$G = (V_N, V_T, x_0, \mathcal{P})$ $u \to v \in \mathcal{P}$ $u, v \in V_G^*$, u conține minim un neterm.

V_G^*, $\Rightarrow$ derivare directă

$p \underset{G}{\Rightarrow} q \overset{def}{\Longleftrightarrow} p = xuy$, $q = xvy$, $u \to v \in \mathcal{P}$ $u, v, x, y \in V_G^*$

– închiderea tranzitivă și reflexivă $\Rightarrow$

$\overset{*}{\Rightarrow}$ derivare

$p \overset{*}{\Rightarrow} q$ dd $\exists\, p_1, \dots, p_k$, $k \geq 2$ ai $p = p_1 \Rightarrow p_2 \Rightarrow \dots \Rightarrow p_k = q$

sau

$p = q$

l_p derivării – nr. de derivări directe

$L(G) = \{ p \in V_T^* \mid x_0 \overset{*}{\Rightarrow} p \}$

$p \overset{+}{\Rightarrow} q$ derivări de $l_p \geq 1$

$G_1 = (\{A\}, \{0,1\}, A, \{A \to 0A \mid A1 \mid \lambda\})$

$A \Rightarrow \lambda \in L(G)$

$A \Rightarrow 0A \Rightarrow 0 \in L(G)$

$A \Rightarrow A1 \Rightarrow 1 \in L(G)$

$A \Rightarrow 0A \Rightarrow 00A \overset{*}{\Rightarrow} 0^k A \Rightarrow 0^k A1 \overset{*}{\Rightarrow} 0^k A 1^t \Rightarrow 0^k 1^t$

$L(G_1) = \{0^n 1^m \mid n,m \geq 0\}$

Ex 2: $G_2 = (\{X,Y,Z\}, \{0,1\}, X, \{X \to YZ, Y \to 0Y \mid \lambda, Z \to 1Z \mid \lambda\})$

$X \Rightarrow YZ \Rightarrow Z \Rightarrow \lambda \qquad X \overset{*}{\Rightarrow} \lambda \in L(G)$

$X \Rightarrow YZ \Rightarrow 0YZ \Rightarrow 00YZ \overset{*}{\Rightarrow} 0^k YZ \Rightarrow 0^k Z \Rightarrow 0^k 1Z \Rightarrow 0^k 11Z \overset{*}{\Rightarrow}$

$\overset{*}{\Rightarrow} 0^k 1^t Z \Rightarrow 0^k 1^t \qquad L(G_2) = \{0^n 1^m \mid m,n \geq 0\} \qquad \boxed{G_1 \approx G_2}$

Clasificarea CHOMSKY

$\mathcal{G}_0$ – tip 0 – fără restricții asupra regulilor

$\mathcal{G}_1$ – tip 1 – dependente de context

$uAv \to upv$, $A \in V_N$, $u, p, v \in V_G^*$, $p \neq \lambda$

admitem $S \to \lambda$, $S \notin dr.$

$\mathcal{G}_2$ – tip 2 – independente de context

$A \to p$, $A \in V_N$, $p \in V_G^*$

$\mathcal{G}_3$ – tip 3 – regulate

$$\begin{cases} A \to pB \\ C \to q \end{cases} \quad A, B, C \in V_N, \; p, q \in V_T^*$$

$$\mathcal{L}_3 \underset{\neq}{\subset} \mathcal{L}_2 \underset{\neq}{\subset} \mathcal{L}_1 \underset{\neq}{\subset} \mathcal{L}_0$$

Ierarhia Chomsky

G_1: $A \to 0A \mid A1 \mid \lambda \in \mathcal{G}_2$

G_2: $\begin{cases} X \to YZ \\ Y \to 0Y \mid \lambda \\ Z \to 1Z \mid \lambda \end{cases} \in \mathcal{G}_2$

$G_3 : \begin{cases} A \to 0A \mid B \\ B \to 1B \mid \lambda \end{cases} \in \mathcal{G}_3$ $\qquad$ $\cdot \begin{cases} A \to pB \\ C \to q \end{cases}$ $p, q \in \underline{\underline{V_T^*}}$ $A, B, C \in V_N$

$A \Rightarrow B \Rightarrow \lambda$, $A \stackrel{*}{\Rightarrow} \lambda \in L(G)$

$A \Rightarrow 0A \Rightarrow 00A \stackrel{*}{\Rightarrow} 0^k A \Rightarrow 0^k B \Rightarrow 0^k 1 B \stackrel{*}{\Rightarrow} 0^k 1^p B \Rightarrow 0^k 1^p$

$L(G_3) = \{0^m 1^n \mid m, n \geq 0\} \in \mathcal{L}_3$ $\quad G_3 \sim G_2 \sim G_1$

$G_4 : \mathcal{G}_2 \ni S \to 0\underline{\underline{S}}1 \mid 01$, $L(G_4) = \{0^n 1^n \mid n \geq 1\}$

$L(G_4) \subseteq \{0^n 1^n \mid n \geq 1\}$ $\quad$ Fie $p \in L(G_4) \Rightarrow \exists$ derivare

$S \stackrel{*}{\Rightarrow} p$.

Arăt că $p \in \{0^n 1^n\}$

$S \overset{*}{\Rightarrow} p \in \{0,1\}^*$ $\qquad$ $S \to 0S1 \mid 01$

Inductie după lungimea derivării l

$l = 1$: $\qquad S \Rightarrow 01 \in \{0^n 1^n \mid n \geq 1\}$ q.e.d.

Pres. că afirm. este adevărată ptr derivări de lungime maxim l.

Consider o derivare de lg $l+1$

$l+1$: $\qquad S \overset{(l+1)}{\Rightarrow} p$ $\quad$ Evidentiez primul pas

$S \Rightarrow 0S1 \overset{(l)}{\Rightarrow} p$ $\quad$ Deci. $p = 0p'1$ $\quad$ și $S \overset{(l)}{\Rightarrow} p'$

din ip ind $\Rightarrow p' = 0^n 1^n \Rightarrow p = 00^n 1^n 1 = 0^{n+1} 1^{n+1} \in \{0^n 1^n \mid n \geq 1\}$.

$\subset$ $\qquad$ $\supset$

$\#$ $\qquad$ $\#$

$$G_5: \begin{cases} x_0 \to I x_1 \mid O x_2 \\ x_1 \to O x_3 \\ x_2 \to A x_4 \\ x_3 \to H \\ x_4 \to H x_5 \\ x_5 \to A \end{cases} \in \mathcal{G}_3$$

$L(G_5) \subset \{I, O, H, A\}^*$

$L(G) = \{p \in V_T^* \mid x_0 \overset{*}{\Rightarrow} p\}$

$L(G_5) = \{IOH, OAHA\}$

$x_0 \Rightarrow I x_1 \Rightarrow IO x_3 \Rightarrow IOH$

$x_0 \Rightarrow O x_2 \Rightarrow OA x_4 \Rightarrow OAH x_5 \Rightarrow OAHA$

$x_0 \to IOH \mid OAHA \in \mathcal{G}_3$

$$G_6: \quad \mathcal{P}_0 \ni \begin{cases} x_0 \to abc \mid aAbc \\ Ab \to bA \\ Ac \to bBcc \\ bB \to Bb \\ aB \to aa \mid aaA \end{cases}$$

$\{x_0, A, B\}$ $\{a, b, c\}$

$\underbrace{u}\underbrace{A}\underbrace{v} \to \underbrace{u}\underbrace{p}\underbrace{v}$, $A \in V_N$, $u, p, v \in V_G^*$

$p \neq \lambda$

$\underbrace{A}_{u=\lambda}\underbrace{b}_{v=b} \to \underbrace{b}_{u}\underbrace{A}_{p}\ \underbrace{}_{v?}$ nu se păstrează contextul

$\underbrace{A}_{\lambda}\underbrace{c}_{c} \to bBcc \in \mathcal{P}_1$

$L(G_6) = ?$

$L(G_6) = \{abc, a^2b^2c^2, \ldots\} = \{a^n b^n c^n \mid n \geq 1\}$.

$x_0 \Rightarrow abc$

$x_0 \Rightarrow a\underline{Ab}c \Rightarrow ab\underline{Ac} \Rightarrow ab\underline{bB}cc \Rightarrow a\underline{bB}bcc \Rightarrow$

$\Rightarrow \underline{aB}bbcc \Rightarrow aabbcc$

G_4: $A \to 0A \mid 1A \mid 1 \in \mathcal{G}_3$

$A \Rightarrow 1 \in L(G_4)$

$A \Rightarrow 0A \Rightarrow 01 \in L(G_4)$

$A \Rightarrow 0A \Rightarrow 00A \overset{*}{\Rightarrow} 0^k A \Rightarrow 0^k 1$

$L(G_4) \supset \{1, 01, 0^k 1, 1^k, \dots \}$
$k \geq 1$

$L(G_4) = \{$ nr. impare în scriere binară $\}$

$= \{ p1 \mid p \in \{0,1\}^* \} = \{ p \in \{0,1\}^* \mid p$ se termină cu $1 \}$