PHYSICAL HUMAN-ROBOT INTERACTION

Modeling DC motors

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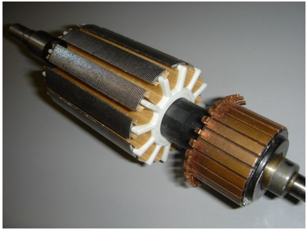
Outline

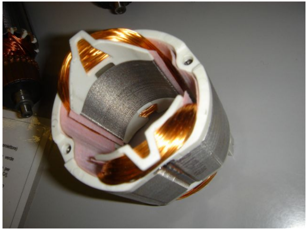
- Why DC motor?
- How a DC motor works
- Modeling
- Block diagram
- Motor, gear box and load

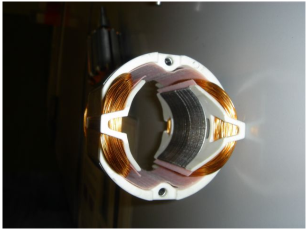
DC motor

Rotor (windings, commutator, shaft)



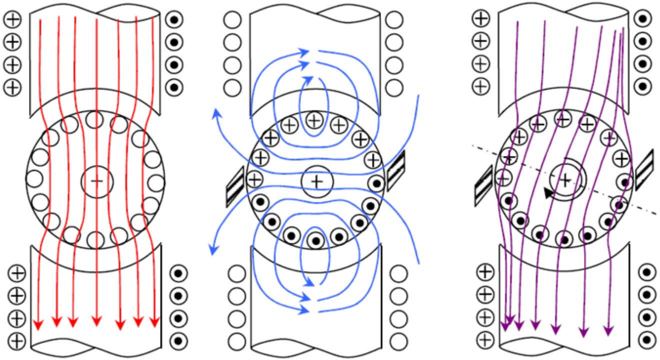
Stator (windings)





DC motor

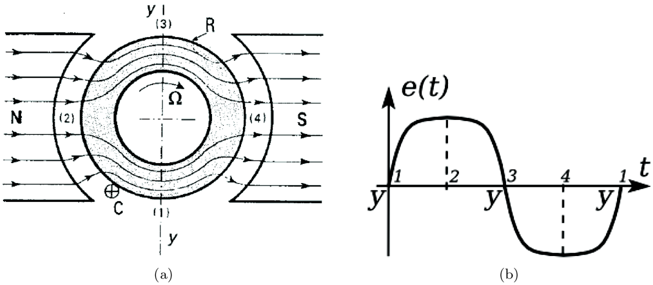
Permanent magnet



(Internet)

Red: magnetic field due to the rotor windings
Blue: magnetic field due to the stator windings
Purple: superposition of the two magnetic fields

Permanent magnet
A coil on the rotor
The rotor is moved externally

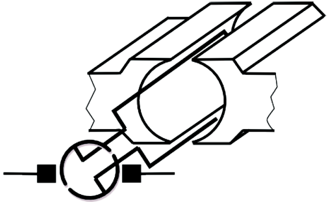


(Internet)

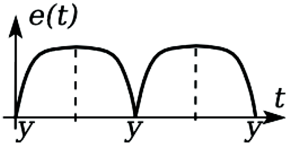
The stator field induces the *back electromotive force (back EMF) voltage* $e(t)$

How does it work?

Permanent magnet
A coil on the rotor
The rotor is moved externally
Commutator connected to the coil



(a)



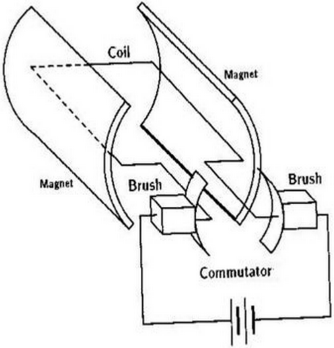
(b)

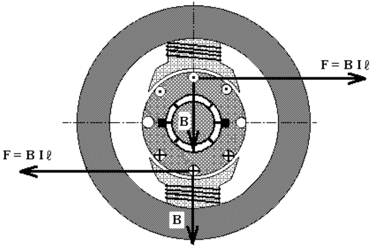
(Internet)

The stator field induces the *back electromotive force (back EMF) voltage* $e(t)$

How does it work?

What happens when the single coil is connected to a generator?
(the current in the rotor windings is not zero)

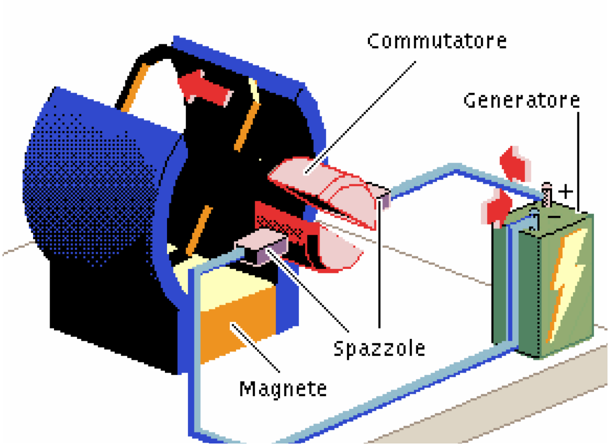




(Internet)

Equilibrium of forces (sum equal to zero)
Torques are not in equilibrium: *the coil rotates*

What happens when the single coil is connected to a generator?
(the current in the rotor windings is not zero)



(Internet)

What happens when the single coil is connected to a generator?
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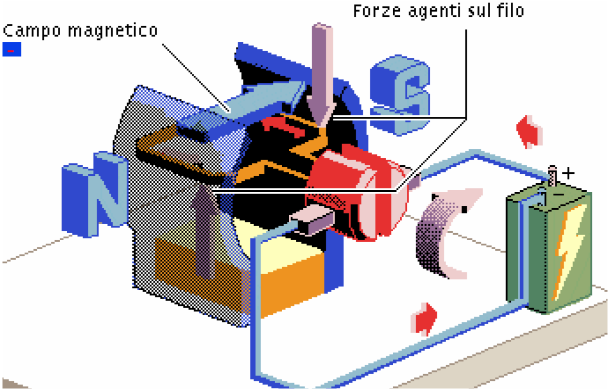
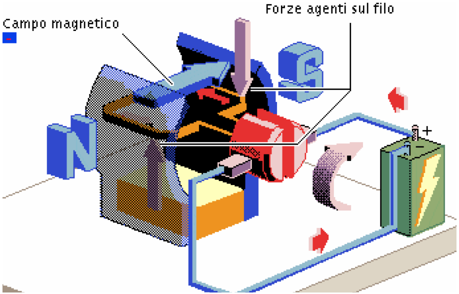


(Internet)

How does it work?



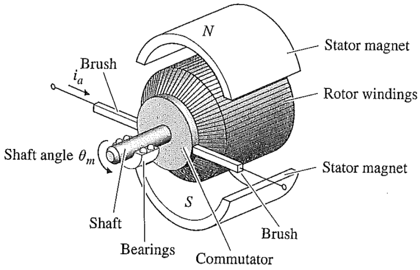
Torque:

$$\tau = I A \Phi \sin(\alpha)$$

I, current (rotor),
Φ, magnetic flux (stator),
$A \sin(\alpha)$, area

How does it work?

Permanent magnet DC motor ($\Phi =$ const.)



If the number if coils within the rotor windings is large and only one coil is connected at a time to the commutator, we have $\sin(\alpha) \simeq 1$ and

torque $\tau = K_m I$

where K_m is the torque constant (or mechanic constant of the motor)

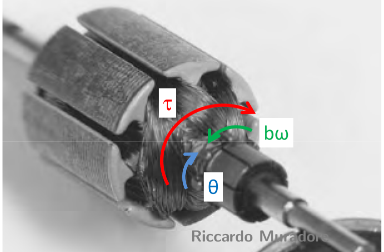
Assumption: no load

J: rotor inertia
b: viscous friction coefficient
θ: rotor angular position

The angular velocity $\dot{\theta}$ satisfies the differential equation

$$J\frac{d\dot{\theta}}{dt} + b\dot{\theta} = K_m I \qquad (J\ddot{\theta} + b\dot{\theta} = K_m I) \tag{1}$$

where the applied torque $\tau = K_m I$ is obtained as the product of the torque constant K_m and the rotor current I



Electrical equation

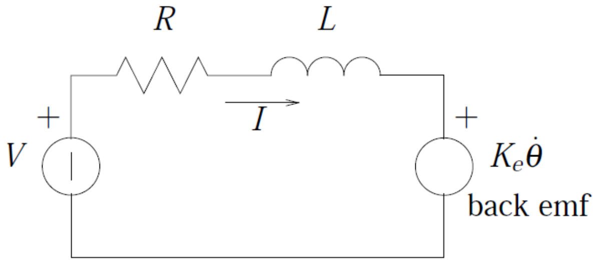
Assumption: no load

The current I satisfies the differential equation

$$L\dot{I} + RI = V - K_e\dot{\theta} \tag{2}$$

where R and L are the resistance and inductance of the electric windings, V is the input voltage and K_e is the electric constant.

The product $K_e\dot{\theta}$ is the back emf voltage.



Overall model

The overall model of a permanent magnet DC motor is given by the two coupled differential equations

$$
\begin{aligned}
J\ddot{\theta} + b\dot{\theta} &= K_m I && (3)\\
L\dot{I} + RI &= V - K_e\dot{\theta} && (4)
\end{aligned}
$$

Computing the Laplace transform, we end up with the following transfer function mapping input voltage into position

$$
P(s) = \frac{\hat{\theta}(s)}{\hat{V}(s)} = \frac{K_m}{s\left[(Js+b)(Ls+R)+K_mK_e\right]} \tag{5}
$$

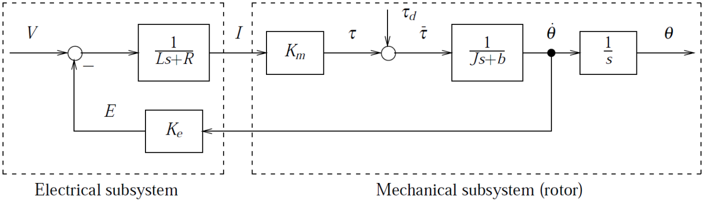
Remark The parameters are positive.

Block diagram

The coupled differential equations

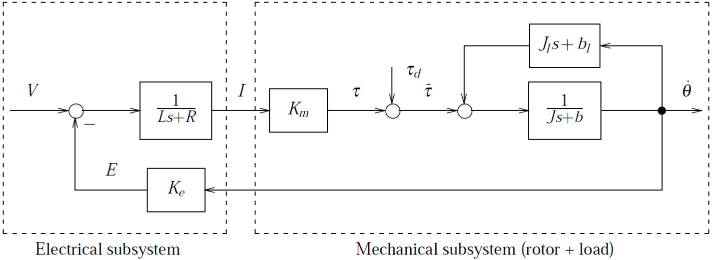
$$J\ddot{\theta} + b\dot{\theta} = K_m I \quad (6)$$
$$L\dot{I} + RI = V - K_e\dot{\theta} \quad (7)$$

describe the following block diagram

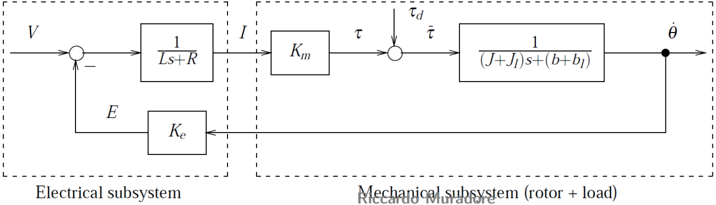


Block diagram with external load

What happens when there is an external load?



The previous block diagram is equivalent to the following one



Block diagram with gear box

What happens when there is a (ideal) gear box?

$$\theta_l = \frac{1}{N}\theta_r \tag{8}$$

(r: rotor, l: load)

The work is constant:

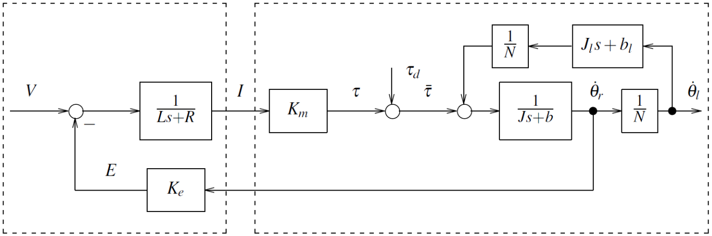
$$\tau_r\theta_r = \tau_l\theta_l \tag{9}$$

The power is constant:

$$\tau_r\dot{\theta}_r = \tau_l\dot{\theta}_l \tag{10}$$

When $N > 1$, the rotor speed is larger than the angular velocity after the gear box, whereas the motor torque is smaller than the torque after the gear box

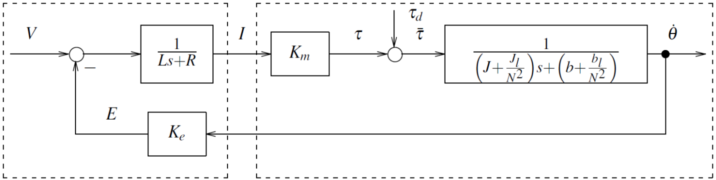
Block diagram with load and gear box



Electrical subsystem

Mechanical subsystem (rotor + load + gearbox)

Equivalent block diagram



Electrical subsystem

Mechanical subsystem (rotor + load + gearbox)

Block diagram with gear box

If N is large, the effect of the load on the motot is negligible

$$\frac{1}{\left(J + \frac{J_l}{N^2}\right) s + \left(b + \frac{b_l}{N^2}\right)}$$

From the Internet

How does it work?

Brushed DC motor
https://www.youtube.com/watch?v=LAtPHANEfQo

Brushless DC motor
https://www.youtube.com/watch?v=bCEiOnuODac