ALI Exam 2024 Part 1

Assignment 1

a) $tr(A) = \Sigma \lambda_i \Rightarrow 12 = 2 \cdot 2 + 3 + \lambda_3 \Rightarrow$

$\lambda_3 = 5$

b)

i = False

ii = False

iii = False

iv = True

v = True

c)

$a_3 = -2a_1 - 2a_2$

$a_4 = 3a_1 + 2a_2$

d)

Rank A = 10

↳ 10 non-zero eigenvalues

e)

Det A = 0

↳ $\lambda = 0 \Rightarrow$ Det A = 0

Assignment 2:

a) $A_{3\times 3} \times B_{3\times 2} \rightarrow$ C is a 3x2

b) $C_{21} = 2\cdot 2 + 0\cdot 0 + 1\cdot 2 = 6$

c) $(B^T A)_{1,2} = 2\cdot 7 + 0\cdot 0 + 2\cdot 1 = 16$

Assignment 3:

a) $\begin{bmatrix} -1-3 & & \\ & -3-3 & \\ & & 11-3 \end{bmatrix} = \begin{bmatrix} -4 & -6 & 8 \\ -4 & -6 & 8 \\ -4 & -6 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\rightarrow$ geometric Multiplicity $= 2$

b) From (a):

$$\boxed{\left\{ \begin{bmatrix} -3/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\} \text{ or } \left\{ \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\}}$$

c) $\det A = 3^2 \cdot 1 = 9$

d)
i. since geom. mult = algeb. mult. A is diag.
ii. Yes since $\det A \neq 0$
iii. Since $\lambda_i \neq 0 \Rightarrow$ Rank$=3$ so 3 pivots.
iv. dim Nul $= n -$ Rank $= 3-3 = 0$

Assignment 4:

$$\begin{cases} (1/\sqrt{2})a + (1/\sqrt{2})b = 0 \\ \sqrt{(a/\sqrt{2})^2 + (b/\sqrt{2})^2} = 1 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} a + b = 0 \\ a^2 + b^2 = 2 \end{cases}$$

$$\Updownarrow$$

$$(-b)^2 + b^2 = 2$$

$$\Updownarrow$$

$$2b^2 = 2 \Rightarrow b = 1$$

$$a + 1 = 0 \Rightarrow a = -1$$

so $a = -1$, $b = 1$, $c = \sqrt{2}$

og omvendt fortegn.

Assignment 5:

a)

i. $\{\bar{v}_1, \bar{v}_2\}$ Ind since $\bar{v}_2 \neq k \cdot \bar{v}_1$

ii. $\{\bar{v}_2, \bar{v}_3\}$ Ind since $\bar{v}_3 \neq k \cdot \bar{v}_2$

iii. $\{\bar{v}_2, \bar{v}_3, \bar{v}_4\}$ Ind since (ii) and $\bar{v}_4 \neq k \cdot \bar{v}_3 + m \cdot \bar{v}_2$

iv. $\{\bar{v}_5\}$ Ind by definition

v. $\{\bar{v}_3, \bar{v}_5, \bar{v}_6\}$ Dep since $\bar{v}_6 = \bar{v}_3 + \bar{v}_5$

vi. $\{\bar{v}_1, \ldots, \bar{v}_6\}$ Dep since case (v) is a subset

b)

i. Spans $\mathbb{R}^4$ since set is ind.

ii. Does not span $\mathbb{R}^4$ since (a.v)

iii. Spans $\mathbb{R}^4$ since (i) is a subset.

Assignment 6

$$\det(A^3) = (\det A)^3 = (-2)^3 = -8$$

$$\det(A^T B^{-1}) = \det(A^T)\cdot\det(B^{-1}) = (-2)\,\frac{1}{5} = -2/5$$

$$\det(2B) = 2^4\cdot\det B = 2^4\cdot 5 = = 80$$

$$\det((AB)^2) = (\det(AB))^2 = (\det A\cdot\det B)^2$$

$$= (-2\cdot 5)^2 = 100$$

$$\det(2BA^{-3}) = \det(2B)\cdot\det(A^{-3}) = 80\cdot\left(-\frac{1}{2^3}\right) = -10$$

Assignment 7:

$$A^2 x = b \Rightarrow x = A^{-2}\cdot b = (A^{-1})^2\cdot b$$

$$= \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 10 & -9 \\ 6 & -5 \end{bmatrix}\begin{bmatrix} 5 \\ -2 \end{bmatrix} = \boxed{\begin{bmatrix} 68 \\ 40 \end{bmatrix}}$$

Assignment 8:

$$P = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix},\ D = \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix},\ P^{-1} = \frac{1}{7}\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$$

$$A = \frac{1}{7}\begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 2 & 15 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -28 & 21 \\ 14 & 7 \end{bmatrix} = \boxed{\begin{bmatrix} -4 & 3 \\ 2 & 1 \end{bmatrix}}$$