$\boxed{T}$ $\mathcal{L}_i, i=\overline{0,3}$ sunt închise la operațiile regulate.

$\boxed{\cup}$ $G_1=(V_{N_1}, V_{T_1}, x_{0_1}, P_1) \in \mathcal{G}_i$, $L(G_1)=L_1$ $\quad V_{N_1} \cap V_{N_2} = \phi$ $\quad i=\overline{0,3}$

$G_2=(V_{N_2}, V_{T_2}, x_{0_2}, P_2) \in \mathcal{G}_i$ $\quad L(G_2)=L_2$

$$G_\cup=(V_{N_1}\cup V_{N_2}\cup\{x_0\}, V_{T_1}\cup V_{T_2}, x_0, \{x_0 \to x_{0_1} | x_{0_2}\}\cup P_1 \cup P_2) \in \mathcal{G}_i, L(G_\cup)=L_1\cup L_2$$

Ex:

$G_1: \begin{cases} A \to iB \\ B \to oC \\ C \to n \end{cases}$ $\quad L(G_1)=\{ion\}$ $\quad G_1 \in \mathcal{G}_3$

$G_2: \begin{cases} X \to aT \\ Y \to nZ \\ Z \to a \end{cases} \in \mathcal{G}_3$ $\quad L(G_2)=\{ana\}$

$L=\{ion, ana\}$. $L(G)=L$. $\quad G?$

$L(G_\cup)=\{ion, ana\}$.

$G_\cup = \begin{cases} x_0 \to A | X \\ A \to iB \\ B \to oC \\ C \to n \end{cases}$ $\quad \begin{matrix} X \to aT \\ T \to nZ \\ Z \to a \end{matrix} \in \mathcal{G}_3$

Obs: Trebuie să respectăm ipoteza $V_{N_1} \cap V_{N_2} = \phi$

$G_1: \begin{cases} A \to iB \\ B \to oC \\ C \to n \end{cases}$ $\quad G_2: \begin{cases} A \to aB \\ B \to nC \\ C \to a \end{cases}$ $\quad$ Aplic rețeta

$G_\cup: \begin{cases} x_0 \to A | A \\ A \to iB | aB \; ?? \\ B \to oC | nC \\ C \to n | a \end{cases}$ $\quad x_0 \Rightarrow A \Rightarrow iB \Rightarrow inC \Rightarrow ina \in \{ion, ana\}$

Produs Tip 2.

$$G_\cdot = (V_{N_1}\cup V_{N_2}\cup\{x_0\}, V_{T_1}\cup V_{T_2}, x_0, \{x_0 \to x_{0_1}x_{0_2}\}\cup P_1\cup P_2) \in \mathcal{G}_2$$

Ex: $L=\{a^n bb \mid n \geq 1\}$

Observ că $L = \underbrace{\{a^n \mid n\geq 1\}}_{L_1}\underbrace{\{bb\}}_{L_2}$

G_2: $A \to bb$ $\quad L(G_2)=L_2$ $\quad G_2 \in \mathcal{G}_2$

G_1: $A' \to aA' | a \in \mathcal{G}_2$ $\quad L(G_1)=\{a^n \mid n \geq 1\} = L_1$

$G_\cdot : \begin{cases} x_0 \to A'A \\ A \to bb \\ A' \to aA' | a \end{cases} \in \mathcal{G}_2$ $\quad L(G_\cdot) = L$

Tip 3 $\quad G_\cdot = (V_{N_1}\cup V_{N_2}, V_{T_1}\cup V_{T_2}, x_{0_1}, P_1' \cup P_2)$

P_1' se obține din P_1 prin înloc. regulilor ce termină derivări $C \to p \in P_1$ cu $C \to p x_{0_2} \in P_1'$

G_1: $A' \to aA' | a$ $\quad L_1=\{a^n \mid n\geq 1\}$ $\quad G_1, G_2 \in \mathcal{G}_3 \to G_\cdot \in \mathcal{G}_3$

G_2: $A \to bb$ $\quad L_2=\{bb \mid n \geq 1\}$

$G_\cdot : \begin{cases} A' \to aA' | aA \\ A \to bb \end{cases} \in \mathcal{G}_3$

$A' \Rightarrow aA' \Rightarrow aaA' \Rightarrow aaaA' \Rightarrow aaaaA \Rightarrow aaaabb$

$\boxed{*}$ Tip 2 $\quad L^* = \bigcup_{k\geq 0} L^k = \{\lambda\} \cup L \cup L^2 \cup L^3 \cup \dots$

$$G_* = (V_{N_1}\cup\{x_0\}, V_{T_1}, x_0, \{x_0 \to \lambda | x_0 x_{0_1}\}\cup P_1) \in \mathcal{G}_2.$$

$L=\{a\}$ $\quad G_1: X \to a \in \mathcal{G}_2$.

$G_*: \begin{cases} x_0 \to \lambda | x_0 X \\ X \to a \end{cases} \in \mathcal{G}_2.$ $\quad L^* = \{a\}^* = \{a^n \mid n \geq 0\}$

$x_0 \Rightarrow \lambda \in L^*$

$x_0 \Rightarrow x_0X \Rightarrow x_0XX \Rightarrow x_0XXX \Rightarrow XXX \stackrel{*}{\Rightarrow} aaa$

Tip 3 $\quad G_* = (V_{N_1}\cup\{x_0\}, V_{T_1}, \{x_0 \to \lambda | x_{0_1}\}\cup P_1 \cup P_1')$ $\quad P_1' \leftarrow P_1$ înloc.

$L=\{a\}$ $\quad G_1: X \to a \in \mathcal{G}_3$

$G_*: \begin{cases} x_0 \to \lambda | X \\ X \to a \\ X \to aX \end{cases} \in \mathcal{G}_3.$

Ex:

$L=\{a,b\}$ $\quad G: X \to a|b \in \mathcal{G}_3.$

$\underline{L^* = \{a,b\}^*}$ $\quad G_*: \begin{cases} x_0 \to \lambda | X \\ X \to a|b \\ X \to aX | bX \end{cases} \in \mathcal{G}_3.$ $\quad G' \sim G_*$

G': $X \to aX | bX | \lambda$

$L(G')=\{a,b\}^*$

Problemă: Construiți o gram de tipul 2 pt generarea limbajului

$L=\{p \in \{0,1\}^* \mid p \text{ conține subșirul } 11\}$

Observ că $L = \{w11w' \mid w, w' \in \{0,1\}^*\} = L_1L_2L_1$, unde $L_1=\{0,1\}^*$ $L_2=\{11\}$

Știu G_1: $X \to 0X | 1X | \lambda \in \mathcal{G}_2$ $\quad L(G_1) = \{0,1\}^* = L_1$

G_2: $A \to 11 \in \mathcal{G}_2$ $\quad L(G_2) = \{11\} = L_2$

Aplic rețeta pt L_1L_2

$G_{12}: \begin{cases} x_0 \to XA \\ X \to 0X | 1X | \lambda \\ A \to 11 \end{cases} \in \mathcal{G}_2.$ și $L(G_{12}) = L_1L_2$ $\quad$ ip $V_{N_1} \cap V_{N_2} = \phi$.

Aplic rețeta pt $(L_1L_2)L_1$

$G_{121}: \begin{cases} Y \to x_0 Z \\ x_0 \to XA \quad X \to 0X | 1X | \lambda \quad A \to 11 \end{cases}$ $\quad Z \to 0Z | 1Z | \lambda$

LEMA DE ELIMINARE A REGULILOR DE ȘTERGERE (pt. gr. independente de context)

$\forall G \in \mathcal{G}_2$ a.î. $L(G) \not\ni \lambda$ $\exists G'$ echivalentă fără reguli de ștergere

$\boxed{xxx|1|1|xxx}$

Ex: $G: \begin{cases} S \to X11X \\ X \to 0X \mid 1X \mid \lambda \end{cases} \in \mathcal{G}_2$ $\quad L(G) = \{ w11w' \mid w, w' \in \{0,1\}^* \} =$
$= \{ w \in \{0,1\}^* \mid w \text{ conține subșirul } 11 \}$

$L(G) \not\ni \lambda$

Procedeu de eliminare

1) Găsesc neterminale ce pot produce λ prin derivare

$$NULL(G) = \{ x \in V_N \mid x \overset{*}{\underset{G}{\Rightarrow}} \lambda \}$$

Construim un șir ascendent de mulțimi astfel

$$\begin{cases} U_0 = \{ x \in V_N \mid x \to \lambda \in P \} \\ U_{k+1} = U_k \cup \{ x \in V_N \mid x \to p \in P,\ p \in U_k^* \} \end{cases}$$

Se poate arăta că $NULL(G)$ este mulțimea maximală din șirul U_k

$$U_0 \subset U_1 \subset \dots \subset U_f = U_{f+1} = \dots \subset V_N$$

$$U_f = NULL(G)$$

2) Elimin regulile de ștergere din G și modific și dreapta tuturor regulilor prin eliminarea în toate modurile posibile a neterminalelor din $NULL(G)$.

Exemplu. $G: \begin{cases} S \to X11X \\ X \to 0X \mid 1X \mid \lambda \end{cases}$ $\quad NULL(G) = \{X\}$

$G': \begin{cases} S \to X11X \mid 11X \mid X11 \mid 11 \\ X \to 0X \mid 0 \mid 1X \mid 1 \end{cases}$ $\quad G' \approx G$

$p = \underline{\underline{11}}01 \in L(G)$

$G: \quad S \Rightarrow X11X \Rightarrow 11X \Rightarrow 110X \Rightarrow 1101X \Rightarrow 1101 \in L(G).$

$G': \quad S \Rightarrow 11X \Rightarrow 110X \Rightarrow 1101 \in L(G')$

Ex: $L = \{a,b\}^*$

$G: \quad S \to aS \mid bS \mid \lambda \in \mathcal{G}_2.$ $\quad$ dar $G \notin \mathcal{G}_1$.

Construim o prom. acceptată la tipul 1 astfel.

Aplic lema de eliminare a regulilor de ștergere.

$NULL(G) = \{S\}$

$G': \quad S \to aS \mid a \mid bS \mid b \in \mathcal{G}_1.$

$L(G') = \{a,b\}^+$

Constr. urm. prom.

$G'': \begin{cases} x_0 \to \lambda \mid S \\ S \to aS \mid bS \mid a \mid b \end{cases} \in \mathcal{G}_1 \Rightarrow L(G'') = \{a,b\}^* \subset \mathcal{L}_1$

Problemă

$G: \begin{cases} S \to aS \mid bQ \mid b \\ Q \to aQ \mid bS \mid a \end{cases} \in \mathcal{G}_3$ $\quad L(G) = ?$

$L(G) = \{ p \in \{a,b\}^* \mid p \text{ conține un nr. impar de simboluri } b \}$

$S \Rightarrow aS \Rightarrow abQ \Rightarrow$
$\Rightarrow abaQ \Rightarrow$
$\Rightarrow ababS \Rightarrow$
$\Rightarrow ababb$

$b, ab, ba, \dots$

0,5 p.

$\cancel{a^n b^m}$

$\cancel{a^n b^m} a^k$ $\quad n \geq 0,\ m \geq 1,\ k \geq 0$

$bb \notin$

$S \Rightarrow bQ \Rightarrow \underline{bbS}$

$\cancel{a^{n_1} b^{m_1} a^{n_2} b^{m_2}}$

$n_1, m_1, n_2, m_2 \geq 0.$

λ?