

$$\boxed{A1} \quad A = \begin{pmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 4 & 4 & 0 \end{pmatrix}$$

(a)

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 2 & 1 & 0 & 0 & 4 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) -2I$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & -5 & -2 & -2 & -8 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \begin{array}{l} +5 \cdot II \\ -4 \cdot II \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & -2 & 8 & 2 \\ 0 & 0 & 4 & -8 & 4 \end{array} \right) (-2)$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & -4 & -1 \\ 0 & 0 & 4 & -8 & 4 \end{array} \right) -4 \cdot III$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 6 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & -4 & -1 \\ 0 & 0 & 0 & 8 & 8 \end{array} \right) :8$$

$$x_4 = 1$$

$$III: x_3 - 4 = -1 \quad | +4 \\ x_3 = 3$$

$$II: x_2 + 2 = 2 \quad | -2 \\ x_2 = 0$$

$$I: x_1 + 3 + 1 = 6 \quad | -4 \\ x_1 = 2$$

$$x_a = \begin{pmatrix} 2 \\ 0 \\ 3 \\ 1 \end{pmatrix}$$

(b)

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 2 & 1 & 0 & 0 & 1 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) -2I$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & -5 & -2 & -2 & -15 \\ 0 & 4 & 4 & 0 & 12 \end{array} \right) \begin{array}{l} +5 \cdot II \\ -4 \cdot II \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & -2 & 8 & 20 \\ 0 & 0 & 4 & -8 & -16 \end{array} \right) (-2)$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 1 & -4 & -10 \\ 0 & 0 & 4 & -8 & -16 \end{array} \right) -4 \cdot III$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 8 \\ 0 & 1 & 0 & 2 & 7 \\ 0 & 0 & 1 & -4 & -10 \\ 0 & 0 & 0 & 8 & 24 \end{array} \right) :8$$

$$x_4 = 3$$

$$III: x_3 - 12 = -10 \quad | +12 \\ x_3 = 2$$

$$II: x_2 + 6 = 7 \quad | -6 \\ x_2 = 1$$

$$I: x_1 + 3 + 2 + 3 = 8 \quad | -8 \\ x_1 = 0$$

$$x_b = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix}$$

A3

a)

$$A = \begin{pmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ -2 & 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \end{array} \right) + 2 \cdot \text{I}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & -3 & 2 & 0 & 3 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & -3 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 & 1 \end{array} \right) + 3 \cdot \text{II}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & -4 & 3 & 6 \\ 0 & 0 & 1 & -2 & 1 \end{array} \right)$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -4 & 3 & 6 \end{array} \right) + 4 \cdot \text{III}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 1 & 0 & 1 \\ 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & -5 & 10 \end{array} \right) \quad | : (-5)$$

$$x_4 = -2$$

$$\text{III: } x_3 + 4 = 1 \quad | -4$$

$$x_3 = -3$$

$$\text{II: } x_2 + 6 - 2 = 1 \quad | -4 \quad \text{I: } x_1 + 6 - 3 = 1 \quad | -3$$

$$x_2 = -3$$

$$x_1 = -2$$

$$x = \begin{pmatrix} -2 \\ -3 \\ -2 \\ -2 \end{pmatrix}$$

Es werden wieder nur drei Diagonale mit Einträgen ungleich null gesetzt. Die restlichen Diagonalen beinhalten nur Nullen.

A4

$$A = \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 5 & -5 & 5 \\ 1 & -5 & 4 & -4 \\ -1 & 5 & -4 & 30 \end{pmatrix}$$

$$A = L \cdot L^T$$

$$l_{11} = \sqrt{a_{11}} = \sqrt{1} = 1$$

$$l_{12} = \frac{a_{12}}{l_{11}} = -1$$

$$\begin{array}{c|ccc} 1 & & & \\ -1 & 5 - (-1) \cdot (-1) & & \\ 1 & -5 - (-1) \cdot 1 & 4 - 1 \cdot 1 & \\ -1 & 5 - (-1) \cdot (-1) & -4 - 1 \cdot (-1) & 30 - (-1) \cdot (-1) \end{array}$$

$$\begin{array}{c|cc} 1 & & \\ -1 & 4 & \\ 1 & -4 & 3 \\ -1 & 4 & -3 \end{array} \quad \begin{array}{c} \\ \\ 29 \\ 29 \end{array}$$

$$\begin{array}{c|cc} 1 & & \\ -1 & 2 & \\ 1 & -2 & 3 - (-2) \cdot (-2) \\ -1 & 2 & -3 - (-2) \cdot 2 \end{array} \quad \begin{array}{c} \\ \\ 29 - 2 - 2 \\ 29 - 2 - 2 \end{array}$$

$$\begin{array}{c|cc} 1 & & \\ -1 & 2 & \\ 1 & -2 & -1 \\ -1 & 2 & 1 \end{array} \quad \begin{array}{c} \\ \\ 25 \\ 25 \end{array}$$

da $l_{33} = \sqrt{-1}$ & \mathbb{R} ist, geht die Zerlegung nicht
 $\rightarrow A$ ist nicht spd.