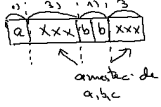


# REȚETE DE GRAMATICE (gramaticale)

- 1) Lb. finite  $L = \{pqr, udmr\}$   $S \rightarrow pqr | udmr \in \mathcal{G}_3$
- 2) Lb. repetiție unei litere  $L = \{a^n | n \geq 0\}$   $S \rightarrow aS | \lambda \in \mathcal{G}_3$
- 3) Lb. amănăci de litere  $L = \{a, b\}^*$   $S \rightarrow aS | bS | \lambda \in \mathcal{G}_3$
- 4) Lb.  $\{a^n b^n | n \geq 1\}$   $S \rightarrow aSb | ab \in \mathcal{G}_2$

Ex1: Găsiți o gram. pt. generarea lb.  $L = \{pca, b, c\}^*$  p începe cu a și se termină cu b



$$\beta_1 \rightarrow \beta_1^1$$

$$C_2 \in \beta_1 \rightarrow C_2 \in \beta_2^1$$

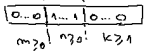
$$G_1: \begin{cases} S \rightarrow XTZT \\ X \rightarrow a \\ Y \rightarrow aT | bT | cT | \lambda \\ Z \rightarrow bb \\ T \rightarrow aT | bT | cT | \lambda \end{cases} \in \mathcal{G}_2 \quad L(G_1) = L$$

$$G_2: \begin{cases} S \rightarrow aXbbX \\ X \rightarrow aX | bX | cX | \lambda \end{cases} \in \mathcal{G}_3$$

$$G_3: \begin{cases} S \rightarrow aA \\ A \rightarrow aAb | cA | B \\ B \rightarrow bbC \\ C \rightarrow aC | bC | cC | \lambda \end{cases}$$

$$G_1 \approx G_2$$

Ex2:  $L = \{0^m 1^n 0^k | m, n \geq 0 \text{ și } k \geq 1\}$



$$G_1: \begin{cases} S \rightarrow XTZ \\ X \rightarrow 0X | \lambda \\ Y \rightarrow 1Y | \lambda \\ Z \rightarrow 0Z | 0 \end{cases} \in \mathcal{G}_2$$

$$G_2: \begin{cases} S \rightarrow 0S | B \\ B \rightarrow 1B | X \\ X \rightarrow 0X | 0 \end{cases} \in \mathcal{G}_3$$

$$G_1 \approx G_2$$

Ex3:  $L = \{a^n b^{n+2} | n \geq 1\}$

$$G_1: \begin{cases} S \rightarrow XT \\ X \rightarrow aX | a \\ Y \rightarrow bY | b^3 \end{cases} \in \mathcal{G}_2 \quad L(G_1) = L$$

$$S \rightarrow XT \Rightarrow a^3 Y \Rightarrow a^3 b^3 \notin L. \text{ incorect}$$

$$a^n b^{n+2} = a^n b^n b^2$$

$$a^n b^{n+2} = a^n b b b^n$$

$$G_2: \begin{cases} S \rightarrow XT \\ X \rightarrow aXb | ab \\ Y \rightarrow bb \end{cases} \in \mathcal{G}_2$$

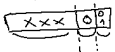
$$G_1: \begin{cases} S \rightarrow aSb | b^3 \\ L(G_1) \neq L \end{cases}$$

$$S \Rightarrow aSb \Rightarrow a^2 S b^2$$

$$G_2: \begin{cases} S \rightarrow aSb | ab^3 \\ L(G_2) = L \end{cases}$$

NU PUTEM GĂSI O GRAM pt.  $L$

Ex4:  $L = \{w \in \{0,1\}^* | w \text{ are în penultima poziție cifra 0}\}$



$$G_1: \begin{cases} S \rightarrow XTZ \\ X \rightarrow 0X | 1X | \lambda \\ Y \rightarrow 0 \\ Z \rightarrow 0 | 1 \end{cases} \in \mathcal{G}_2$$

$$G_2: \begin{cases} S \rightarrow 0S | 1S | T \\ T \rightarrow 0S \\ S \rightarrow 0 | 1 \end{cases} \in \mathcal{G}_3$$

$$G_1: \begin{cases} S \rightarrow X001X01 \\ X \rightarrow 0X | 1X | \lambda \end{cases} \in \mathcal{G}_2$$

Ex5:  $L = \{w \in \{a,b,c\}^* | w \text{ conține cel puțin una a și cel puțin un c}\}$



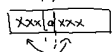
$$G_1: \begin{cases} S \rightarrow XaXcX \\ X \rightarrow aX | bX | cX | \lambda \end{cases} \in \mathcal{G}_2 \quad L(G_1) = L$$

$$G_1: \begin{cases} S \rightarrow X | Y \\ X \rightarrow AaAc^* \in \mathcal{G}_2 \\ A \rightarrow aAb | cA | \lambda \text{ auză} \\ Y \rightarrow cZcZ \\ Z \rightarrow aZ | bZ | cZ | \lambda \end{cases}$$

$$G_2: \begin{cases} S \rightarrow XaXcX | XcXaX \\ X \rightarrow aX | bX | cX | \lambda \end{cases} \in \mathcal{G}_2 \quad G_1 \approx G_2$$

$$G_3: \begin{cases} S \rightarrow X | T \\ X \rightarrow aX | bX | cX | A \\ A \rightarrow aB \\ B \rightarrow aB | bB | cB | b \\ B \rightarrow aE \\ E \rightarrow aE | bE | cE | \lambda \\ Y \rightarrow aT | bT | cT | U \\ U \rightarrow cE \\ E \rightarrow aE | bE | cE | T \\ T \rightarrow aS \\ S \rightarrow aS | bS | cS | \lambda \end{cases} \in \mathcal{G}_3$$

Ex6:  $L = \{w \in \{a,b,c\}^* | w \text{ conține cel mult un a}\}$



$$G_1: \begin{cases} S \rightarrow XT \\ X \rightarrow bX | cX | \lambda \\ T \rightarrow a | \lambda \end{cases} \in \mathcal{G}_2$$

$$\begin{cases} S \rightarrow X | T \\ X \rightarrow bX | cX | S \\ S \rightarrow aT \\ T \rightarrow bT | cT | \lambda \\ T \rightarrow bT | cT | \lambda \end{cases} \in \mathcal{G}_3$$

$$\text{osp} \begin{cases} S \rightarrow 0X1 \\ X \rightarrow 0X1 | 0X1 \\ Y \rightarrow 1Y1 \end{cases} \Rightarrow 0^m 1^n 0^m \Rightarrow 0^m 1^n 0^m$$

Ex7:  $L = \{0^n 1^m 0^n | m, n \geq 1\}$

$$0^n 1^m 0^n = 0^n 1^m 0^n$$

$$S \Rightarrow aSb \quad S \Rightarrow a^2 S b^2$$

$$S \Rightarrow 0S0 \quad S \Rightarrow 0^2 S 0^2$$

$$G_2: \begin{cases} S \rightarrow X10S0 \\ X \rightarrow 1X | 010^2 \end{cases} \quad L(G_2) = L$$

$$G_1: \begin{cases} S \rightarrow 0S0 | 0X0 \\ X \rightarrow 1X | 10 \end{cases} \quad G_1 \approx G_2$$

Ex8:  $L = \{0^n 1^m | 1 < n < m\}$  osp.

$$0^n 1^m = 0^n 1^m \quad S \rightarrow XT, X \rightarrow 0X | 0^2 X | 10^2 X | \lambda$$

$$L = \{p \in \{0,1\}^* | p \text{ are în poziția } a_3 = a \text{ și } a_4 = 0\}$$

$$p \in \{0,1\}^* \quad p \in \{0,1\}^*$$