ALI Exam 2024 Part 2

Assignment 1

$$\begin{bmatrix} 10000 & 5000 & 2500 & 300000 \\ 20000 & 20000 & 10000 & 1000000 \\ 120000 & 80000 & 50000 & 5000000 \end{bmatrix}$$

rref $\sim$ $\begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 60 \end{bmatrix}$ → A = 10, B = 10, C = 60

50 80

Assignment 2:

$$-2(AB-C)X = AC \Rightarrow X = (-2(AB-C))^{-1} \cdot AC,$$

$$\det(X) = \det((-2(AB-C))^{-1} \cdot AC) = \frac{1}{60}$$

Assignment 3:

$$\begin{cases} 4A - 2B = C_1 \\ -5A + 7B = C_2 \end{cases} \Rightarrow 2A - B = \frac{1}{2}C_1 \Rightarrow B = 2A - \frac{1}{2}C_1$$

$$-5A + 7(2A - \tfrac{1}{2}C_1) = C_2 \Rightarrow 9A = \tfrac{7}{2}C_1 + C_2$$

$$A = \tfrac{1}{9}(\tfrac{7}{2}C_1 + C_2)$$

$$B = 2A - \tfrac{1}{2}C_1$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 3 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & 2 \\ -3 & -1 & -5 \end{bmatrix}$$

Assignment 4:

a) $\bar{v}_1 \cdot \bar{v}_2 = 0$ } Need to show both!

$\|\bar{v}_1\| = 1 \wedge \|\bar{v}_2\| = 1$

b) $\dim W = 2 \rightarrow$ # ind. vec.

c) $\dim W^\perp = n - \dim W = 4 - 2$

$= 2$

d) $\text{proj}_W \bar{v} = \frac{\bar{v} \cdot \bar{v}_1}{\bar{v}_1 \cdot \bar{v}_1} \cdot \bar{v}_1 + \frac{\bar{v} \cdot \bar{v}_2}{\bar{v}_2 \cdot \bar{v}_2} \cdot \bar{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix}$

e) $\text{dist}(\bar{v}, W) = \|\bar{v} - \text{proj}_W \bar{v}\| = 4$

Assignment 5:

a) Find intersection:

$$\left.\begin{aligned} x - y + 2z &= 0 \\ 3x + 2y + z &= 0 \end{aligned}\right\} \quad \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$$

Nullspace $\sim \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

b) $\bar{x} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$

$\bar{y}: \bar{v} \wedge \bar{y} \cdot \bar{x} = 0$

$\bar{y} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Python ↓

$, \bar{z}: \bar{v} \wedge \bar{z} \cdot \bar{x} = 0, \bar{z} = \frac{1}{\sqrt{21}} \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix} = \begin{bmatrix} \sqrt{42}/42 \\ -\frac{2\sqrt{42}}{21} \\ \frac{5\sqrt{42}}{42} \end{bmatrix}$

Assignment 6:

$$X_1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 1 & 1 & 1 \\ x & x^2 & x^3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ x \end{bmatrix}$$

$e_1 = 4.80$

$e_2 = 3.88$

$e_3 = 5.24$

e_2 is best fit

Assignment 7:

$$y_f = -\frac{197}{4}\begin{bmatrix} -1/2 \\ 1 \end{bmatrix} e^{-t/10} + \frac{201}{4}\begin{bmatrix} 3/2 \\ 1 \end{bmatrix} e^{-t/50}$$

a) $t = 20$

b) $y_1 = 54$, $y_2 = 27$

c) $\frac{y_1}{y_2} \to \frac{3}{2}$ for $n \to \infty$