



Decomposing real and nominal yield curves



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ABSTRACT

Inflation-indexed and nominal yield curves capture investors' expectations of real short rates and inflation as well as their required compensation for bearing liquidity, inflation, and real interest rate risk. We estimate an affine term structure model that allows us to decompose real and nominal bond yields into these components and use the model to study the transmission of monetary policy. The model decompositions imply that the Federal Reserve's announcements of LSAPs lowered yields primarily by reducing real term premia. Changes in real term premia also account for the strong response of long-term real forward rates to federal funds rate surprises.

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1. Introduction

The evolution of inflation expectations is an important input to monetary policy decisions and is closely watched by financial market participants. Breakeven inflation—the difference between nominal yields from Treasuries and real yields from Treasury Inflation-Protected Securities (TIPS) for a given maturity—reflects inflation expectations, but is subject to two important distortions. First, TIPS are often perceived to be less liquid than Treasuries, especially in times of financial market stress. Second, breakeven inflation incorporates an inflation risk premium, the compensation investors require for bearing inflation risk.

This paper presents a Gaussian affine term structure model (ATSM) for the joint pricing of the Treasury and TIPS yield curves that adjusts for the relative illiquidity of TIPS and generates estimates of the inflation risk premium. Our model adjusts for TIPS illiquidity in a transparent way. Specifically, an index of TIPS liquidity using observable measures is constructed and included as a pricing factor in the model. The illiquidity component is sizable for TIPS, especially during the financial crisis. The estimated model features six pricing factors: three principal components from the cross-section of Treasury yields, two principal components extracted from orthogonalized TIPS yields, and the liquidity factor.

An important finding from the model estimates relates to the decomposition of far in the future breakevens into expected inflation, the inflation risk premium and a liquidity component, as shown in Fig. 1. It has long been argued by market observers that variations of far in the future forward rates mainly reflect changes in risk premia. The model confirms

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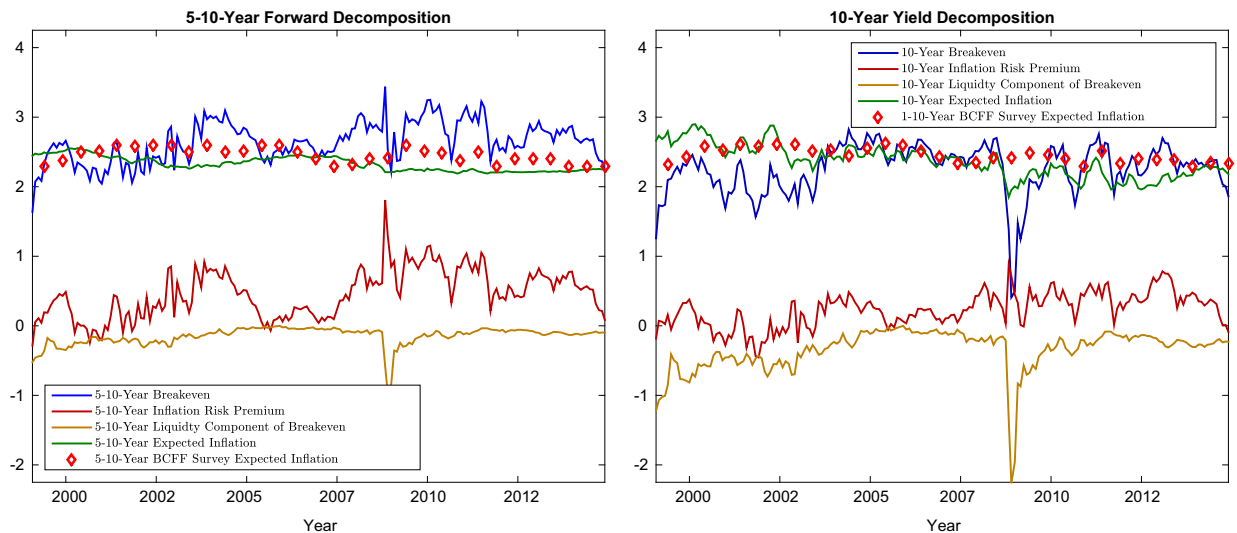


Fig. 1. Inflation breakeven decomposition. This figure shows the decomposition of breakeven inflation rates into the model-implied expected inflation, the inflation risk premium as well as the liquidity component. The left-hand panel shows this decomposition for 5–10 year forward breakeven inflation whereas the right-hand panel displays the decomposition for the 10 year horizon. Each graph also shows the corresponding consensus CPI forecast from the Blue Chip Financial Forecasts (BCFF) Survey.

that conjecture. The left-hand panel of Fig. 1 shows that model implied expected 5–10 year forward inflation is stable, while the variation in the forward breakeven rate mainly captures variation in the estimated inflation and liquidity risk premia. For ten year breakeven inflation, the decomposition is different: both expected inflation and the inflation risk premium vary considerably (the right-hand panel). The liquidity adjustment is quantitatively important for both the 5–10 year and the ten year maturities in the first few years after the inception of the TIPS program and especially during the fall of 2008.

Importantly, the model produces long-term, risk-adjusted inflation expectations that align very closely with surveys of professional forecasters (red diamonds in the charts), despite the fact that no survey data is incorporated in the estimation. In fact, both the model implied and the survey expectation for the 5–10 year forward expectation of inflation are centered around a level of about 2.5 percent and exhibit little variation over time. In contrast, breakeven inflation that is unadjusted for risk premia varies considerably. Averaged over the next ten years, model-implied expected inflation exhibits somewhat stronger time variation, but is still substantially less volatile than unadjusted breakeven inflation.

After documenting that the model-implied risk premiums are economically meaningful and showing that risk-adjusted breakeven inflation provides better forecasts of inflation than unadjusted breakeven inflation, the model is used to assess the reaction of the expectations and risk premium components of the nominal and real yield curve to conventional monetary policy shocks and announcements of the Federal Reserve's large-scale asset purchases (LSAPs).

Correlating the model estimates of the inflation risk premium with macroeconomic and financial indicators shows that the inflation risk premium tends to increase when option-implied Treasury volatility increases and when disagreement rises among professional forecasters about future inflation. The inflation risk premium also correlates positively with the unemployment rate and negatively with consumer confidence, in line with countercyclical risk premia. These correlations indicate that the decomposition of breakeven inflation into expected inflation, liquidity, and the inflation risk premium is economically meaningful, even though the model does not explicitly use any macroeconomic pricing factors.

Breakeven inflation rates are the primary market based measure of inflation expectations and are therefore of considerable interest to policymakers and market participants. Do model-implied risk-adjusted breakeven rates predict future inflation better than the observed breakeven inflation rates? Comparing model-implied average expected inflation for horizons up to three years with those implied by actual observed zero-coupon breakeven inflation rates as well as those implied by a simple random walk for inflation, we find that the affine model outperforms in-sample and out-of sample, showing that the risk-premium adjustment indeed improves inflation forecasts.

The model facilitates a decomposition of the reaction of nominal and real Treasury yields to conventional monetary policy shocks. Using the identification of monetary policy shocks as in Kuttner (2001) and Bernanke and Kuttner (2005), we find that both nominal and real forwards significantly comove with unanticipated monetary policy changes, and that for far into the future real forwards these effects are exclusively due to changes in real term premia. This result is consistent with Hanson and Stein (2014) and provides support for the notion that an important transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy, as also proposed by the recent literature on the "risk taking channel" of monetary policy (see Adrian and Shin, 2010a; Borio and Zhu, 2012; Dell'Ariccia et al., 2014).

The negative response of breakeven inflation rates to contractionary monetary policy shocks is almost exclusively driven by inflation risk premia rather than expected inflation. As monetary policy shocks tend to drive consumption growth and

inflation in the same direction, this finding is consistent with structural models of bond pricing which imply a negative relationship between the inflation risk premium and the covariance between consumption growth and inflation (e.g., Piazzesi and Schneider, 2007).

The decompositions of the TIPS and Treasury yield curves are also used to revisit the effects of the Federal Reserve's large-scale asset purchases and the maturity extension program (MEP). These operations were a major part of the central bank response to the recent financial crisis after the level of the federal funds rate was pushed close to its lower bound. An event study methodology similar to Gagnon et al. (2011) and Krishnamurthy and Vissing-Jorgensen (2011) uncovers that the real and nominal term premium declined due to the LSAP and the MEP announcements, while the risk-adjusted expectations of real and nominal rates rose slightly. These findings are fully consistent with the “duration risk” and the “preferred habitat” channels of asset purchases, as highlighted by Gagnon et al. (2011). In contrast, the results do not support the “signaling channel” of the purchase programs that has been emphasized by Bauer and Rudebusch (2014). Combined, these findings suggest that the purchase programs primarily reduced the price of real interest rate risk.

The remainder of the paper is organized as follows. Section 2 introduces the joint model for Treasury and TIPS yields. Section 3 discusses our empirical specification as well as estimation and fit of the model. Section 4 uses the model to parse the effects of monetary policy. Section 5 reviews the related literature and Section 6 concludes. Detailed derivations are left to a Supplementary Appendix.

2. The model

As is common in Gaussian term structure models,¹ it is assumed that a $K \times 1$ vector of pricing factors evolves under the physical measure (\mathbb{P}) according to the autoregression

$$X_{t+1} - \mu_X = \Phi(X_t - \mu_X) + \nu_{t+1} \quad (1)$$

where ν_t are *i.i.d.* Gaussian with $\mathbb{E}_t[\nu_{t+1}] = 0_{K \times 1}$ and $\mathbb{V}_t[\nu_{t+1}] = \Sigma$. It is also assumed that assets are priced by the stochastic discount factor

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t' \lambda_t - \lambda_t' \Sigma^{-1/2} \nu_{t+1}\right), \quad (2)$$

where r_t is the nominal short rate. Following Duffee (2002) the $K \times 1$ price of risk vector λ_t takes the essentially affine form $\lambda_t = \Sigma^{-1/2}(\lambda_0 + \lambda_1 X_t)$. Further define:

$$\tilde{\mu} = (I_K - \Phi)\mu_X - \lambda_0, \quad \tilde{\Phi} = \Phi - \lambda_1. \quad (3)$$

These parameters govern the dynamics of the pricing factors under the risk-neutral or pricing measure (\mathbb{Q}) and feature prominently in the recursive pricing relationships derived below.

In Gaussian ATSMs the log price, $P_t^{(n)}$, of a risk-free discount bond with remaining time to maturity n follows $\log P_t^{(n)} = A_n + B_n' X_t$, which implies that $r_t = \delta_0 + \delta_1' X_t$. Imposing no-arbitrage restrictions gives rise to the following relationship between parameters²:

$$A_n = A_{n-1} + B_{n-1}' \tilde{\mu} + \frac{1}{2} B_{n-1}' \Sigma B_{n-1} - \delta_0, \quad A_0 = 0 \quad (4)$$

$$B_n' = B_{n-1}' \tilde{\Phi} - \delta_1', \quad B_0 = 0_{K \times 1}. \quad (5)$$

The ordinary Gaussian ATSM framework is expanded to allow for the pricing of inflation-indexed securities jointly with nominal securities so that both yield curves are affine in the state variables. Similar models have been studied before in both continuous and discrete time, and Section 5 discusses the relationship of our model to this literature.

Let Q_t be a price index at time t , and let $P_{t,R}^{(n)}$ denote the price at time t of an inflation-indexed bond with face value 1, paying out the quantity $\frac{Q_{t+n}}{Q_t}$ at time $t+n$. The price of such a bond satisfies

$$P_{t,R}^{(n)} = \mathbb{E}_t^{\mathbb{Q}} \left[\exp(-r_t - \dots - r_{t+n-1}) \frac{Q_{t+n}}{Q_t} \right], \quad (6)$$

where $\mathbb{E}_t^{\mathbb{Q}}$ denotes the expectation under the pricing measure. Let one period log inflation be $\pi_t = \ln\left(\frac{Q_t}{Q_{t-1}}\right)$, so that

$$\frac{Q_{t+n}}{Q_t} = \exp\left(\sum_{i=1}^n \pi_{t+i}\right). \quad (7)$$

Log prices of inflation-indexed bonds are affine in the pricing factors:

$$\log P_{t,R}^{(n)} = A_{n,R} + B_{n,R}' X_t. \quad (8)$$

This implies that one-period inflation is also a linear function of the pricing factors as $\pi_t = \pi_0 + \pi_1' X_t$, where π_0 is a scalar and

¹ See Piazzesi (2010) and Singleton (2006) for overviews.

² See also Ang and Piazzesi (2003) among many others.

π_1 a vector of length K . We can derive pricing recursions for inflation-indexed bonds by rewriting Eq. (6) in terms of an indexed bond purchased one period ahead:

$$P_{t,R}^{(n)} = \mathbb{E}_t^Q \left[\exp(-r_t + \pi_{t+1}) P_{t+1,R}^{(n-1)} \right]. \quad (9)$$

Solving this equation and matching coefficients we then find that the coefficients in Eq. (8) are determined by the following system of difference equations:

$$A_{n,R} = A_{n-1,R} + B_{n-1,R}' \tilde{\mu} + \frac{1}{2} B_{n-1,R}' \Sigma B_{n-1,R}^\pi - \delta_{0,R}, \quad A_{0,R} = 0, \quad (10)$$

$$B_{n,R}' = B_{n-1,R}' \tilde{\Phi} - \delta_1', \quad B_{0,R} = 0_{K \times 1} \quad (11)$$

where $\delta_{0,R} = \delta_0 - \pi_0$ and $B_{n,R}^\pi = (B_{n,R} + \pi_1) \forall n$.

2.1. Expected inflation

The model can be used to compute expected inflation under both the risk-neutral and the physical measure at any horizon. For a given forecast horizon n , model-implied average expected inflation under the pricing measure is given by the breakeven inflation rate, i.e., the difference between the yields on a nominal and an inflation-indexed bond with maturity n :

$$\pi_t^{(n)} = y_t^{(n)} - y_{t,R}^{(n)} = -\frac{1}{n} \left[A_n + B_n' X_t - (A_{n,R} + B_{n,R}' X_t) \right]. \quad (12)$$

Model-implied expected average inflation under the physical measure is then simply obtained by replacing the parameters $\tilde{\mu}$ and $\tilde{\Phi}$ with their risk-adjusted counterparts $(I_K - \Phi)\mu_X$ and Φ in Eqs. (4), (5), (10), and (11). Absent any liquidity premia, the difference between the two measures of inflation expectations constitute the inflation risk premium—the compensation investors require for bearing inflation risk.

2.2. TIPS liquidity premia

Pflueger and Viceira (2016) document that the liquidity of TIPS relative to Treasuries appears to be systematically priced. The relative liquidity of TIPS from their inception until 2003, when the Treasury reaffirmed its commitment to the TIPS program, and in the aftermath of the Lehman bankruptcy in late 2008, which resulted in its considerable TIPS inventory being released into the market, have been discussed in Sack and Elsasser (2004) and Campbell et al. (2009) among others. Liquidity premia have also been found in nominal Treasuries, see for example Gurkaynak et al. (2007) and Fontaine and Garcia (2012).

It is straightforward to model the impact of liquidity on nominal and real yields in our framework. Let L_t be an observable factor capturing systematic changes in liquidity. One can simply expand the state space so that the vector of factors X_t includes this liquidity factor. The pricing coefficients remain subject to the recursive restrictions given in Eqs. (4) and (5) as well as (10) and (11) above. In the empirical results documented in Section 3, the “liquidity-adjusted” estimates of the inflation risk premium and of expected inflation will be reported by first subtracting the liquidity components associated with the factor L from nominal and inflation-indexed bond yields prior to computing the inflation-related indicators. The construction of the liquidity factor is discussed in Section 3.1.

3. Empirical specification

This section discusses the empirical specification and provides details on the estimation and fit of the joint term structure model. First the data sources as well as the choice of pricing factors are discussed. Next, estimation results for the model including the model fit for both the Treasury and the TIPS curve are presented. Finally, results for the decomposition of nominal and real Treasury yields into their expectations and risk premium components are analyzed.

3.1. Data and factor construction

Zero coupon bond yields from the Gurkaynak et al. (2007, 2010) datasets (GSW hereafter) form the basis of the empirical analysis and are available at a daily frequency on the Board of Governors of the Federal Reserve's research data page.³ The GSW real and nominal yield curves are based on fitted Nelson–Siegel–Svensson curves, the parameters of which are published along with the estimated zero coupon curve. These parameters are used to back out the cross-section of real and nominal zero-coupon yields for maturities up to 10 years for TIPS and Treasuries, using end-of-month values from 1999:01 to 2014:11 for a total of $T = 191$ monthly observations. In the estimation, a cross-section of $N_N = 11$ one-month excess

³ See <http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html> and <https://www.federalreserve.gov/pubs/feds/2008/200805/200805-abs.html>.

holding period returns for nominal Treasuries with maturities $n = 6, 12, 24, \dots, 120$ months and $N_R = 9$ excess returns on TIPS with maturities $n = 24, \dots, 120$ months is used.⁴ The effective federal funds rate is used as the nominal risk free rate. The price index Q_t used to calculate TIPS payouts is seasonally unadjusted CPI-U, which is available from the Bureau of Labor Statistics website.

As discussed in Section 2.2, liquidity considerations play an important role in the pricing of TIPS securities. To this end, a composite factor of relative TIPS liquidity is constructed by averaging two indicators. The first indicator is the average absolute TIPS yield curve fitting error from the Nelson–Siegel–Svensson model of Gurkaynak et al. (2010) obtained from the Board of Governors of the Federal Reserve.⁵ Because large fitting errors may imply stress in the market and investors' inability to take advantage of mispricing, this measure is a good proxy for time-variation of liquidity conditions in the TIPS market. Notably, the series shows a sharp spike in late 2008 after the failure of Lehman Brothers, which severely compromised market functioning for inflation-indexed securities for a period of time.

The second liquidity indicator is the 13-week moving average of the ratio of primary dealers' nominal Treasury transaction volumes relative to TIPS transaction volumes. This series is obtained from the Federal Reserve Bank of New York's FR2004 "Weekly Release of Primary Dealer Positions, Transactions, and Financing." This second indicator captures the liquidity of TIPS relative to nominal Treasuries in the first few years after the introduction of the TIPS program particularly well. To construct the composite liquidity indicator, both series are standardized before computing their equal weighted average. To ensure the positivity of the index the negative of the time series minimum is added to the index. Positivity of the index ensures that illiquidity can only result in higher, not lower, yields.

Following Adrian et al. (2013) and various other authors (e.g., Joslin et al., 2011; Wright, 2011), principal components are extracted from yields and used as pricing factors in the model. Specifically, two sets of principal components are used. First, K_N principal components are extracted from nominal Treasury yields of maturities $n = 3, \dots, 120$ months. Then additional factors are obtained as the first K_R principal components from the residuals of regressions of TIPS yields of maturities $n = 24, \dots, 120$ months on the K_N nominal principal components as well as the liquidity factor. This orthogonalization step is done so as to reduce the unconditional collinearity among the pricing factors. In sum, there are a total of $K = K_N + K_R + 1$ model factors. The next subsection discusses the choices of K_N and K_R .

The only financial market data that is used for the estimation are the zero coupon bond prices of TIPS and Treasury securities. While, in principle, inflation swaps represent an alternative source of market information on expected inflation, this analysis relies on TIPS and nominal Treasury securities as a gauge of market-based inflation compensation for the following reasons. First, TIPS were introduced in the late 1990s while inflation swaps started trading quite a bit later. Hence, using TIPS ensures a longer time span of data available. Second, while the amount of outstanding inflation swaps has been growing in recent years, volume is still comparatively low. For example, Fleming and Sporn (2013) report that, in 2010, there were just a little over 2 inflation swap trades per day, with an average trade size of U.S. \$25 million. Moreover, in their sample, trading activity was fairly strongly concentrated at the ten year tenor, potentially reducing the informativeness of inflation swaps at other maturities. Third, comparing liquidity in the TIPS and inflation swaps markets, Christensen and Gillan (2012) document that bid–ask spreads in zero-coupon inflation swap contracts were substantially higher and more volatile than those in (on-the-run) TIPS issues. Finally, as most investors typically seek protection against inflation risk, counterparties in the inflation swaps market (generally broker-dealers) need to be induced to take the other side and sell protection through inflation swaps. In order to hedge these positions, inflation sellers often take the opposite position in Treasury securities to eliminate their own inflation exposure. However, such hedging activities in the cash markets require balance sheet capacity and thus compensation for the opportunity cost of using the balance sheet to engage in other activities. As the marginal cost of balance sheet capacity changes over time, inflation swap spreads likely reflect a time-varying premium for this cost. In the TIPS market, the Treasury represents a large seller of inflation protection who is not subject to such costs.

3.2. Model selection and estimation

In order to price the time series and cross-section of nominal and real Treasury bonds with a high level of precision, one needs to use model factors that span the same space as that spanned by both sets of yields. It has often been documented (see, e.g., Scheinkman and Litterman, 1991; Garbade, 1996, Chapter 16) that the cross-sectional variation of nominal Treasury yields is almost fully captured by three principal components which are commonly referred to as level, slope, and curvature. For parsimony, $K_N = 3$ nominal pricing factors are chosen as the first three principal components of the nominal Treasury yield curve.⁶

⁴ We start at a maturity of two years because short maturities tend to be distorted by the "carry adjustment" due to the CPI indexation lag. In fact, GSW do not publish maturities below 24 months.

⁵ The absolute yield curve fitting errors first appeared as a measure of liquidity in Fleming (2000) and Hu et al. (2013) first used the measure in an asset pricing context.

⁶ Cochrane and Piazzesi (2005, 2008), Duffee (2011), Joslin et al. (2014), Adrian et al. (2013) and others show that factors with negligible contemporaneous effects on the yield curve can have strong predictive power for future excess returns. Consistent with that literature, Section 3.3 presents findings that the additional factors in our joint model significantly affect excess returns on nominal bonds.

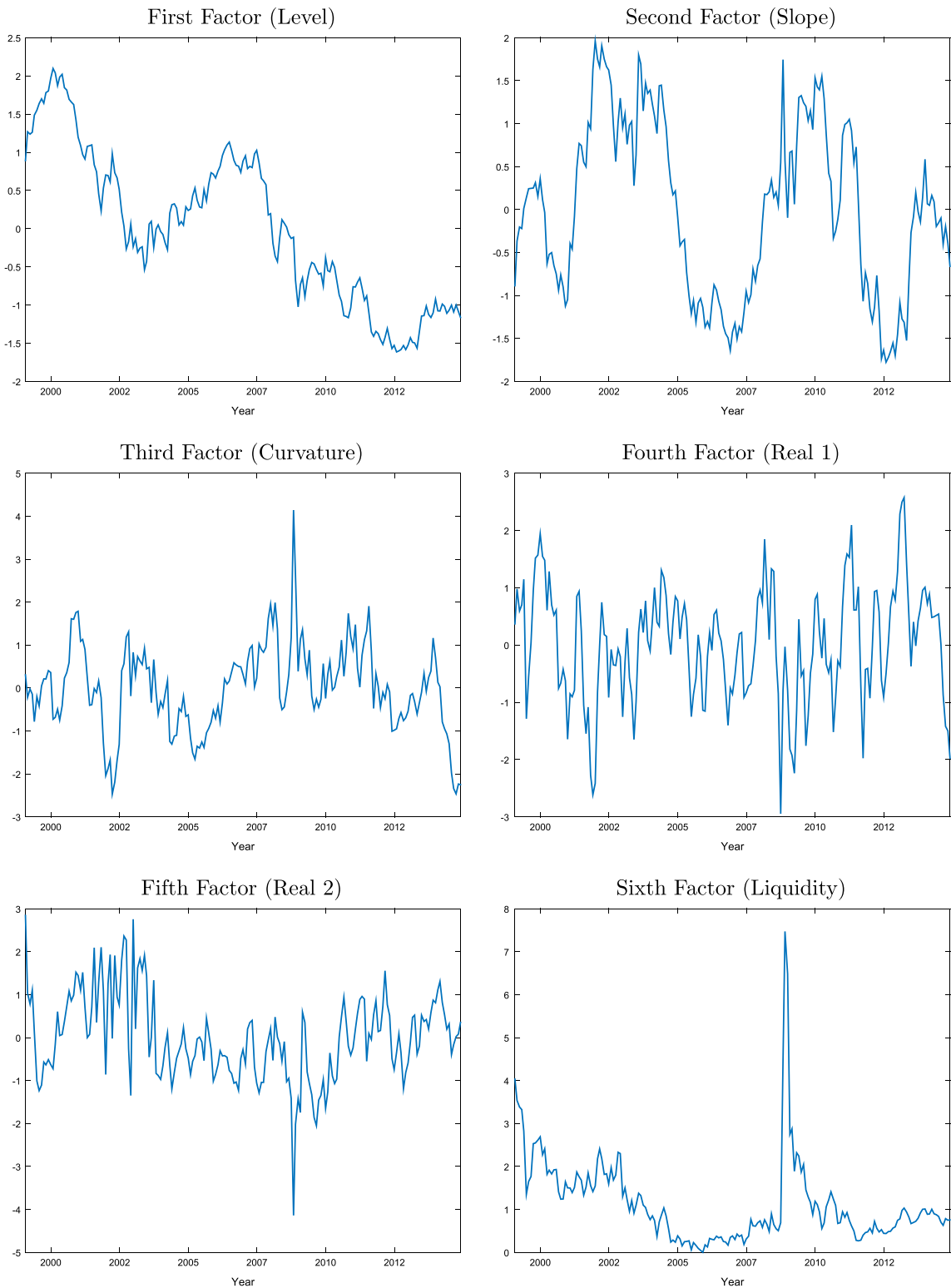


Fig. 2. Pricing factors: observed time series. This figure plots the time series of the factors of our model. These are the first three principal components extracted from the cross-section of end-of-month observations of nominal Treasury yields of maturities $n = 3, \dots, 120$ months. The fourth and fifth factors are the first two principal components extracted from the cross-section of orthogonalized real yields of maturities $n = 24, \dots, 120$ months, the residuals from regressing real yields on the first three principal components of the nominal Treasury yield curve as well as the liquidity factor. The sixth factor is the liquidity factor described in [Section 3.1](#).

Table 1

Treasuries: fit diagnostics. This table summarizes the time series properties of the pricing errors implied by our benchmark model. “Mean”, “std”, “skew”, and “kurt” refer to the sample mean, standard deviation, skewness, and kurtosis of the errors; $\rho(1)$, $\rho(6)$ denote their autocorrelation coefficients of order one and six. Panel 1 reports properties of the yield pricing errors and Panel 2 reports properties of the excess return pricing errors. The sample period is 1999:01–2014:11.

	$n=12$	$n=24$	$n=36$	$n=60$	$n=84$	$n=120$
<i>Panel 1: Yield pricing errors</i>						
mean	0.010	0.010	−0.008	−0.005	0.012	−0.021
std	0.028	0.047	0.024	0.024	0.020	0.040
skew	0.774	−0.907	−0.999	0.939	0.913	−1.062
kurt	6.190	4.554	5.424	4.574	5.406	5.899
$\rho(1)$	0.618	0.759	0.824	0.754	0.846	0.841
$\rho(6)$	0.279	0.313	0.374	0.306	0.400	0.385
<i>Panel 2: Return pricing errors</i>						
mean	0.041	−0.006	−0.041	0.023	0.009	−0.192
std	0.580	0.947	0.717	1.001	1.005	2.657
skew	−0.977	−0.463	−0.764	−0.393	−0.219	−0.230
kurt	4.980	5.738	5.715	5.582	6.642	9.034
$\rho(1)$	0.586	0.143	0.316	−0.007	0.173	0.042
$\rho(6)$	0.316	0.176	0.160	0.197	0.167	0.062

Relatively little is known about the factor structure of the joint cross section of TIPS and Treasury yields. While [Gurkaynak et al. \(2010\)](#) document that three principal components suffice to explain almost all of the cross-sectional variation in nominal yields, TIPS yields, or breakeven inflation rates separately, they do not study the extent to which the principal components extracted from the three sets of yields are cross-correlated. Moreover, they do not explicitly consider the role of liquidity as another source of comovement across TIPS yields. The number K_R of real pricing factors is chosen in the following simple way. First, TIPS yields are regressed on the liquidity factor and the first three principal components of the nominal yield curve. Then principal components from the residuals of these regressions are extracted. A variance decomposition shows that about 90 percent of the cross-sectional variation is explained by the first and that more than 99 percent are accounted for by the first two principal components. Therefore, we use $K_R = 2$ principal components extracted from these orthogonalized TIPS residuals as additional pricing factors. Combined with the liquidity factor and the three nominal factors, the model therefore has a total of six pricing factors. [Section 4.2](#) shows that this model does well at predicting inflation both in-sample and out-of-sample.

[Fig. 2](#) provides time series plots of all six model factors. The first three factors are the typical level, slope, and curvature of the nominal Treasury yield curve. The fourth and fifth factors represent the first two principal components extracted from the components of TIPS yields that are orthogonal to the nominal principal components and the liquidity factor. These two components do not have a clear-cut economic interpretation. The sixth factor is our composite liquidity indicator. As the chart shows, the illiquidity of the TIPS market relative to nominal Treasuries was high in the early years of the TIPS program, then receded markedly when the Treasury reaffirmed its commitment to the TIPS program, and shot up in the wake of the Lehman bankruptcy.

The model parameters are estimated using a maximum likelihood approach.⁷ Excess holding period Treasury and TIPS returns, along with the short rate and inflation, are assumed to be observed with additive Gaussian errors which are serially uncorrelated and independent of the innovations to the factors. This assumption is in contrast to the vast majority of the term structure literature which assumes serially uncorrelated measurement error in yields rather than returns.⁸ When maximizing the likelihood a set of nonlinear constraints is imposed on the parameters which ensure that model-implied factors (i.e., factors constructed as described above but based on fitted rather than observed yields) are equal to our observed factors. Using initial conditions based on closed-form estimators, estimation of this model via numerical optimization converges rapidly. See the Supplementary Appendix for full details.

3.3. Parameter estimates and model fit

This section shows that the preferred specification fits both yield curves precisely and gives rise to substantial time variation in bond risk premia. We first discuss specification tests for the choice of pricing factors by assessing the relative importance of each of the model factors in explaining cross-sectional variation of nominal Treasury returns, TIPS returns, and their joint cross-section. Table 6 in the Supplementary Appendix provides Wald statistics and their associated p -values for tests of whether the risk factor exposures associated with individual pricing factors are jointly different from zero. Not surprisingly, nominal Treasury returns are significantly exposed to all three principal components extracted from nominal Treasury yields. However, nominal Treasury returns do not comove with the two principal components extracted from

⁷ For an alternative approach to estimating the model see [Abrahams et al. \(2015\)](#).

⁸ See [Adrian et al. \(2013\)](#) for a full discussion of the merits of using returns rather than yields to estimate ATSMs.

Table 2

TIPS: fit diagnostics. This table summarizes the time series properties of the pricing errors implied by our benchmark model. “Mean”, “std”, “skew”, and “kurt” refer to the sample mean, standard deviation, skewness, and kurtosis of the errors; $\rho(1)$, $\rho(6)$ denote their autocorrelation coefficients of order one and six. Panel 1 reports properties of the yield pricing errors and Panel 2 reports properties of the excess return pricing errors. The sample period is 1999:01–2014:11.

	<i>n</i> =24	<i>n</i> =36	<i>n</i> =60	<i>n</i> =84	<i>n</i> =120
<i>Panel 1: Yield pricing errors</i>					
mean	0.021	−0.003	−0.005	0.009	−0.015
std	0.154	0.018	0.043	0.016	0.051
skew	−2.091	−0.237	0.780	−0.028	−0.474
kurt	10.304	3.507	4.208	3.345	3.220
$\rho(1)$	0.745	0.785	0.804	0.845	0.843
$\rho(6)$	0.403	0.476	0.445	0.551	0.516
<i>Panel 2: Return pricing errors</i>					
mean	−0.030	−0.049	0.025	0.020	−0.159
std	2.815	0.639	1.682	0.936	3.435
skew	0.132	−0.572	0.047	−0.120	0.134
kurt	11.213	4.569	5.811	3.500	4.786
$\rho(1)$	−0.091	0.316	−0.103	0.134	−0.142
$\rho(6)$	−0.022	0.079	−0.026	0.100	−0.161

orthogonalized TIPS yields and only little with the liquidity factor. Hence, the cross-sectional variation of nominal Treasuries in the model is largely captured by the nominal principal components. In contrast, TIPS returns comove strongly with innovations to all six pricing factors of the model, including the liquidity factor. Moreover, considering the joint cross-section of nominal Treasury and TIPS returns, all six model factors are associated with significant risk exposures.

We next assess the fit of the model for Treasury and TIPS yields. Table 1 reports the time series properties of the yield pricing errors for the nominal Treasury curve implied by the model. The average yield pricing errors are very small, not exceeding two and a half basis points in absolute value. They also exhibit little variability as the standard deviations of Treasury yield fitting errors are below five basis points across all maturities. Table 2 reports analogous results for yield and return fitting errors of TIPS. The model fits of TIPS yield about equally well as nominal Treasuries with a maximum average yield pricing error of about 2 basis points in absolute value across the maturity spectrum. The variability of TIPS pricing errors is also similar to that of nominal Treasuries, ranging from about two to five basis points with the exception being the two-year maturity which is a bit more volatile.

Figs. 3 and 4 provide a number of different visual diagnostics of the time series fit of the model, showing that both nominal and real Treasury yields and returns are precisely matched. This is also documented by the upper two panels of Figs. 8 and 9 (in the Supplementary Appendix) which demonstrate that the model fits the unconditional first and second moments of both yield curves very well.

The lower left panels of Figs. 8 and 9 provide plots of the estimated yield loadings $-\frac{1}{n}B_n$ for Treasuries and $-\frac{1}{n}B_{n,R}$ for TIPS. These provide an interpretation of the model factors according to their respective loadings on different sectors of both yield curves. In line with the previous literature, the first principal component clearly represents a level factor for both the Treasury and the TIPS term structure. Similarly, the second nominal principal component represents a slope factor featuring negative loadings on short maturities and positive loadings on long maturities in both curves. The charts further reveal that the two real factors as well as the liquidity factor have essentially no impact on nominal yields and affect TIPS yields somewhat more for intermediate than for longer maturities.

The lower right panels of both figures display the corresponding excess return loadings $B'_n\lambda_1$ for Treasuries and $B'_{n,R}\lambda_1$ for TIPS. These show that the slope factor is an important driver of expected excess returns on nominal Treasuries, consistent with the evidence presented in, e.g., Campbell and Shiller (1991) and Adrian et al. (2013). In addition, the second principal component extracted from the components of TIPS yields that are orthogonal to nominal yields and the liquidity factor also shows a strong impact on expected excess returns on nominal bonds. A look at the lower-right panel of Fig. 9 reveals that a somewhat different set of factors is largely responsible for the dynamics of expected excess returns on TIPS. In particular, the liquidity factor, the slope of the nominal yield curve as well as the first principal component from the orthogonalized TIPS curve are the most important drivers of expected excess returns on TIPS.

Given the pricing factors and the estimated model parameters, breakeven inflation rates at any horizon can be decomposed into its three constituents: expected inflation, the inflation risk premium, as well as a liquidity premium. Fig. 1 in the Introduction showed the time series of these components for both the ten year breakeven inflation as well as the 5–10 year forward breakeven inflation rate. These charts highlight the two main conclusions from the model. First, while expected inflation explains some of the variation of average inflation over the next ten years, it is stable at long forward horizons. This implies that the bulk of the variability of long-term forward breakeven inflation rates is driven by risk premia. Second, while liquidity effects have played only a minor role from around 2004 through the first half of 2008, they have strongly contributed to the dynamics of breakeven rates in the early years of the TIPS program and in the recent financial

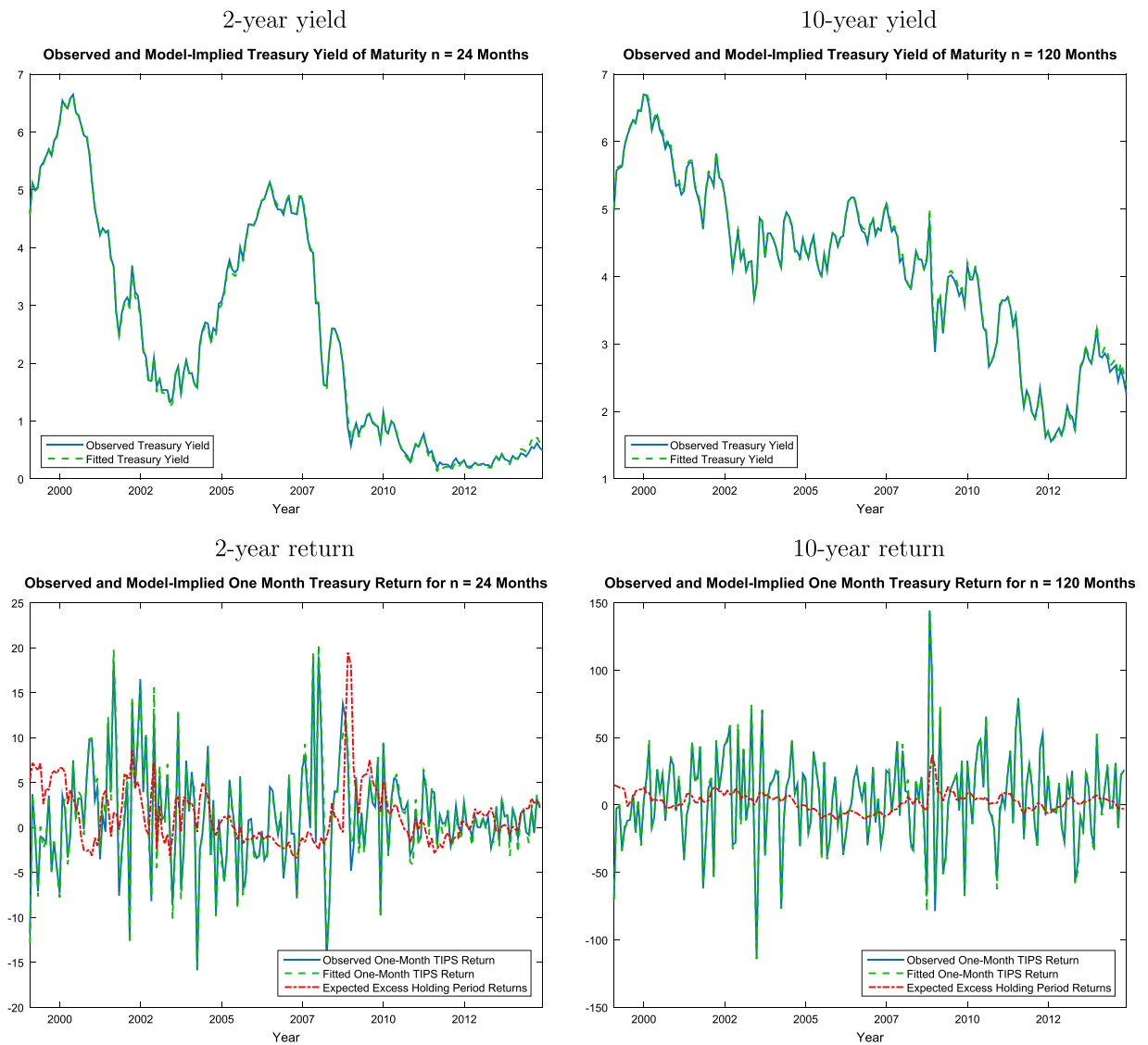


Fig. 3. Treasuries: observed and model-implied time series. This figure provides time series plots of observed and model-implied Treasury yields and excess returns. The upper panels plot zero-coupon Treasury yields at two-year and ten-year maturities and the bottom panels plot green excess holding period returns at two-year and ten-year maturities. The observed yields and returns are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields and returns. Dashed red lines in the lower panels are model-implied expected excess holding period returns.

crisis. In particular, the model largely attributes the collapse of long-term forward breakeven inflation rates in the crisis to liquidity effects rather than changes in underlying inflation expectations.

Fig. 5 provides a more detailed picture of term premia for the 5–10 year forward horizon. The upper-left chart shows the decomposition of the 5–10 year Treasury forward into risk-adjusted future nominal short rates, the nominal forward term premium and the liquidity premium. The latter is essentially zero, but featured a small uptick in the financial crisis. While the nominal 5–10 year forward rate had fallen to levels below three percent in 2011 and 2012 in response to the large-scale asset purchase programs by the Federal Reserve, it increased sharply in the spring and summer of 2013. This episode is sometimes referred to as the “taper tantrum” as the sudden rise in term premia and yields was largely attributed to speculation about a tapering of asset purchases by the Federal Reserve. More recently long-term nominal forward rates have declined substantially but remain above the low levels seen in 2012. The model indicates that the sudden rise and subsequent decline of long-term forward rates is primarily driven by movements in long-term nominal forward premia while expectations of long-term nominal short rates have remained steady around two percent.

The upper-right chart provides the equivalent decomposition of the 5–10 year TIPS yield into a risk-adjusted expected real short rate, the real term premium and the liquidity component of real yields. The liquidity component amounted to almost one percent at the beginning of our sample, but was close to zero between 2004 and mid-2008 before it spiked up to

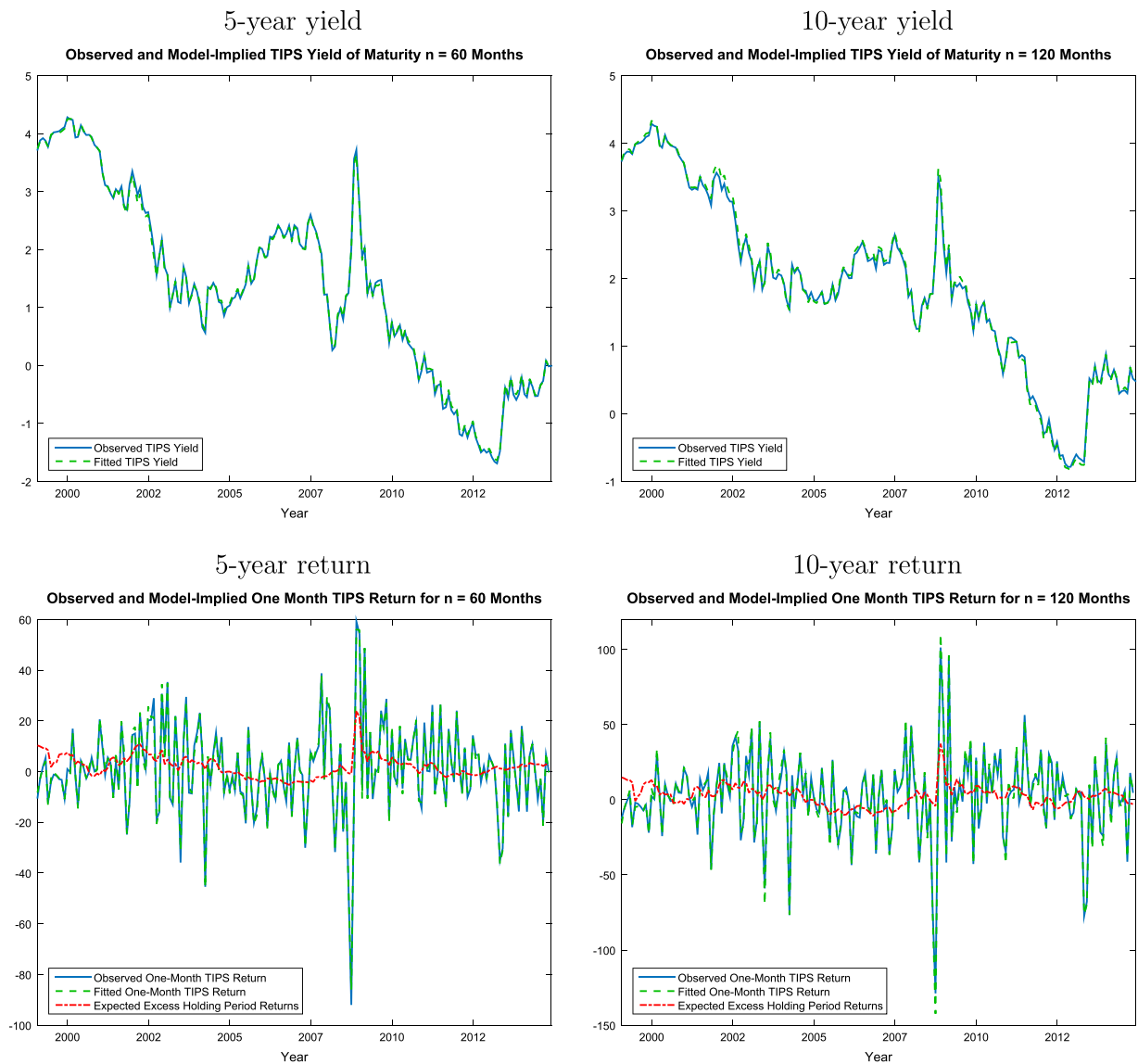


Fig. 4. TIPS: observed and model-implied time series. This figure provides time series plots of observed and model-implied TIPS yields and excess returns. The upper panels plot zero-coupon TIPS yields at five-year and ten-year maturities and the bottom panels plot excess holding period returns at five-year and ten-year maturities. The observed yields and returns are plotted by solid blue lines, whereas dashed green lines correspond to model-implied yields and returns. Dashed red lines in the lower panels are model-implied expected excess holding period returns.

about two percent in September 2008. The estimated real term premium declined from about three percent to around one percent in 2011 and 2012 and spiked up sharply during the taper tantrum episode in 2013. For much of the sample it is estimated to have been higher than the corresponding real forward rate, implying that risk-adjusted expected real short rates were negative at the 5–10-year forward horizon.

The lower-left chart in Fig. 5 superimposes the two estimated forward term premia, as well as their difference, the forward inflation risk premium. The chart shows that at low and intermediate frequencies real term premia account for the bulk of the variation in nominal term premia.⁹ Instead, inflation risk premia only capture a relatively small share of the variation in nominal term premia. Hence, according to our model the compensation investors require for bearing real interest rate risk—the risk that real short rates do not evolve as they expected—is the main driving force behind movements in nominal term premia.

⁹ This finding is consistent with the results in Crump et al. (2016) who obtain nominal and real term premia based on the difference between observed nominal and real yields and survey expectations of nominal and real short rates.

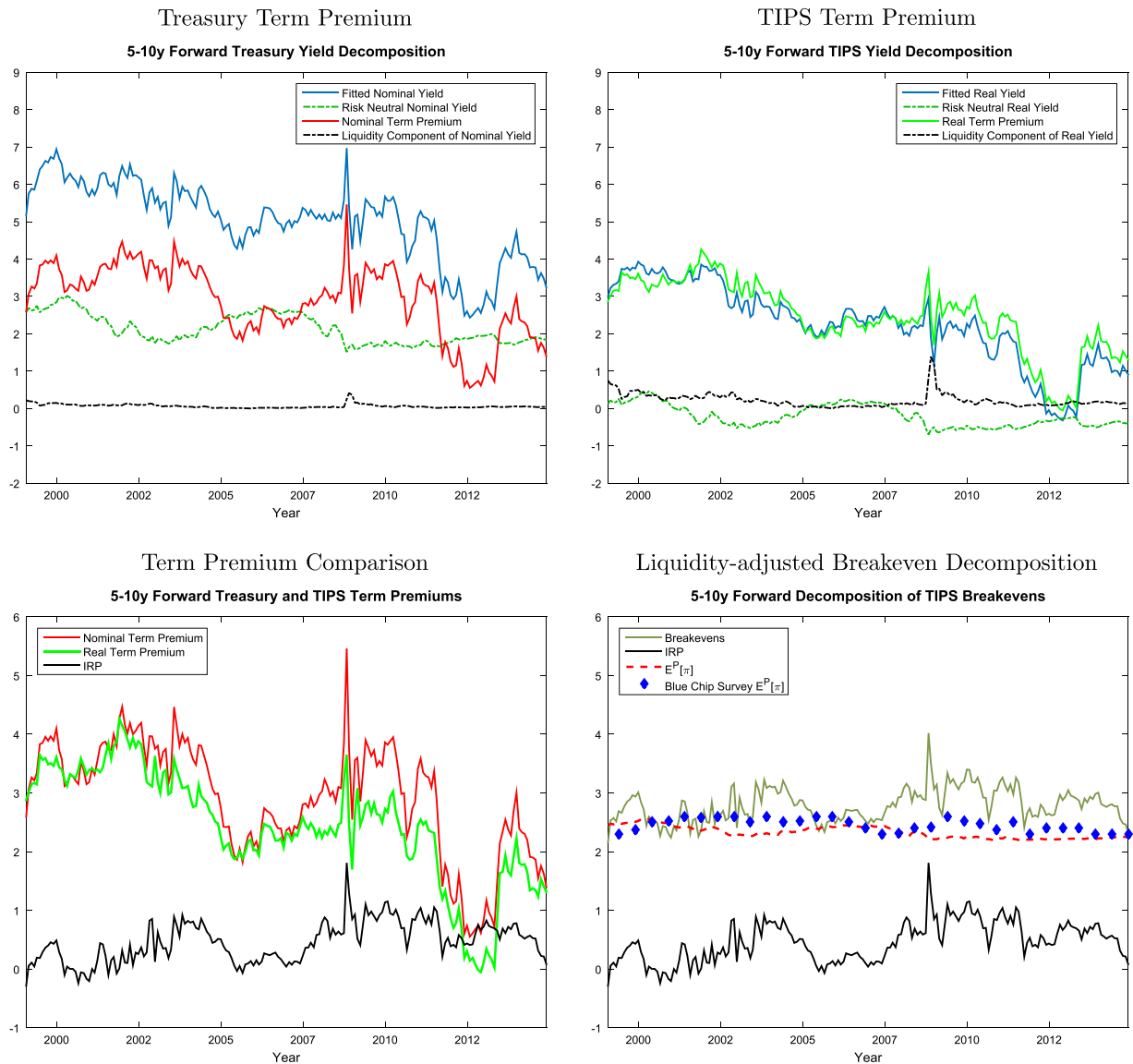


Fig. 5. Treasury and TIPS term premia. This figure provides plots of the Treasury term premium and TIPS term premium across maturities. The upper panels plot decompositions of 5–10 year forward Treasury and TIPS rates into the expected future short rates and the respective nominal and real term premium. A black dotted line represents the liquidity component of each series. The lower left panel plots the 5–10 year forward Treasury and TIPS term premia together. The difference between the measures is the inflation risk premium. The lower right panel plots the decomposition of liquidity-adjusted breakevens into expected inflation in red and the inflation risk premium in black.

Finally, the lower-right chart visualizes the decomposition of liquidity-adjusted 5–10 year forward breakevens into risk-adjusted expected average future inflation over the next five to ten years and the inflation risk premium. According to the model, the inflation risk premium accounts for most of the variability of forward breakeven rates while average expected future inflation has been quite stable between 2.1 and 2.5 percent and has only declined slightly in recent years. Importantly, the persistence of long-term inflation expectations implied by the model is consistent with corresponding survey measures of inflation expectations. For example, the 5–10 year forward consensus inflation forecast from the BCF survey which is observed twice a year and superimposed in the chart has also hovered around 2.5 percent since the late 1990s. In sum, the estimates implied by our model suggest that movements in long-term forwards mainly reflect changes in term premia rather than investors' expectations, and that risk-adjusted inflation expectations implied by our model are fully consistent with available survey data.

4. Studying the effects of monetary policy

In this section, we use our the model to extract quantities of interest to policymakers and market participants. First, the dynamics of the estimated inflation risk premium are compared to relevant financial and economic time series. It is then documented that breakeven inflation rates adjusted for risk premia provide a better predictor of the level of future inflation than observed breakevens. Next, the model decomposition is used to assess the reaction of the nominal and real forward curves to unexpected changes in monetary policy.¹⁰ Finally, the model is used to assess the channels through which large scale asset purchase programs by the Federal Reserve have impacted nominal and real Treasury yields.

4.1. Interpreting the estimated inflation risk premium

In the previous section it was documented that the inflation risk premium accounts for the predominant share of the variation in long-term breakeven forwards. It is thus interesting to study the economic driving forces of the inflation risk premium as well as of the nominal and real term premium.

All risk premia implied by the model are given by linear combinations of the model's pricing factors. Therefore, a direct economic interpretation of their dynamics is not possible. Instead, the estimated risk premia are correlated with a number of observable macroeconomic and financial variables the choices of which are motivated by economic theory. In particular, the following variables are considered: (1) the three-month swaption implied Treasury volatility from Merrill Lynch (SMOVE); (2) the cross-sectional standard deviation of individual inflation forecasts four quarters ahead from the BCFF survey (DISAG); (3) consumer confidence as measured by the Conference Board survey (CONF); (4) the unemployment rate (UNEMP); (5) the nominal trade-weighted exchange value of the U.S. Dollar (DOLLAR); and (5) year-over-year core CPI inflation (CPI).

Fig. 6 plots the estimated 5–10 year inflation risk premium along with each of these series. The top left chart shows that the inflation risk premium (or “IRP” in short) co-moves strongly with the SMOVE index at medium frequencies. This suggests that the expected volatility of Treasury securities at least to some extent reflects movements in the required compensation for bearing inflation risk demanded by investors. The top right chart documents that the IRP also correlates strongly with forecaster disagreement about future inflation. This implies that market participants command higher inflation compensation at times when there is broad disagreement about the inflation outlook. The estimated IRP also exhibits positive co-movement with the unemployment rate and negative co-movement with consumer confidence (middle two graphs). This is consistent with the notion that risk premia are countercyclical which likely also explains the negative relationship between the IRP and core inflation shown in the bottom right. Finally, there is a marginally negative relationship between the IRP and the trade-weighted exchange value of the dollar.

In sum, this analysis shows that the estimated inflation risk premia have an economically meaningful interpretation. As they are quite volatile, however, one may find it difficult to reconcile them with time-varying risk premia implied by consumption-based asset pricing models. That said, the tight linkage between risk premia and volatility could suggest that intermediary asset pricing theories which feature value-at-risk constraints may be a more promising economic foundation, as such theories directly link the pricing of risk to the equilibrium level of volatility (e.g., [Adrian and Shin, 2010b](#); [Adrian and Boyarchenko, 2012](#)).

4.2. Inflation forecasting

Breakeven inflation rates are the primary market based measure of inflation expectations and are therefore of considerable interest to policymakers and market participants alike. However, as discussed before, breakeven inflation dynamics may be influenced by changes in bond market liquidity as well as inflation risk premia. Since the model allows the separation of these effects, it is natural to assess its ability to precisely capture risk-adjusted inflation expectations by using it to predict future inflation. We compute model-implied average expected inflation over the next 6, 12, 24 and 36 months as described in [Section 2.1](#) and contrast these predictions with those implied by actual observed zero-coupon breakeven inflation rates as well as those resulting from a simple random walk forecast for monthly CPI inflation.

[Table 3](#) documents the in-sample (upper panel) and out-of-sample (lower panel) predictive power in terms of root mean squared errors (RMSE). In-sample, the model forecast strongly outperforms the unadjusted breakevens at all horizons, with the relative improvement being of the order of 30–40 percent at the two and three year forecast horizons, respectively.¹¹ Adjusted breakevens also outperform a simple random walk forecast substantially across all forecast horizons, with gains ranging from about 30 percent at the one year to about 35 percent at the three year horizon.

The out-of-sample inflation prediction performance is assessed by recursively reestimating the model each year beginning in January 2004. The results, shown in the lower panel of [Table 3](#), document that the risk-adjusted breakevens

¹⁰ The methodology is also used to analyze the real and nominal term structures of U.K. gilt yields. The results are provided in the Supplementary Appendix.

¹¹ Note that the actual breakeven inflation rates at the six and twelve month horizon are subject to substantial volatility resulting from issues related to the fitting of zero coupon TIPS yields by the Nelson–Siegel–Svensson methodology. Consequently, unadjusted breakeven forecasts at these horizons are omitted.

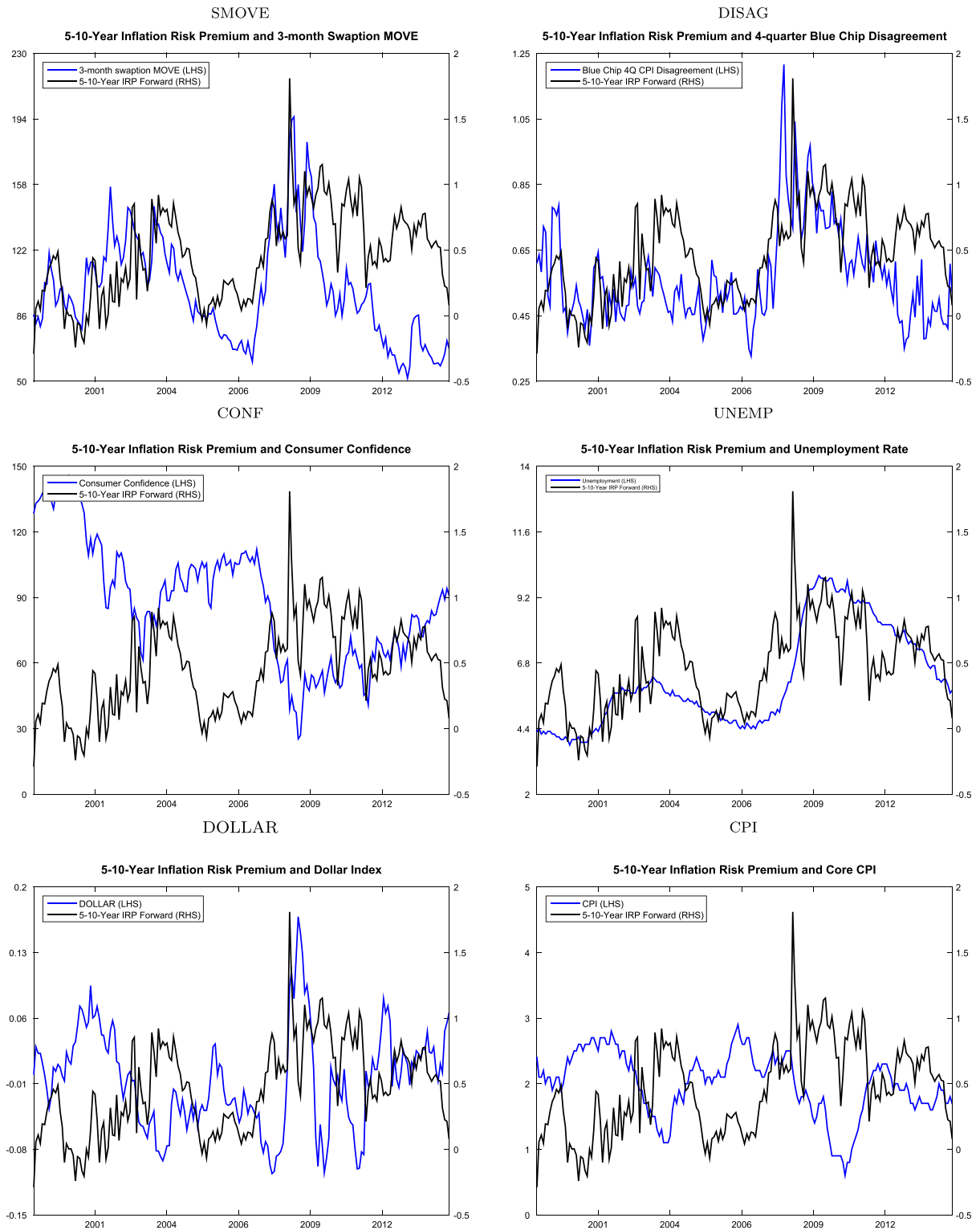


Fig. 6. Interpretation of inflation risk premium. This figure plots the 5–10 year forward inflation risk premium generated by our model against several observable variables. We consider (1) the three-month swaption implied Treasury volatility from Merrill Lynch (SMOVE); (2) the cross-sectional standard deviation of individual inflation forecasts four quarters ahead from the Blue Chip Financial Forecasts survey (DISAG); (3) the unemployment rate (UNEMP); (4) consumer confidence as measured by the Conference Board survey (CONF); (5) year-over-year core CPI inflation (CPI); and (6) the nominal trade-weighted exchange value of the U.S. Dollar (DOLLAR).

Table 3

Inflation forecasting. This table compares the root mean squared error of three models for predicting future inflation. The first uses the model-implied inflation expectations derived in Section 2.1. These represent breakeven inflation rates adjusted for liquidity and risk premia. The second method takes unadjusted TIPS breakevens as a predictor of future inflation. The third is a simple random walk forecast, i.e., it takes average realized inflation over the prior n months as a prediction of average inflation over the next n months. Forecasts are performed over horizons from 6 to 36 months, and forecasting errors are computed using overlapping observations. The first panel reports in-sample results for the full sample from 1999:01 to 2014:11. The second panel reports out-of-sample results, using a five-year “learning period,” forecasting over the period 2004:01–2014:11.

	6m	12m	24m	36m
<i>Panel 1: In sample</i>				
Model forecast	2.428	1.411	0.883	0.667
Breakevens			1.246	1.137
Random walk forecast	4.054	1.972	1.340	1.026
<i>Panel 2: Out of sample</i>				
Model forecast	2.986	1.841	1.330	1.133
Breakevens			1.379	1.198
Random walk forecast	4.643	2.210	1.437	1.100

implied by the model continue to outperform the random walk at the six-month to two-year horizons, with RMSE gains of 7–35 percent, while at the three-year horizon the random walk does slightly better. More importantly, the model also beats the unadjusted breakevens with forecast improvements of the order of 5 percent. In sum, the results in this section show that using the model to adjust breakeven inflation for liquidity and inflation risk premia increases their predictive value for future realized inflation.

4.3. Assessing the effects of conventional monetary policy

The model allows for the decomposition of changes in the nominal and real term structures into expected future real short rates, expected future inflation, real term premia, inflation risk premia and liquidity premia. The model can then parse out the differential effects of policy changes on the various components of the yield curve. While estimation of the model is done at the monthly frequency, daily decompositions can be obtained because the factors are observed at the daily frequency (see Adrian et al., 2013 for further details).¹²

This section considers the effects of surprise changes in conventional U.S. monetary policy. Until late 2008 those consisted primarily in adjusting the target federal funds rate. We thus use the surprise component of federal funds rate futures on FOMC announcement days to measure these surprises (see, e.g., Kuttner, 2001; Bernanke and Kuttner, 2005).¹³ Other studies have used alternative proxies of monetary policy shocks. Nakamura and Steinsson (2013) estimate a significant response of real yields to near-term monetary policy surprises using the heteroskedasticity-based identification approach proposed by Rigobon (2003) and Rigobon and Sack (2004). In another paper, Hanson and Stein (2014) use the two-day change of the two-year nominal Treasury yield on days of announcements by the Federal Open Market Committee (FOMC) as a gauge of near and medium term monetary policy surprises. They find that long-term real forward rates show a sizable reaction to this measure of monetary policy shocks and conjecture that this is driven by movements in term premia rather than changes in long-term expected real forward rates.

In the analysis, we regress changes in the various components of the one-year forward curve onto the Kuttner policy shock measure on days of FOMC announcements. The sample period is restricted to 1999:01–2008:09 in order to capture the effects of surprises about conventional monetary policy and to avoid conceptual problems with the surprise measure in the zero lower bound period at the end of our sample. The results are provided in Fig. 7. The upper two charts show the regression coefficients of liquidity-adjusted nominal and real one-year forward rates on the policy shocks along with the 95 percent confidence interval. At the 2–3 year forward horizon, both nominal and real yields feature large and statistically significant responses to the policy shock of about 60–80 basis points in reaction to a 100 basis point surprise change in the federal funds rate. Interestingly, while the response of nominal forward rates declines strongly with increasing maturity, real forward rates show sizable and significant reactions to policy shocks even at very long maturities of above ten years. These results are quantitatively and qualitatively consistent with Hanson and Stein (2014) despite the use of a different policy shock measure and a somewhat different sample period.

Which components of forward rates account for these strong reactions to policy shocks? This question can be addressed using the model-based decompositions. The middle and bottom panels of Fig. 7 display the decomposition of these forward rates into expectations and risk premium components. The middle panel shows that a substantial portion of the real forward responses at maturities up to about five to six years is due to changes in risk-adjusted expected future real short rates. Hence, our model indicates relatively persistent real effects of monetary policy surprises, consistent with Nakamura and Steinsson (2013). At the same time, the strong response of the real term premium to monetary policy shocks at medium and

¹² Because the relative volume component of the liquidity factor is available at a weekly frequency, it is assumed to be constant throughout each week.

¹³ We use the daily rate changes implied by the first (and second) Federal funds futures contract in the first two-thirds (last third) of the month.

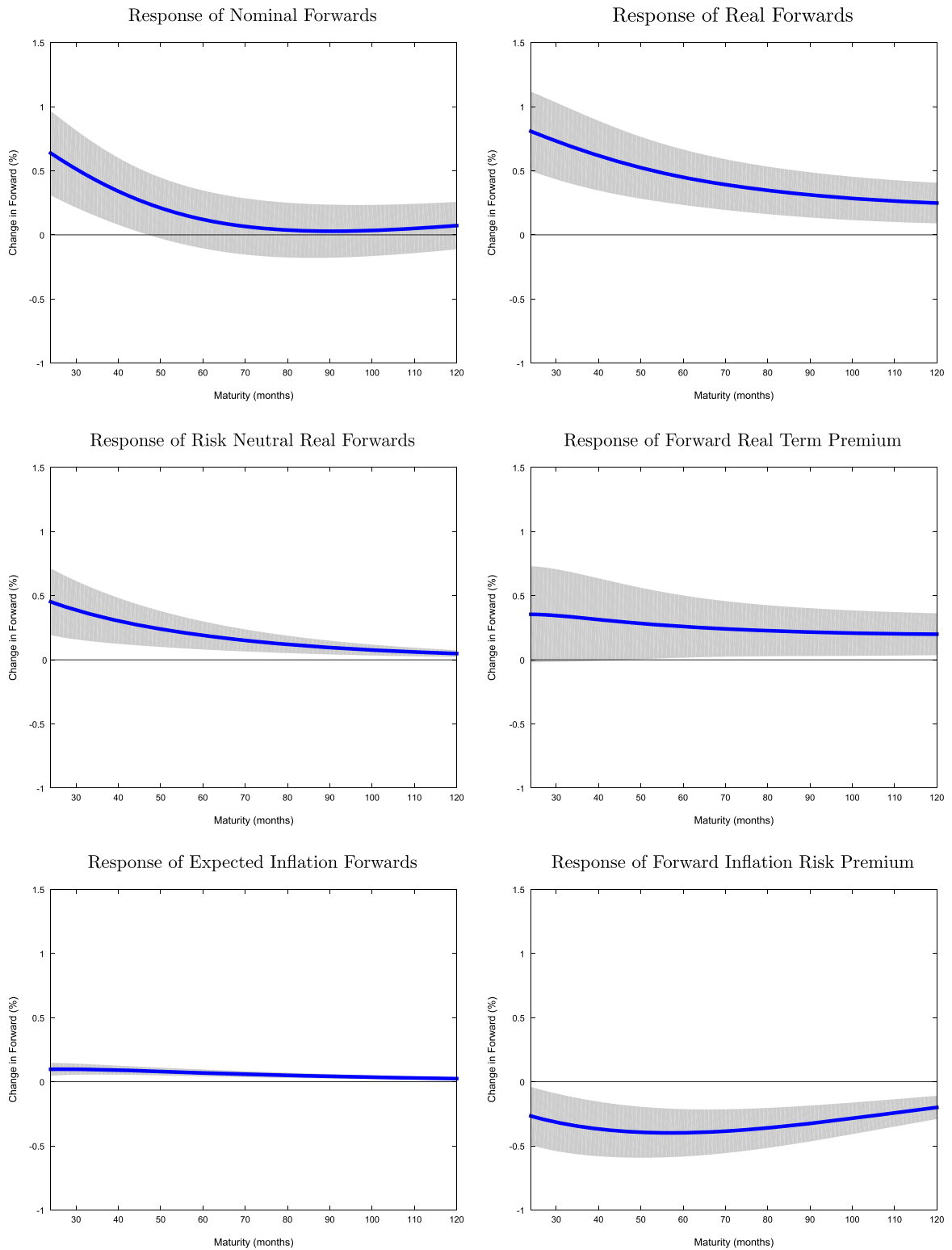


Fig. 7. Long-term forward regressions: fed funds shocks. This figure plots the coefficient $\beta^{(n)}$ from the equation $\Delta f_t^{(n)} = \alpha^{(n)} + \beta^{(n)} z_t + \epsilon_t^{(n)}$, where $\Delta f_t^{(n)}$ is a one-day change in forwards and z_t is a fed funds shock around FOMC announcement days. 95 percent confidence intervals are given by the gray bands. The sample period is 1999:01–2008:09.

longer maturities appears consistent with the “risk taking channel” of monetary policy, which implies that an important transmission channel of interest rate policy is via the equilibrium pricing of risk in the economy (see [Borio and Zhu, 2012](#); [Adrian and Shin, 2010a](#); [Dell’Ariccia et al., 2014](#)). In fact, according to our model longer-term real Treasury forwards are primarily affected by monetary policy shocks through a repricing of real interest rate risk.

The responses of risk-adjusted average forward inflation expectations and the inflation risk-premium to the two shocks are provided in the bottom panels of [Fig. 7](#). They show a statistically significant but economically negligible increase of inflation expectations, but a sizable negative response of the inflation risk premium, particularly at intermediate horizons. Hence, a surprise increase in the federal funds rate compresses the compensation investors require to bear inflation risk, especially in the medium term. This also explains why intermediate maturity nominal forward rates react substantially less strongly to policy shocks than their corresponding real counterparts. The negative response of the inflation risk premium to monetary policy surprises is consistent with structural models of the term structure in which the representative agent’s marginal utility comoves with the payoffs of government bonds (see, e.g., [Piazzesi and Schneider, 2007](#); [Rudebusch and Swanson, 2012](#)). To see this, note that disinflationary shocks increase the real payoff of nominal bonds. When output growth and inflation are positively correlated, nominal bonds thus act as a hedge against bad times making them attractive investments that risk-averse investors would pay a premium to hold. As monetary policy shocks tend to increase the covariance between real consumption growth and inflation, they should compress the inflation risk premium, consistent with our empirical results. A similar argument has also been made in [Campbell et al. \(2013\)](#).

4.4. Assessing the effects of unconventional monetary policy

Beginning in early 2009 the Federal Reserve instituted a series of purchases of longer-maturity Treasury securities. These are commonly referred to as large-scale asset purchases (LSAPs). In addition, in the latter half of 2011, the Federal Reserve also carried out a maturity extension program (MEP) which consisted of purchases of longer-maturity nominal Treasury securities and sales of shorter-maturity nominal Treasury securities. These operations, alongside forward guidance about the future path of monetary policy, were a major part of the Federal Reserve’s response to the recent financial crisis once the federal funds rate was pushed close to the zero lower bound.

A number of recent papers have provided estimates of the market impact of these purchases (and sales) using an event study methodology (see, for example, [Gagnon et al., 2011](#); [Krishnamurthy and Vissing-Jorgensen, 2011](#)). A similar approach is taken here, relying on the model estimates to decompose the nominal and real yield curves into their constituent components and show their one-day changes around days of major Federal Reserve announcements on LSAPs and the MEP. These dates are taken from Appendix A of [Woodford \(2012\)](#) and are comprised of eight days pertaining to LSAP1, four days pertaining to LSAP2, and two days pertaining to MEP announcements. In addition, two days related to LSAP3 announcements are also included (September 13 and December 12, 2012).

The effects of these announcements on the components of real and nominal Treasury yields for maturities of two and ten years are studied. The one-day changes in yields and their components are regressed on announcement date dummy variables which take on the value of one for the dates chosen. The reported coefficients are rescaled so they can be interpreted as the cumulated change in each component on the set of LSAP announcement dates.

[Table 4](#) displays the results for the two-year maturity. In total, the two-year nominal and real yields declined by about 70 and 110 basis points, respectively, on the 16 days relating to these balance sheet actions. Decomposing these changes, it can be seen that the two-year nominal and real term premia dropped more sharply on these days, by a total of about 81 basis points and 147 percentage points, respectively. Consequently, the inflation risk premium—the difference between these two series—rose by 65 basis points. Meanwhile, risk-adjusted expected average nominal and real short-term interest rates over the following two years rose only modestly and by similar magnitudes around these announcement dates. Accordingly, model-implied average expected inflation over the next two years was essentially unchanged by the announcements.

Table 4

LSAP event 1-day change regressions: 2-year, yield decompositions. The upper panel gives a regression estimate of the coefficient β and corresponding t -statistics from the equation $\Delta \tilde{y}_t^{(2)} = \beta \mathbb{I}(\text{LSAP}_t) + \varepsilon_t^{(2)}$ where $\Delta \tilde{y}_t^{(2)}$ is the one-day change in the two-year yield or component of the two-year yield and $\mathbb{I}(\text{LSAP}_t)$ is an indicator function for an LSAP announcement date. The lower panel gives the coefficients from the regression $\Delta \tilde{y}_t^{(2)} = \beta_1 \mathbb{I}(\text{LSAP1}_t) + \beta_2 \mathbb{I}(\text{LSAP2}_t) + \beta_{\text{MEP}} \mathbb{I}(\text{MEP}_t) + \beta_3 \mathbb{I}(\text{LSAP3}_t) + \varepsilon_t^{(2)}$. Reported results are product of estimated coefficient and number of event dates. t -Statistics are provided in parentheses.

	y_N	TP_N	$E[\tilde{y}]$	y_R	TP_R	$E[r]$	Liq^R	BEI	IRP	$E[\pi]$	Liq^{BEI}
<i>Panel 1: All LSAP events</i>											
LSAP	−0.72	−0.81	0.08	−1.10	−1.47	0.10	0.27	0.38	0.65	−0.02	−0.26
(t -stat)	(−3.22)	(−6.46)	(0.37)	(−3.33)	(−6.03)	(0.50)	(1.37)	(1.24)	(5.04)	(−0.06)	(−1.37)
<i>Panel 2: Individual LSAP events</i>											
LSAP 1	−0.76	−0.63	−0.13	−0.81	−1.14	0.05	0.28	0.06	0.50	−0.18	−0.27
LSAP 2	−0.04	−0.08	0.03	−0.16	−0.20	0.04	−0.00	0.12	0.12	−0.00	0.00
MEP	0.10	−0.03	0.13	0.10	0.03	0.10	−0.02	−0.01	−0.06	0.03	0.02
LSAP 3	−0.02	−0.07	0.05	−0.23	−0.16	−0.09	0.02	0.21	0.09	0.14	−0.02

Table 5 shows the same set of results for the ten-year maturity. The nominal and real ten-year Treasury yields cumulatively fell by about 100 and 140 basis points on these announcement days, respectively. Nominal and real term premia, however, both declined by 114 and 175 basis points on all announcement days. Hence, the yield declines are more than accounted for by declines of real term premia which were only partly offset by an increase in the inflation risk premium. According to our estimates, the purchase programs thus primarily achieved their effects on yields through a reduction of uncertainty about future real rate prospects. Consistent with this interpretation, both the risk-adjusted nominal and real expected average future short rates over the next ten years increased somewhat on these days. Note that our finding that expected average nominal and real short rates rose on days of purchase program announcements are in contrast with the results of Bauer and Rudebusch (2014) who argue that the yield effects of purchases are due to a signaling effect about future monetary policy. Instead, our findings are consistent with the notion that the reduction in duration risk leads to a compression in term premia with the bulk accounted for by a reduction in real term premia.

These results are also consistent with those obtained for conventional monetary policy shocks discussed in the previous section. They suggest that the risk premium channel was a key transmission mechanism not only for changes in the federal funds target before the zero lower bound was attained but also for the various purchase programs conducted by the Federal Reserve since 2009. Gagnon et al. (2011) find a similar result in their study of the effects of LSAP1. While these authors only consider nominal term premia, our results suggest that the purchase programs reduced real term premia by more than nominal term premia, consequently increasing inflation risk premia.

5. Related literature

The literature extracting inflation expectations from the joint pricing of TIPS and Treasury yield curves is growing rapidly. Early papers on the topic include Chen et al. (2010), Grishchenko and Huang (2013), and Hördahl and Tristani (2010).

A number of papers have pointed out that TIPS have been less liquid than off the run Treasury securities until about 2003 (see Sack and Elsasser, 2004; Dudley et al., 2009) which generated a liquidity premium in breakeven inflation. Pflueger and Viceira (2016) document that the liquidity of TIPS relative to Treasuries appears to be systematically priced. D'Amico et al. (2014) model this type of liquidity as a latent factor in an ATSM. In contrast, we use an observable factor to adjust TIPS yields for their relative liquidity.

Christensen et al. (2010) report estimates from an ATSM model with three nominal factors (level, slope and curvature) and one real factor (level). Their model is parsimonious, as the prices of risk are restricted so as to be consistent with a Nelson and Siegel (1987) yield curve (see Christensen et al., 2011 for the relation between the ATSM models and the Nelson–Siegel curve). Andreasen et al. (2016) present a model of the TIPS liquidity premium that combines an arbitrage free model of individual TIPS securities and a zero coupon Treasury yield curve. This approach gives rise to a liquidity premium that is comparable to ours in magnitude and time variation.

Haubrich et al. (2012) present a model that uses inflation swaps, actual inflation, and survey inflation in addition to the TIPS and Treasury yield curves. Similar to an earlier paper by Adrian and Wu (2009), Haubrich et al. (2012) allow for heteroskedasticity explicitly by estimating a GARCH model for the yield processes. Prices of risk are restricted to be functions of these estimated second moments. While the model of Haubrich et al. (2012) combines the different data sources elegantly, the resulting inflation risk premium differs sharply from our estimates. In fact, the inflation risk premium is close to constant over time, implying that movements of far in the future breakeven forward rates reflect changes in inflation expectations. In contrast, in our model, the inflation risk premium varies substantially over time, while far in the future

Table 5

LSAP event 1-day change regressions: 10-year, yield decompositions. The upper panel gives a regression estimate of the coefficient β and corresponding t -statistics from the equation $\Delta \tilde{y}_t^{(10)} = \beta \mathbb{I}(\text{LSAP}_t) + \varepsilon_t^{(10)}$ where $\Delta \tilde{y}_t^{(10)}$ is the one-day change in the ten-year yield or component of the ten-year yield and $\mathbb{I}(\text{LSAP}_t)$ is an indicator function for an LSAP announcement date. The lower panel gives the coefficients from the regression $\Delta \tilde{y}_t^{(10)} = \beta_1 \mathbb{I}(\text{LSAP1}_t) + \beta_2 \mathbb{I}(\text{LSAP2}_t) + \beta_{\text{MEP}} \mathbb{I}(\text{MEP}_t) + \beta_3 \mathbb{I}(\text{LSAP3}_t) + \varepsilon_t^{(10)}$. Reported results are product of estimated coefficient and number of event dates. t -Statistics are provided in parentheses.

	y_N	TP_N	$E[\tilde{r}]$	y_R	TP_R	$E[r]$	Liq^R	BEI	IRP	$E[\pi]$	Liq^{BEI}
<i>Panel 1: All LSAP events</i>											
LSAP	−1.03	−1.14	0.11	−1.42	−1.75	0.21	0.12	0.39	0.61	−0.10	−0.12
(t -stat)	(−4.14)	(−5.09)	(0.89)	(−7.36)	(−7.15)	(2.25)	(1.37)	(2.56)	(3.95)	(−1.50)	(−1.37)
<i>Panel 2: Individual LSAP events</i>											
LSAP 1	−0.97	−0.95	−0.02	−1.16	−1.39	0.10	0.13	0.19	0.43	−0.12	−0.12
LSAP 2	−0.01	−0.03	0.02	−0.17	−0.20	0.03	−0.00	0.16	0.17	−0.01	0.00
MEP	−0.06	−0.13	0.08	−0.03	−0.08	0.06	−0.01	−0.03	−0.05	0.01	0.01
LSAP 3	0.01	−0.02	0.03	−0.07	−0.09	0.01	0.01	0.08	0.06	0.03	−0.01

expected inflation is essentially constant. We view our finding as a desirable feature, implying that variations in 5–10 year forward breakevens mainly reflect changes in risk and liquidity premia—consistent with the survey evidence.

Chernov and Mueller (2012) present an ATSM model where a “hidden factor” is extracted from inflation surveys. They show that this hidden inflation survey factor is a significant price of risk factor. While we do not incorporate any survey inflation expectations, our framework would allow the introduction of their hidden factor as an unspanned factor in a straightforward manner. Such unspanned factors would affect the pricing of risk, but not the cross sectional fit of the yield curve (i.e., it would change the \mathbb{P} -dynamics, but not the \mathbb{Q} -dynamics).

Haubrich et al. (2012) and Grishchenko and Huang (2013) also incorporate survey inflation expectations in their estimates of the inflation risk premium. However, those papers consider the inflation forecasts as true probability assessments, while Chernov and Mueller (2012) consider the forecasts of inflation to be subjective and possibly different from the ATSM implied inflation estimate. In contrast, we do not use any survey information in our estimation, but find model-implied inflation expectations that track survey forecasts closely. Gospodinov and Wei (2016) incorporate inflation swaps and oil future prices in an affine model of the TIPS and Treasury term structure and report improved out of sample performance.

A number of recent papers (Grishchenko et al., 2012; Christensen et al., 2016; Kitsul and Wright, 2013; Fleckenstein et al., 2014) have used the fact that final payouts on TIPS include an embedded option¹⁴ to extract risk-neutral deflation probability forecasts. In unreported results we follow a similar approach as in Grishchenko et al. (2012) and find that with our parameter estimates the value of the option is generally small but was sizable during the financial crisis.

6. Conclusion

We present a joint Gaussian affine term structure model for the cross section of TIPS and nominal Treasury securities that has a number of desirable features relative to the existing literature. We explicitly adjust for the relative illiquidity of TIPS by using observable liquidity indicators in a model-consistent way. Relative to other models in the literature, our pricing errors are small, providing decompositions of breakeven inflation into an inflation risk premium, expected inflation, and a liquidity premium with little fitting error. Importantly, we find that the volatility of far into the future forward breakeven inflation rates is primarily due to variations in risk and liquidity premia, while long-term inflation expectations are relatively stable.

The estimated inflation risk premium is highly correlated with observable macroeconomic and financial variables such as disagreement about future inflation among professional forecasters, consumer confidence and measures of option-implied Treasury volatility. Furthermore, breakevens adjusted for risk and liquidity premia outperform unadjusted breakevens and a random walk in predicting realized inflation in-sample and out-of-sample. Using the estimates to investigate the impact of the Federal Reserve's large-scale asset purchases on the term structure of interest rates, we find that their primary effect is attributable to a decline in real term premia while they resulted in smaller increases of the inflation risk premium and risk neutral real and nominal short rates. These results are consistent with the “duration risk” and the “preferred habitat” channels of asset purchases, but not with the “signaling channel.” In addition, surprises about conventional interest rate policy are associated with statistically and economically significant changes in the real term premium and the inflation risk premium. In sum, these results provide strong support for the notion that an important transmission channel of monetary policy is via the equilibrium pricing of risk in the economy.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at <http://dx.doi.org/10.1016/j.jmoneco.2016.10.006>.

¹⁴ At maturity, the value of the bond will be the greater of the nominal principal (\$100,000) or the principal adjusted for cumulative CPI-U inflation since issuance. TIPS coupon payments are not subject to this optionality.

References

- Abrahams, M., Adrian, T., Crump, R.K., Moench, E., 2015. Decomposing Real and Nominal Yield Curves. Staff Report 570, Federal Reserve Bank of New York.
- Adrian, T., Boyarchenko, N., 2012. Intermediary Leverage Cycles and Financial Stability. Staff Report 567, Federal Reserve Bank of New York.
- Adrian, T., Crump, R.K., Moench, E., 2013. Pricing the term structure with linear regressions. *J. Financ. Econ.* 110 (1), 110–138.
- Adrian, T., Shin, H.S., 2010a. Financial intermediaries and monetary economics. In: *Handbook of Monetary Economics*, vol. 3A. Elsevier, North-Holland, pp. 601–650 (Chapter 12).
- Adrian, T., Shin, H.S., 2010b. Liquidity and leverage. *J. Financ. Intermed.* 19 (3), 418–437.
- Adrian, T., Wu, H., 2009. The Term Structure of Inflation Expectations. Staff Report 362, Federal Reserve Bank of New York.
- Andreasen, M., Christensen, J., Cook, K., Riddell, S., 2016. The TIPS Liquidity Premium. Federal Reserve Bank of San Francisco Working Paper.
- Ang, A., Piazzesi, M., 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *J. Monet. Econ.* 50 (May (4)), 745–787.
- Bauer, M.D., Rudebusch, G.D., 2014. The signaling channel for Federal Reserve bond purchases. *Int. J. Cent. Bank.* 10 (September (3)), 233–289.
- Bernanke, B.S., Kuttner, K.N., 2005. What explains the stock market's reaction to Federal Reserve policy? *J. Financ.* 60 (3), 1221–1257.
- Borio, C., Zhu, H., 2012. Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism? *J. Financ. Stud.* 8 (1), 236–251.
- Campbell, J.Y., Shiller, R.J., 1991. Yield spreads and interest rate movements: a bird's eye view. *Rev. Econ. Stud.* 58 (3), 495–514.
- Campbell, J.Y., Shiller, R.J., Viceira, L.M., 2009. Understanding inflation-indexed bond markets. *Brook. Pap. Econ. Act.* 40 (1), 79–138.
- Campbell, J.Y., Sunderam, A., Viceira, L.M., 2013. Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds. Working Paper 09-088, Harvard Business School, January.
- Chen, R.-R., Liu, B., Cheng, X., 2010. Inflation, Fisher equation, and the term structure of inflation risk premia: theory and evidence from TIPS. *J. Empir. Financ.* 17 (4), 702–721.
- Chernov, M., Mueller, P., 2012. The term structure of inflation expectations. *J. Financ. Econ.* 106 (2), 367–394.
- Christensen, J.H., Diebold, F.X., Rudebusch, G.D., 2011. The affine arbitrage-free class of Nelson–Siegel term structure models. *J. Econom.* 164 (1), 4–20.
- Christensen, J.H., Gillan, J.M., 2012. Could the U.S. Treasury benefit from issuing more TIPS? Working Paper 2011–16, Federal Reserve Bank of San Francisco.
- Christensen, J.H.E., Lopez, J.A., Rudebusch, G.D., 2010. Inflation expectations and risk premiums in an arbitrage-free model of nominal and real bond yields. *J. Money Credit Bank.* 42 (1), 143–178.
- Christensen, J.H.E., Lopez, J.A., Rudebusch, G.D., 2016. Pricing deflation risk with U.S. Treasury yields. *Rev. Financ.* 20 (3), 1107–1152.
- Cochrane, J., Piazzesi, M., 2005. Bond risk premia. *Am. Econ. Rev.* 95 (1), 138–160.
- Cochrane, J., Piazzesi, M., 2008. Decomposing the Yield Curve. Working Paper.
- Crump, R.K., Eusepi, S., Moench, E., 2016. The Term Structure of Expectations and Bond Yields. Staff Report 775, Federal Reserve Bank of New York.
- D'Amico, S., Kim, D.H., Wei, M., 2014. Tips from TIPS: The Informational Content of Treasury Inflation-Protected Security Prices. Finance and Economics Discussion Series 2014-024, Board of Governors of the Federal Reserve.
- Dell'Ariccia, G., Laeven, L., Marquez, R., 2014. Real interest rates, leverage, and bank risk-taking. *J. Econ. Theory* 149, 65–99.
- Dudley, W.C., Roush, J., Ezer, M.S., 2009. The case for TIPS: an examination of the costs and benefits. *Econ. Pol. Rev.* 15 (1), 1–17.
- Duffee, G., 2002. Term premia and interest rate forecasts in affine models. *J. Financ.* 57 (1), 405–443.
- Duffee, G.R., 2011. Information in (and not in) the term structure. *Rev. Financ. Stud.* 24 (9), 2895–2934.
- Fleckenstein, M., Longstaff, F.A., Lustig, H., 2014. Deflation Risk. Working Paper.
- Fleming, M.J., 2000. The benchmark U.S. Treasury market: recent performance and possible alternatives. *Econ. Policy Rev.* 6 (1), 129–145.
- Fleming, M.J., Sporn, J.R., 2013. Trading activity and price transparency in the inflation swap market. *Econ. Policy Rev.* 19 (1), 45–57.
- Fontaine, J.-S., Garcia, R., 2012. Bond liquidity premia. *Rev. Financ. Stud.* 25 (4), 1207–1254.
- Gagnon, J., Raskin, M., Remache, J., Sack, B., 2011. Large-scale asset purchases by the Federal Reserve: did they work? *Econ. Policy Rev.* 17 (1), 41–59.
- Garbade, K., 1996. *Fixed Income Analytics*. MIT Press, Cambridge.
- Gospodinov, N., Wei, B., 2016. Forecasts of Inflation and Interest Rates in No-Arbitrage Affine Models. Working Paper 2016-3, Federal Reserve Bank of Atlanta.
- Grishchenko, O.V., Huang, J., 2013. The inflation risk premium: evidence from the TIPS market. *J. Fixed Income* 22 (4), 5–30.
- Grishchenko, O.V., Vanden, J.M., Zhang, J., 2012. The Information Content of the Embedded Deflation Option in TIPS. Finance and Economics Discussion Series 2013-24, Board of Governors of the Federal Reserve.
- Gurkaynak, R.S., Sack, B., Wright, J.H., 2007. The U.S. Treasury yield curve: 1961 to the present. *J. Monet. Econ.* 54 (8), 2291–2304.
- Gurkaynak, R.S., Sack, B., Wright, J.H., 2010. The TIPS yield curve and inflation compensation. *Am. Econ. J.: Macroecon.* 2 (1), 70–92.
- Hanson, S.G., Stein, J.C., 2014. Monetary policy and long-term real rates. *J. Financ. Econ.* 115 (3), 429–448.
- Haubrich, J., Pennacchi, G., Ritchken, P., 2012. Inflation expectations, real rates, and risk premia: evidence from inflation swaps. *Rev. Financ. Stud.* 25 (5), 1588–1629.
- Hördahl, P., Tristani, O., 2010. Inflation Risk Premia in the U.S. and the Euro Area. Technical Report 325, Bank for International Settlements Working Paper.
- Hu, X., Pan, J., Wang, J., 2013. Noise as information for illiquidity. *J. Financ.* 68 (6), 2341–2382.
- Joslin, S., Priebsch, M., Singleton, K.J., 2014. Risk premiums in dynamic term structure models with unspanned macro risks. *J. Financ.* 69 (3), 1197–1233.
- Joslin, S., Singleton, K.J., Zhu, H., 2011. A new perspective on Gaussian dynamic term structure models. *Rev. Financ. Stud.* 24 (3), 926–970.
- Kitsul, Y., Wright, J.H., 2013. The economics of options-implied inflation probability density functions. *J. Financ. Econ.* 110 (3), 696–711.
- Krishnamurthy, A., Vissing-Jorgensen, A., 2011. The effects of quantitative easing on interest rates: channels and implications for policy. *Brook. Pap. Econ. Act.* 43 (2), 215–287.
- Kuttner, K.N., 2001. Monetary policy surprises and interest rates: evidence from the fed funds futures market. *J. Monet. Econ.* 47 (3), 523–544.
- Nakamura, E., Steinsson, J., 2013. High Frequency Identification of Monetary Non-Neutrality. Working Paper.
- Nelson, C.R., Siegel, A.F., 1987. Parsimonious modeling of yield curves. *J. Bus.* 60 (4), 473–489.
- Pflueger, C.E., Viceira, L.M., 2016. Return predictability in the Treasury market: real rates, inflation, and liquidity. In: Veronesi, P. (Ed.), *Handbook of Fixed-Income Securities*. Wiley, Hoboken, N.J., pp. 191–209 (Chapter 10).
- Piazzesi, M., 2010. Affine term structure models. In: Ait-Sahalia, Y., Hansen, L.P. (Eds.), *Handbook of Financial Econometrics*, vol. 1. Elsevier, Oxford, UK, pp. 691–766 (Chapter 12).
- Piazzesi, M., Schneider, M., 2007. Equilibrium yield curves. In: Acemoglu, D., Rogoff, K., Woodford, M. (Eds.), *NBER Macroeconomics Annual 2006*, vol. 21. National Bureau of Economic Research, pp. 389–472 (Chapter 6).
- Rigobon, R., 2003. Identification through heteroskedasticity. *Rev. Econ. Stat.* 85 (4), 777–792.
- Rigobon, R., Sack, B., 2004. The impact of monetary policy on asset prices. *J. Monet. Econ.* 51 (8), 1553–1575.
- Rudebusch, G.D., Swanson, E.T., 2012. The bond premium in a DSGE model with long-run real and nominal risks. *Am. Econ. J.: Macroecon.* 4 (January (1)), 105–143.
- Sack, B., Elsas, R., 2004. Treasury inflation-indexed debt: a review of the U.S. experience. *Econ. Policy Rev.* 10 (1), 47–63.
- Scheinman, J.A., Litterman, R., 1991. Common factors affecting bond returns. *J. Fixed Income* 1 (1), 54–61.
- Singleton, K.J., 2006. *Empirical Dynamic Asset Pricing*. Princeton University Press, Princeton, New Jersey.
- Woodford, M., 2012. Methods of policy accommodation at the interest-rate lower bound. In: 2012 Federal Reserve Bank of Kansas City Jackson Hole Symposium.
- Wright, J.H., 2011. Term premia and inflation uncertainty: empirical evidence from an international panel dataset. *Am. Econ. Rev.* 101 (4), 1514–1534.