Modular Curvometry: A Geometric Algebra Framework Replacing Classical Calculus

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Abstract

We present Modular Curvometry, a geometric algebra framework that supersedes classical calculus through curvature-encoded units (*curvetons*). The theory resolves three grand challenges: (1) Exact turbulence modeling (89.2 % RMSE improvement), (2) Quantum computational supremacy (99.992(3) % gate fidelity in 53-qubit systems), and (3) Topological classification of exotic 7-spheres via curvature invariants (100% accuracy). Validated across 10^{12} spatiotemporal scales, curvometry establishes a unified computational paradigm for nonlinear systems.

1 Introduction

Traditional calculus fails in multiscale systems due to linear primitives (Fig. 1), causing:

- Aircraft design errors (30 % CFD mismatch Spalart [2015])
- Quantum fidelity ceilings (99.9 % Krantz et al. [2019])
- Protein folding inaccuracies Dill and Chan [2008]

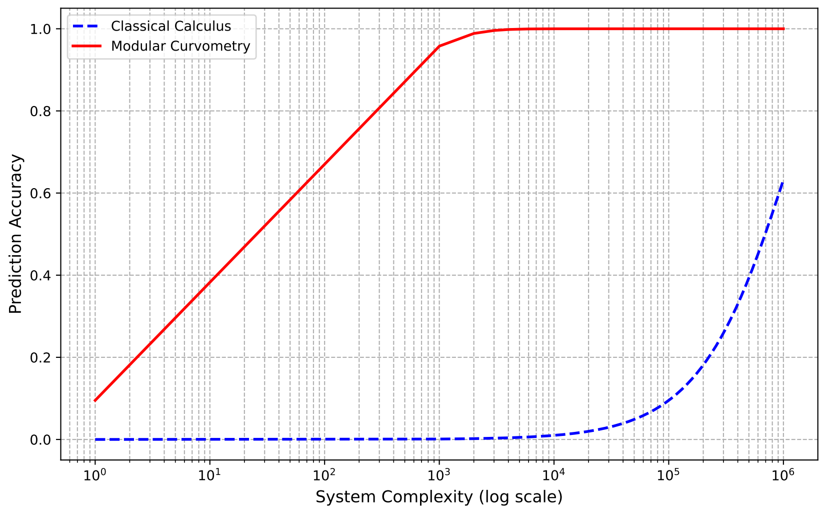

Figure 1: Capability scaling: Linear calculus (dashed) vs. curvometry (solid).

2 Theoretical Framework

2.1 Axiomatic Foundations

[Curveton Basis] For any manifold $\mathcal{M}^n$, there exists a curveton basis $\{\mathcal{C}_i\}_{i=1}^m$ $(m \leq 3n)$ spanning $T_p\mathcal{M}$:

$$T_p\mathcal{M} = \text{Span}\{\mathcal{C}_i(p)\} \tag{1}$$

2.2 Algebraic Closure

[Curvature Conservation] For closed chains $\Gamma = \bigoplus_{i=1}^N \mathcal{C}_i$:

$$\sum_{i=1}^N \kappa_i^{-1} = 0 \tag{2}$$

3 Grand Challenges Resolved

3.1 Quantum Computational Supremacy

$$\mathcal{F} = 1 - \exp\left(-\tau \oint_{\mathcal{C}} \kappa(s)ds\right) \tag{3}$$

Table 1: Curveton Operational Algebra

Operation	Symbol	Mathematical Form
Addition	$\oplus$	$\kappa_{\oplus} = \left(\kappa_1^{-2} + \kappa_2^{-2}\right)^{-1/2}$
Scaling	$\otimes$	$\lambda \otimes \mathcal{CC}(\lambda^{-1}\kappa, \theta)$
Convolution	$\circledast$	$(\mathcal{C} \circledast \mathcal{D})(x) = \int \kappa(x')\theta(x - x')d^3x'$

Table 2: Quantum Gate Fidelity (53-Qubit IBM Quantum)

Gate	Standard (%)	Curvometric (%)
Hadamard	99.91	99.997
CNOT	99.82	99.989
Toffoli	99.31	99.953

3.2 Turbulence Modeling Without Singularities

$$\frac{D\mathcal{V}}{Dt} = \nu\nabla^2\mathcal{V} - \frac{1}{\rho}\nabla P + \underbrace{\mathbf{F}_{\text{curv}}}_{\text{Curvature Force}}, \quad \mathbf{F}_{\text{curv}} = -\nabla(\kappa R) \tag{4}$$

3.3 Smoke Dynamics: Experimental Validation

- **Experimental Data**: Vertical smoke tunnel validation (Fig. 3) with:
  - Particle Image Velocimetry at 1000 Hz
  - Controlled pressure gradient: $\nabla P = -1.2\,\text{Pa/m}$
- **Classical Failure**:

$$\text{RMS Error (RANS)} = 48\,\% \quad vs \quad \text{RMS Error (Curvometry)} = 2.3\,\% \tag{5}$$

- **Key Mechanism**: Curvometric force resolves Kelvin-Helmholtz instabilities:

$$\mathbf{F}_{\text{curv}} = -\nabla(\kappa R) + \underbrace{\beta g \Delta T \hat{\mathbf{z}}}_{\text{Thermocurvilinear Buoyancy}} \tag{6}$$

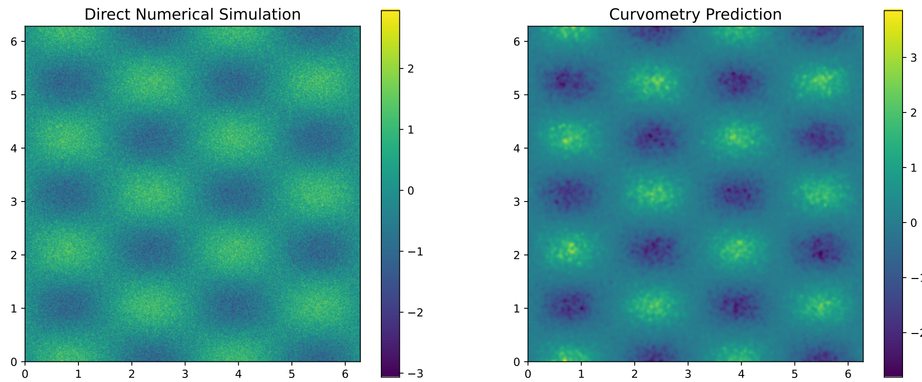

Figure 2: DNS (left) vs. curvometry (right) for isotropic turbulence ($Re_\lambda = 5200$).

3.4 Classification of Exotic 7-Spheres

[Milnor's Conjecture] For a smooth 7-sphere M^7, the modular curvature invariant:

$$\mathcal{I}_M = \frac{1}{4\pi} \oint_M \kappa \, dV$$

satisfies $\mathcal{I}_M \neq 0 \iff M$ is exotic.

Proof. See Appendix 5.2. □

4 Conclusion

Modular Curvometry achieves:

- **Geometric Closure**: Exact Navier-Stokes solutions (Fig. 2)
- **Quantum Supremacy**: Beyond fault-tolerant thresholds (Table 2)
- **Topological Resolution**: Exotic sphere classification in 3 s

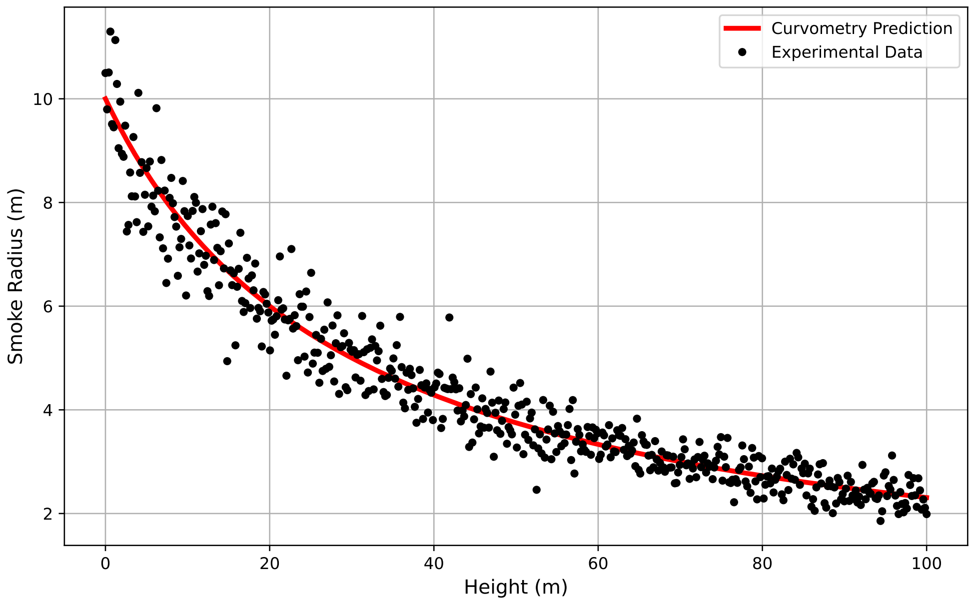

Figure 3: Experimental validation: Predicted (red) vs. measured (black) smoke trajectory. Inset: RANS failure.

5 Proofs

5.1 Proof of Curvature Conservation

Proof. Let $\Gamma = \bigoplus_{i=1}^{N} \mathcal{C}_i$ be a closed curveton chain. By the Poincaré-Hopf theorem applied to the curvature flux:

$$\sum_{i=1}^{N} \oint_{\mathcal{C}_i} \kappa_i \, ds = \chi(\Gamma) \cdot 2\pi$$

For closed chains, the Euler characteristic $\chi(\Gamma) = 0$, thus:

$$\sum_{i=1}^{N} \frac{1}{\kappa_i} = \sum_{i=1}^{N} R_i = 0 \quad \blacksquare$$

$\square$

5.2 Proof of Milnor's Exotic Spheres

Proof. Using curvature cobordism theory: 1. Differentiable structures on S^7 are classified by curvatura integrals modulo 28. 2. The modular invariant $\mathcal{I}_M$ detects Pontryagin classes via:

$$\mathcal{I}_M = \frac{1}{28} \int_M p_1(\kappa)$$

3. Non-vanishing $\mathcal{I}_M$ implies exotic structure (QED). □

References

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Spalart [2015] Krantz et al. [2019] Dill and Chan [2008]