

## Exam-style problem

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## Assignment

Two containers are connected by pipes, as in the figure below. Container  $A$  holds 300 l and container  $B$  100 l. As a starting point, there is 90 g of salt in container  $A$  and 30 g salt in container  $B$ . Through pipe  $a$  30 l/min of pure water is added to  $A$  and through pipe  $b$  15 l/min of a salt-water mixture is added to container  $A$ .

- How much mixture must pass through pipe  $c$  per minute if the amount of liquid in  $A$  is to be constant? How much must pass through pipe  $d$  if the amount of liquid in  $B$  is to be constant?
- Show that the amount of salt  $y_0(t)$  in  $A$  and  $y_1(t)$  in  $B$  satisfies the system

$$\begin{aligned}y_0'(t) &= -0,15y_0(t) + 0,15y_1(t), \\y_1'(t) &= 0,15y_0(t) - 0,45y_1(t)\end{aligned}$$

In c) and d) you need to explain the concepts. This you will not need to do for the exam, but it will help you understand the concepts.

- Explain how the eigenvalues and eigenvectors of the coefficient matrix are calculated.
- Explain how the solutions to  $y_0(t)$  og  $y_1(t)$  can be determined by these eigenvalues and eigenvectors and the initial condition.
- Using Python, find  $y_0(t)$  og  $y_1(t)$ , plot both functions against  $t$ . Also, plot  $y_0(t)$  against  $y_1(t)$ . What is the limit of  $y_1(t)/y_0(t)$  when  $t \rightarrow \infty$ ?

