

Dissolving the Scandal of Propositional Logic?

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In his very interesting response¹ to my lecture² on the scandal of propositional logic Jeroen Valk argues that what I have identified as a deep scandal for propositional logic is actually not a scandal at all. Valk maintains that it can be sufficiently explained “while retaining modern logic’s supremacy” by (i) pointing out how *material implication* in propositional logic should be interpreted, and (ii) by choosing a more natural formalization of the relevant statements than the one I have offered in my lecture. Although I do find Valk’s analysis intriguing and helpful, his alleged dissolution of the scandal fails. Or so I shall argue below. Let me first revisit what I take to be the scandal by considering these two statements in natural language:

- [1] If Brigitte has yellow paint and blue paint, then Brigitte can mix green,
- [2] Brigitte can mix green with yellow paint or Brigitte can mix green with blue paint.

Clearly, no one wants to hold that [2] is a logical consequence of [1]. The argument having [1] as premise and [2] as conclusion should definitely be logically invalid. Now, the *logical form* of this argument is to be formally rendered in the calculus of propositional logic as follows:

- [1*] $P \wedge Q \rightarrow R$,
- [2*] $(P \rightarrow R) \vee (Q \rightarrow R)$.

Here P = “Brigitte has yellow paint”, Q = “Brigitte has blue paint” and R = “Brigitte can mix green”. As I show in my lecture, the calculus of propositional logic does in fact render the argument form [1*]-[2*] and thus the argument [1]-[2] logically valid. This is how I arrived at the scandal of propositional logic. After all, it is certainly a real scandal that the calculus of propositional logic forces us to accept argument [1]-[2] as a valid logical consequence.

Valk points out that on the above formalization of [1]-[2] it should come as no surprise to us that [1]-[2] is logically valid. For it is straightforward to see that [1*]-[2*] must be logically valid if we take into account that the truth table of material implication (\rightarrow) is defined in such a way that $A \rightarrow B$ is false if and only if A is true and B is false. In other words, the material implication is to be understood as expressing *merely* a connection of dependence between A and B . On this thin *adjunctive interpretation* of material implication it is indeed no wonder that [1*]-[2*] and therefore [1]-[2] is logically valid. It is precisely what we expect.

¹ Valk, “Propositional Paradoxes” on composix.blogspot.nl

² Rutten, “Het schandaal van de propositie logica” on gjerutten.nl

Now, I do agree with Valk that the adjunctive interpretation³ of material implication forces us to accept that [1*]-[2*] and therefore [1]-[2] is logically valid. After all, this interpretation of material implication is directly linked to the characteristic truth table of material implication, which I invoke during my lecture to reveal the scandal. The problem however is that material implication was never introduced in propositional logic to capture merely some connection of dependence between two statements. It was introduced to capture the notion of *implication* or *logical consequence* in natural language. So having to rely on an adjunctive interpretation of material implication is not part of the solution. It is part of the problem. Or more apt: it is the problem. The deep scandal of propositional logic obtains precisely because the adjunctive interpretation of material implication fails hopelessly to capture the notion of implication or logical consequence of natural language. So, pointing out that in the above formalization of [1]-[2] material implication is to be interpreted adjunctively does not solve the scandal. Quite to the contrary, it points us to the scandal. Having to resort to an adjunctive interpretation of material implication reveals the scandal of propositional logic instead of dissolving it.

In my lecture I consider two attempts to dissolve the scandal. The first is to try to adjust the truth table of material implication. But this attempt fails, as I show in my lecture. Valk seems to agree here. The second is to point out that material implication $A \rightarrow B$ means actually *only* $\neg A \vee B$ so that it is no surprise at all that [1*]-[2*] and thus [1]-[2] is logically valid. In my lecture I conclude that this attempt fails as well. After all, pointing out that $A \rightarrow B$ means only $\neg A \vee B$ comes down to admitting that material implication deeply fails to capture the notion of implication or logical consequence of natural language. Now, this second attempt is in fact not that different from Valk's attempt above. For he similarly points out that $A \rightarrow B$ means *only* that there is a connection of dependence between A and B. By doing so he similarly admits that material implication does not capture natural language's notion of implication or logical consequence. But then I have in fact already refuted this attempt of Valk in my lecture.

However, he has a second more interesting strategy for trying to dissolve the scandal. Valk asserts that the above described formalization [1*]-[2*] is actually not the most natural one to formalize the argument [1]-[2]. He proposes to formalize [1]-[2] by using the metalanguage of predicate logic. His proposal then leads to the following formalization of [1]-[2]:

[1**] For all propositional formulas P, Q and R: If $P \wedge Q \rightarrow R$ is provable, then $P \rightarrow R$ is provable or $Q \rightarrow R$ is provable.

Since [1**] is false, Valk concludes that the scandal is dissolved. For he seems to hold that the falsehood of [1**] leads us to the desired result that [1]-[2] is logically invalid. Is [1**] indeed false? It surely is. Valk leaves it as an exercise to the reader to show why. Let me gently fulfill

³ Valk attributes this terminology in his response to Hans Reichenbach.

his request and complete this exercise. Let A and B be two propositional atoms. Further, let P=A, Q=¬A and R=B. It follows that $P \wedge Q \rightarrow R$ is provable since from a contradiction ($A \wedge \neg A$) everything (including B) follows. However, $P \rightarrow R$ is not provable since atom B cannot be derived from atom A. Similarly $Q \rightarrow R$ is not provable. So we have found three propositional formulas P, Q and R that make the antecedent of [1**] true and the consequent of [1**] false. These formulas therefore amount to a counterexample of [1**]. But then [1**] is indeed false.

Should we now conclude that, contrary to his first attempt, Valk's second attempt to dissolve the scandal succeeds? No. For I do not agree with Valk that [1**] is a proper formalization of [1]-[2]. For [1**] is much too strong. Is someone who asserts [1]-[2] really committed to the claim that for all formulas P, Q and R it is the case that if $P \wedge Q \rightarrow R$ is a tautology, $P \rightarrow R$ or $Q \rightarrow R$ is a tautology? It seems to me that this is not the case. This claim is much too strong to be a proper formalization of [1]-[2]. To see this, note that if [1**] is a formalization of [1]-[2], [1]-[2] is an *instance* of [1**]. The relevant instance of [1**] is the following statement:

[1***] If “If Brigitte has yellow paint and blue paint, then Brigitte can mix green” is provable, then “Brigitte can mix green with yellow paint” is provable or “Brigitte can mix green with blue paint” is provable.

But [1***] is not what someone who asserts [1]-[2] intends to say. He or she might readily accept that the statement “If Brigitte has yellow paint and blue paint, then Brigitte can mix green” is provable. For this seems indeed obviously true in all possible worlds. But from this it clearly does not follow that “Brigitte can mix green with yellow paint” is true in all possible worlds. It clearly does not follow that “Brigitte can mix green with blue paint” is true in all possible worlds either. The only thing that might follow is that the *disjunction* “Brigitte can mix green with yellow paint or Brigitte can mix green with blue paint” is true in all possible worlds. In fact, asserting that this is indeed the case if “If Brigitte has yellow paint and blue paint, then Brigitte can mix green” is true in all possible worlds is what is really meant by someone who asserts [1]-[2]. Hence [1***] is indeed too strong and therefore an *inadequate* formalization of [1]-[2]. Valk's second attempt to dissolve the scandal of propositional logic thus fails. Since his first attempt has been refuted above and in my previous lecture as well, I conclude that Valk does not succeed in “retaining the supremacy of modern logic”.