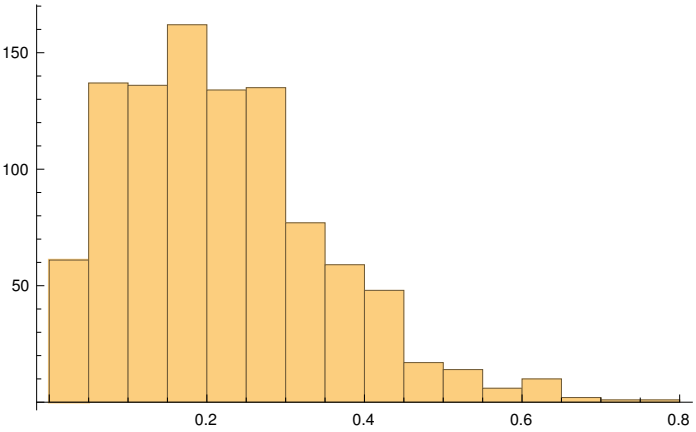
Finding the parameters a,b of a beta distribution via likelihood maximization

First we simulate from a beta with known parameters

In[40]:=
```
a = 2; b = 7;
betasim = Table[RandomVariate[BetaDistribution[a, b]], {i, 1, 1000}];
```

In[41]:=
```
Histogram[betasim]
```

Out[41]=



the likelihood can be written as:

In[42]:=
```
ll[alpha_, beta_] := Plus @@ (Map[Log[PDF[BetaDistribution[alpha, beta], #]] &, betasim])
```

one might try to maximize directly with

In[43]:=
```
Maximize[ll[alpha, beta], {alpha, beta}]
```

NMaximize : The function value −1003.57 − 3141.59 *i* is not a real number at {alpha, beta} = {−0.535769, −0.13703}.

Out[43]=

$$\text{Maximize}\left[\text{Log}\left[\frac{0.499063^{-1+\text{alpha}}\times 0.500937^{-1+\text{beta}}}{\text{Beta}[\text{alpha},\text{beta}]}\right]+\text{Log}\left[\frac{\cdots 1\cdots}{\text{Beta}[\text{alpha},\text{beta}]}\right]+\right.$$
$$\left.\cdots 996\cdots+\text{Log}[\cdots 1\cdots]+\text{Log}\left[\frac{0.0017676^{-1+\text{alpha}}\cdots 1\cdots}{\text{Beta}[\text{alpha},\text{beta}]}\right],\ \cdots 1\cdots\right]$$

large output | show less | show more | show all | set size limit...

but this would not work well because alpha and beta must be bigger than 0. Hence the correct optimization is

In[44]:=
```
Maximize[ll[alpha, beta], alpha > 0 && beta > 0, {alpha, beta}]
```

Out[44]=
```
{703.514, {alpha → 1.98048, beta → 6.9965}}
```

Alternatively, instead of writing explicit boundaries in the **Maximize** call one can rewrite the likelihood in terms of two auxiliary variables (which are not bound to be positive) **alpha1=Log[alpha]** and **beta1=Log[beta]** as follows:

In[45]:=
```
ll2[alpha1_, beta1_] :=
 Plus @@ (Map[Log[PDF[BetaDistribution[Exp[alpha1], Exp[beta1]], #]] &, betasim])
```

In[46]:=
```
Maximize[ll2[alpha1, beta1] , {alpha1, beta1}]
```

Out[46]=
```
{703.514, {alpha1 → 0.683342, beta1 → 1.94541}}
```

In[50]:=
```
Exp[0.6833415077491054` ]
```

Out[50]=
```
1.98048
```

In[51]:=
```
Exp[1.9454095481839093` ]
```

Out[51]=
```
6.9965
```