## Mean of binomial distribution

The mean of a binomial distribution for $N$ trials with probability of success $\mu$ is $N\mu$. This can be derived either by writing the binomial ditribution as a sum of Bernoulli random variables each with probability of success $\mu$ or by computing explicitly the sum

$$\sum_{k=0}^{N} k \binom{N}{k} \mu^k (1-\mu)^{N-k}$$

In this second case the derivation is as follows:

$$\begin{aligned}
\sum_{k=0}^{N} k \binom{N}{k} \mu^k (1-\mu)^{N-k} &= \sum_{k=1}^{N} k \binom{N}{k} \mu^k (1-\mu)^{N-k} \\
&= N \sum_{k=1}^{N} \binom{N-1}{k-1} \mu^k (1-\mu)^{N-k} \text{ [using } \binom{N}{k} = \frac{N}{k}\binom{N-1}{k-1}\text{]} \\
&= N \sum_{t=0}^{s+1} \mu \binom{s}{t} \mu^t (1-\mu)^{s-t} \text{ [with } s = N-1, t = k-1\text{]} \\
&= N\mu
\end{aligned}$$

A good list of properties for manipulating binomials is chapter 1 (p.55) of Knuth's TAOCP (Volume 1).