Item 1

It is possible to obtain 10 points in this assignment, divided equally between each assignment. In each assignment, it is specified how you should answer. No documentation is required for this assignment.
Consider the following system

$$
\begin{aligned}
2x - 3y + 7z &= -11 \\
4x - 7y + 13z &= -25 \\
6x - 8y + 22z &= -30
\end{aligned}
$$

Let *A* denote the coefficient matrix of this system such that the system takes the form $A\vec{x} = \vec{b}$.

a. Translate this system into an augmented matrix. State your answers as integers between 0 and 99. Note all and any negative sign have been pre-printed.

$$\begin{bmatrix} \square & -\square & \square & -\square \\ \square & -\square & \square & -\square \\ \square & -\square & \square & -\square \end{bmatrix}$$

Correct answers:

$$\begin{bmatrix} 2 & -3 & 7 & -11 \\ 4 & -7 & 13 & -25 \\ 6 & -8 & 22 & -30 \end{bmatrix}$$

In the rest of the assignment you can use the fact that the row reduced echelon form of the system above is

$$\begin{bmatrix} 1 & 0 & 5 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b. Assuming that $\vec{a}_i$ denotes the columns of *A*, express $\vec{a}_3$ as a linear combination of the preceding columns. State all inputs as integers between 0 and 99.

$$\vec{a}_3 = \square\,\vec{a}_1 + \square\,\vec{a}_2$$

Correct answers:

$$\vec{a}_3 = 5\vec{a}_1 + 1\vec{a}_2$$

c. State the general solution to the system above in parametric vector form. State all inputs as integers between 0 and 99. Remember to fill in the subscript of the free variable.

$$\vec{x} = \begin{bmatrix} -\square \\ \square \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} -\square \\ -1 \\ \square \end{bmatrix}$$

Correct answers:

$$\vec{x} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix}$$

d. Determine the truth value of the statements below.

	True	False
The columns of *A* are linearly independent	○	○ ✔
A basis for the column space of *A* spans $\mathbb{R}^3$	○	○ ✔
Matrix *A* is singular	○ ✔	○
Rank of $A = 4 - 1 = 3$	○	○ ✔
A is diagonalizable	○	○ ✔

Item 2

It is possible to obtain 5 points in this assignment. You must document how you obtained the result.

Given

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Find the values below.

$\det A = -$ 1 $\square$

Correct answers:

1 1

$$A^{-1} = \begin{bmatrix} -1 & \square \\ \square & -1 \end{bmatrix}$$

Correct answers:

$$A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\left(A^T\right)^{-1} = \begin{bmatrix} -1 & \square \\ \square & -1 \end{bmatrix}$$

Correct answers:

$$\left(A^T\right)^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$$

Item 3

It is possible to obtain 5 points in this assignment. You must document how you obtained the result.

Find the value below. State your answer as a positive integer. Please note that a negative sign has been pre-printed.

$$\begin{vmatrix} -1 & 0 & 0 & -2 \\ 1 & 0 & 5 & -5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -5 & 0 \end{vmatrix} = - \quad 1 \ \square$$

Correct answers:

1 35

Item 4

It is possible to obtain 5 points in this assignment. You must document how you obtained the result.

Find a vector function of the form

$$\bar{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_n(t) \end{bmatrix} = \sum_{i=1}^{n} C_i \bar{v}_i e^{\lambda_i t}$$

such that such that $y_1(0) = 0$ and $y_2(0) = -4$ and such that they are related by the linear system of differential equations,

$$y_1' = y_1 + 2y_2$$
$$y_2' = 3y_1 + 2y_2$$

State your answers as integers between 0 and 99 (note negative signs have been pre-printed).

$$\bar{y}(t) = -\frac{\square}{5}\begin{bmatrix} -1 \\ \square \end{bmatrix} e^{-t} - \frac{\square}{5}\begin{bmatrix} \square \\ 3 \end{bmatrix} e^{\square t}$$

Correct answers:

$$\bar{y}(t) = -\frac{8}{5}\begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} - \frac{4}{5}\begin{bmatrix} 2 \\ 3 \end{bmatrix} e^{4t}$$

Item 5

It is possible to obtain 10 points in this assignment, 3 points for the first two and four for the last assignment. You must document how you obtained the result.

Consider the matrix product *AB*, where

$$A = \begin{bmatrix} 1 & 0 & 4 & 3 & 3 & 2 \\ 1 & 8 & 1 & 9 & 6 & 0 \\ 0 & 1 & 3 & 3 & 8 & 9 \\ 0 & 8 & 3 & 8 & 6 & 3 \\ 7 & 9 & 4 & 0 & 2 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & 5 & 3 \\ 2 & 6 & 9 \\ 4 & 8 & 4 \\ 6 & 9 & 0 \\ 3 & 0 & 7 \\ 2 & 2 & 5 \end{bmatrix}$$

Indexing rows and columns from 1, find the value of entry (3, 2) in *AB*, i.e. $(AB)_{3,2}$. State your answer as an integer between 0 and 99.

$(AB)_{3,2} =$ 1 ☐

Correct answers:

1 75

b. Given the matrices:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ -1 & 4 \end{bmatrix}, \quad \vec{d} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix},$$

Which of the following matrix expressions are defined? Check the box besides those that are defined.

- ☐ $A\vec{d}$ ✔
- ☐ $AB + C$
- ☐ $A + C^T$ ✔
- ☐ $C^T C$ ✔
- ☐ BC ✔
- ☐ $(\vec{d})^T B$ ✔
- ☐ $C\vec{d}$
- ☐ $(\vec{d})^T \vec{d}$ ✔
- ☐ $\vec{d}(\vec{d})^T$ ✔

c. Let A, B, and C be square matrices. Solve the following matrix equation for C.

$3A + 2B = 2(B - A + C)$

State your inputs as integers between 0 and 99 such that the fraction below is irreducible.

$$C = \frac{\square}{\square} A$$

Correct answers:

$C = \frac{5}{2} A$

Item 6

It is possible to obtain 5 points in this assignment, divided equally between the two assignments. No documentation is needed for this assignment.

Below you see a basis for the null space of a 3 x 5 matrix *A*. Determine the values below. State all inputs as integers between 0 and 99.

$$\left(\begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right)$$

Nullity of A = 1 [ ]

Rank of A = 2 [ ]

Dim Row A = 3 [ ]

Dim Nul A^T = 4 [ ]

Correct answers:

1 2 2 3 3 3 4 0

Consider the matrix

$$A = \begin{bmatrix} -2 & -1 & -4 & 5 \\ 0 & -8 & 6 & -6 \\ 0 & 0 & -81 & 13 \\ 0 & 0 & 0 & -604 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Determine the truth value of the statements below.

	True	False
A is invertible.	○	○ ✔
A is on echelon form.	○ ✔	○
For every $\vec{b} \in \mathbb{R}^5$ the system of equations Ax = b is consistent.	○ ✔	○
The null space of A is a subspace of $\mathbb{R}^5$.	○	○ ✔
The dimension of the column space is 5.	○	○ ✔

Item 7

It is possible to obtain 5 points in this assignment. You must document how you obtained the result.

Assume

$$A^{-1} = \begin{bmatrix} -2 & 3 & -3 \\ -2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Find the solution to $A^2\vec{x} = \vec{b}$. State your inputs as integers between 0 and 99.

$$\vec{x} = \begin{bmatrix} \square \\ \square \\ -1 \end{bmatrix}$$

Correct answers:

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$$

Item 8

Assume

$\vec{u} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are basis vectors of eigenspace E_{-1}, E_5, respectively (the subscripts denote the respective eigenvalues). Find A. State all inputs as integers between 0 and 99.

$$A = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Correct answers:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$