum_{n=1}^{\infty} \frac{2}{\pi n} \operatorname{sen} 2n\pi t \quad 0 < t < 1$$

2. Encontrar la STF de cada una de las señales en el intervalo de $-\pi$ a π

2.1 $T = 2\pi$ $\quad \omega_0 = 1$ $\quad f(t)\begin{cases} 0 & ; -\pi < t < -\frac{\pi}{2} \\ A\cos t & ; -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} < t < \pi \end{cases}$

Es PAR

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt = \frac{1}{\pi}\int_0^{\pi} A\cos t\,dt = \frac{A}{\pi}\int_0^{\pi}\cos t\,dt = \frac{A}{\pi}\Big[\operatorname{sen} t\Big]_0^{\pi}$$

$$= \frac{A}{\pi}\Big[\cancel{\operatorname{sen}}\overset{1}{\pi} - \cancel{\operatorname{sen}}\overset{0}{0}\Big] = \frac{A}{\pi}$$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos n\omega_0 t\,dt = \frac{2}{\pi}\int_0^{\pi} A\cos t \cdot \cos t n\,dt$$

$$= \frac{2A}{\pi}\int_0^{\pi}\cos t\cdot\cos nt\,dt = \frac{2A}{\pi}\left[\frac{1}{2}\int_0^{\pi}(\cos(nt+t) + \cos(nt-t))\,dt\right]$$

$\cancel{dt} = \frac{1}{n+1}$

$u = (n+1)t$

$$= \frac{2A}{\pi}\left[\frac{1}{2}\int_0^{\pi}\cos(n+1)t\,dt + \frac{1}{2}\int_0^{\pi}\cos(n-1)t\,dt\right]$$

$du = (n+1)dt$

$u = (n-1)t$

$du = (n-1)dt$

$dt = \frac{1}{n-1}$

$$= \frac{2A}{\pi}\left[\frac{1}{2}\left[\frac{\operatorname{sen}(n+1)t}{n+1} + \frac{\operatorname{sen}(n-1)t}{n-1}\right]\right]$$

$$= \frac{2A}{\pi}\left[\frac{\operatorname{sen}(n+1)t}{2(n+1)} + \frac{\operatorname{sen}(n-1)t}{2(n-1)}\right] = \Bigg|_0^{\pi}\frac{2A}{\pi}\left[\frac{n(\cos t\operatorname{sen} nt - \operatorname{sen} t\cos nt)}{n^2-1}\right]$$

$$= \frac{2A}{\pi}\left[\left(\frac{n\cancel{\cos}\pi\operatorname{sen}\pi - \cancel{\operatorname{sen}}\overset{1}{\pi}\cos n\pi}{n^2-1}\right) - \left(\frac{n\cos 0\operatorname{sen} 0 - \operatorname{sen} 0\cos 0}{n^2-1}\right)\right]$$

$$a_n = \frac{2A}{\pi}\left[\frac{\cos n\pi}{n^2-1}\right] = \frac{2A\cos n\pi}{\pi(n^2-1)}$$

$\forall n \neq 1$ //

$$a_1 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} A\cos t \cdot \cos t\, dt = \frac{2A}{\pi}\int_0^{\frac{\pi}{2}} \cos^2 t\, dt = \frac{2A}{\pi}\left[\frac{\cos t \cdot \operatorname{sen} t}{2} \ldots\right.$$

$$\left.\ldots + \frac{1}{2}\int_0^{\frac{\pi}{2}} dt\right] = \frac{2A}{\pi}\left[\frac{\cos t \cdot \operatorname{sen} t + t}{2}\right]_0^{\pi/2} = \frac{A}{2}$$

$$\therefore f(t) = \frac{A}{\pi} + \frac{A}{2}\cos t + \sum_{n=2}^{\infty} \frac{2A\cos\frac{n\pi}{2}}{\pi(1-n^2)} \cdot \cos nt$$

$-\pi < t < \pi$

2.2 $T = 2\pi$ $\quad W_0 = 1$ $\quad$ Es par.

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t$$

$$a_0 = \frac{2}{2\pi}\int_0^{\pi} \frac{A}{\pi}(t-\pi)\,dt = \frac{A}{\pi^2}\int (t-\pi)\,dt = \frac{A}{\pi^2}\left[\int_0^{\pi} t\,dt - \pi\int_0^{\pi} dt\right]$$

$$= \frac{A}{\pi^2}\left[\frac{t^2}{2} - \pi t\right]_0^{\pi} = \frac{A}{\pi^2}\left[\left(\frac{\pi^2}{2} - \frac{0}{2}\right) - (\pi(\pi) - \pi(\emptyset))\right]$$

$$= \frac{A}{\pi^2}\left[\frac{\pi^2}{2} - \pi^2\right] = \frac{A}{\pi^2}\left[\frac{\pi^2 - 2\pi^2}{2}\right] = \frac{A}{\pi^2}\left[-\frac{\pi^2}{2}\right] = -\frac{A}{2} //$$

$$a_n = \frac{4}{2\pi}\int_0^{\pi} \frac{A}{\pi}(t-\pi)\cos nt\,dt$$

① $u = t$ $\quad du = dt$

$v = \frac{\operatorname{sen} nt}{n}$ $\quad dv = \cos nt\, dt$

$$= \frac{2A}{\pi^2}\left[\underbrace{\int_0^{\pi} t\cos nt\,dt}_{①} - \pi\underbrace{\int_0^{\pi}\cos nt\,dt}_{②}\right.$$

② $u = nt$ $\quad dt = \frac{1}{n}$

$\frac{du}{dt} = n$

$\int \operatorname{sen} u \, du = -\cos u$

$$a_n = \frac{2A}{\pi^2}\left[\frac{t \operatorname{sen} nt}{n} - \int_0^{\pi} \frac{\operatorname{sen} tn}{n} dt - \frac{\pi \operatorname{sen} nt}{n}\right]$$

$$= \frac{2A}{\pi^2}\left[\frac{t \operatorname{sen} nt}{n} + \frac{\cos nt}{n^2} - \frac{\pi \operatorname{sen} nt}{n}\right]_0^{\pi}$$

$$= \frac{2A}{\pi^2}\left\{\left[\cancel{\frac{\pi \operatorname{sen} n\pi}{n}} + \frac{\cos n\pi}{n^2} - \cancel{\frac{\pi \operatorname{sen} n\pi}{n}}\right] - \left[\frac{0 \operatorname{sen} 0}{n} + \frac{\overset{1}{\cos 0}}{n^2} - \frac{0 \operatorname{sen} 0}{n}\right]\right\}$$

$$= \frac{2A}{\pi^2}\left[\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right] = \frac{2A(\cos n\pi - 1)}{\pi^2 n^2}$$

$$f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{2A(\cos n\pi - 1)}{\pi^2 n^2} \cdot \cos nt$$

2.3 $T = 2\pi \quad W_0 = 1$

$$f(t) = \begin{cases} A; & -\pi < t < 0 \\ -A; & 0 < t < \pi \end{cases}$$ Es impar

$$f(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen} n W_0 t \; ; \; b_n = \frac{4}{2\pi}\left[\int_{-\pi}^{0} A \operatorname{sen} nt \, dt + \int_0^{\pi} -A \operatorname{sen} nt \, dt\right]$$

$$b_n = \frac{2A}{\pi}\left[\int_{-\pi}^{0} \operatorname{sen} nt \, dt - \int_0^{\pi} \operatorname{sen} nt \, dt\right] = \frac{2A}{\pi}\left[-\frac{1}{n}\cos nt \Big|_{-\pi}^{0} + \frac{1}{n}\cos nt \Big|_0^{\pi}\right]$$

$$= \frac{A}{2n\pi}\left\{\left[-\overset{1}{\cos 0} - \cos n\pi\right] + \left[\cos n\pi - \overset{1}{\cos 0}\right]\right\} = \frac{A}{2n\pi}\left[-1 + (-1)^n + (-1)^n - 1\right]$$

$$= \frac{[(-1)^n - 1]2A}{n\pi} \; ; \; f(t) = \sum_{n=1}^{\infty} \frac{2A[(-1)^n - 1] \operatorname{sen} nt}{n\pi}$$

$-\pi < t < \pi$

2.4 $T = 2\pi$

$w_0 = \frac{2\pi}{T} = 1$

$$f(t) \begin{cases} A\,\text{sen}\,t & ; -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0 & ; \text{ en otro caso} \end{cases}$$

IMPAR

$$f(t) = \sum_{n=1}^{\infty} b_n \,\text{Sen}\, n w_0 t \;;\; b_n = \frac{4}{2\pi}\int_{-\pi/2}^{\frac{\pi}{2}} A\,\text{sen}\,t \cdot \text{sen}\,n t\,dt$$

$$b_n = \frac{\cancel{2}A}{\pi}\left[\frac{\text{sen}(1-n)t}{2(1-n)} - \frac{\text{sen}(1+n)t}{\cancel{2}(1+n)}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \frac{A}{\pi}\left\{\left[\frac{\text{sen}(1-n)\frac{\pi}{2}}{(1-n)} - \frac{\text{sen}(1+n)\frac{\pi}{2}}{(1+n)}\right]\right\}$$

$$= \frac{A}{\pi(1-n^2)}\left[(1+n)(\overset{1}{\cancel{\text{sen}\frac{\pi}{2}}})\cos\frac{n\pi}{2} - \underset{0}{\cancel{\cos\frac{\pi}{2}}}\,\text{sen}\,\frac{n\pi}{2} - (1-n)(\text{sen}\,\frac{\pi}{2}\dots\right.$$

$$\left.\dots\cdot\cos\frac{n\pi}{2} - \overset{0}{\cancel{\cos\frac{\pi}{2}}}\,\text{sen}\,\frac{n\pi}{2})\right]$$

$$= \frac{A}{\pi(1-n^2)}\left[(1+n)(\cos\frac{n\pi}{2}) - (1-n)\cos\frac{n\pi}{2}\right] \;\forall n \neq 1$$

$$b_1 = \frac{2A}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \text{sen}\,t \cdot \text{sen}\,n t\,dt = \frac{2A}{\pi}\left[\frac{\text{sen}\,0\,t}{2(0)} - \frac{\text{sen}\,2t}{2(2)}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{2A}{\pi}\left[\overset{0}{\cancel{\text{sen}(2)\frac{\pi}{2}}} + \overset{0}{\cancel{\text{sen}\,\pi}}\right] = \underline{0}\,//$$

$$\Rightarrow f(t) = A\sum_{n=2}^{\infty}\frac{(1+n)(\cos\frac{n\pi}{2}) - (1-n)(\cos\frac{n\pi}{2})}{\pi(1-n^2)}\,\text{sen}\,n t$$

Problema 3. Determinar la STF de cada una de las señales periódicas.

3.1

$$f(t) = g(t) + 1$$

$T = 2;\ \omega_0 = \frac{2\pi}{T}$

$\omega_0 = \pi$

$$g(t) = \begin{cases} \frac{At}{2} + \frac{A}{2} & ; -1 < t < 0 \\ \frac{At}{2} - \frac{A}{2} & ; 0 < t < 1 \\ g(t+2) & ; \text{otro caso} \end{cases}$$

Impar

$$g(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen} n\pi t$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t) \operatorname{sen} n\pi t\, dt = \frac{4}{2}\int_0^1 f(t) \operatorname{sen} n\pi\, dt$$

$$= 2\int_0^1 \left(\frac{At}{2} - \frac{A}{2}\right)\operatorname{sen} n\pi t\, dt = A\int_0^1 t \operatorname{sen} n\pi t\, dt - A\int_0^1 \operatorname{sen} n\pi t\, dt$$

$u = t \quad dv = \operatorname{sen} n\pi\, dt$

$du = dt \quad v = -\frac{\cos n\pi t}{n\pi}$

$$b_n = A\left[-\frac{1}{n\pi}\cos n\pi t\Big|_0^1 + \frac{1}{n\pi}\int_0^1 \cos n\pi t\, dt\right] + \left[\frac{A}{n\pi}\cos n\pi t\right]\Big|_0^1$$

$$= A\left[-\frac{1}{n\pi}\cos n\pi t\Big|_0^1 + \frac{1}{n^2\pi^2}(\underbrace{\operatorname{sen} n\pi - 0}_{0})\right] + \left[\frac{A}{n\pi}(\cos n\pi - 1)\right]$$

$$= -\frac{A}{n\pi}\cos n\pi + \frac{A}{n\pi}\cos n\pi - \frac{A}{n\pi} = -\frac{A}{n\pi}$$

$$g(t) = -\frac{A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\operatorname{sen} n\pi t$$

$$\Rightarrow f(t) = 1 - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\operatorname{sen} n\pi t$$

$$3.2 \quad g(t) = \begin{cases} \phi & ; -2\pi < t < \pi \\ -A\,\text{sen}\, t & ; -\pi < t < \pi \\ \phi & ; \pi < t < 2\pi \end{cases}$$

$T = 4\pi \; ; \quad w_0 = \frac{2\pi}{T} = \frac{1}{2}$ Impar

$$b_n = \frac{\cancel{4}}{\cancel{4}\pi}\int_0^{2\pi} f(t)\,\text{sen}\,\frac{nt}{2}\,dt = \frac{-A}{\pi}\int_0^{2\pi} \text{sen}\, t \cdot \text{sen}\,\frac{nt}{2}\,dt$$

$$= -\frac{A}{\pi}\left[-\frac{\text{sen}\left(t+\frac{nt}{2}\right)}{2\left(1+\frac{n}{2}\right)}\Bigg|_0^{\pi} + \frac{\text{sen}\left(t-\frac{nt}{2}\right)}{2\left(1-\frac{n}{2}\right)}\Bigg|_0^{\pi}\right]$$

$$= \frac{-A}{\pi}\left[-\frac{\text{sen}\left(1+\frac{n}{2}\right)\pi}{2\left(1+\frac{n}{2}\right)} + \cancel{\frac{\text{sen}\left(1+\frac{n}{2}\right)0}{2\left(1+\frac{n}{2}\right)}} + \frac{\text{sen}\left(1-\frac{n}{2}\right)\pi}{2\left(1-\frac{n}{2}\right)} - \cancel{\frac{\text{sen}\left(1-\frac{n}{2}\right)0}{2\left(1-\frac{n}{2}\right)}}\right]$$

$$= -\frac{A}{\pi}\left[-\frac{\text{sen}\left(\pi+\frac{n\pi}{2}\right)}{2+n} + \frac{\text{sen}\left(\pi-\frac{n\pi}{2}\right)}{2-n}\right]$$

$$= -\frac{A}{\pi}\left[-\left(\frac{\overset{0}{\cancel{\text{sen}\,\pi\cos\frac{n\pi}{2}}} + \overset{-1}{\cancel{\cos\pi}}\,\text{sen}\,\frac{n\pi}{2}}{2+n}\right) + \left(\frac{\overset{0}{\cancel{\text{sen}\,\pi\cos\frac{n\pi}{2}}} - \overset{-1}{\cancel{\cos\pi}}\,\text{sen}\,\frac{n\pi}{2}}{2-n}\right)\right]$$

$$= -\frac{A}{\pi}\left[\frac{\text{sen}\,\frac{n\pi}{2}}{2+n} + \frac{\text{sen}\,\frac{n\pi}{2}}{2-n}\right] = -\frac{A}{\pi}\left[\frac{(2-n)\,\text{sen}\,\frac{n\pi}{2} + (2+n)\,\text{sen}\,\frac{n\pi}{2}}{(n^2-4)}\right]$$

$$= \frac{A}{\pi}\left[\frac{\text{sen}\,\frac{n\pi}{2}\,(2\cancel{-n}+2\cancel{+n})}{(n^2-4)}\right] = \frac{A}{\pi}\left[\frac{4\,\text{sen}\,\frac{n\pi}{2}}{(n^2-4)}\right]$$

$$= \frac{4A}{\pi(n^2-4)}\cdot\text{sen}\,\frac{n\pi}{2} \quad \forall n \neq 2$$

$$b_2 = -\frac{4A}{4\pi}\int_0^{\pi} \operatorname{sen} t \operatorname{sen}\frac{2t}{2}\,dt = -\frac{A}{\pi}\int_0^{\pi} \operatorname{sen} t \operatorname{sen} t\,dt$$

$$= -\frac{A}{\pi}\left[\frac{t}{2} - \frac{1}{4}\operatorname{sen} 2t\right]\Big|_0^{\pi} = -\frac{A}{\pi}\left[\left(\frac{\pi}{2} - \frac{0}{2}\right) - \frac{1}{4}\left(\overset{0}{\operatorname{sen} 2\pi} - \overset{0}{\operatorname{sen} 0}\right)\right]$$

$$= -\frac{A}{\pi}\left[\frac{\pi}{2}\right] = -\frac{A}{2}$$

$$g(t) = -\frac{A}{2}\operatorname{sen} t + \frac{4A}{\pi}\sum_{\substack{n=1,3\\ n\neq 2}}^{\infty} \frac{\operatorname{sen} n\frac{\pi}{2}}{n^2-4} \cdot \operatorname{sen}\frac{nt}{2}$$

$$f(t) = g(t-\pi) = -\frac{A}{2}\operatorname{sen}(t-\pi) + \frac{4A}{\pi}\sum_{\substack{n=1\\ n\neq 2}}^{\infty} \frac{\operatorname{sen}\frac{n\pi}{2}}{n^2-4} \cdot \operatorname{sen} n\frac{(t-\pi)}{2}$$

3.3

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{A-0}{0-(-1)} = \frac{A}{1} = A$$

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-A)}{1-0} = \frac{A}{1} = A$$

$$y - y_1 = m(x - x_1) \rightarrow y - A = A(x-0)$$
$$y = Ax + A$$

$$y - y_1 = m(x - x_1) \rightarrow y + 0 = A(x-1)$$
$$y = Ax - A$$

$$f(t) = \begin{cases} 0 \ ; & -\frac{3}{2} < t < -1 \\ Ax + A \ ; & -1 < t < 0 \\ Ax - A \ ; & 0 < t < 1 \\ 0 \ ; & 1 < t < \frac{3}{2} \\ f(t-3) \ ; & \text{otro caso} \end{cases}$$

IMPAR

$T = 3$

$w_0 = \frac{2}{3}\pi$

$$f(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen}\frac{2n\pi t}{3}$$

$$bn = \frac{4}{T}\int_0^{T/2} f(t)\,sen\,\frac{2n\pi t}{3}\,dt = \frac{4}{3}\int_0^{3/2} f(t)\,sen\,\frac{2n\pi}{3}t\,dt$$

$$= \frac{4}{3}\int_0^1 (At+A)\,sen\,\frac{2n\pi t}{3}\,dt = \frac{4A}{3}\int_0^1 t\,sen\,\frac{2n\pi t}{3}\,dt + \frac{4A}{3}\int_0^1 sen\,\frac{2n\pi t}{3}\,dt$$

$u = t \quad dv = sen\,\frac{2n\pi}{3}t\,dt$

$du = dt \quad v = -\frac{3}{2n\pi}\cos\frac{2n\pi}{3}t$

$$bn = \frac{4A}{3}\left[-\frac{3t}{2n\pi}\cos\frac{2n\pi}{3}t + \frac{3}{2n\pi}\int_0 \cos\frac{2n\pi t}{3}\,dt\right]\Bigg|_0^1 + \frac{12A}{6n\pi}\cos\frac{2n\pi t}{3}\Bigg|_0^1$$

$$= \frac{4A}{3}\left[-\frac{3t}{2n\pi}\cos\frac{2n\pi}{3}t + \frac{9}{4n^2\pi^2}\,sen\,\frac{2n\pi}{3}t\right]\Bigg|_0^1 + \frac{2A}{n\pi}\cos\frac{2n\pi}{3}\Bigg|_0^1$$

$$= \frac{4A}{3}\left[-\frac{3}{2n\pi}\cos\frac{2n\pi}{3} + \frac{9}{4n^2\pi^2}\left(sen\,\frac{2n\pi}{3} - 0\right)\right] + \frac{2A}{n\pi}\left[\cos\frac{2n\pi}{3} - 1\right]$$

$$= -\frac{2A}{n\pi}\cos\frac{2n\pi}{3} + \frac{3A}{n^2\pi^2}\,sen\,\frac{2n\pi}{3} + \frac{2A}{n\pi}\cos\frac{2n\pi}{3} - \frac{2A}{n\pi}$$

$$= \frac{3A}{n^2\pi^2}\,sen\,\frac{2n\pi}{3} - \frac{2A}{n\pi} \;//$$

$$f(t) = \sum_{n=1}^{\infty}\left(\frac{3A\,sen\frac{2n\pi}{3}}{n^2\pi^2} - \frac{2A}{n\pi}\right) sen\,\frac{2n\pi t}{3} \;//$$

3.4 $T = 2\pi$

$w_0 = \frac{2\pi}{T} = 1 \;//$

$$f(t) = \begin{cases} A\,sen\,t & ; \; 0<t<\pi \\ 0 & ; \; \pi<t<2\pi \\ f(t+2\pi) & ; \; \text{otro caso.} \end{cases}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n\,sen\,n w_0 t$$

$$\boxed{a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,dt = \frac{A}{2\pi}\int_0^{\pi} \operatorname{sen} t\,dt = \frac{-A}{2\pi}\Big[\cos t\Big]_0^{\pi}}$$

$$a_0 = \frac{-A}{2\pi}\Big[\overset{-1}{\cos \pi} - \overset{1}{\cos 0}\Big] = \frac{-A}{2\pi}[-2] = \frac{2A}{2\pi} = \frac{A}{\pi} \,//$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n w_0 t\,dt = \frac{2A}{2\pi}\int_0^{\pi} \operatorname{sen} t \cdot \cos nt\,dt; \quad \begin{array}{l} u = \operatorname{sen} t \\ du = \cos t\,dt \\ dv = \cos nt\,dt \\ v = \frac{\operatorname{sen} nt}{n} \end{array}$$

$$= \frac{A}{\pi}\left[\frac{\operatorname{sen} nt}{n}\cdot \operatorname{sen} t\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi} \operatorname{sen} nt \cdot \cos t\,dt\right] \quad \begin{array}{l} u = \cos t \quad du = -\operatorname{sen} t\,dt \\ dv = \operatorname{sen} nt\,dt \quad v = -\frac{\cos nt}{n} \end{array}$$

$$= \frac{A}{\pi}\left[\frac{1}{n}\Big(\underset{0}{\operatorname{sen} n\pi} \cdot \underset{0}{\operatorname{sen}\pi} - \operatorname{sen} 0 \cdot \operatorname{sen} 0\Big) - \frac{1}{n}\left[-\frac{1}{n}\cos t \cdot \cos nt\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi} \operatorname{sen} t \ldots \right.\right.$$

$$\left.\left. \ldots \cos nt\,dt\right]\right]$$

$$a_n = \frac{A}{\pi}\left[\frac{1}{n^2}\Big(\cos n\pi\,\overset{-1}{\cos \pi} - \overset{1}{\cos 0}\,\overset{1}{\cos 0}\Big)\right] + \frac{1}{n^2}\int_0^{\pi} \operatorname{sen} t \cos nt\,dt\Big]$$

$$= \frac{A}{\pi}\left[\frac{1}{n^2}(-\cos n\pi - 1) + \frac{1}{n^2}\int_0^{\pi} \operatorname{sen} t \cos nt\,dt\right]$$

$$= \frac{-A}{n^2\pi}[\cos n\pi + 1] + \frac{A}{n^2\pi}\int_0^{\pi} \operatorname{sen} t \cos nt\,dt$$

$$= \frac{-A(\cos n\pi + 1)(n^2\pi)}{n^2\pi(An^2 - A)}$$

$$= \frac{\cos n\pi + 1}{\pi(1-n^2)}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\, \text{sen}\, nt\, dt = \frac{2A}{2\pi}\int_0^{\pi} \text{sen}\, t \cdot \text{sen}\, nt\, dt$$

$$= \frac{A}{\pi}\int_0^{\pi} \text{sen}^2 t\, dt = \frac{A}{\pi}\int_0^{\pi} \frac{1}{2}(1-\cos 2t)\, dt = \frac{A}{2\pi}\int_0^{\pi} dt - \frac{A}{2\pi}\int_0^{\pi} \cos 2t\, dt$$

$$= \frac{A}{2\pi}\left[t\right]\Big|_0^{\pi} - \frac{A}{2\pi}\left[\text{sen}\, 2t\right]\Big|_0^{\pi} = \frac{A}{2\pi}\left[\pi - 0\right] - \frac{A}{2\pi}\left[\overset{0}{\text{sen}\, 2\pi} - \overset{0}{\text{sen}\, 0}\right]$$

$$= \frac{A}{2} \Rightarrow f(t) = \frac{A}{\pi} + \sum_{\substack{n=2 \\ n\neq 1}}^{\infty}\left[\frac{\cos n\pi + 1}{\pi(1-n^2)} \cdot \cos nt\right] + \frac{A}{2}\, \text{sen}\, nt$$

3.5

$P_1(-1,0)\ ;\ P_2(-\frac{1}{2}, A)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{A}{-\frac{1}{2}+1} = \frac{A}{\frac{1}{2}} = 2A$$

$y - y_1 = m(x - x_1)$

$y - A = 2A(x + \frac{1}{2})$

$y = 2A(x+1)$

Función PAR

$P_1(\frac{1}{2}, A)\quad P_2(1,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-A}{1-\frac{1}{2}} = -2A$$

$y - y_1 = m(x - x_1)$

$y - A = 2A(x - \frac{1}{2})$

$y = -2A(x-1)$

$$f(t) = \begin{cases} 0 & ; -\frac{3}{2} < t < -1 \\ 2A(t+1) & ; -1 < t < -\frac{1}{2} \\ A & ; -\frac{1}{2} < t < \frac{1}{2} \\ -2A(t-1) & ; \frac{1}{2} < t < 1 \\ 0 & ; 1 < t < \frac{3}{2} \end{cases}$$

$T = 3$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n\, \text{sen}\, n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{3}\int_{-3/2}^{-1} 0\,dt + \int_{-1}^{-1/2}(2At+2A)\,dt + \int_{-1/2}^{1/2} A\,dt - \int_{1/2}(2At-2A)\ldots$$

$$\ldots + \int_{1}^{3/2} 0\,dt$$

$$= \frac{1}{3}\left[At^2 + 2At\Big|_{-1}^{-1/2} + At\Big|_{-1/2}^{1/2} - At^2 + 2At\Big|_{1/2}^{1}\right]$$

$$= \frac{1}{3}\Big[(A(-1/2)^2 + 2A(-1/2) - A(-1)^2 + 2A(-1)) + (A(1/2) - A(-1/2)) - \ldots$$

$$(A(1)^2) + 2A(1) - A(1/2)^2 + 2(1/2))\Big]$$

$$= \frac{1}{3}\left[\left(\frac{1}{4}A - A - A + 2A\right) + \left(\frac{1}{2}A + \frac{1}{2}A\right) - A - 2A + \frac{1}{4}A + A\right]$$

$$= \frac{1}{3}\left[\frac{9}{4}A - 2A + A - 3A + \frac{5}{4}A\right] = \frac{1}{3}\left[-\frac{1}{2}A\right] = -\frac{A}{6}$$

$$a_n = \frac{4}{T}\int_{T_0}^{t_0+T} f(t)\cos n\omega_0 t\,dt = \frac{4}{3}\int_0^{3/2} f(t)\cos n\omega_0 t\,dt$$

$$= \frac{4A}{3}\int_0^{1/2}\cos 2\frac{n\pi}{3}\,dt - \frac{8A}{3}\int_{1/2}^{1}(t-1)\cos 2\frac{n\pi}{3}t\,dt$$

$$= \frac{\cancel{4}A}{3}\left[\frac{\cancel{3}}{2n\pi}\operatorname{sen} 2\frac{n\pi}{3}t\right]_0^{1/2} - \frac{8A}{3}\left[\frac{3}{2n\pi}(t-1)\operatorname{sen} 2\frac{n\pi}{3}t\Big|_{1/2}^{1} - \frac{3}{2n\pi}\int_{1/2}\operatorname{sen}\frac{2n\pi t}{3}\,dt\right]$$

$$= \frac{2A}{n\pi}\left[\operatorname{sen}\frac{2n\pi}{3}\left(\frac{1}{2}\right) - \cancel{\operatorname{sen} 0}^{\,0}\right] - \frac{8A}{3}\left[\frac{3}{2n\pi}(0 - (-\tfrac{1}{2})\operatorname{sen}\frac{2n\pi}{3}(1)\right] + \ldots$$

$$\ldots \frac{3^2}{2^2n^2\pi^2}\cos\frac{2n\pi}{3}t\Big]\Big|_{1/2}^{1}$$

$$a_n = \frac{2A}{n\pi} \cdot \operatorname{sen} \frac{n\pi}{3} - \frac{2A}{n\pi} \cdot \operatorname{sen} \frac{n\pi}{3} - \frac{6}{n^2\pi^2} \cdot \cos 2\frac{n\pi}{3} \cos\frac{n\pi}{3}$$

$$= \frac{6}{n^2\pi^2}\left[\cos\frac{n\pi}{3}\ \cos 2\frac{n\pi}{3}\right]$$

$$\Rightarrow f(t) = -\frac{A}{6} + \sum_{n=1}^{\infty}\left[-\frac{6}{n^2\pi^2}\left(\cos\frac{n\pi}{3} \cdot \cos 2\frac{n\pi}{3}\right)\cos\frac{2n\pi}{3}t\right]$$

3.6

$$f(t) = \begin{cases} -Ae^{-t/10} & ; -\pi \le t < 0 \\ Ae^{-t/10} & ; 0 \le t < \pi \\ f(t-2\pi) & ; \text{otro caso} \end{cases}$$

$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = 1$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{-A}{2\pi}\left[\int_{-\pi}^{0} e^{-t/10}\,dt - \int_{0}^{\pi} e^{-t/10}\,dt\right]$$

$u = -t/10$ $\quad dt = -10du$

$du = -\frac{1}{10}dt$

$$a_0 = \frac{10A}{2\pi}\left[\int_{-\pi}^{0} e^{u}\,du - \int_{0}^{\pi} e^{u}\,du\right]$$

$$= \frac{5A}{\pi}\left\{\left[e^{0} - e^{\pi/10}\right] - \left[e^{-\pi/10} - e^{0}\right]\right\} = \frac{5A}{\pi}\left\{1 - e^{\pi/10} - e^{-\pi/10} + 1\right\}$$

$$= \frac{5A}{\pi}\left(2 - e^{\pi/10} - e^{-\pi/10}\right)$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega_0 t\,dt = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt$$

$$= \frac{-A}{\pi}\left[\int_{-\pi}^{0} e^{-t/10}\cos nt\,dt - \int_{0}^{\pi} e^{-t/10}\cos nt\,dt\right]$$

$u = e^{-t/10}$

$du = -\frac{1}{10}e^{-t/10}\,dt$

$dv = \cos nt\,dt \quad v = \frac{\operatorname{sen} nt}{n}$

$$a_n = -\frac{A}{\pi}\left\{\left[\frac{e^{-t/10}}{n}\operatorname{sen} nt\Big|_{-\pi}^{0} + \frac{1}{10n}\left[\frac{-e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{1}{10n}\int_{-\pi}^{0} e^{-t/10}\cos nt\,dt\right]\right]\right.$$

$$\left. - \left[\left(\frac{e^{-t/10}}{n}\operatorname{sen} nt\right)\Big|_{0}^{\pi} + \frac{1}{10n}\left(\frac{-e^{-t/10}}{n}\cos nt\right)\Big|_{0}^{\pi} - \frac{1}{10n}\int_{0}^{\pi} e^{-t/10}\cos nt\,dt\right]\right\}$$

$$= -\frac{A}{\pi}\left\{\left[\frac{e^{-t/10}}{n}\operatorname{sen} nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{10n^2}\cos nt\Big|_{-\pi}^{0} + \frac{1}{100n^2}\int_{-\pi}^{0} e^{-t/10}\cos nt\,dt\right]\right.$$

$$\left. - \left[\left(\frac{e^{-t/10}}{n}\operatorname{sen} nt\right)\Big|_{0}^{\pi} - \frac{e^{-t/10}}{10n^2}\cos nt\Big|_{0}^{\pi} - \underbrace{\frac{1}{100n^2}\int_{0}^{\pi} e^{-t/10}\cos nt\,dt}_{1}\right]\right\}$$

1 $$\left(1+\frac{1}{100n^2}\right)\int_{-\pi}^{0} e^{-t/10}\cos nt\,dt = \frac{e^{-t/10}}{n}\operatorname{sen} nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{10n^2}\cos nt$$

$$= \frac{100n^2}{100n^2+1}\left[\left(\frac{e^{-t/10}}{n}\operatorname{sen} nt\right)\Big|_{-\pi}^{0} - \left(\frac{e^{-t/10}}{10n^2}\cos nt\right)\Big|_{-\pi}^{0}\right]$$

$$= \frac{100ne^{-t/10}}{100n^2+1}\operatorname{sen} nt\Big|_{-\pi}^{0} - \frac{10e^{-t/10}}{100n^2+1}\cos nt\Big|_{-\pi}^{0}$$

$$a_n = -\frac{A}{\pi}\left\{\left[\frac{100ne^{-t/10}}{100n^2+1}\cdot\operatorname{sen} nt\Big|_{-\pi}^{0} - \frac{10e^{-t/10}}{100n^2+1}\cos nt\Big|_{-\pi}^{0}\right]\right.$$

$$\left. - \left[\left(\frac{100ne^{-t/10}}{100n^2+1}\operatorname{sen} nt\Big|_{0}^{\pi} - \frac{10e^{-t/10}}{100n^2+1}\cos nt\Big|_{0}^{\pi}\right)\right]\right\}$$

$$a_n = \frac{20A}{\pi(100n^2+1)} - \frac{10e^{\pi/10}(-1)^n A}{\pi(100n^2+1)} - \frac{10e^{-\pi/10}(-1)^n A}{\pi(100n^2+1)}$$

$$= \frac{A\left[20 - 10e^{\pi/10}(-1)^n - 10e^{-\pi/10}(-1)^n\right]}{\pi(100n^2+1)}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\,\text{sen}\,nt\,dt = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\,\text{sen}\,nt\,dt$$

$u = e^{-t/10}$ $du = -\frac{e^{-t/10}}{10}dt$

$$= \frac{A}{\pi}\left[-\int_{-\pi}^{0} e^{-t/10}\,\text{sen}\,nt\,dt + \int_{0}^{\pi} e^{-t/10}\,\text{sen}\,nt\,dt\right]$$

$dv = \text{sen}\,nt\,dt$; $v = \frac{-\cos nt}{n}$

$$= \frac{A}{\pi}\left[-\left(\frac{-e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{1}{10n}\int_{-\pi}^{0} e^{-t/10}\cos nt\,dt\right) + \left(-\frac{e^{-t/10}}{n}\cos nt\Big|_{0}^{\pi} - \frac{1}{10n}\int_{0}^{\pi} e^{-t/10}\cos nt\,dt\right)\right]$$

$u = e^{-t/10}$ $du = \cos nt\,dt$

$du = -\frac{1}{10}e^{-t/10}dt$ $v = \frac{1}{n}\,\text{sen}\,tn$

$$= \frac{A}{\pi}\left[-\left(-\frac{e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{10n^2}\,\text{sen}\,nt\Big|_{-\pi}^{0} - \frac{1}{100n^2}\int_{-\pi}^{0} e^{-t/10}\,\text{sen}\,nt\,dt\right)\right.$$

$$\left. + \left(\frac{-e^{-t/10}}{n}\cos nt\Big|_{0}^{\pi} - \frac{e^{-t/10}}{10n^2}\,\text{sen}\,nt\Big|_{0}^{\pi} + \frac{1}{100n^2}\int_{0}^{\pi} e^{-t/10}\,\text{sen}\,nt\,dt\right)\right]$$

$$= \left(1 + \frac{1}{100n^2}\right)\int_{-\pi}^{0} e^{-t/10}\,\text{sen}\,nt\,dt = \frac{-e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{n^2}\,\text{sen}\,nt\Big|_{-\pi}^{0}$$

$$= \int_{-\pi}^{0} e^{-t/10}\,\text{sen}\,t\,dt = \frac{-e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{n^2}\,\text{sen}\,nt\Big|_{-\pi}^{0}$$

$$= -\frac{e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{10n^2}\,\text{sen}\,tn\Big|_{-\pi}^{0}$$

$$\frac{100n^2+1}{100n^2}$$

$$= \frac{100n^2}{100n^2+1}\left[-\frac{e^{-t/10}}{n}\cos nt\Big|_{-\pi}^{0} - \frac{e^{-t/10}}{10n^2}\,\text{sen}\,nt\Big|_{-\pi}^{0}\right]$$

$$= \frac{A}{\pi}\left\{-\left[-\frac{100n}{100n^2+1} + \frac{e^{\pi/10}(-1)^n}{100n^2+1}\right] + \left[\frac{-100n\,e^{-\pi/10}(-1)^n}{100n^2+1} + \frac{100n}{100n^2+1}\right]\right\}$$

$$= \frac{A\,200n}{\pi(100n^2+1)} - \frac{A\,100n\,e^{\pi/2}(-1)^n}{\pi(100n^2+1)} - \frac{A\,100n\,e^{-\pi/10}(-1)^n}{\pi(100n^2+1)}$$

$$= \frac{A(200n - 100ne^{\pi/10}(-1)^n - 100ne^{-\pi/10}(-1)^n)}{\pi(100n^2+1)}$$

$$\Rightarrow f(t) = \frac{A(10-5e^{\pi/10}-5e^{-\pi/10})}{\pi}\sum_{n=1}^{\infty}\left(\frac{A(20-10e^{-\pi/10}(-1)^n-10e^{\pi/10}(-1)^n)}{\pi(100n^2+1)}\right)$$

$$\ldots\cdot\cos nt + \left(\frac{A(200n-100ne^{\pi/10}(-1)^n-100ne^{-\pi/10}(-1)^n)}{\pi(100n^2+1)}\right)\text{sen}\,nt$$

4. Calcula la serie exponencial de Fourier de cada una de las señales periódicas y graficar magnitud y fase.

a) $T = \frac{7}{2}$ $\quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\frac{7}{2}} = \frac{4\pi}{7}$

$$f(t) \begin{cases} 0 & ; -7/4 < t < -1/2 \\ A & ; -1/2 < t < 1/2 \\ 0 & ; 1/2 < t < 7/4 \\ f(t+7/2) & ; \text{otro caso} \end{cases}$$

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{in\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) e^{-in\omega_0 t} dt = \frac{1}{\frac{7}{2}}\int_{-7/4}^{7/4} f(t) e^{-in\frac{4\pi}{7}t} = \frac{2A}{7}\int_{-1/2}^{1/2} e^{-in\frac{4\pi t}{7}} dt$$

$$= \frac{2A}{7}\left(\frac{-7}{4n\pi i}\right) e^{-i\frac{4n\pi}{7}t}\Big|_{-1/2}^{1/2} = \frac{A}{2in\pi}\left[e^{-i\frac{2n\pi}{7}} - e^{i\frac{2n\pi}{7}}\right]$$

$$= \frac{A}{n\pi}\left[\frac{e^{i\frac{2n\pi}{7}} - e^{-i\frac{2n\pi}{7}}}{2i}\right] = \frac{A}{n\pi} \operatorname{sen}\left(\frac{2n\pi}{7}\right) = \frac{A\cdot\frac{2}{7}}{\frac{2}{7}n\pi} \cdot \operatorname{sen}\frac{2n\pi}{7}$$

$$= \frac{2A}{7} \operatorname{sen}\left(\frac{2n\pi}{7}\right) \qquad |C_n| = -A$$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty} \frac{2A}{7} \operatorname{sen}\left(\frac{2n\pi}{7}\right) e^{i\frac{4n\pi}{7}t}$$

b) $T=4,\ \omega_0=\frac{2\pi}{T}=\frac{\pi}{2}$; $f(t)\begin{cases} \emptyset & ; -2<t<-1 \\ A(t+1) & ; -1<t<0 \\ -A(t-1) & ; 0<t<1 \\ \emptyset & ; 1<t<2 \end{cases}$

$P_1(-1,0)$ $\quad P_2(0,A)$ $\quad P_1(0,A)\ P(1,0)$

$m=-\frac{A}{1}=-A$

$m=\frac{y_2-y_1}{x_2-x_1}=\frac{A}{1}$

$y-0=-A(x-1)$

$f(t)=\sum_{-\infty}^{\infty} C_n e^{in\omega_0 t}$

$m=A$

$y-y_1=m(x-x_1)$

$y=(x+1)A$

$$C_n=\frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-in\omega_0 t}dt=\frac{1}{4}\int_{-2}^{2} f(t)e^{-in\frac{\pi}{2}t}dt=\overbrace{\frac{A}{4}\int_{-1}^{0}(t+1)e^{-in\frac{\pi}{2}t}dt}^{①}\ldots\underbrace{-\frac{A}{4}\int_{0}^{1}(t-1)e^{-in\frac{\pi}{2}t}dt}_{②}$$

$u=t\pm1 \quad dv=e^{-in\frac{\pi}{2}t}dt$

$du=dt \quad v=\frac{-2}{in\pi}e^{-in\frac{\pi}{2}t}$

$$①=\frac{A}{4}\left[-\frac{2}{in\pi}(t+1)e^{-in\frac{\pi}{2}t}\Big|_{-1}^{0}+\frac{2}{in\pi}\int_{-1}^{0}e^{-in\frac{\pi}{2}t}dt\right.$$

$$=\frac{A}{4}\left[\frac{-2}{in\pi}\left(\cancel{e^{0}}^{1}-(\cancel{0})e^{-\frac{in\pi}{2}}\right)-\frac{4}{i^2n^2\pi^2}e^{-in\frac{\pi}{2}t}\Big|_{-1}^{0}\right]$$

$$=\frac{A}{4}\left[\frac{-2}{in\pi}-\frac{4}{n^2\pi^2}\left(\cancel{e^{0}}^{1}-e^{in\frac{\pi}{2}}\right)\right]=\frac{A}{4}\left[-\frac{2}{in\pi}+\frac{4}{n^2\pi^2}\left(1-e^{in\frac{\pi}{2}}\right)\right]$$

$$=\frac{-A}{2in\pi}+\frac{A}{n^2\pi^2}\left(1-e^{in\frac{\pi}{2}}\right)$$

$$②=-\frac{A}{4}\left[\frac{-2}{in\pi}(t-1)e^{-in\frac{\pi}{2}t}\Big|_{0}^{1}+\frac{2}{in\pi}\int_{0}^{1}e^{-in\frac{\pi}{2}t}dt\right]$$

$$= -\frac{A}{4}\left[-\frac{2}{in\pi}\left((\cancel{1})e^{-\frac{in\pi}{2}t} - (-1)e^{0}\right) - \frac{4}{i^2n^2\pi^2}e^{-in\frac{\pi}{2}t}\Big|_0^1\right]$$

$$= -\frac{A}{4}\left[-\frac{2}{in\pi} + \frac{4}{n^2\pi^2}\left(e^{-\frac{in\pi}{2}} - e^{0}\right)\right] = \frac{-A}{2in\pi} + \frac{A}{n^2\pi^2}\left(e^{-\frac{in\pi}{2}} - 1\right)$$

$$C_n = \frac{A}{n^2\pi^2} - \frac{A}{n^2\pi^2}e^{-\frac{in\pi}{2}} - \cancel{\frac{A}{2in\pi}} + \cancel{\frac{A}{2in\pi}} - \frac{A}{n^2\pi^2}\cdot e^{-\frac{in\pi}{2}} + \frac{A}{n^2\pi^2}$$

$$= \underbrace{\frac{2A}{n^2\pi^2} - \frac{A}{n^2\pi^2}\left(e^{-\frac{in\pi}{2}} - e^{\frac{in\pi}{2}}\right)}_{3} = \frac{1}{2i}\left[3\right]$$

$$= \frac{\cancel{2}A}{\cancel{2}in^2\pi^2} + \frac{2iA}{n^2\pi^2}\left(\frac{e^{-\frac{in\pi}{2}} - e^{\frac{in\pi}{2}}}{2i}\right) = \frac{A}{in^2\pi^2} + \frac{2iA}{n^2\pi^2}\operatorname{sen}\left(\frac{in\pi}{2}\right)$$

$$= \frac{Ai}{i^2n^2\pi^2} - \frac{A}{\frac{in^2\pi^2}{2}}\operatorname{sen}\frac{in\pi}{2} = \frac{-Ai}{in^2\pi^2} - \frac{A}{n\pi}\operatorname{sen}\left(in\frac{\pi}{2}\right)$$

$$C_1 = \lim_{n\to 1} C_n = \frac{\frac{d}{dn}-A_i}{\frac{d}{dn}n^2\pi^2} - \frac{\frac{\partial}{\partial n}A\operatorname{sen}\left(\frac{in\pi}{2}\right)}{\frac{\partial}{\partial n}\frac{in^2\pi^2}{2}}$$

$$= \lim_{n\to 1}\frac{0}{2n\pi^2} - \frac{Ai\frac{\pi}{2}\cos\frac{in\pi}{2}}{in\pi^2} = \lim_{n\to 1}\frac{-A\frac{i\pi}{2}\cos\left(\frac{in\pi}{2}\right)}{in\pi^2}$$

$$= \lim_{n\to 1}\frac{-A\frac{i\pi}{2}\cos\frac{in\pi}{2}}{i\pi^2} = -\frac{iA\pi}{2i\pi^2}\cos\left(\frac{i\pi n}{2}\right)$$

$$= -\frac{A}{2\pi}\cos\left(\frac{i\pi n}{2}\right);\quad f(t) = \sum_{\substack{-\infty \\ n\neq 1}}^{\infty}\left[\left(-\frac{Ai}{n^2\pi^2} - \frac{A}{n\pi}\operatorname{sen}\left(\frac{in\pi}{2}\right)\right)e^{\frac{in\pi}{2}t}\ldots\right.$$

$$\left.\ldots\cdot\frac{-A}{2\pi}\cos\frac{i\pi n}{2}\right\}$$

c) $T=1 \quad W_0 = \frac{2\pi}{T} = 2\pi \quad f(t) = \begin{cases} -A(t+1) & ; 0<t<1 \\ f(t+1) & ; \text{otro caso} \end{cases}$

$P_1(0,A) \quad P_2(1,0)$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{inw_0 t}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{A}{1}$$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t) e^{-in2\pi t} dt$$

$m = -A$

$y - y_1 = m(x - x_1)$

$y - A = -A(x)$

$y = -A(x+1)$

$$= -A\int_0^1 (t+1) e^{-i2\pi n t} dt$$

$u = t+1 \quad du = dt$

$dv = e^{-i2\pi n t} dt$

$v = -\frac{e^{-i2\pi n t}}{i2\pi n}$

$$= \frac{(t+1)Ae^{-i2\pi n t}}{i2\pi n}\Big|_0^1 - \frac{A}{i2\pi n}\int_0^1 e^{-i2\pi n t} dt$$

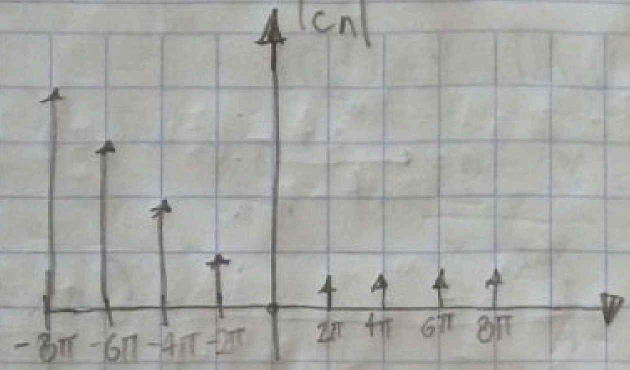
$$C_n = \frac{A}{i2\pi n}\left[2e^{-i2\pi n} - e^{0}\right] + \frac{A}{i^2 n^2 \pi^2} \cdot e^{-i2\pi n t}\Big|_0^1$$

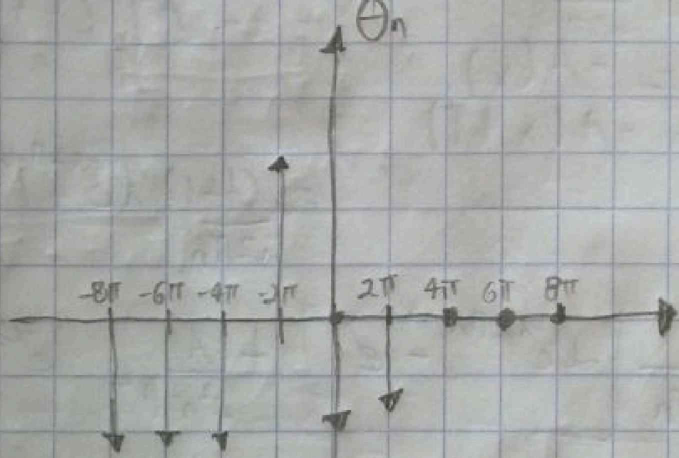
$$= \frac{A}{i2\pi n}\left[2e^{-i2\pi n} - 1\right] - \frac{A}{4n^2\pi}\left[e^{-i2\pi n} - e^{0}\right]$$

$$= \frac{A}{i2\pi n}\left[2e^{-i2\pi n} - 1\right] - \frac{A}{4n^2\pi^2}\left[e^{-i2\pi n} - 1\right]$$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty}\left[\frac{A}{i2\pi n}\left[2e^{-i2\pi n} - 1\right] - \frac{A}{4n^2\pi^2}\left(e^{-i2\pi n} - 1\right)\right] \cdot e^{i2\pi n t}$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
-4	-8π	453.98A	-1.57
-3	-6π	34.87A	-1.57
-2	-4π	3.39A	-1.57
-1	-2π	0.61A	1.55
0	0	∅	-0.785
1	2π	0.133A	0.00015
2	4π	0.0737A	∅
3	6π	0.050A	∅
4	8π	0.058A	∅





D) $T=\pi$ $\quad \omega_0 = \frac{2\pi}{T} = 2$ $\quad f(t) = \begin{cases} A\cos t \; ; & 0<t<\frac{\pi}{2} \\ 0 \; ; & \frac{\pi}{2}<t<\pi \end{cases}$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \; ; \quad C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-in\omega_0 t}\, dt$$

$$C_n = \frac{1}{\pi}\int_0^{\pi/2} A\cos t\, dt = \frac{A}{\pi}\int_0^{\pi/2} \cos t \cdot e^{i2nt}\, dt$$

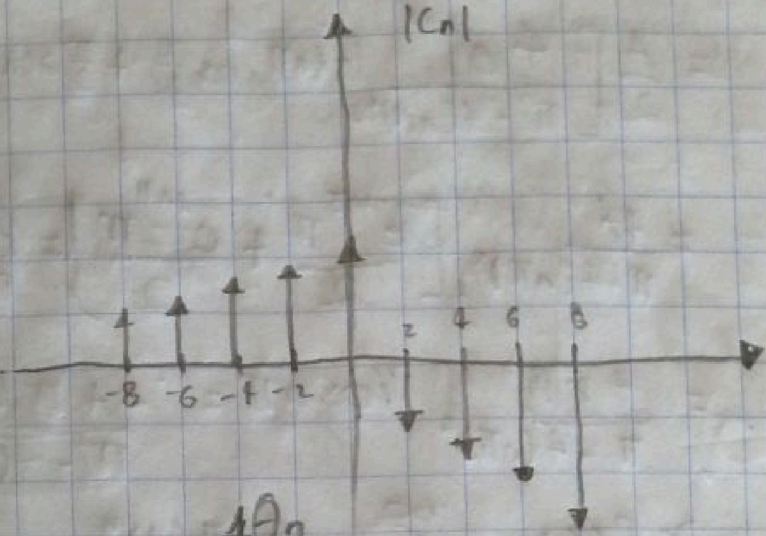
$$= \frac{A}{\pi}\left[\frac{e^{-i2nt}}{1+(-in^2)^2}\left(-in^2\cos t + \operatorname{sen} t\right)\right]\Bigg|_0^{\pi/2}$$

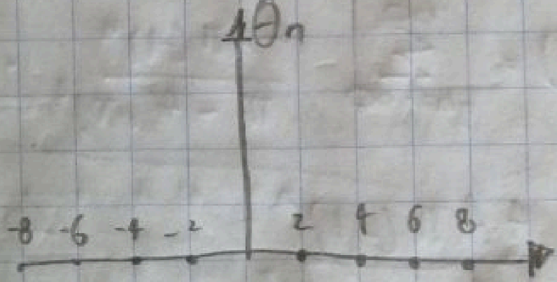
$$= \frac{A}{\pi}\left[\frac{e^{-in2\frac{\pi}{2}}}{1-(n^2)^2}\left(-in^2 \overset{0}{\cos\frac{\pi}{2}} + \overset{1}{\cancel{\operatorname{sen}\frac{\pi}{2}}}\right)\right] - \left[\frac{e^{0}}{1-(n^2)^2}\left(-in^2 2\pi \cancel{\cos 0} + \cancel{\operatorname{sen} 0}\right)\right]$$

$$= \frac{A}{\pi}\left[\frac{e^{-in\pi}}{1-(n^2)^2}\left(\frac{-in^2}{1-(n^2)^2}\right)\right] = \frac{A}{\pi}\left[\frac{e^{-in\pi} + in^2}{1-(n^2)^2}\right]$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{A}{\pi}\left[\frac{e^{-in\pi} + in^2}{1-(n^2)^2}\right] e^{i2nt}$$

n	$n\omega_0$	$\lvert C_n \rvert$	θ_n
-4	-8	0.07A	∅
-3	-6	.052A	∅
-2	-4	.084A	∅
-1	-2	.207A	∅
0	0	.31A	∅
1	2	-2.661A	∅
2	4	-11.44A	∅
3	6	-112.75A	∅
4	8	-1448.86	∅





$$\lvert C_n \rvert = \frac{A}{\pi}\left(\frac{e^{i\pi} + n^2}{1-(n^2)^2}\right)$$

$$\theta_n = \emptyset$$

e) $T = 2$
$\omega_0 = \frac{2\pi}{T} = \pi$

$$f(t) = \begin{cases} A\cos\frac{\pi t}{2} \; ; & -1 < t < 1 \end{cases}$$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega_0 t} \; ; \; C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-in\omega_0 t}\, dt$$

$$C_n = \frac{1}{2}\int_{-1}^{1} A\cos\frac{\pi}{2}t\, e^{-in\pi t}\, dt = \frac{A}{2}\int_{-1}^{1} \cos\frac{\pi}{2}t \cdot e^{-in\pi t}\, dt$$

$$= \frac{A}{2}\left[\frac{e^{-in\pi t}}{(\frac{\pi}{2})^2 + (-in\pi)^2}\left(-in\pi\cos\frac{\pi}{2}t + \frac{\pi}{2}\sin\frac{\pi}{2}t\right)\right]\Bigg|_{-1}^{1}$$

$$C_n = \frac{A}{2}\left[\frac{e^{-in\pi}}{\left(\frac{\pi}{4}\right)-(n\pi)^2}\left(-in\pi\cos\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\frac{\pi}{2}\right)-\frac{e^{in\pi}}{\left(\frac{\pi}{4}\right)-(n\pi)^2}\left(-in\pi\cos\frac{\pi}{2}+\frac{1}{4}\operatorname{sen}\frac{\pi}{2}\right)\right]$$

$$= \frac{A}{2}\left[\frac{4e^{-in\pi}}{\pi-4(n\pi)^2}\left(-in\pi\cos\frac{\pi}{2}+\frac{\pi}{2}\operatorname{sen}\frac{\pi}{2}\right)-\frac{4e^{in\pi}}{\pi-4(n\pi)^2}\left(-in\pi\cos\frac{\pi}{2}-\frac{\pi}{2}\operatorname{sen}\frac{\pi}{2}\right)\right]$$

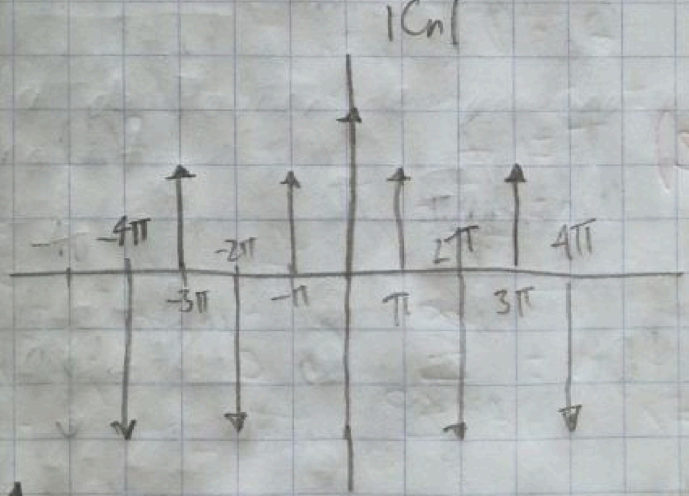
$$= \frac{4A}{\pi-4(n\pi)^2}\left[\frac{e^{-in\pi}\pi}{2}+\frac{e^{in\pi}\pi}{2}\right] = \frac{4A\pi}{\pi-4(n\pi)^2}\left[\frac{e^{-in\pi}+e^{in\pi}}{2}\right]$$

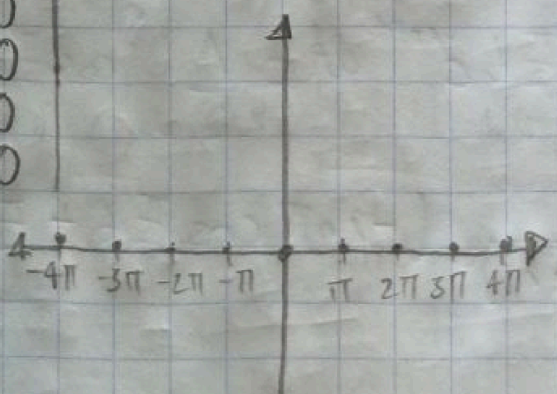
$$= \frac{4A\pi}{\pi-4(n\pi)^2}\cos n\pi = \frac{4A\pi(-1)^n}{\pi-4(n\pi)^2};$$

$$\Rightarrow f(t) = \sum_{n=-\infty}^{\infty}\frac{4A\pi(-1)^n}{\pi-4(n\pi)^2}\cdot e^{in\pi t}$$

$$|C_n| = \frac{4A\pi(-1)^n}{\pi-4(n\pi)^2} \qquad \theta = 0$$

n	$n\omega_0$	$\lvert C_n\rvert$	θ_n
-4	-4π	-.019A	0
-3	-3π	0.035A	0
-2	-2π	-.81A	0
-1	$-\pi$	.34A	0
0	0	4	0
1	π	0.34A	0
2	2π	-.81A	0
3	3π	.035A	0
4	4π	-.019A	0





f) $T=2$ $\qquad f(t)=\begin{cases} At^2 & ; 0<t<1 \\ 0 & ; 1<t<2 \end{cases}$

$W_0 = \frac{2\pi}{T} = \pi$

$$f(t)=\sum_{n=-\infty}^{\infty} C_n e^{inw_0 t} \; ; \; C_n = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-inw_0 t}\, dt$$

$$C_n = \frac{1}{2}\int_0^1 At^2 e^{-in\pi t} dt = \frac{A}{2}\int_0^1 t^2 \cdot e^{-in\pi t} dt$$

$$= \frac{A}{2}\left[\frac{-t^2 e^{-in\pi t}}{in\pi} - \frac{2te^{-in\pi t}}{(in\pi)^2} - \frac{2e^{-in\pi t}}{(in\pi)^3}\right]\Big|_0^1$$

$$C_n = \frac{A}{2}\left[\frac{-e^{-in\pi}}{in\pi} - \frac{2e^{-in\pi}}{(in\pi)^2} - \frac{2e^{-in\pi}}{(in\pi)^3} + \frac{0}{in\pi} + \frac{0}{(in\pi)^2} + \frac{2e^{0}}{(in\pi)^3}\right]$$

$$= \frac{A}{2}\left[\frac{-e^{-in\pi}}{in\pi} + \frac{2e^{-in\pi}}{(n\pi)^2} - \frac{2e^{-in\pi}}{i(n\pi)^3} - \frac{2}{i(n\pi)^2}\right]$$

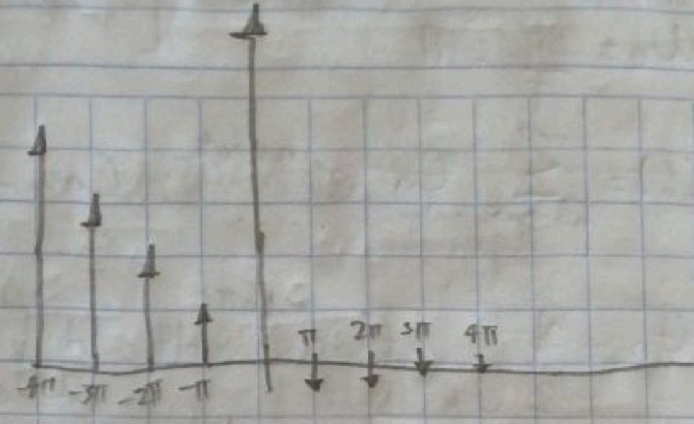
$$= \frac{A}{2i(n\pi)^3}\left[-n\pi^2 e^{-in\pi} + 2ie^{in\pi}(n\pi) + 2e^{-in\pi} - 2\right]$$

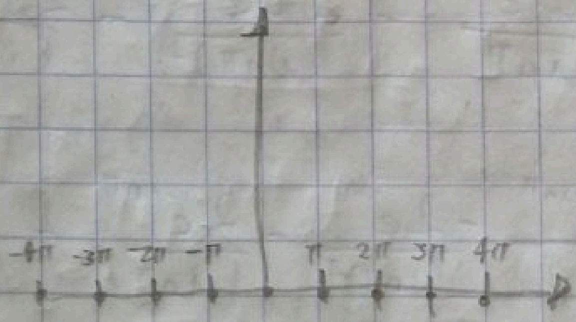
$$f(t) = \sum_{n=-\infty}^{\infty} \frac{A}{2i(n\pi)^3}\left[-(n\pi)^2 e^{-in\pi} + 2ie^{-in\pi}(n\pi) + 2e^{-in\pi} - 2\right] e^{in\pi t}$$

$$|c_n| = \frac{A}{2i(n\pi)^3}\left[-(n\pi^2) e^{-in\pi} + 2ie^{-in\pi}(n\pi) + 2e^{-in\pi} - 2\right]$$

$\theta = 0$

n	$n\omega_0$	$\lvert c_n \rvert$	θ_n
-4	-4π	13080.3A	0
-3	-3π	782.09A	0
-2	-2π	54.01A	0
-1	$-\pi$	5.24A	0
0	0	Ø	0
1	π	.031A	0
2	2π	.0039A	0
3	3π	.0011A	0
4	4π	.0005A	0





Problema 5.

Obtener la transformada de Fourier de cada una de las señales de la figura 4.

a)
$$f(t)\begin{cases} -\frac{4}{3}t-\frac{8}{3} & ; \quad -2<t<-3/2 & ① \\ 4t+4 & ; \quad -3/2<t<0 & ② \\ -4t+4 & ; \quad 0<t<3/2 & ③ \\ \frac{4}{3}t-\frac{8}{3} & ; \quad 3/2<t<2 & ④ \end{cases}$$

$$\mathcal{F}\{f(t)\}=F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$$

$$\int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt \; ; \quad ① \int_{-2}^{-3/2}\left(-\frac{4}{3}t-\frac{8}{3}\right)e^{-i\omega t}\,dt$$

$$=\left[\frac{-\left(\frac{4}{3}t+\frac{8}{3}\right)e^{-i\omega t}}{i\omega}-\frac{\left(\frac{4}{3}\right)e^{-i\omega t}}{\omega^2}\right]\Bigg|_{-2}^{-3/2}$$

$$= \left[\frac{\left(\frac{4}{3}\left(-\frac{3}{2}\right)-\frac{8}{3}\right)e^{+i\frac{3}{2}w}}{\frac{\frac{-12}{6}+\frac{16}{6}}{w}} - \frac{\left(\frac{4}{3}\right)e^{\frac{3iw}{2}}}{w^2} - \frac{\left(\frac{4}{3}(-2)+\frac{8}{3}\right)e^{2iw}}{iw} + \frac{\left(\frac{4}{3}\right)e^{-2iw}}{w^2} \right]$$

$$= \left[\frac{\left(\frac{-12}{6}+\frac{8}{3}\right)e^{i\frac{3w}{2}}}{iw} - \frac{4e^{i\frac{3w}{2}}}{3w^2} + \frac{4e^{2iw}}{3w^2} \right] = \left[\frac{4e^{i\frac{3w}{2}}}{6iw} - \frac{4e^{i\frac{3w}{2}}}{3w^2} + \frac{4e^{2iw}}{5w^2} \right]$$

② $$\int_{-3/2}^{0} (4t+4)e^{-iwt}\,dt = 4\left[\frac{-(t+1)e^{-iwt}}{iw} + \frac{e^{-iwt}}{w^2} \right]\Bigg|_{-3/2}^{0}$$

$$= 4\left[\frac{-e^{-iw0}}{iw} + \frac{e^{0}}{w^2} + \frac{\left(-\frac{3}{2}+1\right)e^{-iw\left(\frac{3}{2}\right)}}{iw} - \frac{e^{\frac{3}{2}iw}}{w^2} \right]$$

$$= 4\left[\frac{1}{iw} + \frac{1}{w^2} - \frac{e^{i\frac{3}{2}w}}{2iw} - \frac{e^{-i\frac{3w}{2}}}{w^2} \right]$$

③ $$\int_{0}^{3/2} (-4t+4)e^{-iwt}\,dt = 4\left[\frac{-(-t+1)e^{-iwt}}{iw} - \frac{e^{-iwt}}{w^2} \right]\Bigg|_{0}^{3/2}$$

$$= 4\left[-\frac{(-3/2+1)e^{-i\frac{3w}{2}}}{iw} - \frac{e^{-i\frac{3w}{2}}}{w^2} + \frac{1}{iw} + \frac{1}{w^2} \right]$$

$$= 4\left[\frac{e^{-i3w}}{2iw} - \frac{e^{-i\frac{3}{2}w}}{w^2} + \frac{1}{iw} + \frac{1}{w^2} \right]$$

④ $$\int_{3/2}^{2} \left(\frac{4}{3}t-\frac{8}{3}\right)e^{-iwt}\,dt = \left[\frac{-\left(\frac{4}{3}t-\frac{8}{3}\right)e^{iwt}}{iw} + \frac{4e^{iwt}}{5w^2} \right]_{3/2}^{2}$$

$$= \frac{4e^{2iw}}{3w^2} + \frac{\left(\frac{4}{3}\left(\frac{3}{2}\right)-\frac{8}{3}\right)e^{i\frac{3}{2}w}}{iw} - \frac{4e^{i\frac{3}{2}w}}{3w^2}$$

(4) $= \frac{4e^{2iw}}{3w^2} - \frac{4e^{i\frac{3w}{2}}}{6iw} - \frac{4e^{i\frac{w3}{2}}}{3w^2}$

$\Rightarrow$ Sumando (1), (2), (3) y (4)

$$f(t) = \frac{8}{w^2} - \frac{18e^{-i\frac{3}{2}w}}{3w^2} + \frac{8e^{2iw}}{3w^2}$$

b) $f(t) = \begin{cases} A\cos t & ; -\frac{\pi}{2} < t < \frac{\pi}{2} \\ \emptyset & ; \text{otro caso} \end{cases}$

$$\mathcal{F}\{f(t)\} = F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} A\cos t \cdot e^{-iwt} dt = A\left[\frac{e^{-iwt}}{(-iw)^2+1}(-iw)\cos t + \operatorname{sen} t\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= A\left[\frac{e^{-iw\frac{\pi}{2}}}{-w^2+1}(1) - \frac{e^{-iw\frac{\pi}{2}}}{-w^2+1}(-1)\right] = A\left[\frac{e^{-iw\frac{\pi}{2}} + e^{iw\frac{\pi}{2}}}{1-w^2}\right]$$

$$= \frac{2A}{1-w^2}\cos w\frac{\pi}{2} \;;\; \mathcal{F}\{f(t)\} = \frac{2A}{1-w^2}\cos w\frac{\pi}{2}$$

c) $f(t) = \begin{cases} A & ; -6 < t < -4 \\ 0 & ; -4 < t < 4 \\ A & ; 4 < t < 6 \end{cases}$ $\quad \mathcal{F}\{f(t)\} = F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$

$$A\left[\int_{-6}^{-4} e^{-iwt} dt + \int_{4}^{6} e^{-iwt} dt\right] = A\left[\frac{-e^{-iwt}}{iw}\Big|_{-6}^{-4} - \frac{e^{-iwt}}{iw}\Big|_{4}^{6}\right]$$

$$= \frac{-A}{iw}\left(e^{4iw} - e^{6iw} + e^{-4iw} + e^{-6iw}\right)$$

$$= \frac{2A}{iw}\left(\frac{e^{4iw} + e^{-4iw} + (e^{-6iw} - e^{6iw})}{2}\right)$$

$$= \frac{-2A}{iw}\left(\cos 4w - \operatorname{sen} 6w\right]; \ \mathcal{F}\{f(t)\} = \frac{-2A}{iw}(\cos 4w - \operatorname{sen} 6w)$$

D) $f(t) = \begin{cases} t^2 A; & 0 < t < 1 \\ 0; & 1 < t < 2 \\ t^2 A; & 2 < t < 3 \end{cases}$ $\quad \mathcal{F}\{f(t)\} = F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$

$$\int_0^1 A t^2 e^{-iwt} dt + \int_2^3 A t^2 e^{-iwt} dt = A\left[\left(\frac{-t^2 e^{-iwt}}{iw} - \frac{2te^{-iwt}}{(iw)^2} - \frac{2e^{-iwt}}{(iw)^3}\right)\Big|_0^1 \dots\right.$$

$$\dots + \left.\left(\frac{-t^2 e^{-iwt}}{iw} - \frac{2te^{-iwt}}{(iw)^2} - \frac{2e^{-iwt}}{(iw)^3}\right)\right]\Big|_2^3$$

$$= A\left[\frac{-e^{-iwt}}{iw} - \frac{2e^{-iw}}{(iw)^2} - \frac{2e^{-iw}}{(iw)^3} + \frac{2}{(iw)^3} - \frac{9e^{-i3w}}{iw} - \frac{6te^{-i3t}}{(iw)^2} - \dots\right.$$

$$\dots \frac{2e^{-i3w}}{(iw)^3} + \frac{4e^{-i2w}}{iw} + \frac{4e^{-2iw}}{(iw)^2} + \frac{2e^{-2iw}}{(iw)^3}$$

e) $f(t) = \begin{cases} \frac{A}{2}t + 5A; & -10 < t < 8 \quad (1) \\ \frac{A}{2}t + 3A; & -8 < t < -6 \quad (2) \\ \frac{A}{2}t - 3A; & 6 < t < 8 \quad (3) \\ \frac{A}{2}t - 5A; & 8 < t < 10 \quad (4) \end{cases}$ $\quad \mathcal{F}\{f(t)\} = F(w) = \int_{-\infty}^{\infty} f(t) e^{-iwt} dt$

① $\int_{-10}^{-8} \left(\frac{A}{2}t + 5A\right)e^{-i\omega t}\,dt = \left[\frac{-\left(\frac{A}{2}t + 5A\right)e^{-i\omega t}}{i\omega} - \frac{Ae^{-i\omega t}}{2(i\omega)^2}\right]_{-10}^{-8}$

$= \left[-\frac{(-4A + 5A)e^{8i\omega}}{i\omega} - \frac{Ae^{8i\omega}}{(i\omega)^2} + \frac{\cancel{(-5A + 5A)}e^{10i\omega}}{i\omega} + \frac{Ae^{10i\omega}}{2(i\omega)^2}\right]$

$= -\frac{Ae^{i8\omega}}{i\omega} - \frac{Ae^{i8\omega}}{2(i\omega)^2} + \frac{Ae^{i10\omega}}{2(i\omega)^2}$

② $\int_{-8}^{-6} \left(\frac{A}{2}t + 3A\right)e^{-i\omega t}\,dt = \frac{-\left(\frac{A}{2}t + 3A\right)e^{-i\omega t}}{i\omega} - \frac{Ae^{-i\omega t}}{2(i\omega)^2}$

$= \left[\frac{-\cancel{(-3A + 3A)}e^{6i\omega}}{i\omega} - \frac{Ae^{6i\omega}}{2(i\omega)^2} + \frac{(-4A + 3A)e^{8i\omega}}{i\omega} + \frac{Ae^{8i\omega}}{2(i\omega)^2}\right]$

$= -\frac{Ae^{6i\omega}}{2(i\omega)^2} - \frac{Ae^{8i\omega}}{i\omega} + \frac{Ae^{8i\omega}}{2(i\omega)^2}$

③ $\int_{6}^{8} \left(\frac{A}{2}t + 3A\right)e^{-i\omega t}\,dt = \frac{-Ae^{-i\omega 8}}{i\omega} - \frac{Ae^{-i\omega 8}}{2(i\omega)^2} + \frac{Ae^{-i\omega 6}}{2(i\omega)^2}$

④ $\int_{8}^{10} \left(\frac{A}{2}t + 3A\right)e^{-i\omega t}\,dt = -\frac{Ae^{-10i\omega}}{2(i\omega)^2} + \frac{(4A - 5A)e^{-i8\omega}}{i\omega} + \frac{Ae^{-i8\omega}}{2(i\omega)^2}$

$\Rightarrow \mathcal{F}\{f(t)\} = \frac{-A}{i\omega}\cos 8\omega + \frac{Ai}{\omega^2}\operatorname{sen} 6\omega + \frac{A}{i\omega^2}\operatorname{sen} 10\omega$

f) $\begin{cases} 1 ; & 0 < t < 1 \quad ① \\ 2 ; & 1 < t < 2 \quad ② \\ 3 ; & 2 < t < 3 \quad ③ \\ 2 ; & 3 < t < 4 \quad ④ \\ 1 ; & 4 < t < 5 \quad ⑤ \end{cases}$ $\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$

① $\int_0^1 e^{-i\omega t} dt = \left[\frac{-e^{-i\omega t}}{i\omega} \right]_0^1 = \left(\frac{-e^{-i\omega}}{i\omega} + \frac{1}{i\omega} \right)$

② $2\int_1^2 e^{-i\omega t} dt = 2\left[\frac{-e^{-2i\omega}}{i\omega} + \frac{e^{-i\omega}}{i\omega} \right]$

③ $3\int_2^3 e^{-i\omega t} dt = 3\left[\frac{-e^{-3i\omega}}{i\omega} + \frac{e^{-2i\omega}}{i\omega} \right]$

④ $2\int_3^4 e^{-i\omega t} dt = 2\left[\frac{-e^{-i\omega 4}}{i\omega} + \frac{e^{-3i\omega}}{i\omega} \right]$

$\mathcal{F}\{f(t)\} = \frac{1}{i\omega} + \frac{e^{-i\omega}}{i\omega} + \frac{e^{-2i\omega}}{i\omega} - \frac{e^{-i3\omega}}{i\omega} - \frac{e^{-i4\omega}}{i\omega} - \frac{e^{-i5\omega}}{i\omega}$

6. Determinar cada una de las señales $f(t)$ cuya transformada de Fourier

a) $F(\omega) = Ae^{-i\frac{\omega}{2}}$ $\quad f(t) = A\int_{-\omega}^{\omega} e^{-i\frac{\omega}{2}} \cdot e^{i\omega t} dt = A\int_{-\omega}^{\omega} e^{\frac{2i\omega t - i\omega}{2}} dt$

$f(t) = 2Ai\, e^{\frac{i}{2}(2t-1)\omega} \Big|_{-\omega}^{\omega}$

$f(t) = \frac{-i4A\, \mathrm{sen}\left(\frac{1}{2}\omega(1-2t)\right)}{1-2t}$

b) $F(\omega) = Ae^{-i\frac{\omega}{2}}$
$= \pi e^{-i\frac{\pi\omega}{2}}$

$$f(t) = \frac{1}{2\pi}\int_{-\omega}^{\omega} \pi e^{-\frac{i\pi\omega}{2}} \cdot e^{i\omega t}\, d\omega = \left[\frac{e^{i\omega(t-\frac{\pi}{2})}}{2\pi t - i\pi}\right]\Bigg|_{-\omega}^{\omega}$$

$$f(t) = \frac{e^{i\omega(t-\frac{\pi}{2})} - e^{-i\omega(t-\frac{\pi}{2})}}{2it - i\pi} \Rightarrow f(t) = \frac{1}{t-\pi}\operatorname{sen}(\omega(t-\tfrac{\pi}{2}))$$

c)

$$F(\omega) = \begin{cases} \frac{A}{a+b}(b+\omega) \; ; & -b < \omega < -a \\ \frac{A}{b-a}(b-\omega) \; ; & b < \omega < a \\ A \; ; & -a < \omega < b \end{cases}$$

$$f(\omega) = \frac{A}{2\pi}\int_{-b}^{-a} \frac{b+\omega}{a+b} \cdot e^{i\omega t}\, d\omega + \frac{A}{2\pi}\int_{-a}^{a} e^{i\omega t}\, d\omega + \dots$$

$$\dots \frac{A}{2\pi}\int_{a}^{b} \frac{b-\omega}{b-a} \cdot e^{i\omega t}\, d\omega$$

$$= \frac{A}{2\pi}\left[\frac{1}{a+b}\left(\frac{(b+\omega)e^{i\omega t}}{it} - \frac{e^{i\omega t}}{(it)^2}\right]\right|_{-b}^{-a}$$

$$= \frac{A}{2\pi}\left[\frac{1}{a+b}\left(\frac{(b-a)e^{-ia\omega}}{it} - \frac{e^{-iat}}{(it)^2}\right) - \frac{1}{a+b}\left(\frac{(b-b)}{it}e^{-ibt} - \frac{e^{-ibt}}{(it)^2}\right)\right]$$

$$\frac{A}{2\pi}\int_{a}^{a} e^{-i\omega t}\, d\omega = \frac{A}{2\pi}\left(\frac{e^{i\omega t}}{it}\right)\Bigg|_{-a}^{a} = \frac{A}{2\pi}\left(\frac{e^{iaa}}{it} - \frac{e^{-iaa}}{it}\right)$$

$$= \frac{A}{\pi t} \operatorname{Sen} Wa$$

$$\frac{A}{2\pi}\int_a^b \frac{(b-w)}{(b-a)} e^{iwt}\,dw = \frac{A}{2\pi}\left[\frac{1}{b-a}\left(\frac{(b-w)e^{iwt}}{it} + \frac{e^{iwt}}{(it)^2}\right)\right]\Bigg|_a^b$$

$$= \frac{A}{2\pi}\left[\frac{1}{b-a}\left(\frac{e^{itb}}{(it)^2} - \frac{(b-a)e^{ita}}{it} - \frac{e^{ita}}{(it)^2}\right)\right]$$

$$f(t) = \frac{A}{2\pi(b-a)}\left[\frac{e^{itb}}{(it)^2} - \frac{(b-a)e^{ita}}{it} - \frac{e^{ita}}{(it)^2}\right] + \frac{A\operatorname{sen} Wa}{\pi t} \ldots$$

$$\ldots + \frac{A}{2\pi(b+a)}\left[\frac{(b-a)e^{-ita}}{it} - \frac{e^{-iat}}{(it)^2} - \frac{e^{-ibt}}{(it)^2}\right]$$

e) $F(w) = \begin{cases} 8\delta(w+1) & ; -1 < w < 0 \\ 8\delta(w-1) & ; 0 < w < 1 \end{cases}$

$$f(t) = 8\int_{-1}^{0} \delta(w+1)e^{iwt}\,dw + 8\int_0^1 \delta(w-1)e^{iwt}\,dw$$

$$= 8\int_{-1}^{0} e^{iwt}\,dw + 8\int_0^1 e^{iwt}\,dw = 8\left[\frac{e^{iwt}}{it}\Bigg|_{-1}^{0} + \frac{e^{iwt}}{it}\Bigg|_0^1\right]$$

$$= 8\left[\frac{1}{it} - \frac{e^{-it}}{it} + \frac{e^{it}}{it} - \frac{1}{it}\right] = 8\left(\frac{e^{it} - e^{-it}}{it}\right)$$

$$= \frac{16 \operatorname{sen} t}{t}$$

Problema 7. Por medio de la propiedad de muestreo de la función impulso, calcular las siguientes integrales.

a) $\int_{-\infty}^{\infty} \delta(t-5)\,\text{sen}\,2t\,dt$ Si $t=5$

$\text{sen}(2\cdot 5) = \text{sen}\,10$

$\Rightarrow \int_{-\infty}^{\infty} \delta(t-5)\,\text{sen}\,2t\,dt = \text{sen}\,10$

b) $\int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt$ Si $t=2$

$((2)^5-3)$

$32-3=29$ $\Rightarrow \int_{-\infty}^{\infty} \delta(2-t)(t^5-3)\,dt = 29$

c) $\int_{-\infty} e^{-x^2}\delta(x)\,dx = 0$ $x=0$

d) $\int_{-\infty}^{\infty} \delta(t-2)\cos[\pi(t-3)]\,dt$ Si $t=2$

$\cos[\pi(2-3)] = \cos\pi = \underline{-1}$

e) $\int_{-\infty}^{\infty} \delta(t+2)e^{-2t}\,dt$ Si $t=-2$

$e^{-2(-2)} = e^4$ $\Rightarrow \int_{-\infty}^{\infty} \delta(t+2)e^{-2t}\,dt = e^4$

f) $\int_{-\infty}^{\infty} e^{\cos t}\delta(t-\pi)\,dt$ Si $t=\pi$

$e^{\cos\pi} = e^{-1} = \frac{1}{e}$ $\Rightarrow \int_{-\infty}^{\infty} e^{\cos t}\delta(t-\pi)\,dt = \frac{1}{e}$

g) $\int_{1}^{100} \log_{10}(t)\,\delta(t-10)\,dt$ Si $t=10$

$\log_{10}(10) = 1 \Rightarrow \int_{-\infty}^{\infty} \log_{10}(t)\,\delta(t-10)\,dt = 1$

Problema 8. Considerando que $f(t)$ y $F(\omega)$ forman un par de transformadas, usando las propiedades de la transformada, encontrar la transformada de Fourier de las siguientes expresiones.

a) $f(2-t)$

$f(2-t) \leftrightarrow ?$

$f(t) \leftrightarrow F(\omega)$ → p. de desplazamiento en t

$f(t) \,/\!\!\leftrightarrow$

$t = 2-t$

$f(2-t) \leftrightarrow F(\omega)e^{-i2\omega}$

b) $f[(t-3)-3]$; $f[(t-3)-3] \leftrightarrow ?$; $f(t) \leftrightarrow F(\omega)$

$f(t) \,/\!\!\leftrightarrow$ $t = t-3$; $f(t-3) \leftrightarrow F(\omega)e^{-i3\omega}$

$f[(t-3)-3] \leftrightarrow F(\omega)e^{-i6\omega}$

$\therefore f(t-6) \leftrightarrow F(\omega)e^{-i6\omega}$

c) $\left(\frac{df(t)}{dt}\right)(\operatorname{sen} t)$; si $\omega_0 = 1$

$\left(\frac{df(t)}{dt}\right)\operatorname{sen} t \leftrightarrow ?$; $f(t)\operatorname{sen}\omega_0 t \leftrightarrow \frac{i}{2}[F(\omega+\omega_0) - F(\omega-\omega_0)]$

$f(t)\operatorname{sen} t \leftrightarrow \frac{i}{2}[F(\omega+1) - F(\omega-1)]$

$\frac{df(t)}{dt}\operatorname{sen} t \leftrightarrow -\frac{\omega}{2}[F(\omega+1) - F(\omega-1)]$

d) $\frac{d}{dt}[f(-2t)]$; $\frac{d}{dt}[f(-2t)] \longleftrightarrow ?$

$f(t) \longleftrightarrow F(w)$; $f(-2t) \longleftrightarrow \frac{1}{2} F(-\frac{w}{2})$

$\frac{d}{dt}[f(-2t)] \longleftrightarrow \frac{iw}{2} F(-\frac{w}{2})$

e) $t f(3t)$; $t f(3t) \longleftrightarrow ?$; $f(t) \longleftrightarrow F(w)$

$f(3t) \longleftrightarrow \frac{1}{3} F(\frac{w}{3})$

$(-it) f(3t) \longleftrightarrow \frac{d}{dw} \frac{1}{3} F(\frac{w}{3})$ * P. diferenciación en frecuencia

$(i)(-it) f(3t) \longleftrightarrow \frac{1}{3} \frac{d}{dw} F(\frac{w}{3})(i)$ * p. linealidad

$t f(3t) \longleftrightarrow i \frac{d}{dw} \frac{1}{3} F(\frac{w}{3})$

f) $(t-5) f(t)$; $(t-5) f(t) \longleftrightarrow ?$

$f(t) \longleftrightarrow F(w)$ * P. escalar

$f(t+5) \longleftrightarrow \frac{1}{5} F(\frac{w}{5})$

$(-it) f(t+5) \longleftrightarrow \frac{1}{5} \frac{d}{dw} F(\frac{w}{5})$ * P. de diferenciación en frecuencia

$(i)(-it) f(t+5) \longleftrightarrow \frac{1}{5} \frac{d}{dw} F(\frac{w}{5})(i)$ * P. de linealidad

$t f(t+5) \longleftrightarrow i \frac{d}{dw} \frac{1}{5} F(\frac{w}{6})$

$(t-5) f(t) \longleftrightarrow i \frac{d}{dw} \frac{1}{25} F(-\frac{w}{25})$

g) $(t-3)\,f(-3t)$; $(t-3)\,f(-3t) \longleftrightarrow ?$

$f(t) \longleftrightarrow F(\omega)$ * P. escalar

$$f(-3t) \longleftrightarrow \frac{1}{3}\,F\left(-\frac{\omega}{3}\right)$$

$$(t-3)\,f(-3t) \longleftrightarrow i\frac{d}{d\omega}\,\frac{1}{3}\,F(\omega) - 3F\left(-\frac{\omega}{3}\right)$$

h) $t\dfrac{df(t)}{dt}$; $t\dfrac{d\,f(t)}{dt} \longleftrightarrow ?$

$f(t) \longleftrightarrow F(\omega)$ * P. de diferenciación en tiempo

$$\frac{df}{dt}(t) \longleftrightarrow (i\omega)\,F(\omega)$$

$$(i)(-it)\frac{d}{dt}f(t) \longleftrightarrow \frac{d}{d\omega}\,(i\omega)\,F(\omega)(i)$$ * P. de diferenciación de frecuencia y linealidad

$$t\,\frac{d\,f(t)}{dt} \longleftrightarrow i\frac{d}{d\omega}\,(i\omega)\,F(\omega)$$

i) $f(6-t)$

$f(t) \longleftrightarrow F(\omega)$ * P. de desplazamiento en tiempo

$$f(6-t) \longleftrightarrow F(\omega)\,e^{-i6\omega}$$

j) $(2-t)\,f(8-t)$; $(2-t)\,f(8-t) \longleftrightarrow ?$

$f(t) \longleftrightarrow F(\omega)$ * P. de desplazamiento en tiempo

$$f(8-t) \longleftrightarrow F(-\omega)\,e^{-i8\omega}$$

$2\,f(8-t) \longleftrightarrow 2\,F(-\omega)\,e^{-i8\omega}$ * P. de linealidad

$$f(8-t) \leftrightarrow F(-\omega)e^{-i8\omega}$$

$$(-i)(-i\omega)f(8-t) \leftrightarrow (-i)\frac{d}{d\omega}F(-\omega)e^{-i8\omega}$$

* P. de diferenciación en frecuencia y linealidad

$$-tf(8-t) \leftrightarrow -i\frac{d}{d\omega}F(-\omega)e^{-i8\omega}$$

$$(2-t)\delta(8-t) \leftrightarrow 2F(-\omega)e^{-i8\omega} - i\frac{d}{d\omega}F(-\omega)e^{-i8\omega}$$

Problema 9. Completa en tiempo o frecuencia el par de transformadas solicitado, usando las propiedades de la transformada de Fourier.

a) $5\delta(t-1) \leftrightarrow ?$; Si $\delta(t) \leftrightarrow 1$

$\delta(t-1) \leftrightarrow e^{-i\omega}$ * desplazamiento en tiempo

$5\delta(t-1) \leftrightarrow 5e^{-i\omega}$ * linealidad

b) $? \leftrightarrow 8\delta(\omega+1) + 8\delta(\omega-1)$

Si $\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\,\delta(-\omega)$ * Simetría

$\frac{1}{2\pi} \leftrightarrow \delta(\omega)$; $\frac{4}{\pi} \leftrightarrow 8\,\delta(\omega)$

$$\frac{4}{\pi}e^{-i\omega} \leftrightarrow 8\delta(\omega+1)$$

$$\frac{4}{\pi}e^{i\omega} + \frac{4}{\pi}e^{-i\omega} \leftrightarrow 8\delta(\omega-1) + 8\delta(\omega+1)$$

c) $t \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(\omega)$ * simetría

$(-it) \longleftrightarrow 2\pi \frac{d}{d\omega} \delta(\omega)$; $t \longleftrightarrow \frac{2\pi}{i} \frac{d}{d\omega} \delta(\omega)$

d) $t^2 \longleftrightarrow ?$; Si $\delta(t) \longleftrightarrow 1$

$\Rightarrow$ $1 \longleftrightarrow 2\pi\delta(\omega)$ * simetría

$(-it^2) \longleftrightarrow 2\pi \frac{d^2}{d^2\omega} \delta(\omega)$; $t^2 \longleftrightarrow -2\pi \frac{d^2}{d^2\omega} \delta(\omega)$

e) $2G_2(t) \cos 1000t \longleftrightarrow ?$; $A=2$; $d=2$

Si $AG_d(t) \longleftrightarrow Ad\, Sa \frac{\omega d}{2}$

$2G_2(t) \longleftrightarrow 4 Sa(\omega)$

$2G_2(t) \cos 1000t \longleftrightarrow \frac{1}{2}[4Sa(\omega+1000) + 4Sa(\omega-1000)]$

$2G_2(t) \cos 1000t \longleftrightarrow 2Sa(\omega+1{,}000) + 2Sa(\omega-1000)$

f) $? \longleftrightarrow \cos 1{,}000\,\omega$

Si $\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(\omega)$ * simetría

$\cos 1{,}000\,\omega \longleftrightarrow \frac{1}{2}[2\pi\delta(\omega+1000) + 2\pi\delta(\omega-1000)]$

*modulación

$\cos 1{,}000\,\omega \longleftrightarrow \pi\delta(\omega+1{,}000) + \pi\delta(\omega-1{,}000)$

g) $? \leftrightarrow 5\omega$; si $\delta(t) \leftrightarrow 1$

$(-i)\frac{d}{dt}\delta(t) \leftrightarrow i\omega(-i)$

$-i\frac{d}{dt}\delta(t) \leftrightarrow \omega$; $-5i\frac{d}{dt}\delta(t) \leftrightarrow 5\omega$

h) $? \leftrightarrow \delta(\omega)e^{-i5\omega}$; Si $\delta(t) \leftrightarrow 1$

$1 \leftrightarrow 2\pi\delta(\omega)$ * simetría

$\frac{1}{2\pi} \leftrightarrow \delta(\omega)$; $\frac{1}{2\pi} - 5 \leftrightarrow \delta(\omega)e^{-i5\omega}$

Problema 10. A partir de los siguientes pares de transformadas:

$\delta(t) \leftrightarrow 1$; $A\,C_d(t) \leftrightarrow Ad\,Sa\left(\frac{\omega d}{2}\right)$; $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{i\omega}$

$\mathrm{sgn}(t) \leftrightarrow \frac{2}{i\omega}$

Encuentre:

a) $\mathrm{sgn}(t) \leftrightarrow \frac{2}{i\omega}$; $-\frac{2}{i} \leftrightarrow 2\pi\left(\frac{2}{i\omega}\right)(\omega)$ *simetría

$\mathrm{sgn}(4t-2) \leftrightarrow \frac{1}{4}\left(\frac{8}{i\omega}\right)e^{-2i\omega}$ * desplazamiento en el tiempo

$\mathrm{sgn}(4t-2) \leftrightarrow \frac{2}{i\omega}e^{-2i\omega}$

$\frac{2}{it}e^{-2it} \leftrightarrow 2\pi\,\mathrm{sgn}(2-4\omega)$

$-\frac{2}{it}e^{-2it} \leftrightarrow 2\pi\,\mathrm{sgn}(4\omega-2)$

$$-\frac{3}{2\pi}\cdot\frac{2}{it}\cdot e^{-2it} \longleftrightarrow \frac{3}{2\pi}\cdot 2\pi\,\mathrm{sgn}(4w-2)$$ * linealidad

$$-\frac{3}{i\pi t}\cdot e^{-2it} \longleftrightarrow 3\,\mathrm{sgn}(4w-2)$$

b) $C_2(\frac{2}{3}t) \longleftrightarrow ?\ ;\ C_2(t) \longleftrightarrow 2Sa(w)$

$$C_2(\tfrac{2}{3}t) \longleftrightarrow \frac{3}{2}\,Sa\left(\frac{3w}{2}\right)$$

c) $2C_2(t)\cos 250t \longleftrightarrow ?$

$$2C_2(t) \longleftrightarrow 4Saw$$

$$2C_2(t)\cos 250t \longleftrightarrow 2[Sa(w+250)+Sa(w-250)]$$

* desplazamiento en frecuencia

d) $U(10t-1)\,t \longleftrightarrow ?$

Si $U(t) \longleftrightarrow \pi\delta(w)+\frac{1}{iw}$

$$U(t-1) \longleftrightarrow \left(\pi\delta(w)+\frac{1}{iw}\right)e^{-iw}$$ * desplazamiento en tiempo

$$U(10t-1) \longleftrightarrow \frac{1}{10}\left[\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right]e^{-\frac{iw}{10}}$$ * escalar

$$t\,U(10t-1) \longleftrightarrow \frac{d}{dw}\left[\frac{1}{10}\left(\pi\delta\left(\frac{w}{10}\right)+\frac{10}{iw}\right)e^{-\frac{i5w}{10}}\right]$$

e) $e^{i7t}\,\delta(6t-1)\,t^3\,e^{i5t} \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$$\delta(t-1) \longleftrightarrow e^{-iw}$$ * escalar

$$\delta(6t-1) \longleftrightarrow \frac{1}{6}e^{-6iw}$$

$$e^{i7t}\delta(6t-1) \longleftrightarrow \frac{1}{6}e^{-6i(w-7)}$$

$$e^{i7t}\,\delta(6t-1)\,e^{i5t} \longleftrightarrow \frac{1}{6}\,e^{-6i(\omega-5-7)}$$

$$e^{i7t}\,\delta(6t-1)\,e^{i5t} - i(-it)^3 \longleftrightarrow -i\frac{d^3}{d^3\omega}\,\frac{1}{6}\,e^{-6i(\omega-12)}$$

$$e^{i7t}\,\delta(6t-1)\,e^{i5t}\,t^3 \longleftrightarrow -i\frac{d^3}{d^3\omega}\,\frac{1}{6}\cdot e^{-6i(\omega-12)}$$

f) $? \longleftrightarrow \frac{4}{\pi} Sa(4\omega-2)$ $\quad A=2\,;\ d=2$

Si $C_d(t) \longleftrightarrow Ad\,Sa\left(\frac{\omega d}{2}\right)$

$2C_2(t) \longleftrightarrow 4S_a\omega$

$2C_2(t)\,e^{2it} \longleftrightarrow 4Sa(\omega-2)$ * desplazamiento en frecuencia

$2C_2\left(\frac{t}{4}\right)e^{it/2} \longleftrightarrow 16\,Sa(\omega-2)$

$\frac{1}{2\pi}\,C_2\left(\frac{t}{4}\right)e^{it/2} \longleftrightarrow \frac{4}{\pi}\,Sa(4\omega-2)$

g) $? \longleftrightarrow \left(\pi\,\delta(\omega+\frac{3}{4}) + \frac{1}{i(\omega+\frac{3}{4})}\right)(-\omega)\,e^{i1000\omega}$

Si $u(t) \longleftrightarrow \pi\,\delta(\omega) + \frac{1}{i\omega}$

$u(t)\,e^{-i\frac{3}{4}t} \longleftrightarrow \pi\,\delta(\omega+\frac{3}{4}) + \frac{1}{i(\omega+\frac{3}{4})}$

$-\frac{1}{i}\frac{d}{d\omega}\,u(t)\,e^{-i\frac{3t}{4}} \longleftrightarrow \left[\pi\,\delta(\omega+\frac{3}{4}) + \frac{1}{i(\omega+\frac{3}{4})}\right](-\omega)$

$-\frac{1}{i}\frac{d}{d\omega}\,u(t+1000)\,e^{-i3\frac{(t+1000)}{4}} \longleftrightarrow \left[\pi\,\delta(\omega+\frac{3}{4}) + \frac{1}{i(\omega+\frac{3}{4})}\right]\ldots\cdot(-\omega)e^{-1000\omega}$

h) $C_{\frac{4}{3}}(t-6) \longleftrightarrow ?$; Si $A C_d(t) \longleftrightarrow A d\, Sa\left(\frac{wd}{2}\right)$

$C_{\frac{3}{4}}(t) \longleftrightarrow \frac{4}{3} Sa \frac{w2}{3}$; $C_{\frac{4}{3}}(t-6) \longleftrightarrow \frac{4}{3} Sa \frac{2w}{3} \cdot e^{-i6w}$

i) $(3\delta(t-1) - 3\delta(t+1)) \cdot \cos 18t \longleftrightarrow ?$

Si $\delta(t) \longleftrightarrow 1$

$3\delta(t) \longleftrightarrow 3$ // linealidad

$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow 3e^{-iw} - 3e^{iw}$ *desplazamiento en el tiempo

$3\delta(t-1) - 3\delta(t+1) \longleftrightarrow 3(e^{-iw} - e^{iw})$

$3\delta(t-1) - 3\delta(t+1) \cdot \cos 18t \longleftrightarrow \frac{3}{2}\left[\left(e^{-iw+18} - e^{i(w+18)}\right) + \left(e^{-i6w-18} \ldots - e^{i(w-18)}\right)\right]$

j) $? \longleftrightarrow 2\cos 500w$

Si $\delta(t) \longleftrightarrow 1$; $1 \longleftrightarrow 2\pi\delta(w)$ *simetría

$2\cos 500w \longleftrightarrow 2\left[\frac{1}{2}(2\pi\delta(w+500) + 2\pi\delta(w-500)\right]$

$2\cos 500w \longleftrightarrow \pi\delta(w+500) + \pi\delta(w-500)$

K) $t + t^2 + 1 \longleftrightarrow ?$; $t \longleftrightarrow -\frac{2\pi}{i} \frac{d}{dw}$

$t^2 \longleftrightarrow -2\pi \frac{d}{dw} \delta(w)$

$1 \longleftrightarrow 2\pi\delta(w)$;

$t + t^2 + 1 \longleftrightarrow -\frac{2\pi}{i} \frac{d}{dw} \delta(w)\, \delta(w) - 2\pi \frac{d^2}{d^2w} \delta(w) + 2\pi\delta(w)$

L) $i\frac{5}{t} \longleftrightarrow ?$

Si $sgn(t) \longleftrightarrow \frac{2}{iw}$

$\frac{2}{it} \longleftrightarrow 2\pi\, sgn(-w)$ *simetría

$-i\frac{2}{t} \longleftrightarrow 2\pi\, sgn(-w)$; $i\frac{2\left(\frac{5}{2}\right)}{t} \longleftrightarrow -\left(\frac{5}{2}\right) 2\pi\, sgn(-w)$

$i\frac{5}{t} \longleftrightarrow -5\pi\,sgn(-w)$

m) $? \longleftrightarrow \frac{1}{w}$

Si $sgn(t) \longleftrightarrow \frac{2}{iw}$; $\frac{i}{2} sgn(t) \longleftrightarrow \frac{1}{w}$

n) $? \longleftrightarrow \frac{1}{w}\cdot e^{-i4w}$

Si $sgn(t) \longleftrightarrow \frac{2}{iw}$; $\frac{i}{2} sgn(t) \longleftrightarrow \frac{1}{w}$

$\frac{i}{2} sgn(t-4) \longleftrightarrow \frac{1}{w} e^{-i4w}$

ñ) $5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow ?$

$5e^{-i\frac{7}{8}} \longleftrightarrow 10\pi\,\delta(w-\frac{7}{8})$

$5e^{-i\frac{7}{8}(t-3)} \longleftrightarrow 10\pi\,\delta(w-\frac{7}{8})\,e^{i3w}$

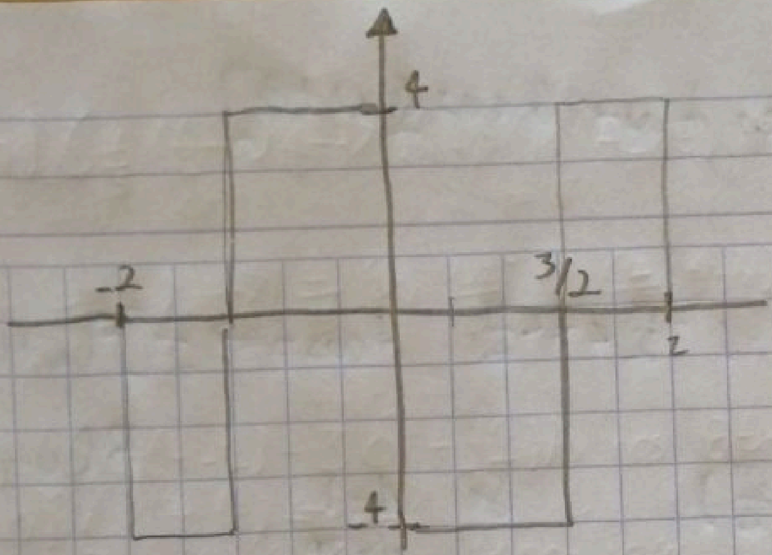
Problema 11. Aplicando las propiedades de la transformada de Fourier, determinar $F(w)$ para cada una de las señales

a) $$f(t) = \begin{cases} -4t-8 \ ; & -2 < t < -3/2 \\ 4t+4 \ ; & -3/2 < t < 0 \\ -4t+4 \ ; & 0 < t < 3/2 \\ 4t-8 \ ; & 3/2 < t < 2 \end{cases}$$

1era derivada :

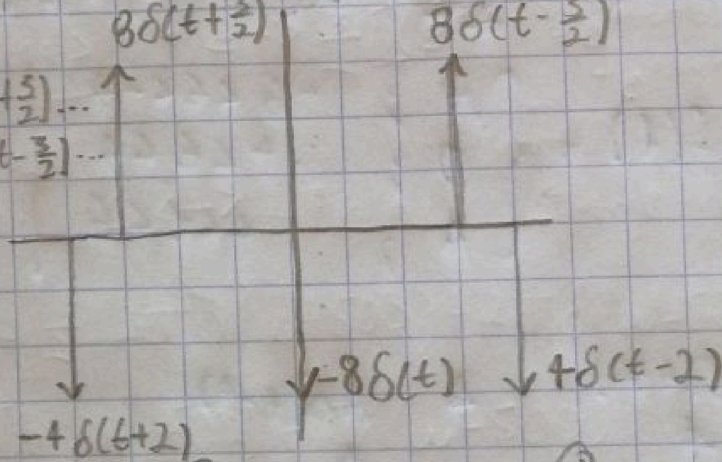
$\cdot \frac{d}{dt} -4t-8 = -4 \quad \cdot \frac{d}{dt} -4t+4 = -4$

$\cdot \frac{d}{dt} 4t+4 = 4 \quad \cdot \frac{d}{dt} 4t-8 = 4$



2° derivada

$$\frac{d^2 f(t)}{d^2 t} = -4\delta(t+2) + 8\delta(t+\tfrac{3}{2}) \dots$$
$$\dots -8\delta(t) + 8\delta(t-\tfrac{3}{2}) \dots$$
$$\dots +4\delta(t-2)$$



$$\mathcal{F}\left\{\frac{d^2}{d^2 t} f(t)\right\} = -4\,\mathcal{F}\{\delta(t+2)\} + 8\,\mathcal{F}\{\delta(t+\tfrac{3}{2})\} - 8\,\mathcal{F}\{\delta(t)\}$$
$$\dots + 8\,\mathcal{F}\{\delta(t-\tfrac{3}{2})\} + 4\,\mathcal{F}\{\delta(t-2)\}$$

① Si $\delta(t) \longleftrightarrow 1$; ② Si $\delta(t) \longleftrightarrow 1$; ③ Si $\delta(t) \longleftrightarrow 1$

$\delta(t+2) \longleftrightarrow e^{i2w}$ $\quad \delta(t+\frac{3}{2}) \longleftrightarrow e^{i\frac{3}{2}w}$

④ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-\frac{3}{2}) \longleftrightarrow e^{-i\frac{3}{2}w}$

⑤ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-2) \longleftrightarrow e^{-i2w}$

$$\mathcal{F}\left\{\frac{d^2}{d^2 t} f(t)\right\} = 4e^{i2w} + 8e^{i\frac{3}{2}w} - 8 + 8e^{-i\frac{3}{2}w} + 4e^{-i2w}$$

$$\frac{d^2}{d^2 t} \longleftrightarrow -8 + 16\cos\frac{3}{2}w - 8\,\mathrm{sen}\,2w$$

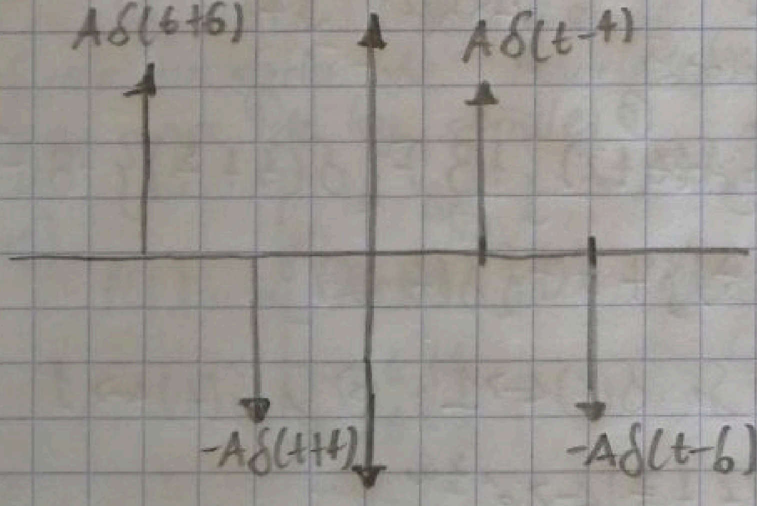
$$F(w) \leftrightarrow \frac{1}{w}\left[8 \operatorname{sen} 2w - 16 \cos \frac{3}{2} w + 8\right]$$

b) $f(t) = \begin{cases} A \cos t \ ; & -\frac{\pi}{2} < t < \frac{\pi}{2} \end{cases}$ $\quad A \cos t \leftrightarrow \frac{A}{2}[F(w+1) + F(w-1)]$

$\sqcap(t) \leftrightarrow \pi \, Sa\left(\frac{w\pi}{2}\right)$ $\quad$ Si $A=1$, $d=\pi$

$$A \sqcap(t) \leftrightarrow \frac{A}{2}\left[\pi Sa\left(\frac{w\pi}{2}+1\right) + \pi Sa\left(\frac{w\pi}{2}-1\right)\right]$$

c) $f(t) = \begin{cases} A \ ; & -6 < t < -4 \\ 0 \ ; & -4 < t < 4 \\ A \ ; & 4 < t < 6 \end{cases}$ $\quad \frac{d}{dt} f(t) = A\delta(t+6) - A\delta(t+4) + A\delta(t-4) - A\delta(t-6)$



$$\mathcal{F}\left\{\frac{d}{dt} f(t)\right\} = A\,\mathcal{F}\{\delta(t+6)\} - A\,\mathcal{F}\{\delta(t+4)\} + A\,\mathcal{F}\{\delta(t-4)\}$$
1 2 3

4
$$- A\,\mathcal{F}\{\delta(t-6)\}$$

① Si $\delta(t) \longleftrightarrow \underline{1}$; ② $\delta(t) \longleftrightarrow 1$; ③ Si $\delta(t) \Leftrightarrow 1$

$\delta(t+6) \longleftrightarrow e^{i6w}$ $\quad$ $\delta(t+4) \longleftrightarrow e^{i4w}$ $\quad$ $\delta(t-4) \longleftrightarrow e^{-i4w}$

④ Si $\delta(t) \longleftrightarrow 1$;

$\delta(t-6) \longleftrightarrow e^{-i6w}$

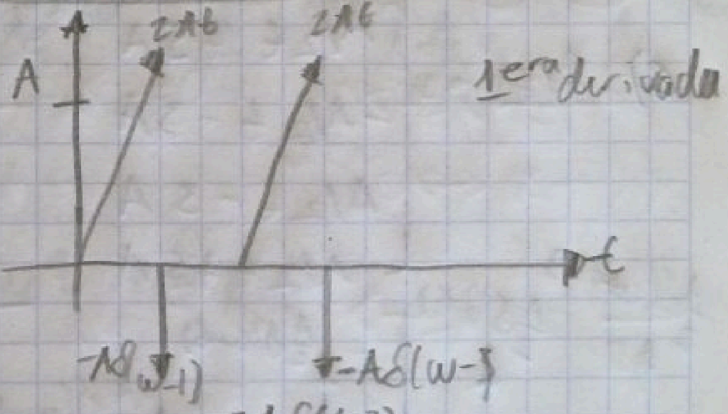
$$\mathcal{F}\left\{\frac{d}{dt} f(t)\right\} = Ae^{i6w} - Ae^{i4w} + Ae^{-i4w} - A^{-i6w}$$

$$\mathcal{F}\left\{\frac{d}{dt} f(t)\right\} = 2A\,\text{sen}\,6w - 2A\,\text{sen}\,4w$$

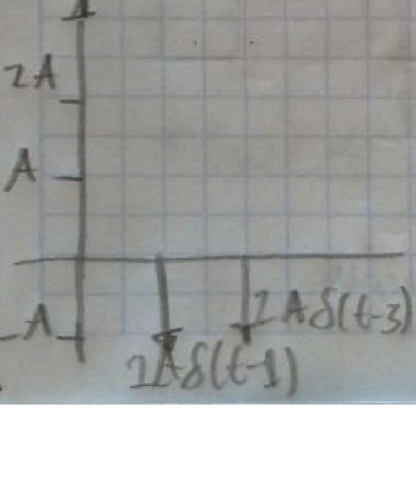
$$\frac{d}{dt} f(t) \longleftrightarrow i\omega F(\omega)$$

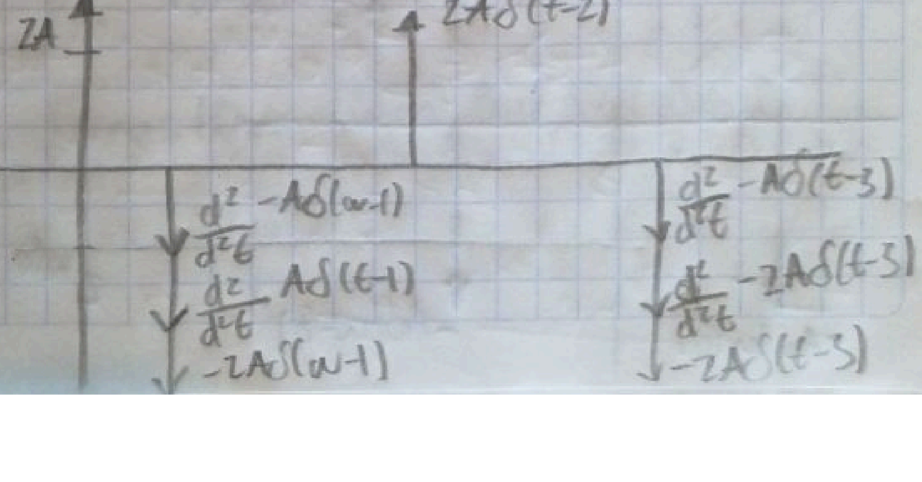
$$F(\omega) \longleftrightarrow \frac{2A}{i\omega}\left[\text{sen}\,6w - \text{sen}\,4\omega\right]$$

d)
$$f(t) = \begin{cases} At^2 \; ; & 0<t<1 \\ 0 \; ; & 1<t<2 \\ At^2 \; ; & 2<t<3 \end{cases}$$



2da derivada





$$\mathcal{F}\left\{\frac{d^3}{d^3t} f(t)\right\} = 2A\mathcal{F}\{\delta(t)\} - A\mathcal{F}\left\{\frac{d^2}{d^2t}\delta(t-1)\right\}$$
$$+2A\mathcal{F}\left\{\frac{d}{dt}\delta(t-1)\right\} - 2A\mathcal{F}\{\delta(\omega-1)\}$$
$$+2A\mathcal{F}\{\delta(t-2)\} - A\mathcal{F}\left\{\frac{d^2}{d^2t}(t-3)\right\} + 2A\mathcal{F}\left\{\frac{d}{dt}\delta(t-3)\right\}$$
$$-2A\mathcal{F}\{\delta(t-3)\}$$

$$\mathcal{F}\left\{\frac{d^3}{d^3t} f(t)\right\} = 2A + A\omega^2 e^{-i\omega} + 2A\omega i e^{-i\omega} - 2Ae^{-i\omega} + 2Ae^{-2i\omega}$$
$$+ A\omega^2 e^{-3i\omega} + 2Ai\omega e^{3i\omega} - 2Ae^{-3i\omega}$$

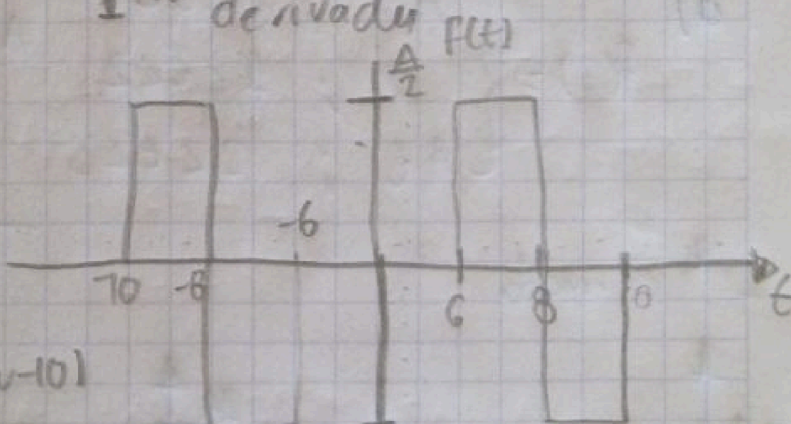
$$\frac{d^3}{d^3t} f(t) \longleftrightarrow (i\omega)^3 F(\omega)$$

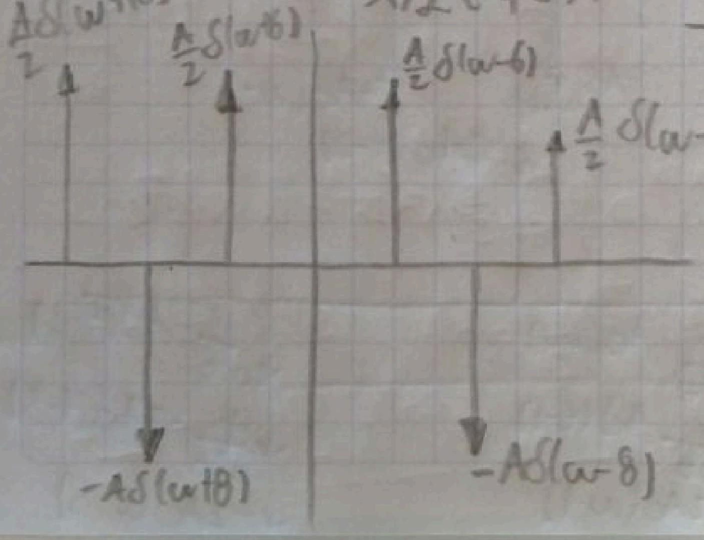
$$F(\omega) \longleftrightarrow \frac{-1}{i\omega^3}\Big[2A + A\omega^2 e^{-i\omega} + 2A\omega i e^{-i\omega} - 2Ae^{-2i\omega} + 2Ae^{-2i\omega}$$
$$+ A\omega^2 e^{-i\omega} + 2A\omega i e^{-3i\omega} - 2Ae^{-3\omega}\Big]$$

---

e) $f(t) = \begin{cases} A/2\, t + 5A \\ -A/2\, t - 3A \\ A/2\, t - 3A \\ -A/2\, t + 5A \end{cases}$

1era derivada





2nda derivada

$$\frac{d^2}{d^2t}f(t) = \frac{A}{2}\delta(t+10) - A\delta(t+8) + \frac{A}{2}\delta(t+6) + \frac{A}{2}\delta(t-6) - A\delta(t-8) + \frac{A}{2}\delta(t-10)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}f(t)\right\} = \frac{A}{2}\mathcal{F}\{\delta(t+10)\} - A\mathcal{F}\{\delta(t+8)\} + \frac{A}{2}\mathcal{F}\{\delta(t+6)\} + \frac{A}{2}\mathcal{F}\{\delta(t-6)\} - A\mathcal{F}\{\delta(t-8)\} + \frac{A}{2}\mathcal{F}\{\delta(t-10)\}$$

① Si $\delta(t) \longleftrightarrow 1$; $\delta(t+10) \longleftrightarrow e^{i10w}$; ② Si $\delta(t) \longleftrightarrow 1$ $\delta(t+8) \longleftrightarrow e^{i8w}$

③ Si $\delta(t) \longleftrightarrow 1$ $\delta(t+6) \longleftrightarrow e^{i6w}$ ④ Si $\delta(t) \longleftrightarrow 1$ $\delta(t-6) \longleftrightarrow e^{-i6w}$

⑤ Si $\delta(t) \longleftrightarrow 1$ $\delta(t-8) \longleftrightarrow e^{-i8w}$ ⑥ Si $\delta(t) \longleftrightarrow$ $\delta(t-10) \longleftrightarrow e^{-i10w}$

$$\mathcal{F}\left\{\frac{d^2}{d^2t}f(t)\right\} = \frac{A}{2}e^{i10w} - Ae^{i8w} + \frac{A}{2}e^{i6w} + \frac{A}{2}e^{-i6w} - Ae^{-i8w} + \frac{A}{2}e^{-i10w}$$

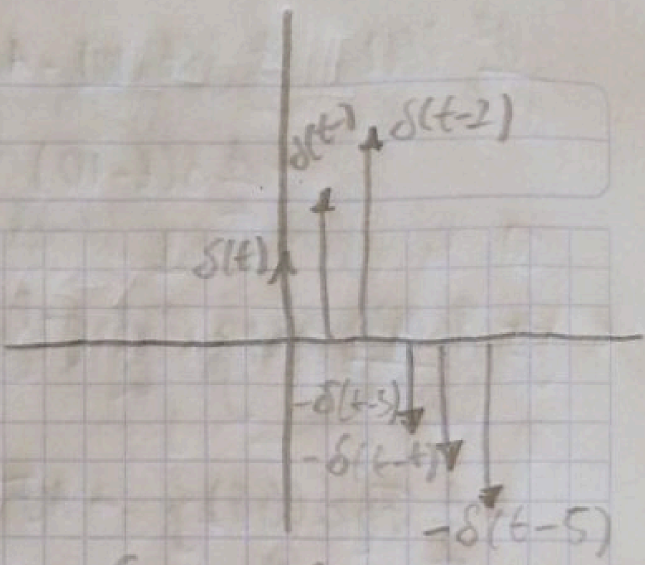
$$= A\cos 10w - 2A\cos 8w + A\cos 6w$$

$$\frac{d^2}{d^2t} \longleftrightarrow (iw)^2 F(w)$$

$$F(w) \longleftrightarrow -\frac{1}{w^2}\left[A\cos 10w - 2A\cos 8w + A\cos 6w\right]$$

f)

$$f(t) = \begin{cases} 1 & ; 0 < t < 1 \\ 2 & ; 1 < t < 2 \\ 3 & ; 2 < t < 3 \\ 2 & ; 3 < t < 4 \\ 1 & ; 4 < t < 5 \end{cases}$$



$$\frac{d}{dt} f(t) = \delta(t) + \underset{①}{\delta(t-1)} + \underset{②}{\delta(t-2)} - \underset{③}{\delta(t-3)} - \delta(t-4) - \underset{④}{\delta(t-5)}$$

$$\mathcal{F}\left\{\frac{d}{dt} f(t)\right\} = \mathcal{F}\{\delta(t)\} + \mathcal{F}\{\delta(t-1)\} + \mathcal{F}\{\delta(t-2)\} - \mathcal{F}\{\delta(t-3)\}$$

$$- \underset{⑤}{\mathcal{F}\{\delta(t-4)\}} - \underset{⑥}{\mathcal{F}\{\delta(t-5)\}}$$

① Si $\delta(t) \longleftrightarrow 1$

② Si $\delta(t) \longleftrightarrow 1$
$\delta(t-1) \longleftrightarrow e^{-i\omega}$

③ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-2) \longleftrightarrow e^{-i2\omega}$

④ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-3) \longleftrightarrow e^{-i3\omega}$

⑤ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-4) \longleftrightarrow e^{-i4\omega}$

⑥ Si $\delta(t) \longleftrightarrow 1$
$\delta(t-5) \longleftrightarrow e^{-i5\omega}$

$$\frac{d}{df} f(t) \longleftrightarrow (i\omega) F(\omega)$$

$$F(\omega) \longleftrightarrow \frac{e^{-i\omega} + e^{-i2\omega} - e^{-i3\omega} - e^{-i4\omega} - i^{5\omega} + 1}{i\omega}$$

Problema 12. Aplicando el teorema de Modulación, encontrar la transformada de cada una de las señales moduladas.

a) $f(t) = A\cos 20t$

$f(t) \longleftrightarrow$

$\cos 20t \longleftrightarrow \frac{1}{2}\left[F(w-20)+F(w+20)\right]$

$A\cos 20t \longleftrightarrow \frac{A}{2}\left[F(w+20)+F(w-20)\right]$

$F(w) = ?$

$f(t) \longleftrightarrow 2\pi \sum_{n=-\infty}^{\infty} C_n \delta(w-nw_0)$; $\quad w_0 = \frac{2\pi}{\frac{2\pi}{5}} = 5$

$$C_n = \frac{5}{2\pi}\int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} f(t)e^{-in5t}dt = \frac{5}{2\pi}\int_{-\frac{\pi}{5}}^{\pi/5} e^{-in5t}dt = \frac{5}{2\pi}\left[-\frac{e^{in5t}}{5in}\right]\Bigg|_{-\frac{\pi}{5}}^{\frac{\pi}{5}}$$

$$= \frac{-5}{2\pi(5in)}\left[e^{in5\frac{\pi}{5}}+e^{-in5\frac{\pi}{5}}\right] = \frac{-5}{5\pi in}\left[\frac{e^{in\pi}+e^{-in\pi}}{2}\right] = \frac{-\cos n\pi}{\pi in}$$

$$= \frac{(-1)^{n+1}}{\pi in} \qquad f(t) \longleftrightarrow 2\pi \sum_{n=-\infty} \frac{(-1)^{n+1}}{\pi in}\delta(w-5n)$$

$$Af(t)\cos 20t \longleftrightarrow \frac{A}{2}\Bigg[2\pi \sum_{n=-\infty} \frac{(-1)^{n+1}}{\pi in}\delta(w-5n+20) + 2\pi \sum_{n=-\infty} \cdots$$

$$\cdots \frac{(-1)^{n+1}}{\pi in}\delta(w-5n+20)\Bigg]$$

b) $f(t) = \begin{cases} \frac{-40A}{9\pi} t \cos 20t ; & \frac{-9\pi}{40} < t < \frac{9\pi}{40} \end{cases}$

$$-\frac{40A}{9\pi} C_{\frac{9}{20}\pi}(t) \cos 20t \longleftrightarrow ?$$

$$A\, C_d(t) \longleftrightarrow A d\, Sa\left(\frac{\omega d}{2}\right)$$

$$C_{\frac{9}{20}\pi}(t) \longleftrightarrow \frac{9}{20}\pi\, Sa\left(\frac{9\pi\omega}{40}\right)$$

$$C_{\frac{9}{20}\pi} \cos 20t \longleftrightarrow \frac{9}{40}\pi\left[Sa\left(\frac{9\pi}{40}(\omega+20)\right) + Sa\left(\frac{9\pi}{40}(\omega-20)\right)\right]$$

$$\frac{40A}{i9\pi}\left[(-it)\, C_{\frac{9}{20}}\pi(t) \cos 20t \longleftrightarrow \frac{9\pi}{40}\frac{d}{d\omega}\left[Sa\,\frac{9\pi}{40}(\omega+20) \ldots\right.\right.$$

$$\left.\left.\ldots + Sa\,\frac{9\pi}{40}(\omega-20)\right]\right]$$

$$\frac{40At}{9\pi} C_{\frac{9}{20}}\pi(t) \cos 20t \longleftrightarrow \frac{A}{i}\frac{d}{d\omega}\left[Sa\,\frac{9\pi}{40}(\omega+20) + Sa\,\frac{9\pi}{40}(\omega-20)\right]$$

c) $f(t) = \begin{cases} A\cos 200\pi t ; & -1 < t < 1 \\ 0 ; & 1 < t < 3 \end{cases}$

$$\cos 200\pi t \longleftrightarrow \frac{1}{2}\left[F(\omega+200\pi) + F(\omega-200\pi)\right]$$

$$C_n = \frac{1}{2}\int_{-1}^{1} e^{-i\pi t n}\, dt = \frac{1}{2}\left[\frac{-e^{i\pi n t}}{i\pi n}\right]\Big|_{-1}^{1} = \frac{-1}{2i\pi n}\left[e^{i\pi n} - e^{-i\pi n}\right]$$

$$\frac{-1}{\pi n}\left[\frac{e^{i\pi n} - e^{-i\pi n}}{2i}\right] = \frac{-1}{\pi n}\, \mathrm{sen}\, n\pi$$

$$f(t) \longleftrightarrow -\frac{2}{n}\sum_{n=-\infty}^{\infty} \operatorname{sen} n\pi\, \delta(\omega-\pi n)$$

$$A\cos(200\pi t) \longleftrightarrow \frac{-2A}{n}\sum_{n=-\infty}^{\infty} \operatorname{sen} n\pi \left[\delta(\omega-\pi n+200\pi)+\delta(\omega \ldots\right.$$

$$\left.\ldots -\pi n-200\pi)\right]$$

d) $$f(t)=\begin{cases}(At+A)\cos 100\pi t \;; & -1<t<0\\ (-At+A)\cos 100\pi t \;; & 0<t<1\end{cases}$$

$$\cos 100\pi t \longleftrightarrow \frac{1}{2}\left[F(\omega+100)+F(\omega-100)\right]$$

$$(At+A)\cos 100\pi t \longleftrightarrow -Ai\frac{d}{d\omega}\left[\frac{1}{2}F(\omega+100)+F(\omega-100)\right]+\ldots$$

$$\ldots \frac{A}{2}\left[F(\omega+100)+F(\omega-100)\right]$$

$$(-At+A)\cos 100\pi t \longleftrightarrow Ai\frac{d}{d\omega}\left[\frac{1}{2}F(\omega+100)+F(\omega-100)\right]+\frac{A}{2}\left[F(\omega+100)+F(\omega-100)\right]$$

$$(At+A)\cos 100\pi t+(-At+A)\cos 100\pi t \longleftrightarrow A\left[F(\omega+100)+F(\omega-100)\right]$$

$$C_n=\frac{1}{2}\int_{-1}^{1} f(t)\, e^{-i\pi n t}\,dt=\frac{1}{2}\left[\frac{-e^{-i\pi n t}}{in\pi}\right]\Big|_{-1}^{1}=\frac{-1}{2in\pi}\left[e^{-in\pi}-e^{in\pi}\right]$$

$$\Rightarrow \frac{2A}{n}\left[\sum_{n=-\infty}^{\infty} \operatorname{sen} n\pi\, \delta(\omega-n\pi+100)+\sum_{n=-\infty}^{\infty} \operatorname{sen} n\pi\, \delta(\omega-n\pi-100)\right]$$