$\boxed{A1}$

ⓐ $A = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$|\Delta A| = \frac{1}{10}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad |\Delta b| \leq \frac{1}{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \tilde{x} = \begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix}$

$\det(A) = 6 \neq 0$

$r = b - A\tilde{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 1{,}8 - 1{,}1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0{,}3 \\ 0 \end{pmatrix}$

$|\Delta A| \cdot |\tilde{x}| + |\Delta b| = \frac{1}{10} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0{,}9 \\ 1{,}1 \end{pmatrix} + \frac{1}{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{10} \cdot \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{1}{10}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \frac{1}{10} \cdot \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$r = \begin{pmatrix} \frac{3}{10} \\ 0 \end{pmatrix} \leq \begin{pmatrix} 3/10 \\ 3/10 \end{pmatrix} = |\Delta A| \cdot |\tilde{x}| + |\Delta b|$

$\Rightarrow \tilde{x}$ kann akzeptiert werden mit: $(A + \Delta A)\tilde{x}_1 = b + \Delta b$

ⓑ $A = \begin{pmatrix} 1 & -1 \\ -1 & 1{,}001 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\Delta A| \leq 5 \cdot 10^{-4} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad |\Delta b| \leq 5 \cdot 10^{-4}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \tilde{x} = \begin{pmatrix} 501 \\ 500 \end{pmatrix}$

$\det(A) = 1 \cdot 0{,}01 = 0{,}01 \neq 0$

$r = b - A\tilde{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -1 & 1{,}001 \end{pmatrix} \cdot \begin{pmatrix} 501 \\ 500 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 501 - 500 \\ -501 + 500{,}5 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}$

$|\Delta A| \cdot |\tilde{x}| + |\Delta b| = 5 \cdot 10^{-4} \cdot \left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 501 \\ 500 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) = 5 \cdot 10^{-4} = \begin{pmatrix} 1002 \\ 1002 \end{pmatrix} \cdot 5 \cdot 10^{-4} = \begin{pmatrix} 0{,}501 \\ 0{,}501 \end{pmatrix}$

$r = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} \leq \begin{pmatrix} 0{,}501 \\ 0{,}501 \end{pmatrix}$

$\Rightarrow \tilde{x}$ kann akzeptiert werden

$\boxed{A3}$

$$A = \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix}$$

$$a = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix} \quad a_{11} > 0 \qquad \|a\|_2 = \sqrt{1+16+64} = \sqrt{81} = 9$$

$$v = a + 9e_1 = \begin{pmatrix} 10 \\ -4 \\ 8 \end{pmatrix} \quad \|v\|_2^2 = 100 + 16 + 64 = 180$$

$$Q_1 = I - \frac{2}{180} \cdot v \cdot v^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{90} \cdot \begin{pmatrix} 10 \\ -4 \\ 8 \end{pmatrix} \cdot (10; -4; 8)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{1}{90} \cdot \begin{pmatrix} 100 & -40 & 80 \\ -40 & 16 & -32 \\ 80 & -32 & 64 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{10}{9} & -\frac{4}{9} & \frac{8}{9} \\ -\frac{4}{9} & \frac{8}{45} & -\frac{16}{45} \\ \frac{8}{9} & -\frac{16}{45} & \frac{32}{45} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{9} & \frac{4}{9} & -\frac{8}{9} \\ \frac{4}{9} & \frac{37}{45} & \frac{16}{45} \\ -\frac{8}{9} & \frac{16}{45} & \frac{13}{45} \end{pmatrix} = \frac{1}{45} \cdot \begin{pmatrix} -5 & 20 & -40 \\ 20 & 37 & 16 \\ -40 & 16 & 13 \end{pmatrix}$$

$$Q_1 A = \frac{1}{45} \cdot \begin{pmatrix} -5 & 20 & -40 \\ 20 & 37 & 16 \\ -40 & 16 & 13 \end{pmatrix} \cdot \begin{pmatrix} 1 & -5 & -20 \\ -4 & 11 & -1 \\ 8 & -4 & 2 \end{pmatrix}$$

$$= \frac{1}{45} \cdot \begin{pmatrix} -5-80-320 & 25+220+160 & 100-20-80 \\ 20-148+128 & -100+407-64 & -400-37+32 \\ -40-64+104 & 200+176-52 & 800-16+26 \end{pmatrix}$$

$$= \frac{1}{45} \cdot \begin{pmatrix} -405 & 405 & 0 \\ 0 & 243 & -405 \\ 0 & 324 & 810 \end{pmatrix} = \begin{pmatrix} -9 & 9 & 0 \\ 0 & \frac{27}{5} & -9 \\ 0 & \frac{36}{5} & 18 \end{pmatrix}$$

$$\|a\|_2 = \sqrt{\left(\frac{27}{5}\right)^2 + \left(\frac{36}{5}\right)^2} = \frac{1}{5} \cdot \sqrt{27^2 + 36^2} = 9$$

$$a_{22} = \frac{27}{5} > 0$$

$$v = a + 9e_1 = \frac{1}{5}\begin{pmatrix} 27 \\ 36 \end{pmatrix} + \begin{pmatrix} 9 \\ 0 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 72 \\ 36 \end{pmatrix} = \frac{36}{5}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{72}{5} \\ \frac{36}{5} \end{pmatrix}$$

$$\|v\|_2^2 = \left(\frac{72}{5}\right)^2 + \left(\frac{36}{5}\right)^2 = \frac{1}{25} \cdot (72^2 + 36^2) = 6480 \cdot \frac{1}{25} = \frac{1296}{5}$$

$$\tilde{Q}_2 = I - \frac{2}{\|v\|_2^2} \cdot vv^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{5}{648} \cdot \frac{36}{5}\begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \frac{36}{5}(2, 1)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{5}{648} \frac{1296}{25} \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{2}{5}\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{8}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{2}{5} \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 1 & \\ & \tilde{Q}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$R = Q_2 \cdot Q_1 \cdot A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \cdot \begin{pmatrix} -9 & 9 & 0 \\ 0 & \frac{27}{5} & -9 \\ 0 & \frac{36}{5} & 18 \end{pmatrix} = \begin{pmatrix} -9 & 9 & 0 \\ 0 & -9 & 9 \\ 0 & 0 & 18 \end{pmatrix}$$

$$Q = (Q_2 \cdot Q_1)^T = Q_1^T \cdot Q_2^T = Q_1 Q_2$$

$$= \frac{1}{45} \cdot \begin{pmatrix} -5 & 20 & -40 \\ 20 & 37 & 16 \\ -40 & 16 & 13 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{3}{5} & -\frac{4}{5} \\ 0 & -\frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$$= \frac{1}{9} \cdot \begin{pmatrix} -1 & 4 & -8 \\ 4 & -7 & -4 \\ -8 & -4 & 1 \end{pmatrix}$$

$\boxed{A4}$

am Beispiel aus $\boxed{A3}$:

$d = (-)\,\|a_1\| = -9$

$v_1 = 1 + 9 = 10$

$\|v\|_2^2 = -2 \cdot 10 \cdot (-9) = 180$ ✓

$$\|v\|_2^2 = -2 \cdot v_1 \cdot d = -2 \cdot (a_{11} - d) \cdot -\operatorname{sign}(a_{11}) \cdot \|a_1\|$$

$$= -2a_{11} \cdot (-\operatorname{sign}(a_{11})) \cdot \|a_1\| + 2d \cdot (-\operatorname{sign}(a_{11})) \cdot \|a_1\|$$

$$= 2 \cdot \|a_1\| \cdot \left(-a_{11} - \operatorname{sign}(a_{11}) + (-\operatorname{sign}(a_{11})) \cdot \|a_1\| \cdot (-\operatorname{sign}(a_{11}))\right)$$

$$= 2 \cdot \|a_1\| \cdot \left(-a_{11} - \operatorname{sign}(a_{11}) + \|a_1\|\right)$$

$$= 2 \cdot \|a_1\| \cdot \left(|a_{11}| + \|a_1\|\right) = 2 \cdot |a_{11}| \, \|a_1\| + 2 \cdot \|a_1\|^2$$

$$= 2 \cdot \left(\|a_1\|^2 + |a_{11}| \cdot \|a_1\|\right) = 2 \cdot \left(\|a_1\| \cdot \|a_1\| + |a_{11}| \cdot \|a_1\|\right)$$

$$= \|a_1\|^2 + 2 \cdot |a_{11}| \, \|a_1\| + \underbrace{\|a_1\|^2}$$

$$= \left(\|a_1\|^2 + 2 \cdot |a_{11}| \cdot \|a_1\| + a_{11}^2\right) + a_{21}^2 + \ldots + a_{n1}^2$$

$$= \left(|a_{11}| + \|a_1\|\right)^2 + a_{21}^2 + \ldots + a_{n1}^2$$

$$= \|v\|_2^2$$

$$\Leftrightarrow \quad v = \begin{pmatrix} |a_{11}| + \|a_1\| \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix} = a + \|a\|_2 \cdot e_1 \quad , a_{11} > 0$$