

# Smart Beta and Beyond: Maximising the Benefits of Factor Investing

February 2018



with the support of



**EDHEC-RISK**  
Institute



# Table of Contents

---

Executive Summary .....	5
1. Introduction .....	13
2. A Taxonomy of Factors .....	17
3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process .....	33
4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia .....	51
5 Conclusions and Perspectives .....	85
Technical Appendix .....	89
References .....	97
About Amundi .....	107
About EDHEC-Risk Institute .....	109
EDHEC-Risk Institute Publications and Position Papers (2015-2018) .....	113

# About the Authors

---



**Lionel Martellini** is Professor of Finance at EDHEC Business School and Director of EDHEC-Risk Institute. He has graduate degrees in economics, statistics, and mathematics, as well as a PhD in finance from the University of California at Berkeley. Lionel is a member of the editorial board of the *Journal of Portfolio Management* and the *Journal of Alternative Investments*. An expert in quantitative asset management and derivatives valuation, his work has been widely published in academic and practitioner journals and he has co-authored textbooks on alternative investment strategies and fixed-income securities.



**Vincent Milhau** is a research director at EDHEC-Risk Institute. He holds master's degrees in statistics and economics (ENSAE) and financial mathematics (Université Paris VII), as well as a PhD in Finance from the University of Nice-Sophia Antipolis. His research areas include static and dynamic portfolio optimisation, factor investing and goal-based investing. He has co-authored a number of articles published in several international finance journals.

# Executive Summary



# Executive Summary

---

Factor investing is an investment paradigm under which an investor decides how much to allocate to various factors, as opposed to various securities or asset classes. Its popularity has been growing since the turn of the millennium, especially after the recognition in 2008 that multiple asset classes can experience severe losses at the same time despite their apparent differences. The term “factor”, however, is used with many different meanings depending on the context and the targeted application. The main goal of this paper is to provide clarification with respect to the various possible definitions of factors that are relevant in investment practice. This paper also develops a framework for allocating to factors in two main contexts, namely allocation decisions at the asset class level, and benchmarking decisions within a given class. For each of these applications, we examine the three most important questions raised by the adoption of a factor investing approach: (i) why think in terms of factors? (ii) what factors should be chosen? and (iii) how do we allocate between them?

**Several definitions for factors co-exist, which differ through their focus on return versus risk, or on cross-sectional differences between assets and the time-series properties of assets. A distinction can be made between (i) asset pricing factors, (ii) strategies that deliver a positive premium in the long run, (iii) common sources of risk in various assets and (iv) state variables that characterise current business conditions.**

Asset pricing theory is concerned with the search of “asset pricing factors”, defined as factors that explain the cross section of expected returns in the following sense: the expected returns of various assets are

completely determined by the exposures of these assets to the factors, the exposures being obtained by running a multivariate regression of asset returns on factor values. The premium of a factor measures the incremental reward received in the form of additional expected return by increasing the exposure. According to the theory, it is driven by the covariance between the factor and the “marginal utility of consumption” of the representative agent, which is the gain in utility for a small increase in consumption. A factor is positively rewarded if it tends to be high in “good times”, defined as scenarios in which consumption is high (and consequently marginal utility is low) and low in “bad times”, defined as scenarios in which consumption is low. This is because bearing exposure to this factor tends to generate a high payoff when it is least needed because consumption is already high, and a low payoff when additional money is most valued, meaning that this exposure is unattractive unless it is rewarded by higher return in the long run.

In investment practice, the notion of a factor is more polysemic. A factor can be a *profitable strategy* that delivers a long-term premium over a benchmark, provided this premium is economically justified as a reward for bearing additional risk, like in asset pricing theory, or as the result of biases in investors’ behaviour that cannot be completely eliminated due to the existence of limits to arbitrage. This definition applies well to the passive equity strategies that select stocks based on some observable characteristic: low size, high value, high momentum, low volatility, high profitability and low investment are sources of long-term returns documented by extensive academic research and backed by sound economic rationale. Capturing these premia at

# Executive Summary

reasonable cost is the goal of "equity factor indices" offered by equity index providers. In non-equity classes, research is more recent, so our understanding of risk premia is comparatively more limited. Value, momentum and carry are three effects that have been reported for bonds and commodity and currency futures, but the list is likely still incomplete, and further research is needed to study the existence and the persistence of rewarded factors, in particular in fixed income securities, which is a major asset class for institutional investors.

Another notion of a factor is that of the risk factor, and it refers to common sources of risk that affect various securities or asset classes. Volatilities and correlations are then mainly explained by the exposures to these factors, and common exposures can result in joint losses in severe bear markets, like in 2008. Several macroeconomic variables such as output, growth and inflation can play this role, but in order to maximise the explanatory power, risk factors are often taken to be implicit, that is they are extracted from asset returns by statistical analysis. In the Barra equity model, *implicit* factors are intended to represent the common sources of risk that affect assets with similar microeconomic characteristics. Specific statistical procedures can also be used to obtain factors with a zero correlation, a property that facilitates the decomposition of the risk of a portfolio. These procedures are named principal component analysis (PCA) and minimum linear torsion (MLT), and we review them in the paper.

Finally, a third possible definition for a factor in practice is as a *state variable* that contributes to explaining time variation in the risk premia, volatilities and correlations of assets. This definition takes a time-series

perspective, unlike the previous ones, which aim to explain cross-sectional properties. The risk and return characteristics of assets can be compared across regimes defined in terms of macroeconomic variables that have an impact on discount rates or expected future cash flows. It is also standard practice to take state variables as the dividend yield as a predictor of stock returns, or use the forward-spot spread to predict bond returns.

It should be noted that financial theory establishes connections between the three practical categories of factors and the notion of the pricing factor: the risk-based explanation for the profitability of passive strategies is that they are exposed to rewarded pricing factors, the Arbitrage Pricing Theory of Ross<sup>1</sup> shows that common risk factors can be pricing factors, and the Intertemporal Capital Asset Pricing Model of Merton<sup>2</sup> implies that state variables that predict changes in investment opportunities are pricing factors.

**At the asset class level, risk factors allow the diversification of a portfolio to be assessed in a more meaningful way than dollar weights, and they are involved in the construction of liability-hedging portfolios by factor matching techniques. Conditioning factors are useful to design performance-seeking portfolios that react to market conditions.**

Modern portfolio theory gives a clear definition of what a "well-diversified" portfolio should be: it should have the highest Sharpe ratio, equal to the reward, measured as expected excess return over the risk-free rate, per unit of risk, measured as volatility. But this prescription is hard to implement in practice, due to the strong uncertainty over expected return

1 – Ross, S. 1976. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory* 13(3): 341–360.

2 – Merton, R. 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica*: 41(5). 867–887.

## Executive Summary

estimates, which research has shown to have a dramatic impact on performance. To alleviate the concern over parameter uncertainty, one may decide to go back to conventional wisdom and diversify by “spreading eggs across baskets”, which hopefully leads to more efficient collection of risk premia across assets. A standard interpretation of this principle is to weight constituents equally, but it opens the door to portfolios with concentrated risk: the risk of a 50%-50% stock-bond portfolio is mostly explained by stocks.

By equating the contributions of assets to risk, the risk parity approach to allocation is a big step towards addressing this issue, but it still misses the fact that constituents are exposed to common sources of risk. To assess the level of diversification of a portfolio in terms of risk factors, we propose in this paper to calculate the *effective number of uncorrelated bets* (ENUB), a quantitative measure of the deconcentration of factor contributions to portfolio volatility that is minimal when risk is concentrated in a single factor, and

maximal when all factors contribute equally to risk. The latter condition defines a *factor risk parity portfolio*.

Factor contributions are easiest to calculate when the factors are uncorrelated from each other because there are then no cross-correlation terms to divide between factors. As introduced earlier, uncorrelated risk factors that completely explain uncertainty in a given universe can be obtained by (at least) two statistical procedures, namely principal component analysis (PCA) and minimum linear torsion (MLT). The latter method was introduced more recently, and it aims to address some of the shortcomings of PCA by minimising the distortion of factors with respect to the original assets: this property facilitates the economic interpretation of factors and enhances robustness across samples.

Exhibit 1 shows an example of the ENUB calculation in a seven-asset class universe mixing equities, bonds, commodities and real estate. The four benchmark

Exhibit 1. Diversification with risk factors.

(a) Effective number of uncorrelated bets for selected portfolios.

	PCA factors	MLT factors
Policy portfolio	1.34	3.40
Equally-weighted	1.08	3.77
Risk parity	2.00	6.00
Minimum variance	2.67	2.28

(b) Composition of risk parity and factor risk parity portfolios (in %).

	US equities	Int'l equities	US Treasuries	US corporate	US TIPS	Commodities	Real estate	Total
Risk parity	7.4	6.3	40.6	16.4	18.5	6.2	4.6	100.0
Factor risk parity - PCA	13.7	-11.7	98.9	-33.3	12.0	15.7	4.7	100.0
Factor risk parity - MLT	15.0	4.3	42.1	18.2	15.5	5.3	-0.4	100.0

Note 1. Constituents are US equities, world ex-US equities, US Treasuries, US corporate bonds, US Treasury Inflation-Protected Securities, commodities and US real estate. Composition of policy portfolio is 30% in US equities, 30% in international equities, 15% in US Treasuries, 15% in US credits, 3.33% in TIPS, 3.33% in commodities and 3.33% in real estate. Sample period goes from April 1997 to September 2017.

Note 2. By construction, the factor risk parity portfolio is not unique, so we select the one with the lowest leverage (sum of absolute values of short positions) in Panel (b).



## Executive Summary

portfolios have ENUBs much lower than the theoretical maximum of seven, which means that their risk is concentrated in a few risk factors, except for the risk parity allocation when MLT factors are employed: indeed, each MLT factor is close to an asset, so the risk parity portfolio should not be exceedingly far from a factor risk parity portfolio. Nevertheless, the true factor risk parity portfolio for MLT factors has a different composition than the risk parity one. With PCA factors, it is virtually impossible to achieve factor risk parity with a long-only allocation, since the first factor will inevitably dominate the others, so sizeable short positions must be taken. This example illustrates the fact that MLT factors are computationally easier to handle.

Risk factors are also naturally involved in a different context, where the objective is not to efficiently diversify across assets, but to replicate a benchmark as closely as possible, like in asset-liability management, where a good liability-hedging portfolio (LHP) is needed. Through the discounting mechanism of future cash flows, interest rate risk is a major source of risk, and often the dominant

one, in liabilities, so aligning the interest rate exposures of assets and liabilities is the first step towards the construction of a LHP. The difficulty here is that exposures are not linear, so linear approximations are needed. The first-order approximation leads to duration matching, which is effective at immunising the funding ratio against small changes in the yield curve, but in order to hedge against the effects of larger changes, finer approximation is required, involving a matching of convexities in addition to duration alignment.

Beyond risk factors, state variables characterising time-varying investment opportunities may prove useful in asset allocation, in order to construct a performance-seeking portfolio that adapts to market conditions. A simple way to define regimes is to look at inflation and growth in Gross Domestic Product and to make a distinction between four regimes, depending on whether inflation and growth are below or above their mean values. The results in Exhibit 2 show that equities do best when inflation is modest and growth is dynamic, while the low growth and high inflation regime is the least favourable to

Exhibit 2. Conditional means and volatilities of asset classes in inflation-growth regimes (in %).

Regime	Average returns				
	High gro. Low inf.	High gro. High inf.	Low gro. Low inf.	Low gro. High inf.	Unconditional
US equities	23.0	10.7	9.0	5.8	11.1
Int'l equities	23.4	13.9	9.4	-0.4	10.4
US Treasuries	4.4	8.0	6.5	9.5	7.4
US Credit Baa	5.7	10.3	10.0	7.4	8.6
Commodities	-3.1	17.6	-2.5	17.0	8.7

Regime	Volatilities				
	High gro. Low inf.	High gro. High inf.	Low gro. Low inf.	Low gro. High inf.	Unconditional
US equities	15.0	14.2	17.0	18.9	16.6
Int'l equities	17.4	15.1	19.6	22.4	19.2
US Treasuries	4.9	5.4	5.6	7.3	6.0
US Credit Baa	4.7	7.1	6.5	2.5	8.6
Commodities	20.1	18.5	23.6	26.6	23.0

Note. Sample period is from January 1973 through December 2016.

## Executive Summary

---

them, both in terms of performance and volatility. Commodities perform better in the high inflation than in the low inflation periods, and Treasuries deliver their best performance in the low growth and high inflation regime, thereby confirming their role of a "safe haven". These results suggest that regimes of growth and inflation can be used to adapt the relative weighting of asset classes as a function of market conditions.

**Within an asset class, theory makes a case for factor investing by showing that the maximum Sharpe ratio (MSR) portfolio of individual securities coincides with the MSR portfolio of pricing factors, no matter how large the original universe is. Empirically, equity factor indices representing the six well-documented factors (size, value, momentum, volatility, profitability and investment) dominate the standard cap-weighted index in terms of risk-adjusted return, especially if they are "smart weighted". Further improvement over the risk-return characteristics of individual factors is achieved by building multi-factor portfolios.**

Going back to the theoretical definition of a well-diversified portfolio as the maximum Sharpe ratio (MSR) portfolio, a theoretical result that we prove in this paper is that the MSR portfolio of any set of assets coincides with the MSR portfolio of factors, provided the latter are pricing factors in the sense of asset pricing theory. This result holds regardless of the number of assets, so it represents a substantial reduction in dimensionality if there are many of them, as is generally the case in benchmarking. It also provides the optimal form of the "two-step process", in which an allocation exercise to multiple securities

is divided into two steps, namely the grouping of securities in benchmarks, and then an allocation to the benchmarks.

This theoretical result cannot be directly applied in practice to calculate the MSR portfolio because a complete set of pricing factors is not known, but the idea of dimension reduction can be exploited with other types of factors, namely risk factors. Indeed, under a factor model, each return can be decomposed into a systematic part that is a sum of factor exposures, plus an idiosyncratic term, and provided idiosyncratic returns are uncorrelated across assets, the number of independent parameters to estimate in the covariance matrix is much smaller than if no factor structure is postulated. Considering for instance a universe of 500 stocks, it is shown in the paper that the number of covariances is 125,250 without a factor model, 3,521 with six factors and 2,006 with three of them. In other words, the use of risk factors alleviates the curse of dimensionality for the estimation of the covariance matrix. This idea is implemented in BARRA models, as explained in the Barra Risk Model Handbook.

Although a comprehensive set of pricing factors has not been uncovered to date, it is well known from a large body of empirical research that at least in the equity class, factors understood as profitable strategies provide a substantial improvement over the standard cap-weighted index in terms of risk-return characteristics. Exhibit 3 summarises this evidence by assuming factors to be a set of long-only factor indices made of stocks with a given characteristic: mid market capitalisation, high book-to-market, high past one-year return, low volatility, high gross profit-to-asset ratio or low total asset growth. The base case version of these indices is

# Executive Summary

Exhibit 3. Effects of selection and weighting in equity benchmarks.

	All stocks			Mid Cap			Value		
	CW	EW	IV	CW	EW	IV	CW	EW	IV
Ann. ret. (%)	10.80	12.76	12.83	13.42	14.13	14.10	12.29	14.54	14.48
Ann. vol. (%)	16.90	16.76	16.12	17.12	17.06	16.29	17.19	16.72	16.16
Sharpe ratio	0.35	0.47	0.49	0.50	0.54	0.56	0.43	0.58	0.59

	High Momentum			Low Volatility			High Profitability		
	CW	EW	IV	CW	EW	IV	CW	EW	IV
Ann. ret. (%)	11.68	13.63	13.63	11.12	13.34	13.26	11.12	13.27	13.34
Ann. vol. (%)	17.28	17.05	16.45	15.41	14.54	14.34	17.03	16.79	16.13
Sharpe ratio	0.39	0.51	0.53	0.40	0.58	0.58	0.37	0.50	0.52

	Low Investment		
	CW	EW	IV
Ann. ret. (%)	12.72	14.20	13.94
Ann. vol. (%)	15.85	15.98	5.38
Sharpe ratio	0.49	0.58	0.59

Note. Index values are from the ERI Scientific Beta database and span the period from June 1970 through December 2015. CW means "cap-weighted", EW stands for "equally-weighted", and IV for "inverse volatility".

cap-weighted, but "smart factor indices" deviate from this weighting scheme in order to better diversify away unrewarded risk. Among the many possible schemes, the table considers equal weighting and inverse volatility weighting. Over the long period considered, all factor indices outperform the broad cap-weighted index and display a higher Sharpe ratio, and the smart versions bring further improvement on these figures.

The equivalence result between the MSR portfolio of securities and the MSR portfolio of pricing factors suggests that it is interesting to combine factors. Thus, the next exercise that we conduct consists in the construction of multi-factor equity portfolios. Exhibit 4 shows statistics for selected allocations. Given that the six factors are long only, they are all exposed to the market equity factor and they have high correlations, greater than 90%, so one may wonder what benefits can be expected from mixing such highly correlated constituents. It turns out that the annualised long-term

return and the Sharpe ratio are only marginally improved with respect to the average properties of the constituents, but the relative analytics, which measure risk and return with respect to the broad cap-weighted index, are much more favourably impacted. This can be attributed to the fact that the relative correlations, that is the correlations between excess returns, are much lower than the absolute correlations and are often negative, so diversification can be expected to be more effective from a relative perspective. In particular, the equally-weighted portfolio has much lower tracking error, a much lower maximum relative drawdown, as well as a much higher information ratio than the average of the constituents.

The choice of the allocation method has important effects on the properties of the multi-factor portfolios. The global minimum variance portfolio achieves its objective even on an out-of-sample basis, but does so at the cost of sizeable relative risk, while the equally-weighted portfolio

# Executive Summary

Exhibit 4. Properties of multi-factor equity portfolios.

	Ann. return (%)	Ann. volatility (%)	Sharpe ratio	Maximum drawdown (%)	Tracking error (%)	Info. ratio	Max. rel. drawdown (%)
<b>Constituents' average</b>	<b>11.62</b>	<b>16.88</b>	<b>0.40</b>	<b>53.74</b>	<b>4.19</b>	<b>0.29</b>	<b>25.92</b>
Equal Weight	11.74	16.45	0.41	52.30	1.97	0.72	13.32
Minimum variance (L/O)	11.25	15.46	0.41	47.84	3.98	0.23	36.90
Risk Parity	11.76	16.36	0.42	51.96	2.09	0.69	14.73
Max ENUB (L/O)	11.53	15.56	0.42	48.30	3.31	0.37	30.80
Max Rel. ENUB (L/O)	11.09	16.81	0.37	53.18	1.12	0.69	5.18

Note. The sample period is from July 1972 through December 2015, and portfolios are rebalanced quarterly. Implicit factors are extracted by minimum linear torsion, and statistics are computed out of sample. "L/O" stands for "long-only".

displays a higher volatility, but better relative analytics. We also calculate two portfolios that maximise diversification in terms of risk factors (subject to a long-only constraint). At this stage, two notions of factors are involved: on the one hand, constituents are profitable passive equity strategies, and on the other hand, the weighting scheme seeks to maximise diversification in terms of underlying risk factors. The latter factors are extracted successively from the covariance matrix of the constituents and from their relative covariance matrix, which collects the covariances of excess returns. Each system of factors gives rise to its own value for the ENUB, and the two ENUBs respectively measure the deconcentration of the volatility and the tracking error. The maximum relative ENUB portfolio has lower relative risk, measured either through the tracking error or the relative drawdown, than its absolute counterpart. The additional backtests conducted in the paper show that this finding is robust to the choice of the sample period.

**As a conclusion, the various notions of factors are not mutually exclusive and can be combined within a comprehensive framework for factor allocation. Further research is needed to improve our understanding of their interactions, especially in the fixed-income class.**

As we argue in the previous empirical illustration, a factor allocation exercise can involve more than one notion of factors. It is possible to use factor strategies as building blocks and to diversify risk across underlying factors, or to seek to exploit knowledge of economic regimes to design portfolios that react to changes in market conditions. After five decades of research on equities, robust sources of profitability are now well identified in this class, but not as well in other classes, especially in fixed-income. Moreover, while past research has mostly focused on finding predictors for the equity market or the bond market as a whole, and while it is recognised that factor strategies have cyclical behaviour, further investigation is needed to quantify the degree of predictability in these factors and to identify relevant predictors.

# 1. Introduction



# 1. Introduction

---

The factor approach to investment recommends that allocation decisions be taken in terms of factors as opposed to physical constituents such as stocks, bonds, futures contracts or fund shares. It is gaining popularity amongst sophisticated institutional investors, especially after the 2008 market downturn, which revealed that multiple asset classes could plunge together despite their apparent differences. The broad objective pursued by adopting a factor perspective is to understand the sources of risk and return of various assets and to better assess the level of diversification of a portfolio than by simply looking at its constituents. This is illustrated by Ang (2014)'s comparison between factors and nutrients (p. 194): "Factors are to assets what nutrients are to food.", and in the same way as different foods can contain identical nutrients, factor exposures can overlap across different assets.

While the relevance of factor investing is now widely accepted, some ambiguity remains, however, with respect to what factors should be used and what exact role they are expected to play in the investment process. In the practitioners' literature, factors are broadly defined as the sources of risk and return of securities, and the existence of a risk premium with strong track record and economic motivation is a key argument in the promotion of a factor product. It is also sometimes argued that in order to be building blocks for factor investing, factors should have lower correlations than asset classes. Finally, academic research on asset pricing models – in particular the work of Fama and French (1992, 1993, 1996, 2006, 2012, 2014) and Carhart (1997) – is often invoked by factor product providers as a scientific endorsement. Overall, factors are generally associated with the following properties: an ability to explain returns, the existence of

a premium, low correlations and academic acceptance. The first objective of this paper is to contribute to the widespread acceptance of factor investing by providing useful pedagogical clarification with respect to the various definitions of factors. To this end, we propose a "taxonomy" to classify the practically relevant notions of factors and we discuss how they are connected with that of the "asset pricing factor", which is the one favoured by the academic literature.

The second objective of this paper is to analyse the three key questions that naturally arise before a factor investment process is undertaken: (i) why adopt a factor approach? (ii) what factors should be used? and (iii) how can it be done? As far as the "why" question is concerned, we make an important distinction between two main types of benefits that can be expected from factor investing. On the one hand, adopting a focus on the allocation perspective, we argue that factor investing allows for a better structuration of the investment process, both from an asset-only perspective and from an asset-liability management perspective. On the other hand, shifting to a focus on the benchmarking perspective, we argue that factor investing allows for a more efficient harvesting of risk premia compared to traditional approaches focusing, for example, on sector decompositions.

We draw the same distinction between the multi-class and the single-class cases for the "what" question. In asset allocation, it is useful to have factors that capture commonalities between asset classes with seemingly heterogeneous properties and factors that summarise the time variation in their expected returns, volatilities and correlations: to date, this is a more reachable objective than to have a comprehensive

# 1. Introduction

---

factor model that explains the differences in average returns between all classes. In benchmarking, the challenge is to find the sources of the various risk premia that exist in a class to efficiently collect them. This task is more advanced in the equity class than in the others, thanks to a very active research that started as early as in the 1970s, when the limitations of the Capital Asset Pricing Model were first acknowledged, and went on with the discovery and the study of a number of empirical patterns in stock returns. The size, the value and the momentum effects are the most traditional examples of these regularities, and they have been more recently completed by the volatility, the investment and the profitability effects: all of these effects have given rise to dedicated factors in the academic literature, and they are commonly represented in the standard offering of factor index providers. Unfortunately, extensive data mining has led to a somewhat confusing situation, with a proliferation of candidate “factors” in the literature: more than 300 of them have been counted by Harvey, Liu, and Zhu (2016) – not to mention jokes such as “factors” based on the company's name. Among this crowd, the aforementioned six factors have become consensual, and we conduct a survey of the literature that has examined their statistical robustness and their economic grounding. The understanding of risk premia in non-equity asset classes is comparatively more limited, although recent literature has uncovered patterns in multiple classes: value and momentum (Asness, Moskowitz, and Pedersen 2013), carry (Koijen et al. 2013) and low beta (Frazzini and Pedersen 2014).

The last piece in a comprehensive factor allocation framework that can be used by institutional investors is a methodology to

allocate to factors. We provide an empirical illustration in an asset allocation context, by constructing factor risk parity portfolios of seven asset classes that cover equities, bonds, commodities and real estate. This portfolio construction method extends the risk parity technique of Maillard, Roncalli, and Teiletche (2010) to the case where the constituents are risk factors as opposed to asset classes. In benchmarking, the “how to allocate” question encompasses two sub-questions, namely how to design a benchmark for a given factor, and how to use these factor strategies as the building blocks of a multi-factor allocation. Again, there is a discrepancy between equities and the other classes: in the equity class, the smart beta industry has developed a variety of factor indices based on systematic stock selection and weighting strategies, and Exchange-Traded Funds (ETFs) tracking some of these indices have been launched, thereby providing equity investors with cost-efficient access to factor premia. Similar products spanning a wide range of factors are still to be developed outside equities. Finally, allocation to multiple factors gives investors access to the benefits of diversification by taking advantage of the imperfect alignment of factor cycles.

Our paper is related to the substantial literature on the identification of factors, the construction of investable proxies for factors and allocation methods to factors. Academic research is mainly concerned with the search for “pricing factors”, that is factors that explain differences in expected returns across assets. It has given rise to a considerable number of publications, both on the theory of factor models and on empirical models. Cochrane (2005) is a standard reference for asset pricing theory, and a survey of the empirical literature is given in Section 4. More detailed reviews



# 1. Introduction

---

can be found in Amenc et al. (2015) and Martellini and Milhau (2015). The three-factor model of Fama and French (1993) is a cornerstone of this literature, with the introduction of the long-short factors capturing the equity size and value premia. Another strand of the literature has focused on the statistical issues posed by the estimation of latent risk factors. The question of the optimal number of factors is discussed in Trzcinka (1986), Connor and Korajczyk (1993) and Bai and Ng (2002), while Carli, Deguest, and Martellini (2014) and Meucci, Santangelo, and Deguest (2015) compare two methods to extract uncorrelated latent factors from a sample of returns: the standard principal component analysis, and a new approach called "minimum linear torsion". The literature on factor investability examines the contribution of the short side in long-short factors (Israel and Moskowitz 2013), the robustness of factor premia to implementation costs (Korajczyk and Sadka 2004; Frazzini, Israel, and Moskowitz 2012; Asness et al. 2014; Novy-Marx and Velikov 2016) and their resistance to "crowding risk", which is the risk that investors eager to capture the premia collectively bid up prices and depress subsequent returns (Amenc and Goltz 2016). The question of allocating to factors is discussed by Roncalli and Weisang (2012) and Deguest, Martellini, and Meucci (2013) – who seek to construct portfolios that are well diversified in terms of factors –, Amenc et al. (2014a) – who compare various allocation methods –, and Amenc, Coqueret, and Martellini (2015) – who introduce a framework for tactical allocation. Also related is the paper by Amenc et al. (2017), who focus on the construction of portfolios that achieve exposure to multiple equity factors.

various definitions of factors, arising from investment practice and theoretical literature. In Section 3, we present a detailed discussion of the role of factors in strategic allocation decisions across asset classes, with a focus on implicit factors that explain cross-sectional differences in risk and return parameters, and/or explicit macroeconomic factors that explain their changes over time. In Section 4, we propose an analysis of the role of factors in benchmarking decisions, with a focus on investable proxies for microeconomic risk premia, and we present an empirical application to the design of multi-factor equity portfolios. Section 5 concludes. For better readability, the paper has been alleviated from most of the mathematical content: all technical details and mathematical derivations are relegated to a Technical Appendix.

The rest of the paper is organised as follows. In Section 2, we review the



## 2. A Taxonomy of Factors



## 2. A Taxonomy of Factors

The term “factor” is used with many different meanings in investment practice and in academic research. This can be a source of confusion, and sometimes disappointment about the benefits that can be expected from factor investing approaches. The main focus of this section is to provide an actionable taxonomy of factors, as well as some conceptual clarification with respect to how the various types of factors may be used in investment practice. To this end, we start with a reminder of the academic definition of factors as sources of risk that explain the cross-sectional differences between the expected returns on different assets. While this notion of a *pricing factor* is not immediately useful in portfolio management, it is important because most of the academic literature on factor models refers to it. To classify the factors used in practice, we make a distinction between three categories: (i) rule-based strategies that deliver a positive expected excess return over the risk-free asset in the long run (also named *priced factors*); (ii) common sources of risk in multiple securities; and (iii) state variables acting as predictors of expected returns and variances. This taxonomy involves a gradual shift from the cross-section to the time-series perspective, as pricing factors focus on differences between assets, while state variables attempt to explain how risk and return parameters for a single asset vary over time. Between the two, both the time series and the cross section perspectives are present in the definition of profitable strategies and risk factors.

### 2.1 Academic Perspective: Factors and the Cross-Section of Expected Returns

Asset pricing theory defines the notion

of *pricing factors*, which, broadly speaking, are variables that contribute to explaining differences in expected returns across assets. A complete presentation of consumption-based asset pricing and factor models can be found in the reference textbook of Cochrane (2005).

#### 2.1.1 The Cross Section of Expected Returns

Let  $F_1, \dots, F_K$  be a set of  $K$  random variables, and, for an asset  $i$ , let  $\beta_{ki}$  be the multivariate beta with respect to factor  $k$ , defined as the coefficient of factor  $k$  in a multivariate regression of the asset on all factors.<sup>1</sup> Formally, a set of factors are said to be *pricing factors* if there exist  $K$  scalar numbers  $\Lambda_1, \dots, \Lambda_K$  such that the expected return of asset  $i$  in excess of the risk-free rate admits the decomposition:

$$\mu_i = \sum_{k=1}^K \beta_{ki} \Lambda_k, \quad (2.1)$$

where the constants  $\Lambda_1, \dots, \Lambda_K$  are called the factor premia.

Of particular interest are the factors for which there exists a perfect factor-replicating portfolio, that is a portfolio with excess returns (with respect to the risk-free rate) that match the factor values in any state of the world. For such perfectly replicable factors, the premium equals the expected excess return of the portfolio. The market factor of the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965a, b) falls in this category, as it is defined as the excess return to the market portfolio, which consists of all available assets weighted by their market values, so the market premium is the weighted sum of the expected returns of all assets. The long-short size and value factors in the three factor model of Fama and French (1993) are also excess

1 – These betas are distinct from the univariate betas with respect to factors taken in isolation, unless all factors are uncorrelated.

## 2. A Taxonomy of Factors

returns: indeed, a zero-dollar portfolio that goes long small or value stocks and goes short large or growth stocks can be regarded as a collateralised investment with the margin capital invested in cash, so the excess return of the portfolio is the difference between the returns to the long and the short legs. Thus, the size premium is the expected excess return of small stocks over large stocks, and the value premium is that of value stocks over growth stocks. A more general interpretation of the premium, which is directly based on Equation (2.1) and holds whether there exists a factor-replicating portfolio or not, is as the marginal change in expected return per unit of change in exposure.

Note that there is no alpha, or abnormal return, in decomposition (2.1): when the true asset pricing model is used, differences in expected returns are entirely explained by differences in factor exposures. As such, a factor model establishes a fully explanatory relationship between ex-ante performance (measured by the  $\mu_i$ ) and risk, where risk is measured by the  $K$  factor exposures. In theory, alphas can arise only when the model is ill-specified or incomplete, but they are zero if all factors are included. Specific risk, which is the standard deviation of the residuals in a regression of asset or portfolio returns on the factors, plays no role at all.

In practice, factor models are often used for performance attribution, where the goal is to separate out the fraction of the performance of a managed fund that can be attributed to manager's skills from the fraction that originates from exposure to systematic sources of risk. If an exhaustive set of pricing factors is used in this exercise, the alpha of any fund with respect to the factors is theoretically

zero, so the fraction of past performance that remains after controlling for the risk exposures is interpreted as deviations from the model, due to temporary mispricing, that the manager was able to exploit. The three-factor model of Fama and French (1993), which includes the market, the size and the value factors, is commonly used to decompose the performance of equity funds. It is sometimes augmented with the momentum factor to form the four-factor model of Carhart (1997).

Of course, the identity of factors to include in the model is a key question, and it is precisely the focus of the literature on empirical asset pricing. Although the ultimate goal of asset pricing models is to explain the expected returns to all securities, whichever asset class they belong to, with a parsimonious set of factors, the literature is largely segmented across asset classes, and equities have been studied in much more detail than the others. At this stage, it turns out that a distinction must be made between the single-class and the multi-class contexts, with a different set of explanatory factors in each case.

### 2.1.2 Explaining and Measuring Factor Premia

Factor premia can be positive, negative or zero. According to consumption-based asset pricing models, a factor commands a positive premium if, and only if, it covaries *negatively* with the marginal utility of consumption of the representative agent. The exact mathematical expression for the ex-ante risk premium of a factor  $F$  reads

$$\Lambda = -\frac{1}{\mathbb{E}[u'(c)]} \text{Cov}[u'(c), F], \quad (2.2)$$

where  $c$  is the next period consumption,  $u$  is the utility function and  $\mathbb{E}$  and  $\text{Cov}$

## 2. A Taxonomy of Factors

are respectively the expectation and the covariance operators. The marginal utility,  $u'$ , is positive (the more the agent consumes, the happier s/he is), so the risk premium is indeed negatively related to the factor's covariance with marginal utility. The premium is negative if the covariance is positive, and a covariance of zero implies a zero premium.<sup>2</sup>

The intuition behind Equation (2.2) is easier to grasp by noting that it also holds for individual assets and portfolios of assets, the  $F$  in the right-hand side being re-interpreted as the asset or portfolio return (or excess return, which does not make any difference in the covariance operator). Risk-averse investors seek to reduce uncertainty over future consumption, although they do not seek to cancel it completely unless they have infinite risk aversion. For the purpose of smoothing consumption across states of the world, they are interested in assets that have high payoffs in "bad times", defined as states of the world where the marginal utility is high: as marginal utility is decreasing, these are states where the investor is relatively poor and has little to consume. In such events, any additional dollar brings a lot of welfare, so these assets are desirable from the perspective of a utility maximiser. Conversely, assets that have high payoffs in "good times", where marginal utility is low and consumption is high, are not attractive for hedging purposes. Eventually, assets are judged according to two criteria: their expected return and their covariance with the marginal utility of consumption. Equation (2.2) tells us that there is a negative linear relationship between the two: a lower covariance must be compensated by higher expected return since the asset is less interesting from a hedging perspective, and conversely, a

high covariance is paid by lower expected return.

In consumption-based models, marginal utility of consumption is the most "fundamental" factor, as it has a clear economic interpretation and it suffices to explain all differences in expected returns across assets. Another formulation of Equation (2.2) for assets and portfolios of assets is in terms of the *stochastic discount factor*, which measures how a dollar payoff is valued in each state of the world. In good times, where marginal utility is low, one additional dollar does not improve welfare much, so it has low monetary value today for the agent. But in bad times, an additional dollar brings a large improvement in utility, so the agent is ready to pay a high price for the promise of receiving this dollar. For example, Cochrane (2005) shows that the stochastic discount factor is proportional to marginal utility. Formally, if  $m$  is the stochastic discount factor,  $x$  is the future payoff of an asset and  $p$  is the current price, then the pricing equation reads

$$p = \mathbb{E}[mx].$$

Unfortunately, marginal utility of consumption is hard if not impossible to measure accurately, because aggregate consumption is a low-frequency macroeconomic indicator, and marginal utility is unobservable. Against this backdrop, factor models substitute ad-hoc factors to proxy for marginal utility. Each factor defines its own set of "good" and "bad" times. For a positively rewarded factor, these are the times where the factor is high. Keeping in mind that the factor covaries negatively with marginal utility, these are states of the world where marginal utility tends to be low, hence the name "good times" for this particular factor: the good times

2 - A negative premium can always be turned into a positive premium by changing the factor sign (shorting the factor if it is investable), which changes the signs of all factor exposures.

## 2. A Taxonomy of Factors

---

defined by a positively rewarded factor tend to coincide with, but do not exactly overlay, those defined by marginal utility. Conversely, the "good times" for a negatively rewarded factor are states of the world where the factor is low.

Empirically, the premium of a factor coincides with its expected value if the factor is the excess return to some portfolio, so it can be measured over a sufficiently long sample. But this equivalence does not carry to factors that are not excess returns: expected inflation is not the inflation risk premium. To estimate factor premia in the general case, researchers look in fact for the values of the  $\Lambda_K$  that bring the two sides of Equation (2.1) as close as possible to each other. To this end, they use a two-step statistical procedure introduced by (Fama and MacBeth 1973): in the first step, the betas of assets with respect to the factors are estimated through time-series regressions, and in the second step, a time series of estimates for the factor premia is obtained by running cross-sectional regressions of returns on exposures. The estimates for the factor premia are the averages of these time series. When a factor is an excess return, the Fama-MacBeth estimate for the premium can be compared to the historical mean, so inconsistencies between the two estimates point to model misspecification. It is precisely one of the tests that the CAPM fails to pass: Fama-MacBeth estimates of the market premium are much closer to zero – and are sometimes negative – than the historical equity risk premium (see Table III of Fama and French (1992) for an example).

### 2.1.3 Pricing versus Priced Factors

An asset or a portfolio strategy is said to be priced if it has a non-zero premium,

and more generally, a random variable is said to be priced if it has non-zero covariance with the marginal utility of consumption: if the random variable is an excess return, this condition is equivalent to saying that its expectation is non zero. Any strategy that earns a non-zero expected excess return can be regarded as a priced factor, but that does not make it an additional pricing factor. Indeed, if we have a set of pricing factors  $F_1, \dots, F_K$ , the expected excess return of any asset is a function of the factor exposures as shown in Equation (2.1), but the exposure to the rewarded portfolio does not appear in this formula. Specifically, a change in the exposure to the rewarded strategy while holding the  $K$  factor exposures has no effect on the expected return, which we express by saying that the candidate factor is irrelevant.

Conversely, the pricing factors  $F_1, \dots, F_K$  are not all necessarily priced. A factor can be a pricing factor without being priced, if it helps price risky assets without having a premium. This means that expected returns are insensitive to a change in the exposure to this factor while keeping other exposures fixed, but the factor is still useful because its presence affects the way the exposures to the other factors are measured.

Strictly speaking, one cannot categorise an isolated random variable as being or not being a pricing factor, because this status depends on the context. If this variable alone, or augmented with other random variables, explains the cross section of expected returns, it is a pricing factor. On the other hand, if there exists a set of pricing factors that does not contain this variable and suffices to explain the cross section, the variable is not a pricing factor relative to the set. For instance,

## 2. A Taxonomy of Factors

---

assume for a moment that the three-factor model of Fama and French (1993) provides a comprehensive description of the cross section of expected returns of stocks. (We know it does not in reality, because there are patterns, like the investment and the profitability effects, which are not explained by the market, size and value factors.) Relative to the three-factor set, minus the size factor is not an additional pricing factor because it is redundant with the usual small-minus-big excess return. But we can construct a pricing model equivalent to the Fama-French model by replacing the standard size factor by its negative. In this alternative model, minus the size factor is a pricing factor.

In fact, not even the set of pricing factors is uniquely defined. Indeed, if we have a set of pricing factors  $F_1, \dots, F_K$  it can be shown that any new set of  $K$  factors obtained by linearly combining the factors is still a set of pricing factors, as long as the combination operation does not introduce redundancies between factors (see Section B of the Technical Appendix). For instance, one may construct an alternative version of the three-factor Fama-French model by keeping the market factor as it is, and replacing the size and the value factors respectively by their sum and their difference. This changes the factor exposures and the factor premia, but the resulting three-factor model is strictly equivalent to the original one in the sense that idiosyncratic risk is the same (residuals are unchanged), and the new model explains the cross section of expected returns just as well: if the alphas in the Fama-French model are zero, so are those in the new model.

### 2.2 Factors as Profitable Strategies

An early notion of a factor commonly used in investment practice is that of a strategy that earns a positive excess return over a given benchmark in the long run. It echoes the theoretical definition of a "priced factor", which is a strategy that outperforms the risk-free asset because of its covariance with the marginal utility of consumption, but it is broader because the benchmark can be another risky strategy and the economic rationale for the premium does not necessarily hinge on higher exposure to consumption risk. Among these strategies, the distinction between long-short and long-only portfolios is important because it relates to investability considerations.

#### 2.2.1 Examples of Profitable Strategies

Risky assets, such as stocks and bonds, are empirically found to outperform a roll-over of short-term money market instruments: according to the figures of Dimson, Marsh, and Staunton (2008) (Chap. 33), US stocks and US bonds have outperformed short-term bills respectively by 6% and 0.7% per year between 1900 and 2000, and positive excess returns are also reported in other countries (UK, Japan, France...). Hence, a straightforward example of a profitable strategy in a multi-class context is the strategy that passively holds an index representative of the asset class: the excess return is called the asset class (e.g., equity or bond) premium. It has long been standard to use cap-weighted indices in the equity class and their debt-weighted or Gross-Domestic-Product-weighted equivalents in the bond class, but this practice is now largely questioned in view of the relative inefficiency of these indices with respect to a variety of "smart beta" and "smart factor" indices (see Section 4).

## 2. A Taxonomy of Factors

---

Within a given class, many other sources of profitability have been identified by practitioners and researchers, some of which correspond to investment styles with a long tradition. An example is value investing, an investment principle that recommends buying securities with a market value that is "low" with respect to some fundamental measure of value, and is rooted in the seminal writings of Graham and Dodd (1934) and Graham (1959). In the equity class, the use of the book-to-market ratio as a value score dates back at least to Stattman (1980), who provided early evidence of the positive relationship between this variable and average stock returns, and has been firmly established since the work of Fama and French (1993), who introduced a "high-minus-low" factor, defined as the excess return to a zero-dollar portfolio that goes long stocks with the highest book-to-market ratios and short stocks with the lowest ratios. The expected excess return of value stocks (those with a high ratio) over growth stocks (those with a low ratio) is the (long-short) value premium.

Another investment style that has led to the introduction of a dedicated long-short factor is trend following, which recommends purchasing stocks that have been doing well in the recent past: many mutual funds employ strategies of this type, with a ranking period ranging from one quarter (Grinblatt, Titman, and Wermers 1995) to one year (Jegadeesh and Titman 1993). The corresponding long-short factor is defined by Carhart (1997) as the excess return to a zero-dollar portfolio that goes long the past year winners and short the past year losers. More generally, long-short factors with positive long-term returns can be constructed by sorting securities

on an observable characteristic that has an impact on average returns, and then taking a long position in the best performing group and a short position in the worst performing one. The long-short factor premium is the excess return of the long portfolio over the short one and can be measured *ex-ante* (expected excess return) or *ex-post* (realised excess return).

Some patterns that have been identified in stock returns have been shown to exist in other asset classes as well. They include long-term reversal and short-term momentum in sovereign bonds, commodities and currencies (Asness, Moskowitz, and Pedersen 2013; Jostova et al. 2013), carry effects (Koijen et al. 2013) as well as low-beta effects in equities, Treasuries, credits, commodities and currency futures (Frazzini and Pedersen 2014). However, this list is likely incomplete and many of these factors have not yet been thoroughly investigated nor adopted in investment practice. In the absence of a consensual inventory of sources of premia and of cost-efficient replicating investment vehicles, the definition of a factor as a profitable strategy is less suited to multi-asset allocation problems than to single-class problems.

### 2.2.2 Investable Proxies for Priced Factors

Many investors face long-only constraints and are thus unable to invest in long-short factors. A natural approach in this case is to take exposure to the long leg only, but this workaround possibly entails the loss of a fraction of the long-short premium. Formally, if  $\mu_L$  and  $\mu_S$  denote the expected returns of the long and short legs and  $\mu_{market}$  is the market expected return, the long-short premium can be



## 2. A Taxonomy of Factors

---

decomposed as

$$\mu_{L/S} = [\mu_L - \mu_{market}] + [\mu_{market} - \mu_S].$$

If both terms in the right-hand side are positive, the expected excess return of the long leg over the market is less than the long-short premium. The question is whether the second bracketed term, which measures the contribution of the short leg to the long-short premium, is about the same magnitude as the first one or whether it is dominant.

The respective importance of the long and the short sides has been examined for the equity momentum factor by Asness et al. (2014), who show that the excess return of the long leg over the market is at least as large as the excess return of the market over the short leg. Israel and Moskowitz (2013) consider the size, value and momentum factors with 86 years of US data and 40 years of data on international equity markets and non-equity classes such as fixed income, commodities and currencies, and they report (i) that the positive alpha (with respect to the market factor) of a long-only small-cap portfolio is greater in magnitude than the negative alpha of the mirror large-cap portfolio, (ii) that the long and short legs of momentum have alphas that are about the same size, and (iii) that the long leg of value has a much greater alpha than the short leg. They conclude that long-only versions of these three factors capture at least 50% of the long-short premium, so these portfolios are still attractive.

Liquidity, turnover and market capacity are also important investability criteria. Liquidity can be improved by screening for sufficiently liquid securities before a selection based on a characteristic like size, book-to-market or past return is

performed. Various liquidity measures are available, including the number of days a security was traded and the trading volume, among other indicators. Higher turnover is associated with higher transaction costs, but it can be reduced by restricting rebalancing operations to situations where the effective weights deviate too much from the target weights. Comparing three methods to reduce transaction costs, Novy-Marx and Velikov (2016) find that a suitable control is effective at preserving the profitability of passive strategies that exploit the size, value and momentum patterns.

The capacity of an index or a passive strategy represents the market's ability to supply securities in sufficient amount to meet the aggregate demand emanating from investors replicating this index or following this strategy. Arnott, Hsu, and Moore (2005) and Johansson and Pekkala (2013) introduce formal measures of the capacity of a portfolio, both of which are based on a comparison between the portfolio weights and the weights of the reference cap-weighted index. Broadly speaking, a portfolio has large capacity if it does not put exceedingly large weights on securities with a small capitalisation. As a consequence, capacity can be improved by limiting the size of deviations between the portfolio weights and the weights proportional to capitalisation.

"Smart factor indices" aim to provide investors with an access to factor premia in the form of passive strategies incurring moderate implementation costs. They have witnessed spectacular development in the equity universe, where they address the well-documented shortcomings of cap-weighted indices, namely their excessive concentration in a few very large stocks and their low or even negative



## 2. A Taxonomy of Factors

exposures to rewarded factors. Section 4 examines the construction of smart factor strategies in more detail. In the fixed-income class, smart factor indices are not yet as developed, but the limitations of traditional debt-weighted indices are getting increasingly acknowledged.

### 2.2.3 Rationale for Positive Performance

The premium of a factor strategy should be statistically robust and backed by convincing economic explanation, in order to ensure that it does not show up "by chance" in a particular dataset, and that it will persist in the future. International evidence is helpful in this regard. For instance, the size and the momentum effects described in Section 4 have been shown to exist not only in the US equity markets, but also in other stock markets. On the other hand, the premium need not be positive in any sub-period, because factor strategies can experience negative short-term returns despite having positive long-term performance. Statistical significance is also a test that strategies should pass, as Harvey, Liu, and Zhu (2016) have recently recalled.

Turning to economic justifications, one typically distinguishes risk-based explanations from behavioural explanations. The risk-based interpretation holds that the performance of a factor strategy is a reward for bearing the risk of enduring losses in "bad times", defined as times where the marginal utility from one additional dollar is high. This view is fully consistent with asset pricing theory (see Section 2.1), where the premium arises when an asset covaries negatively with marginal utility. An alternative explanation for the positive return earned by a strategy is that investors make systematic mistakes,

overbidding certain securities at the expense of others. The risk-based and the behavioural explanations may coexist for a given effect, as shown by the example of the value premium, which has been related to a distress factor (Fama and French 1993) and to investors' tendency to extrapolate past performance and their reluctance to incorporate mean reversion in their predictions (Lakonishok, Shleifer, and Vishny 1994). It should be noted that for a behavioural premium to be persistent, there must exist "limits to arbitrage", which prevent investors from fully exploiting the pricing anomaly and the premium from eventually vanishing.

### 2.3 Factors as Underlying Sources of Risk

Another notion of a factor is that of an underlying source of risk that impacts a large set of securities. The idea that the returns to risky assets can be decomposed into a *systematic* part, which arises because of the exposure to common factors, and an *idiosyncratic* part, which reflects the asset's specific behaviour, is at the heart of commercial factor models such as Barra, APTimum, Quantal, Wilshire, Axioma, etc. Many (but not all) of these risk factors are of the implicit type, meaning that they are not directly observed but their values are inferred from the data.

#### 2.3.1 Systematic and Specific Returns

Consider a universe of  $N$  risky assets with excess returns over the risk-free rate that are  $R_1, \dots, R_N$  and a set of  $K$  risk factors  $F_1, \dots, F_K$ , and run a time-series regression of the excess returns on the factors:

$$R_i = c_i + \sum_{k=1}^K \beta_{ki} F_k + \varepsilon_i. \quad (2.3)$$

## 2. A Taxonomy of Factors

The regression produces estimates for the  $K$  factor exposures  $\beta_{1i}, \dots, \beta_{Ki}$ , the intercept  $c_i$  and the idiosyncratic return  $\varepsilon_i$ . The sum of the intercept and the  $K$  factor contributions is called the systematic part of the return. The variance of the residual is decreasing in the regression R-square.

Equation (2.3) allows one to express the covariance matrix of assets as a function of the covariance matrix of factors and the covariance matrix of the residuals. Letting  $\Sigma$  denote the covariance matrix of assets and  $\beta$  be the matrix of factor exposures,<sup>3</sup> and using the no-correlation property between residuals and the factors, we have

$$\Sigma = \beta' \Sigma_F \beta + \Sigma_\varepsilon. \quad (2.4)$$

In particular, we can decompose the variance of each asset as

$$\sigma_i^2 = \sum_{k,l=1}^N \beta_{ki} \beta_{li} \sigma_{F,k,l} + \sigma_{\varepsilon,i}^2, \quad (2.5)$$

where  $\sigma_{F,k,l}$  is the covariance between the  $k^{\text{th}}$  and the  $l^{\text{th}}$  factors. This equation is the usual variance decomposition, which breaks the total risk of an asset into a systematic risk, arising from factor exposures, and an idiosyncratic risk, that cannot be attributed to factors.

Equation (2.3) is model-free until structure is placed on the residuals. The assumption that residuals are uncorrelated across assets is often imposed to alleviate the curse of dimensionality in the estimation of the covariance matrix of assets, like in the BARRA model (MSCI (2007) p. 11). Indeed, while there are  $N(N+1)/2$  independent variances and covariances between assets to estimate without the factor structure, imposing this structure narrows down this number. There are  $NK$  sensitivities (the betas),  $N$  specific variances and  $K(K+1)/2$  factor variances

and covariances. Considering for instance a universe of  $N = 500$  stocks, this amounts to reducing the number of parameters from 125,250 down to 3,521 if one retains six factors, and to 2,006 if one retains only three of them.

The Arbitrage Pricing Theory (APT) of Ross (1976) connects the concept of the risk factor with that of the pricing factor (see the definition in Section 2.1) by showing that if factors have prices, then the risk premium on any security can be approximated by a linear combination of factor premia provided the idiosyncratic risk is sufficiently small (see Cochrane (2005) Chap. 9.4). Thus, the APT is concerned with finding factors such that idiosyncratic returns are uncorrelated across assets and are small in magnitude.<sup>4</sup>

It is well known that an arbitrarily large explanatory power can be achieved by putting more regressors in the right-hand side of Equation (2.3), but this comes at the cost of a loss in parsimony and a lack of out-of-sample robustness in beta estimates. Formal tests have been developed in the literature to find the appropriate number of factors by analysing the statistical properties of returns.<sup>5</sup>

### 2.3.2 Examples of Implicit Risk Factors

Factor models can be based on *explicit* or *implicit* factors. Explicit factors are those with values that are directly observed and do not require any estimation, while implicit factors are estimated from a sample of returns. The size, value and momentum equity factors are standard examples of explicit factors, since they rely on a sort of stocks on an observable characteristic: the market capitalisation, book-to-market ratio or past performance.

3 - By convention, our matrix  $\beta$  has  $K$  rows and  $N$  columns, so each column in the matrix is the vector of the factor exposures of an asset.

4 - A zero correlation between the idiosyncratic returns of different assets is not a necessary condition for the APT to hold: Chamberlain and Rothschild (1983) introduce an "approximate" factor structure that allows for some degree of correlation.

5 - See Trzcinka (1986); Brown (1989); Connor and Korajczyk (1993); Bai and Ng (2002); Ahn and Horenstein (2013). One of the econometric problems addressed in this literature is how to account for the presence of cross sectional correlation between idiosyncratic returns in an "approximate" factor structure.

## 2. A Taxonomy of Factors

In the Barra equity model, the factors are implicit and are intended to represent the common sources of risk to which assets with similar microeconomic characteristics are exposed. Empirical evidence supporting the idea that certain characteristics, such as country and industry classification, induce comovements beyond those arising from market exposure, was provided by King (1966) and Rosenberg (1974). The Global Equity Model of Barra has around 90 firm characteristics,<sup>6</sup> and each of them is used to measure the sensitivity to a given factor. For instance, each stock has an exposure of 1 to its industry and an exposure of 0 to the others. Other characteristics are continuous variables: the market capitalisation, the book-to-market ratio and the past recent return respectively represent the exposures to the size, value and momentum factors.<sup>7</sup> Hence, all exposures are treated as observable quantities, and it is the factor values that are estimated: at each date, a cross sectional regression of asset returns on exposures is run to extract the factor values. The “global hybrid multi-factor models” of Quantal follow a similar approach, albeit with a smaller number of factors (30 on May 30, 2017, according to their website).

The fundamental factors of the Barra fixed income model include the three standard term structure factors known to explain most of the movements of the yield curve, namely “shift, twist, and butterfly” (MSCI (2007), Chap. 4). Alongside these variables are credit spread factors for non-sovereign issuers. The intended applications of factor models offered by Barra, APTimum, Quantal, Wilshire and Axioma include reporting of the decomposition of portfolio risk and return across the various factors, improved estimation of

the covariance matrix of securities for portfolio optimisation, and the analysis of the performance of a portfolio relative to a benchmark.

For the purpose of identifying distinct underlying sources of risk, it is often convenient to have uncorrelated factors. A zero correlation is often cited as a desirable characteristic of risk factors because it addresses the instability of correlations between asset classes. The approach that is arguably most commonly employed to extract uncorrelated factors from a panel of returns is principal component analysis (PCA). The first factor is the linear combination that best explains variances and covariances, the second is the combination that best explains the variances and covariances of the residuals that remain after purging returns from the influence of the first factor, and so on.

The procedure has shortcomings: Carli, Deguest, and Martellini (2014) point that principal factors are often hard to interpret and that they are unstable across different time periods, and Meucci, Santangelo, and Deguest (2015) show that if all assets have the same volatility and the same pairwise correlation, then an equally-weighted portfolio of the assets is fully invested in the first principal factor (i.e., the one with the largest variance) and that the other factors have zero weight. As a result, the portfolio risk is entirely explained by the first factor, regardless of the value of the common correlation. This property is counter-intuitive, as one would expect uncorrelated factors to have more balanced contributions when the correlation across assets shrinks to zero. To address these issues, Meucci, Santangelo, and Deguest (2015) introduce a competing method known as *minimum*

6 - There are 90 characteristics in the MSCI version and 93 in the FT one.  
7 - Continuous characteristics are transformed into z-scores by subtracting the mean and dividing by the volatility across firms.

## 2. A Taxonomy of Factors

---

*linear torsion*, the objective of which is to extract  $N$  uncorrelated linear combinations of assets that minimise the distance with respect to the original assets, as measured by the sum of the squared tracking errors between the factors and the assets. Section C of the Technical Appendix provides the detailed mathematical solution to this problem.

### 2.3.3 Examples of Explicit Risk Factors

Risk factors can also be explicit, and macroeconomic factors are natural candidates here because intuition suggests that they impact all securities. Specifically, they can have an effect on the future cash flows of an asset and the rate at which these cash flows are discounted. Based on this idea, Chen, Roll, and Ross (1986) introduce a macro factor model with the growth in industrial production, expected inflation, inflation surprises, the ex-post real rate, the credit spread and the term spread as factors. They show that at least some of these variables are priced in the cross-section of stock returns: for instance, it empirically turns out that investors require a lower premium (i.e., set higher prices today) for stocks that are more exposed to changes in the term spread.

But macro factors generally have low explanatory power, as measured by the R-square in linear regressions, when it comes to explaining realised returns, unlike explicit microeconomic factors, defined as excess returns to portfolios. The market factor, which is the excess return to a broad stock portfolio, and the size and the value factors of Fama-French's model (Fama and French 1993), fit in this category. Regression R-squares are much higher with these factors – almost by construction since the factors are combinations of the returns

that serve as dependent variables. The limitation of micro factors is that they are not as straightforward to relate to common sources of risk as macro factors are.

Another problem with macro factors is that they are not investable, unlike the aforementioned micro factors. Hence, capturing their premia requires factor-replicating portfolios, designed by maximising a measure of the portfolio exposure to the factor, or by optimising a criterion subject to an exposure constraint: standard weighting schemes include the maximum deconcentration portfolio (which is the closest approximation to an equally-weighted portfolio compatible with the constraint) and the minimum variance one.

### 2.4 Factors and the Time-Series of Risk and Return Parameters

A third practically relevant notion of a factor is that of a state variable that contributes to defining market conditions. This definition arises from a shift in focus from the cross-sectional perspective to the time-series perspective: as opposed to using factors to try and explain differences in expected return and risk parameters across securities or asset classes, the focus is on using factors to try and explain changes over time of expected return and risk parameters for each security or asset class. In applications, time-varying parameters tend to be introduced at the asset class level rather than at the security level, if only for the purpose of keeping the number of parameters to estimate reasonably low: this means, for instance, that we are interested in modelling variations in the equity or the bond risk premium over time, as opposed to modelling time-

## 2. A Taxonomy of Factors

---

varying expected returns for individual stocks or bonds.

### 2.4.1 Time Variation in Risk and Return Parameters

There is substantial empirical evidence that expected returns, volatilities and even correlations vary over time. A well-known example is volatility clustering in stock markets: "quiet" periods, with moderately large returns, alternate with periods where returns are much larger in magnitude. In statistical terms, this results in a positive autocorrelation of squared returns, which stands in contrast with the close-to-zero autocorrelation of returns. This stylised fact has motivated the use of ARCH and GARCH models, which precisely allow for clustering effects in volatility (Engle 1982; Bollerslev 1986). Correlations also vary over time: a simple rolling-window estimate as well as an estimate coming from a more sophisticated statistical model reveal that the correlation between a broad US equity index and a Treasury bond index has varied between -60% and +60% over the period from 1968 to 2010 (Baele, Bekaert, and Inghelbrecht 2010). Finally, expected returns also appear to depend on time, which is equivalent to saying that some variables have the ability to predict future returns. A substantial amount of empirical evidence has been accumulated against the assumption of the random walk hypothesis for stocks (which states that stock prices behave like a random walk with a constant drift), and against the pure expectation hypothesis for interest rates (which implies that the expected excess return of long-term bonds over short-term bonds is constantly equal to zero). For instance, the dividend yield is often used as a predictor at the stock index level (Fama and French 1989), and several variables have been found to have predictive

power for the excess returns to long-term bonds. Fama and Bliss (1987) identify the forward-spot spread, and a more recent literature has reinforced the evidence for predictability by studying other variables that sometimes have an even higher forecasting power: Dai, Singleton, and Yang (2004) use the principal factors from a principal component analysis of zero-coupon yields; Cochrane and Piazzesi (2005) introduce a "tent factor" which is a combination of forward-spot rates of different maturities; Hellerstein (2011) defines an international return forecasting factor; and Cieslak and Povala (2015) show that a "cycle factor" obtained by taking the residuals in regressions of yields on expected inflation performs even better than the tent factor. The values of conditional risk and return parameters define *investment opportunities* at each point in time.

Time variation in parameters has implications for optimal asset allocation (in the sense of expected utility maximisation), as demonstrated by Merton (1973). First, the maximum Sharpe ratio (MSR) portfolio, which maximises the instantaneous Sharpe ratio, has time-varying weights. Second, it is not optimal to invest only in the MSR portfolio and a money market account, and investors should also take long positions in assets that tend to perform well when investment opportunities deteriorate, and short positions in assets that perform poorly in these conditions. An obvious example is interest rate hedging: a decrease in interest rates means a worsening of the opportunity set, the effects of which can be compensated by investing in long-term bonds, because bond prices rise when interest rates decrease. This motive for holding bonds is entirely independent from the desire to achieve a good Sharpe

## 2. A Taxonomy of Factors

---

ratio in the short run, and it corresponds to a “hedging motive”. More generally, the additional positions in risky assets taken to face the risk of unexpected changes in investment opportunities are called “hedging demands”.

More generally, any asset allocation strategy based on risk and return parameters (including notably the global minimum variance, the risk parity and the MSR portfolios; see Section 3.3) requires a dynamic estimation of these quantities. A rolling-window estimation from past returns is suitable for volatilities, but not for expected returns, because the sample mean is highly unrobust. For first-order moments, an estimate based on a predictor, such as the dividend yield for stocks, should be preferred. Dynamic models, including Markov regime-switching models, can also be fruitfully employed for volatilities and correlations, to replace the backward-looking sample estimates by forward-looking estimates.

### 2.4.2 Macro Factors as Conditioning Information

The expression of an asset price as the expected sum of discounted future dividends suggests that macro factors, which impact expectations of future discount rates and future cash flows (e.g., dividends or coupons), matter for the determination of instantaneous expected returns and volatilities. While macro factors are essentially continuous variables, it is possible to discretise them to define regimes, defined as economic environments where a variable is high, average or low.

For instance, macroeconomic variables like the GDP growth or inflation can be regarded as instruments to define various regimes of economic activity, and these

regimes are relevant for asset allocation if expected performance and risk of assets and liabilities vary across these states of the economy. From the standpoint of first principles and historical evidence, it is clear that both economic growth and inflation have an impact on typical liability cash flows for pension plans. For example, inflation will have an immediate impact on current retirees due to cost-of-living-adjustment rules. The active workers are also likely to experience increases in salary when inflation increases, but with a lag (in most cases). The role of economic growth, on the other hand, depends upon the nature of the growth, its impact on the workforce and the commensurate changes in inflation. Because of time lags and non-linear relationships such as complex cost-of-living-adjustment and catch-up rules, the traditional approach of estimating liability exposures to these factors through linear regressions cannot adequately explain changes in the liability value over a wide range of conditions. On the other hand, economic regimes can provide a context for measuring sensitivities. Thus, it can be envisioned to relate changes in the two macro factors to changes in liabilities through a local duration estimates, as a function of projections of the workforce and its close connection to the population of retirees.

On the asset side, there are advantages to identifying and employing economic regimes within a consistent asset allocation framework. Starting with Ang and Bekaert (2002), much research has focused on the nature of economic environments and markets via regimes (see for example Mulvey and Liu (2016) for a recent reference). With such models, there is a finite number of possible values for each expected return and covariance.



## 2. A Taxonomy of Factors

### 2.4.3 From State Variables to Pricing Factors

The Intertemporal Capital Asset Pricing Model of Merton (1973) and Breeden (1979) connects the time series and the cross section perspectives on asset returns by showing that under certain assumptions, state variables driving time variation in conditional expected returns and variances act as pricing factors. It predicts that the expected excess return of each risky asset over the risk-free rate is a linear function of its market beta (like in the one-period CAPM of Sharpe (1964)) and its betas with respect to the  $K$  "hedging portfolios", each of which maximises the squared correlation with a state variable.<sup>8</sup> The pricing formula can also be written by replacing the hedging portfolios betas with state variables betas themselves.

The consumption-based CAPM of Breeden (1979) allows the ICAPM to be related to the central idea of consumption-based asset pricing models: what determines the risk premium of a risky asset is its covariance with the marginal utility of consumption, with a positive covariance commanding a negative premium and a negative covariance determining a positive premium. The CCAPM replaces the multiple factors of the ICAPM with a single one, namely the aggregate consumption, which makes it clear that the ICAPM fits in the class of consumption-based models.

This fact helps to get the intuition for the pricing formula in the ICAPM: state variables are pricing factors because they affect marginal utility of consumption. To see why marginal utility depends on the state variables, one may use the "envelope condition", which states that at the optimum, marginal utility of consumption

equals marginal utility of wealth: one dollar spent today in consumption goods brings exactly as much utility as one dollar saved for future consumption, for if the two marginal utilities were different, the investor would be better off allocating more to consumption or investment. Marginal utility of wealth is the sensitivity of an investor's welfare with respect to wealth: it is increasing and concave in wealth, but in the multi-period setting, it depends on investment opportunities: if the opportunity set is good (e.g. with high expected returns and low volatilities), then welfare is higher than with poor opportunities. As a result, marginal utility of wealth is a function of the state variables that describe the opportunity set, and so is the marginal utility of consumption.

Factor premia in the ICAPM are determined by the sensitivity of marginal utility of consumption with respect to the factor. If marginal utility is increasing in the factor, then a rise in the factor is associated with bad times (high marginal utility), so the factor commands a positive premium. Conversely, if marginal utility is decreasing in the factor, the predicted premium is negative. Moreover, the sensitivity of marginal utility with respect to the factor can change sign and value over time, which opens the door to time-varying factor premia.

8 - This result is established by Merton (1973) in a continuous-time setting with a single state variable (the risk-free interest rate). An extension to multiple state variables is done by Long (1974) in a discrete-time model, and by Breeden (1979) in the continuous-time case. See also Merton (1992) Chap. 15 for the multivariate case, which extends the original paper of Merton (1973).

## 2. A Taxonomy of Factors

---



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

Factor investing is the process in which investors decide how much to allocate to each *factor* as opposed to each *asset class*, even if factor allocation decisions must of course be eventually translated back into asset weights in order to be implemented. In this section, which has a focus on multi-asset allocation, we argue that the use of implicit factors that explain cross-sectional differences in risk and return parameters as well as the use of explicit macroeconomic factors that explain their changes over time allow for a better structuration of the investment, both from an asset-only perspective and from an asset-liability management perspective. To provide support for this claim, we first analyse the rationale for factor investing (factor investing: why?). We then propose an overview of the various forms of factor investing (factor investing: what?), and we finally discuss how factor investing can be implemented in practice in an allocation context (factor investing: how?).

#### 3.1 Factor Investing and Risk Allocation: Why?

To summarise the factor taxonomy in Section 2, the factors used in investment practice can be categorised as follows:

- Profitable strategies ("priced factors");
- Risk factors: they are the common drivers of returns in seemingly disparate asset classes;
- State variables: they capture the time variation in the expected returns, the volatilities and the correlations of asset classes.

While multiple sources of long-term profitability that span several asset classes have been identified, the search for historically proven and economically justified premia is relatively less

advanced in other asset classes than it is in for the equity class, and the notion of a factor defined as a profitable strategy is consequently less relevant in a multi-class environment. This section thus focuses on the use of risk factors and state variables in asset allocation.

#### 3.1.1 Building Improved Performance-Seeking Portfolios

##### *The Maximum Sharpe Ratio Portfolio*

In modern portfolio theory, efficient portfolios are defined as portfolios that achieve the highest expected return for a given level of variance. The two-fund separation theorem states that any efficient portfolio is a linear combination of the maximum Sharpe ratio (MSR) portfolio and the risk-free asset. This result provides an unambiguous definition of diversification – the risk management technique that seeks the best reward (expected return) per unit of risk (volatility) taken, to efficiently harvest risk premia across risky assets. In the next section, we shall provide a theoretical result (Proposition A in Section 4) showing that the MSR portfolio has zero specific risk, which makes intuitive sense since bearing residual risk only increases total risk without bringing a commensurate reward in the form of additional expected return.

Unfortunately, the straightforward definition of a well-diversified portfolio as a portfolio that maximises the Sharpe ratio is not fully operational because of estimation risk: indeed, the optimal weights depend on the expected returns of the constituents, which are notoriously difficult to estimate (Merton 1980), and estimation errors, which also plague covariance matrix estimates, can outweigh the benefits of scientific diversification out of sample (see Kan and

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

Zhou (2007) and DeMiguel, Garlappi, and Uppal (2009) for recent references).

#### *Diversification: From Assets to Factors*

To alleviate the concern over parameter uncertainty, one may decide to implement a weighting scheme that does not require expected return estimation: a proxy for the true, unknown, MSR portfolio is called a *performance-seeking portfolio* (PSP). The global minimum variance (GMV) portfolio is an alternative portfolio, which coincides with the MSR one when all expected returns are equal. Hence, choosing the GMV is equivalent to constructing the MSR portfolio by ignoring any difference in expected returns. But this weighting rule tends to overweight the least volatile constituents, or those that have the lowest correlations with the others, resulting in concentrated portfolios. The concern is exacerbated if short positions are allowed, because large positions in assets with low volatilities and/or correlations are taken and are compensated by short positions in the most volatile assets. A solution is to impose weight constraints to avoid excessive dispersion of weights across constituents: these constraints can be expressed as bounds on weights (Jagannathan and Ma 2003) or as a lower bound on the sum of squared weights (DeMiguel et al. 2009). As shown in these references, the reason why these methods bring an improvement over variance minimisation is that they reduce the impact of estimation errors in the covariance matrix.

To bypass the estimation of expected returns, one may also replace the MSR portfolio with a "well-diversified portfolio", by applying the conventional wisdom rule to "spread eggs across baskets": obviously, the maximum extent of diversification is achieved when each

basket contains the same number of eggs. But how do we define "eggs" and "baskets" in asset allocation? Equal weighting sounds like the most natural option to diversify across constituents and has the advantage of being completely free from estimation risk because it does not require any parameter estimate. But the risk of an equally-weighted portfolio of stocks and bonds is largely dominated by the risk of stocks, which have a much higher volatility. To address this problem, an alternative weighting scheme known as risk parity was introduced, in which each constituent must have the same contribution to risk (Qian 2005; Maillard, Roncalli, and Teiletche 2010). For two constituents, equating risk contributions is equivalent to weighting constituents by the inverse of their volatilities, but for three or more assets, no closed-form expression for risk parity weights is known, and numerical techniques must be employed (Maillard, Roncalli, and Teiletche 2010; Spinu 2013).

A shortcoming of the risk parity approach, however, is that it can give a misleading picture for highly correlated assets. For instance, an equally-weighted portfolio of two highly correlated bonds with similar volatilities is well diversified in terms of dollar and volatility contributions, but portfolio risk is extremely concentrated in a factor, which is the level of interest rates. This example highlights the need to look through the constituents' glass and identify the drivers of a portfolio's risk to assess their relative importance. One of the key problems experienced by institutional investors during the sub-prime crisis and the severe market downturn that has followed is that even a seemingly well-diversified allocation to multiple asset classes can hide an extremely concentrated set of factor exposures.

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

#### *Using Conditioning Factors*

Because state variables characterise current market conditions (expected returns, volatilities and correlations), they are useful for the construction of dynamic PSPs that react to changes in the economic environment. The dividend yield, which is often used to predict the returns to stock portfolios, and the forward rate, which is used as a predictor of bond returns, are examples of such variables. GDP and inflation are also state variables that serve to define a finite number of economic regimes. Continuous variables define a continuous range of regimes, which can be narrowed down to a finite set by discretising the set of possible values.

#### 3.1.2 Building Improved Liability-Hedging Portfolios

Many, if not all, investors have liabilities or goals. Institutional investors like pension funds and insurance companies have explicit commitments to their beneficiaries, and individual investors have goals that translate into capital requirements: for instance, to secure a minimum stream of income in retirement, an individual must have a certain amount of capital that equals the no-arbitrage price of an annuity delivering the income. These investors face the risk of an unexpected increase in the present value of their liabilities or goals, which will adversely impact their funding ratio, defined as the value of assets divided by the present value of liabilities or goals. Diversification, which aims at extracting the risk premia from assets at the lowest possible (absolute) risk is ineffective at reducing the volatility of the funding ratio.

Hedging is the risk management technique that aims to construct a portfolio with

returns that match those of liabilities, also known as a *liability-hedging portfolio* (LHP). But diversification and hedging are not mutually exclusive principles, and when an investor is concerned with increasing his funding ratio at the lowest possible risk, the optimal approach is to combine a performance-seeking portfolio – theoretically, the MSR portfolio –, a safe LHP and the cash account. Several examples of this three-fund separation theorem have been given in the literature.<sup>9</sup>

A factor approach is natural for the construction of a safe LHP. Indeed, suppose that a set of risk factors that accurately describe liability returns has been identified, and consider a portfolio with factor exposures that match those of liabilities. In mathematical terms, we have a set of factors  $F_1, \dots, F_K$  such that asset returns and liability returns are well explained by the factor exposures. The returns to a portfolio and to the liability process are given by

$$R_p = c_p + \sum_{k=1}^K \beta_{kp} F_k + \varepsilon_p,$$

$$R_L = c_L + \sum_{k=1}^K \beta_{kL} F_k + \varepsilon_L,$$

with the residuals  $\varepsilon_p$  and  $\varepsilon_L$  including any omitted source of risk. When factor exposures are the same on the asset and the liability sides, uncertainty in the excess return of the portfolio over liabilities,  $R_p - R_L$ , is entirely explained by the residual terms. If these terms are small, then so is the variance of the excess return. To sum up, for the purpose of constructing LHPs, we are interested in finding risk factors that explain most of the variance of assets and liabilities.

9 - See examples in Sørensen (1999) and Brennan and Xia (2002), and dynamic allocations with risk budgeting in Teplá (2001) and Martellini and Milhau (2012).

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

If, in addition, the risk factors are pricing factors (hence, are factors in the APT sense), then the differences in the expected returns of assets and liabilities are fully explained by the differences in their exposures. By replicating exposures, one ensures that not only does the excess return of the LHP over liabilities have a low variance, but it also has a zero average return.

#### 3.2 Factor Investing and Risk Allocation: What?

In practice, factors that are useful in multi-asset allocation decisions belong to two of the categories enumerated in Section 2, namely underlying risk factors that explain cross-sectional differences in risks and returns, and conditioning variables that explain their changes over time.

##### 3.2.1 Underlying Risk Factors

Risk factors appear in two applications: for the construction of a well-diversified PSP and for the construction of a safe LHP. They can be categorised as explicit factors, which do not involve any estimation, and implicit factors, which are treated as quantities to be estimated. We review these two classes here.

##### *Explicit Factors*

A rapid examination of the characteristics of asset classes suggests that macroeconomic factors are good candidates: activity impacts the anticipated real future cash flows of stocks, interest rates have a straightforward effect on bond returns and on the discount rates of dividends in stock prices, exchange rates determine the returns to currency futures, inflation obviously affects the returns to inflation-linked bonds, etc. However, as

highlighted previously, macro factors often have limited ability to explain asset returns in the statistical sense, so a portfolio cannot be accurately described as a combination of factor values: from the low regression R-squares reported in Table III of Asness, Moskowitz, and Pedersen (2013), it is apparent that macroeconomic variables like consumption growth and GDP growth have low explanatory power for portfolios of stocks or non-stock constituents. As a result, focusing on the factor contributions to see how balanced they are gives a misleading picture of portfolio risk that ignores a large part of that risk. In LHP construction, matching exposures to factors with small explanatory power leaves substantial replication error.

However, the claim that macro factors have low explanatory power must be moderated. In particular, there is pervasive evidence that such variables go a long way towards explaining yield curve movements. It is standard to describe changes in the location and the shape of the yield curve in terms of three latent factors respectively labelled level, slope and curvature (see Section 4.2.2), but Ang and Piazzesi (2003) show that composite measures of inflation and real activity (made up of several macro series) explain a substantive fraction, up to 85%, of the variance of yield forecasts. This percentage is higher for yields of short maturities than for those of long maturities, due to the stronger presence of inflation effects in the short end of the curve. Moreover, the macro factors absorb a good part of the slope and curvature factors, though they are unable to capture the level factor. The intertwined lagged effects of macro variables on the three latent factors, and the feedback effects of these factors on future macro aggregates

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

are examined by Diebold, Rudebusch, and Aruoba (2006).

As mentioned in Section 2.3.3, another way to better capture time series variation in returns is to consider micro, as opposed to macro, factors. A micro factor is defined as the excess return to a portfolio of assets picked from the universe of assets to explain. This construction method has been extensively applied in the equity universe, where the market factor and the size, value and momentum factors are defined in this way (see Fama and French (1993) and Carhart (1997)). But the procedure can be carried to multi-class portfolios: Asness, Moskowitz, and Pedersen (2013) compute "value and momentum everywhere factors" by combining single-class value and momentum factors, and weighting them by the inverse of their volatility. (The inverse volatility scheme tends to smooth risk contributions across constituents and avoids the strong dominance of the most volatile asset classes, like equities, over the least volatile ones, like Treasuries.) Because such micro factors are endogenous (they are linear combinations of the returns to explain), they are more likely to deliver good R-squares in time-series regressions.

To overcome the non-investability of many macro factors, one can form factor-replicating portfolios, by maximising the correlation with changes in the factor or by selecting a portfolio with a beta of one with respect to the factor and zero betas with respect to other factors (see Section 2.3.3). Interestingly, the second method has a strong connection with the Fama-MacBeth procedure, which is often employed in the literature on empirical asset pricing to measure factor premia. Indeed, the time series of estimates for

the factor premia produced in the second stage of the method coincide with the returns to constrained minimum variance portfolios: the constraint is to have a beta of one with respect to a given factor, and zero betas with respect to the others (see Section D of the Technical Appendix for a proof).

Whichever method is taken, the construction of replicating portfolios for macro factors is complicated by the necessity to correctly measure the exposures of the various securities to these factors. Several of these factors are typically published at a low frequency, which is quarterly for US GDP numbers, or monthly for the US consumer price index: but in that latter case, there is a two-week lag between the end of the month and the release. Other macro factors, like interest rate variables, are available at higher frequencies, for example daily. A bigger concern is the instability of factor exposures over time: a portfolio with an in-sample beta of one may have very different betas out of sample. Ang, Brière, and Signori (2012) give examples of this instability by studying the exposures of stocks with respect to inflation.

#### *Implicit Factors*

Unlike explicit factors, implicit factors are not given ex-ante, but they are functions of the returns of the assets that they are intended to explain. They have a clear advantage for the purpose of being used as risk factors, because they can be chosen to maximise the fraction of variance explained. In the limit, if they fully explain asset returns, with no residual risk, then a portfolio, which is normally a combination of assets, can also be regarded as a package of factors. A second advantage of implicit factors is that they can be defined as uncorrelated



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

variables. This property guarantees that they represent non-overlapping sources of risk, and it facilitates the attribution of portfolio variance to the various sources.

Meucci (2009) and Deguest, Martellini, and Meucci (2013) precisely suggest to evaluate the level of diversification of a portfolio by measuring the contributions of uncorrelated risk factors that fully explain constituents' returns. To this end, they use the two methods introduced in Section 2.3.2, namely principal component analysis and minimum linear torsion (PCA and MLT): both produce a set of uncorrelated factors that exhaust all of the uncertainty over asset returns.

Formally, consider a set of  $N$  risk factors given as non-redundant linear combinations of asset returns. Then, asset returns can be recovered from factor values by inverting the system of equations that defines the factors in terms of assets. The return to a portfolio, which is the weighted sum of asset returns, can be rewritten as a sum of factor returns. Moreover, these factors are uncorrelated by construction. In this case, Equation (2.5) applied to a portfolio  $p$  gives

$$\sigma_p^2 = \sum_{k=1}^N \beta_{kp}^2 \sigma_{F,k}^2, \quad (3.1)$$

where the  $\beta_{kp}$  are the factor exposures of the portfolio and the  $\sigma_{F,k}^2$  are the factor variances. From this equation, and given the absence of correlation terms across factors, it is straightforward to define the contribution of each factor to portfolio risk: this contribution is proportional to the product  $\beta_{kp}^2 \sigma_{F,k}^2$ , and a normalisation by  $\sigma_p^2$  ensures that the  $N$  factor contributions sum up to one. Hence, a portfolio is well-diversified in terms of risk factors if exposure-times-volatility

products are not too dispersed, and the highest level of diversification is attained when the absolute values of exposures are inversely proportional to volatilities: such a portfolio is referred to as a "factor risk parity portfolio". A similar notion is introduced by Roncalli and Weisang (2012), with explicit macroeconomic factors instead of implicit factors.

#### 3.2.2 Conditioning Variables

Section 2.4 has given several examples of state variables that contribute to characterise market conditions. The dividend yield is a standard predictor of stock returns, but predictive regressions with multiple factors have been employed: Campbell, Chan, and Viceira (2003) add the term spread, the nominal T-bill rate, the ex-post T-bill rate and the past excess returns to the stock and the bond. For bond returns, the forward-spot spread, which is related to the slope of the term structure, has predictive ability for the excess return to the bond of the same maturity (Fama and Bliss 1987), and a single factor, known as the "tent factor", has predictive power for several maturities.

As argued in Section 2.4.2, macro factors such as GDP and inflation can also serve to define regimes characterised as phases of the business cycle. In Section 3.4, we provide an empirical illustration with the identification of economic regimes based on the values of inflation and real output growth.

### 3.3 Factor Investing and Risk Allocation: How?

We now discuss how factor investing can be implemented in practice in an asset allocation context. The methods applied depend on the building block to

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

be constructed. For LHP construction, the problem at hand is a risk budgeting one, where one has to match the factor exposures of the liability process. For the purpose of constructing a PSP, the factor exposures are not given ex-ante and they must be optimally chosen, to ensure an optimal level of diversification.

#### 3.3.1 Factor Matching for LHP Construction

The factor approach to LHP construction is particularly interesting in situations where a set of common risk factors is available and the exposures are easily measurable. Nominal bond replication is a typical example: if liabilities consist of a stream of fixed cash flows, their present value is the price of a nominal bond. If the replicating assets are also nominal bonds, the level of interest rates is the main risk factor in asset and liability returns. Moreover, differences in the exposures of bonds with respect to this factor are well proxied by differences in (modified) duration. Hence, a duration-matching portfolio captures most of the risk in the liability process.

However, duration alignment does not imply perfect liability replication, because the exposures of bonds to the level factor are not linear. Modified duration measures the sensitivity of a bond price with respect to a small change in the factor, but it does not accurately measure the sensitivity to a larger change: convexity, which is the second-order sensitivity, improves the approximation. Hence, to immunise a portfolio against large movements in rates, it is necessary to go beyond the first-order approximation and to match convexities in addition to durations. These techniques are routinely used by pension funds, but they are applicable in any situation where liabilities or goals

are represented by fixed cash flows. In theory, the process should be carried on to all other derivatives in order to achieve perfect replication.

The second reason why duration matching is not an exact replication method is because the interest rate level is not the only risk factor that makes bond prices move: the slope and the curvature factors also have marginal contributions, but measuring the corresponding bond exposures is arguably not as straightforward as for the level. But for some types of liabilities, a multi-factor approach to LHP construction may be needed. For instance, pension liabilities are exposed to inflation risk if pension benefits are indexed, and they are also exposed to longevity risk. A perfect hedge against these sources of uncertainty may not be possible due to the difficulty to find assets with forecastable exposures, but even an imperfect hedge will improve replication. More generally, a proper identification of all meaningful sources that have an impact on liability values is a key requirement in the implementation of a sound asset-liability management framework.

#### 3.3.2 Risk Factors and Diversification

The theoretical prescription to hold the MSR portfolio is hard to implement because of substantial uncertainty over parameter values, especially expected returns. In practice, it can be replaced by a more general recommendation to hold a "well-diversified portfolio", but as highlighted above, there are multiple definitions of diversification. Equal weighting corresponds to the highest diversification level in terms of dollars: each constituent has exactly the same dollar weight, which is a version of the "spread eggs across baskets" rule, in which



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

eggs are dollars and baskets are assets. The risk parity portfolio in the sense of Qian (2005) and Maillard, Roncalli, and Teiletche (2010) splits contributions to risk equally across constituents. It is the counterpart of the equally-weighted portfolio, when "eggs" and "baskets" are respectively defined as risk contributions and assets. However, these approaches focus on assets, so they disregard the underlying risk factors.

To have a quantitative measure of diversification in terms of factors, Deguest, Martellini, and Meucci (2013) (henceforth, DMM13) and Carli, Deguest, and Martellini (2014) propose to extend the notion of the effective number of constituents (or ENC), which is defined as the inverse of the sum of squared weights – itself known as the Herfindahl index of the portfolio – and it quantifies the level of diversification in terms of dollars. The minimum possible value for the ENC (at least for a long-only portfolio) is 1 and corresponds to a portfolio invested in a single asset, and the maximum value is the nominal number of constituents, which is attained if, and only if, the portfolio is equally weighted (Deguest, Martellini, and Meucci 2013). The *effective number of uncorrelated bets* (ENUB) introduced by Deguest, Martellini, and Meucci (2013) and Carli, Deguest, and Martellini (2014) is based on the decomposition of portfolio variance across factors, as in Equation (3.1). The relative contribution of each factor equals  $\beta_{kp}^2 \sigma_{F,k}^2 / \sigma_p^2$  (and the contributions sum up to one), so the ENUB is defined as the inverse of the sum of squared contributions to risk:

$$ENUB = \frac{\sigma_p^4}{\sum_{k=1}^N [\beta_{kp}^2 \sigma_{F,k}^2]^2}.$$

Going back to the eggs and baskets analogy, the ENUB measures diversification in a setting where eggs are risk contributions and baskets are risk factors as opposed to being constituents. It attains its maximum of  $N$ , the nominal number of factors or assets, when all factors contribute equally to risk: this property defines a *factor risk parity* (FRP) portfolio: There is no unique FRP portfolio. DMM13 show that for  $N$  assets, there are exactly  $2^{N-1}$  of them. Two of them stand out: the one that minimises the variance and the one that maximises the Sharpe ratio. They can be regarded respectively as the minimum variance and the maximum Sharpe ratio portfolios subject to a factor risk parity constraint. Other choice criteria can be adopted: since many of the FRP portfolios involve short positions, one may focus on the one with the lowest amount of leverage.

It is also possible to use the ENUB to impose a minimum level of diversification across factors in an optimisation program, in the spirit of the work of DeMiguel et al. (2009), who show that imposing a "norm constraint" such as a minimum ENC constraint leads to lower out-of-sample volatility and higher out-of-sample Sharpe ratio for estimated minimum variance portfolios. This finding suggests that heuristic diversification techniques based on the "spread eggs across baskets" rule and scientific diversification rules aiming to minimise variance or to maximise Sharpe ratio should not be regarded as competing methodologies. Introducing some amount of heuristic diversification appears to be an effective way to improve the out-of-sample properties of estimates for scientifically diversified portfolios. Following this principle, a minimum ENUB constraint can be introduced in various optimisation programs to ensure

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

that risk contributions are not overly concentrated in a few factors and to reduce biases inherent to some weighting schemes. For instance, an unconstrained minimum variance portfolio of stocks, Treasuries and corporate bonds is biased towards Treasuries, which are the least volatile asset class, and is thus dominated by interest rate risk. An ENUB constraint allows the relative weight of the interest rate risk factor to be reduced.

#### 3.4 Empirical Illustration

We now present an empirical illustration with the following seven asset classes: US equities, world non-US equities, US Treasuries, US corporate bonds, US Treasury Inflation-Protected Securities (TIPS), commodities and real estate.

##### 3.4.1 Extracting Implicit Risk Factors

Asset classes are represented with the following series: US equities are represented by the S&P 500 index, world non-US equities by the MSCI World Ex-US index, US Treasuries by the Barclays US Treasury index, US corporate bonds by the Barclays US Baa index, US Treasury Inflation-Protected Securities by the Barclays US TIPS index, commodities by the S&P GSCI index and real estate by the Dow Jones US Select REIT index. All index values include dividend and coupon reinvestment. The data is monthly and covers the period from April 1997 to September 2017. Index series are converted to excess returns by subtracting the US Treasury bill secondary market rate of three-month maturity. Principal component analysis and minimum linear torsion are successively applied to extract seven implicit uncorrelated risk factors.

Figure 1 shows the compositions of the statistical factors in terms of assets:

since the variables analysed are excess returns, the coefficients shown in the figure can be interpreted as the weights of long-short portfolios invested in the constituents.

In the MLT procedure, each factor is intended as a proxy of an asset. Panel (b) of Figure 1 clearly shows that each factor is mainly composed of the asset to which it is associated. The positions in the other assets, which are often short positions, are taken in order to eliminate the effects of the inter-asset correlations. In factors 1 and 2, which respectively correspond to US equities and World equities, the short positions in the other equity constituent are sizable, because the two equity indices have a correlation of 86.8% in the sample.

The interpretation of principal components (PCs) is not as straightforward as for MLT factors. Panel (a) of Figure 1 displays the composition of each factor: PC1 loads negatively on almost all asset classes (except for US Treasuries) but has very small loadings on the three bonds: it represents a sort of multi-asset, non-bond, market factor. PC2 loads almost exclusively on commodities and real estate, so it could be described as an alternative asset factor.<sup>10</sup> PC3 is an equity factor, that loads positively on both equity indices and takes short positions in commodities and real estate. PC4 is a factor that captures the differences between the two equity indices by being long US equities and short international equities. It has almost no exposure to the other classes. In contrast with the previous factors, which have nearly zero exposures to bonds, PC5 is mostly exposed to the three bonds, with negative weights: it accounts for the commonalities between the three

10 - The sign of asset weights in a factor is a heuristic convention: all signs could be flipped, changing the factor into its negative, without the explanatory power of the factor being impacted.

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

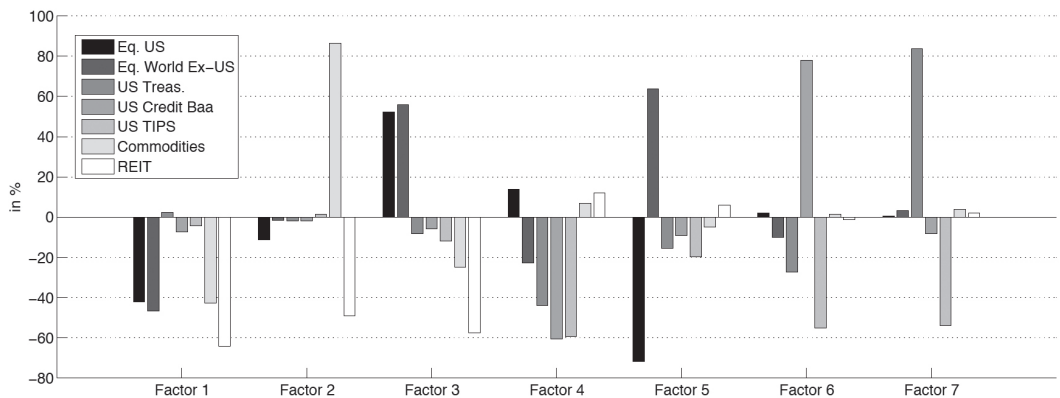
bond classes, and thus corresponds to an interest rate level factor. PC6 is long the US corporate bonds and short the Government bonds (Treasuries and TIPS), so it can be related to credit risk. The last factor, PC7, is long US Treasuries and short TIPS, and accounts for the differences between these securities: it is possibly related to the spread between nominal and real interest rates, which is also known as the breakeven inflation rate and includes contributions from expected inflation and the inflation risk premium.

Figure 2 provides another perspective on the principal factor, by showing which

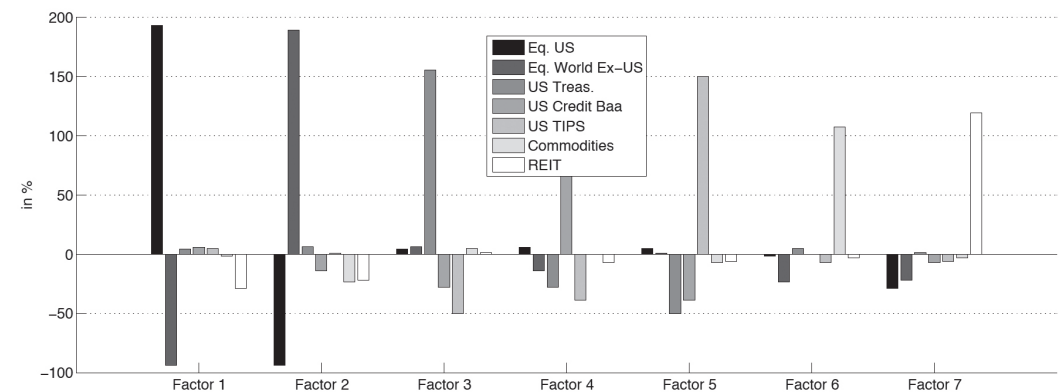
factors are important to explain time variation in the returns of a given asset. For each asset, we compute the tracking error with respect to a linear combination of 1, 2, 3, 4, 5 or 6 factor(s) (the coefficients being the exposures of the asset with respect to the factors). We also report the tracking error with respect to a portfolio with no factor (i.e., a constant), that is the volatility of the asset class. Each of these combinations can be regarded as a portfolio of factors that proxies the asset. By construction, the tracking error decreases when a new factor is added, and it would be zero for a seven-factor portfolio (not shown in the picture). It is seen that eliminating the influence of the

Figure 1. Composition of implicit risk factors in terms of assets; 1997-2017.

(a) Principal component analysis.



(b) Minimum linear torsion.



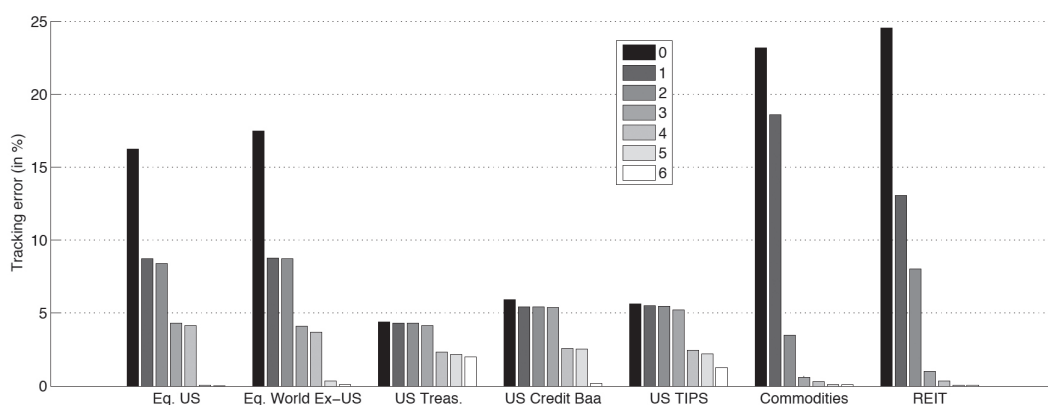
Each picture shows the composition of implicit uncorrelated risk factors in terms of the seven asset classes analysed: US equities, world ex-US equities, US Treasuries, US corporate bonds, US TIPS, commodities and real estate. In Panel (a), factors are extracted by principal component analysis, while in Panel (b), they are obtained through minimum linear torsion. The statistical procedures are applied on monthly excess returns.

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

first factor significantly reduces volatility for all classes except for the two bonds. This is consistent with the interpretation of PC1 as a non-bond market factor. The next factor, PC2, is only useful to explain commodities and real estate: for all other classes, the replication quality is hardly improved by moving from one to two factor(s). PC3 and PC4 bring an improvement for all classes except bonds. The only factors that are relevant for Treasuries and corporate bonds are PC5 and PC6, an observation in line with Figure 1. With six replication factors, all tracking errors are virtually zero, and the marginal contribution of PC7 is hardly noticeable.

The fact that interest rate risks do not show up in the first factors can be attributed to the low volatilities of the bond indices with respect to the other classes: over the sample period, the volatility of excess returns is 16.2% for US equities, 17.5% for international equities, 23.2% for commodities and 24.6% for real estate, but only 4.4%, 5.9% and 5.7% respectively for Treasury bonds, corporate bonds and TIPS. Being far less volatile than the other classes, bonds represent only a small fraction of the aggregate risk of the seven classes, so the PCA procedure does not make the explanation of their returns a priority. Statistically, the first four factors explain about 97% of aggregate risk in this example, which sounds like a large fraction. But Figure 2

Figure 2. Approximation of asset excess returns with linear combinations of principal factors; 1997-2017.



Principal factors are extracted by analysing the monthly excess returns on seven asset classes over the period from April 1997 to September 2017: US equities, world ex-US equities, US Treasuries, US corporate bonds, US TIPS, commodities and real estate. Asset excess returns are then proxied with linear combinations of one to seven factor(s), and the figure shows the annualised tracking error of the factor portfolio with respect to the asset. Bars 0 show asset volatilities over the sample period, and bars 7 have zero height because each asset is perfectly replicated by a portfolio of the seven factors.

Table 1. Effective number of uncorrelated bets with implicit risk factors.

	PCA factors	MLT factors
Policy portfolio	1.34	3.40
Equally-weighted	1.08	3.77
Risk parity	2.00	6.00
Minimum variance	2.67	2.28

Implicit uncorrelated factors are extracted from the monthly excess returns on seven asset classes over the period from April 1997 to September 2017: US equities, world ex-US equities, US Treasuries, US corporate bonds, US TIPS, commodities and real estate. Two statistical procedures are applied: principal component analysis and minimum linear torsion. The table displays the effective number of uncorrelated bets for four portfolios: a policy portfolio with 60% in equities, 30% in bonds and 10% in alternative assets (including TIPS), and an equally-weighted split within each of these group; the equally-weighted portfolio; the risk parity portfolio; and the global minimum variance portfolio.

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

shows that these four factors, while being decently good at describing equities, commodities and real estate, are close to useless when it comes to explaining bond returns: to capture some of the time variation in bond returns, factors 5, 6 and 7 are needed. This example invites to some caution in the decision to eliminate principal factors on the grounds of their marginal statistical contribution. Indeed, an economically important risk, here interest rate risk, can be relegated to the last factors, and omitting it may result in poor replication of some assets.

#### 3.4.2 Assessing Diversification in Terms of Factors

The effective number of uncorrelated bets (ENUB) depends on the chosen risk factors. In Table 1, we compute the ENUB for four portfolios with the two methods: principal component analysis and minimum linear torsion. The first portfolio is a policy portfolio with a 60% weight in equities, 30% in bonds and 10% in alternative assets. Within each class, the portfolio is equally split across constituents: hence, there is a 30% weight in US equities, 30% in international equities, 15% in US Treasuries, 15% in US credits, 3.33% in TIPS, 3.33% in commodities and 3.33% in real estate. The second portfolio is equally weighted across the seven classes. The third one is the risk parity portfolio (RKP) of Maillard, Roncalli, and Teiletche (2010) and the fourth one is the global minimum variance (GMV) portfolio subject to a long-only constraint. The weights in percentage points of the risk parity and the minimum variance portfolios are

$$\begin{aligned} \mathbf{w}_{RKP} &= [7.4 \quad 6.3 \quad 40.6 \quad 16.4 \quad 18.5 \quad 6.2 \quad 4.6], \\ \mathbf{w}_{GMV} &= [10.0 \quad 0 \quad 87.1 \quad 0 \quad 2.9 \quad 0 \quad 0]. \end{aligned}$$

Both portfolios overweight Treasuries, which are the least volatile class, but RKP is less concentrated than GMV.

With seven factors, the maximum possible ENUB is 7 and is attained by factor risk parity portfolios. Only RKP is close to factor parity, with an ENUB of 6, when MLT factors are employed. Indeed, each MLT factor is "close" to a constituent, so a risk parity portfolio is a rough proxy for a factor risk parity portfolio. With PCA factors, the greatest ENUB (2.67) is attained by GMV: this result contrasts with the high concentration of this portfolio in Treasuries, which makes it appear poorly diversified. But the large weight in Treasuries re-equilibrates the allocation towards interest rate risk factors, which are marginal from a statistical standpoint since they correspond respectively to the last three factors. As a result, this portfolio has a higher ENUB than the policy and the equally-weighted portfolios, in which equity factors are overweighted. As a general rule, the ENUB is highly sensitive to the choice of the underlying factors, so it is important to specify what the factors are when mentioning this indicator.

#### 3.4.3 Factor Risk Parity Portfolios

For seven constituents, the number of portfolios that satisfy the factor risk parity condition is  $2^7 - 1 = 64$ , so it is feasible to compute all of them, using the formulas given in DMM13 and recalled in Section E the Technical Appendix. All of them involve at least one short position in a constituent, and the short positions are sometimes sizable, as appears from Panel (a) of Figure 3, where the total leverage of each portfolio is plotted. The total leverage is defined as minus the sum of all negative weights: it is zero for a long-only portfolio, and would be 0.50 for a two-asset portfolio with a 150% weight

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

in one constituent partially financed by a short position of -50% in the other. That said, leverage varies considerably from one portfolio to the other. Some of them have excessively high levels of leverage, as high as 27, or 2,700%, while the minima are much more reasonable, at 45.0% for PCA factors and 0.4% for MLT factors. The corresponding weights (expressed in percentage points) are

$$\mathbf{w}_{PCA,LEV} = [13.7 \quad -11.7 \quad 98.9 \quad -33.3 \quad 12.0 \quad 15.7 \quad 4.7],$$

$$\mathbf{w}_{MLT,LEV} = [15.0 \quad 4.3 \quad 42.1 \quad 18.2 \quad 15.5 \quad 5.3 \quad -0.4].$$

With MLT factors, the FRP portfolio that minimises leverage is almost a long-only one. It can be turned into a strictly long-only portfolio by setting the real estate weight equal to zero and cutting the other weights to bring back their sum to 100%. The composition of this "rescaled" portfolio is

$$\mathbf{w}_2 = [14.9 \quad 4.3 \quad 41.9 \quad 18.1 \quad 15.4 \quad 5.3 \quad 0],$$

and its ENUB with MLT factors is 6.99, which is indeed very close to 7. In details, the percentage contributions to risk of the MLT factor are:

$$[14.3 \quad 14.3 \quad 13.9 \quad 14.0 \quad 14.0 \quad 14.1 \quad 15.4].$$

The dispersion is small, so the long-only portfolio  $\mathbf{w}_2$  is a good approximation for a factor risk parity portfolio.

Panel (b) displays the ex-ante volatilities of the 64 portfolios: the ex-ante volatility is the quantity  $\sqrt{\mathbf{w}'\Sigma\mathbf{w}}$ , where  $\mathbf{w}$  denotes the vector of weights and  $\Sigma$  is the covariance matrix. Rather unsurprisingly, volatility turns out to be strongly related to leverage, as indicated by the similar shapes of the two graphs. In fact, whether PCA or MLT factors are employed, the FRP portfolio that minimises the variance is the same as the one that minimises the sizes of short positions. It has an ex-ante volatility of 5.0% per year with

PCA factors and 4.9% with MLT factors. For comparison purposes, the ex-ante volatility of the GMV portfolio subject to a long-only constraint is 3.8%.

#### 3.4.4 Volatilities and Average Returns Across Growth and Inflation Regimes

We now turn to the calibration of a regime-switching model in which the risk and return parameters of the seven asset classes are functions of the regime, defined with respect to the current values of inflation and real output growth.

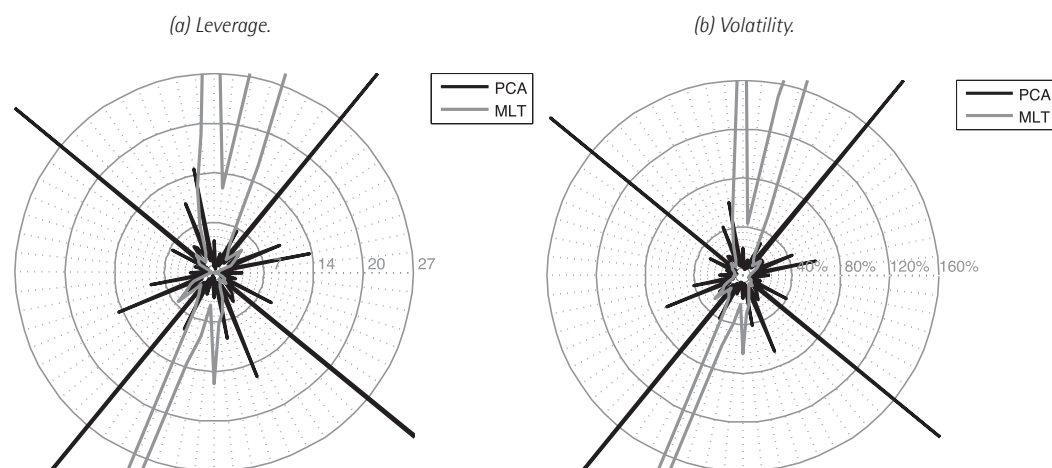
Inflation is represented by the annual growth in the Consumer Price Index for all urban consumers, seasonally adjusted, and output is represented by the real Gross Domestic Product. Both series are obtained from the Federal Reserve of St Louis database, where they are sourced from the US Bureau of Economic Analysis. The growth rate is taken to be the annual growth rate in the GDP. For each of the two variables, we divide the sample into two equal parts by taking the median of annual changes, and we define the four regimes as follows. A quarter is said to be in regime 1 if inflation and growth are respectively below and above their medians; it is in regime 2 if inflation and growth are high; it is in regime 3 if inflation and growth are low; and it is in regime 4 if inflation is high and growth is low. With these explicitly defined regimes, one can easily form the quarterly time series of regime numbers, and estimate the probability of staying in regime  $i$  as the percentage of pairs  $(i, j)$  in the series. Similarly, the probability of switching from regime  $i$  to regime  $j$  is the percentage of pairs  $(i, j)$ .

The longest period over which output and price index data is available ranges from January 1947 to April 2017, but in order



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

Figure 3. Total leverage and ex-ante volatility of factor risk parity portfolios; 2000-2017.



*Implicit factors are extracted by analysing the monthly excess returns on seven asset classes over the period from April 1997 to September 2017: US equities, world ex-US equities, US Treasuries, US TIPS, US corporate bonds, commodities and real estate. Two statistical procedures are applied to extract uncorrelated risk factors: principal component analysis and minimum linear torsion. The 64 factor risk parity portfolios equate the contributions of the implicit factors to portfolio volatility. Panel (a) shows the total leverage of each portfolio, defined as the sum of the absolute values of the negative positions, and Panel (b) shows the annualised ex-ante volatilities of the 64 portfolios. For readability, portfolios with a leverage greater than 27 (8 out of 64) or a volatility greater than 160% (7 out of 64) are not plotted.*

to assess the robustness of the results, we repeat the estimation of transition probabilities over four samples: the full sample, and three samples starting respectively in January 1957, 1967 and 1977. We do not try shorter samples, as the estimation of a regime-switching model requires a dataset spanning multiple regimes. Table 2 shows the results of the estimation. Results are roughly consistent across the samples, with transition probabilities that remain qualitatively similar from one to the other. The highest probabilities are located on the diagonal, meaning that regimes are rather persistent: it is always more likely to stay in a regime than to switch to another. We also compute "discrepancy rates" between samples: for instance, we sort quarters in four regimes based on the breakpoints for the period 1947-2017 and repeat the exercise, but taking into account the breakpoints for the period 1957-2017. The fraction of quarters that are assigned a different regime in the second calibration than in the first is

4.9%. The same comparison is done for all pairs of calibration periods, and it appears from Table 2 that the classification of quarters is rather stable: with discrepancy rates lower than 17%, a quarter that is classified as regime 1, 2, 3 or 4 in a calibration is likely to be the same regime in another calibration. This is of course due to the stability of the median across samples: the median annual inflation varies from 2.9% to 3.4%, and the median annual growth is between 2.8% and 3.1%.

In order to have a longer track record for the asset classes, we use the ERI Scientific Beta Long-Term US cap-weighted index to represent US equities, because the total returns for the S&P 500 index are not available prior to 1988. The availability period depends on the asset class: while the equity, bond and commodity series are available since January 1973, the real estate series starts in January 1978 and the US TIPS series in April 1997. Our US equity series stops in December 2016, so

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

Table 2. Estimation of four-regime model for inflation and growth.

1947 - 2017					
Median inflation and growth (%)		2.9	3.1		
Transition probabilities (%)	To Regime	1	2	3	4
From Regime	1	78.3	5.7	13.0	2.9
	2	10.1	77.1	4.3	8.7
	3	8.7	1.4	76.8	14.5
	4	2.9	15.7	5.8	73.9

1957 - 2017					
Median inflation and growth (%)		3.0	3.1		
Transition probabilities (%)	To Regime	1	2	3	4
From Regime	1	79.7	8.3	11.9	0.0
	2	11.9	73.3	5.1	10.2
	3	6.8	0.0	79.7	15.3
	4	1.7	18.3	3.4	74.6

1967 - 2017					
Median inflation and growth (%)		3.4	3.0		
Transition probabilities (%)	To Regime	1	2	3	4
From Regime	1	74.5	9.6	13.7	0.0
	2	12.8	76.9	2.0	8.5
	3	12.8	0.0	80.4	10.6
	4	0.0	13.5	3.9	80.9

1977 - 2017					
Median inflation and growth (%)		2.9	2.8		
Transition probabilities (%)	To Regime	1	2	3	4
From Regime	1	67.6	8.9	15.9	0.0
	2	17.6	75.6	2.3	8.8
	3	11.8	2.2	77.3	17.6
	4	2.9	13.3	4.5	73.5

Discrepancy rates (%)				
	Shorter period	1957 - 2017	1967 - 2017	1977 - 2017
Longer period	1947 - 2017	4.9	15.8	9.2
	1957 - 2017	-	13.3	10.4
	1967 - 2017	-	-	16.6

In each sample period, the four regimes are defined as follows: Regime 1 = growth above median and inflation below median; Regime 2 = high growth and high inflation; Regime 3 = low growth and low inflation; Regime 4 = low growth and high inflation. The discrepancy rate is the percentage of quarters in the shorter period that are not assigned the same regime as in the longer period when the model is re-estimated. Data is from the US Bureau of Economic Analysis.

we present results for two sub-periods: 1973-2016, where all series but real estate and US TIPS are observed, and 1997-2016, where all series with no exception are available.

Table 3 shows the conditional mean returns and volatilities of the seven classes in the four regimes, as well as the unconditional mean for reference. It is well known that sample means are



### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

Table 3. Conditional means and volatilities of asset classes in inflation-growth regimes (in %, annualised).

	1973 - 2016				
	Average returns				
Regime	1	2	3	4	Unconditional
Eq. US	23.0	10.7	9.0	5.8	11.1
Eq. World Ex-US	23.4	13.9	9.4	-0.4	10.4
US Treas.	4.4	8.0	6.5	9.5	7.4
US Credit Baa	5.7	10.3	10.0	7.4	8.6
US TIPS	-	-	-	-	-
Commodities	-3.1	17.6	-2.5	17.0	8.7
REIT	-	-	-	-	-

	Volatilities				
Regime	1	2	3	4	Unconditional
Eq. US	15.0	14.2	17.0	18.9	16.6
Eq. World Ex-US	17.4	15.1	19.6	22.4	19.2
US Treas.	4.9	5.4	5.6	7.3	6.0
US Credit Baa	4.7	7.1	6.5	12.5	8.6
US TIPS	-	-	-	-	-
Commodities	20.1	18.5	23.6	26.6	23.0
REIT	-	-	-	-	-

	1997 - 2016				
	Average returns				
Regime	1	2	3	4	Unconditional
Eq. US	27.4	1.3	7.1	-7.5	8.7
Eq. World Ex-US	25.6	0.4	5.4	-10.2	6.8
US Treas.	5.6	4.1	4.2	8.3	5.4
US Credit Baa	7.7	0.5	8.0	4.3	6.5
US TIPS	5.9	4.4	4.3	9.0	5.8
Commodities	-6.1	33.7	-6.0	17.8	2.4
REIT	16.7	16.2	10.4	9.1	12.2

	Volatilities				
Regime	1	2	3	4	Unconditional
Eq. US	17.4	5.4	18.5	14.4	17.5
Eq. World Ex-US	18.6	5.4	21.3	19.0	20.0
US Treas.	4.0	4.6	5.5	4.7	4.9
US Credit Baa	3.9	4.0	6.8	4.8	5.6
US TIPS	3.1	4.7	5.7	5.1	4.9
Commodities	22.9	28.2	25.5	28.7	26.2
REIT	15.9	19.4	27.4	20.7	22.5

The four regimes are defined as follows: Regime 1 = growth above median and inflation below median; Regime 2 = high growth and high inflation; Regime 3 = low growth and low inflation; Regime 4 = low growth and high inflation. The regimes are based on the median inflation rate and the median growth rate for the period 1947-2017. Conditional and unconditional means are calculated for the seven asset classes in the sub-periods 1973-2016 and 1997-2016. No values are reported for US TIPS and real estate in the first sub-period, due to the unavailability of TIPS data prior to 1997 and real estate data prior to 1978. The conditional mean and the conditional volatility in a given regime are the average return and the standard deviation of returns in those quarters that belong to the regime.

### 3. Factor Investing and Risk Allocation Decisions: A More Efficient Structuration of the Investment Process

---

highly sample-dependent, so they vary from one sub-period to the other, but some regularities arise. Both US and international equities do by far best when inflation remains modest and growth is dynamic (regime 1), while the low growth and high inflation regime (regime 4) is the least favourable to them. Regime 4 is also associated with high equity volatility. Regime 3 (low growth and low inflation) appears relatively detrimental to equities, since they have lower than average conditional mean, and higher than average volatility. Commodities exhibit marked cyclicalities in their returns, with conditional means that largely exceed the unconditional mean in the high inflation regimes (2 and 4), and negative conditional means in the low inflation regimes (1 and 3). In regime 4, however, the higher expected returns are paid by high volatility. Government bonds, whether nominal or indexed, do best when growth is low and inflation is high (regime 4): this finding supports the notion that they are playing the role of a "safe haven" in a market environment where stocks are adversely affected. Their volatility does not exhibit as much variability across regimes as equity volatility, although it is in the high growth and low inflation regime (1) that it is lowest. Corporate bonds have above average performance in the low growth and low inflation regime (3). Finally, real estate appears to be sensitive to economic growth: it posts its best average returns and its lowest volatilities in the high growth regimes (1 and 2). Inflation has only a small effect on average returns once growth is taken into account. These results provide a simple illustration of the fact that growth and inflation can serve as state variables to identify regimes with higher or lower conditional expected returns and volatilities.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

We now shift the perspective from allocation decisions to benchmarking decisions within a given asset class. Like the previous one, this section starts with an analysis of the rationale for factor investing (factor investing: why?), by showing that forming *pricing factors* is theoretically the best way to group individual securities before combining the groups. It is also useful to group them into priced factors in order to more efficiently collect the available risk premia in a given asset class than by constructing a traditional benchmark, like the broad cap-weighted index in the equity class. We then present the most commonly used risk premia (factor investing: what?). This section has a main focus on equity markets, where the factor investing approach is most mature, but we also discuss fixed-income factors as well as multi-asset factors. Merging the allocation and benchmarking perspectives, we finally demonstrate that further improvements in risk-adjusted performance can be achieved when improved diversification techniques are used not only to construct smart factor index benchmarks, but also to efficiently allocate to such efficient benchmarks (factor investing: how?).

### 4.1 Factor Investing and Benchmarking: Why?

This intuition is justified by a mathematical result on the loss of efficiency in the delegated asset management process. The two-step process is a standard investment approach in delegated portfolio management, in which securities are first grouped into portfolios, and the portfolios subsequently serve as building blocks for an asset allocation exercise.

#### 4.1.1 Limits of the Two-Step Process

Taking the example of a stock-bond allocation, it is common practice to create stock and bond benchmarks in the first step, before combining them in the second step. The default option is a cap-weighted index of all stocks in the equity class, and also in the fixed-income universe, where the market value or the face value of debt is used as an equivalent for the market capitalisation.

Cap-weighted indices have well-known limitations. In the equity class, they are poorly diversified across securities because they load heavily on a few very large stocks. Moreover, they represent bundles of factor exposures that are highly unlikely to be optimal for any investor. For instance, they have implicit tilts towards large and growth stocks, while these stocks are outperformed by small and value stocks in the long run. As a result, they have a poor risk-return profile, which can be improved in a straightforward manner by tilting the portfolio towards certain stocks such as the small ones or the value ones (see e.g. Exhibit 3 in Amenc et al. (2014b)). In the bond class, debt-weighted indices are often used as an equivalent of cap-weighted indices, but they mechanically overweight the most indebted issuers: as noted by Siegel (2003), these issuers are the most likely “to be downgraded or to default”, a concern he refers to as the “bums problem”.

Beyond the intrinsic limitations of cap-weighted indices, the two-step process is in general suboptimal because it fails to deliver the optimal allocation to the individual securities. Indeed, the grouping of securities in the first step results in a narrower opportunity set for the asset manager who allocates to the benchmarks

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

(Sharpe 1981). van Binsbergen, Brandt, and Koijen (2008) point the following reasons for the lack of efficiency of the “decentralised portfolio” obtained by forming the mean-variance efficient portfolio of two mean-variance efficient portfolios: the possible unavailability of a riskless asset for the delegate portfolio managers, who have to replace it with a risky minimum variance portfolio, and its availability at the asset allocation level, and the fact that securities are in general correlated across groups, so optimal allocation decisions cannot be made independently in each group. The authors illustrate with numerical computations of the indirect utility – the maximum expected utility that can be achieved by the fund manager – that the two-step process causes substantial welfare losses.

All this raises the question whether it is possible to have a better articulation of the benchmarking and the allocation steps. Sharpe (1981) and van Binsbergen, Brandt, and Koijen (2008) suggest procedures to increase the expected utility in the second step, in a context where the security groups from which each manager builds their own portfolio are defined *ex-ante*. This situation is commonplace in asset allocation with multiple securities, where investment universes are created *ex-ante*, e.g. by separating stocks and bonds and having one manager for each class. The result that we present in the next section relaxes this restriction, and aims precisely to describe an *optimal* sorting of securities. The assumption that the compositions of groups can be optimised over rather than taken as a constraint is not unrealistic if all securities belong to the same class: after all, an equity manager could decide to choose the optimal grouping of individual stocks, as opposed to deciding

*ex-ante* to sort them on the sector or any other arbitrary criterion.

### 4.1.2 Pricing Factors and Efficient Two-Step Process: A Formal Result

Consider an investment universe made of  $N$  risky securities, and assume that they are grouped in  $K$  benchmarks in the first step. These benchmarks have a  $K \times K$  covariance matrix  $\Sigma_b$  and a  $K \times 1$  vector of expected excess returns  $\mu_b$ . These moments can be related to the covariance matrix  $\Sigma$  and the vector of expected excess returns  $\mu$  of the  $N$  securities, by introducing  $\mathbf{w}$ , the  $N \times K$  matrix of asset weights in the benchmarks: each column of  $\mathbf{w}$  describes the composition of a benchmark in terms of the risky securities. It is not necessary to assume that the weights in each column sum up to 100%: each benchmark can contain some fraction of a risk-free asset (which by definition contributes neither to variance nor risk premium), or involve some leverage. For instance, if the  $N$  securities are stocks, the benchmarks can be long-short portfolios sorted on size, book-to-market ratio or past return and collateralised with cash.

Each risky asset can be regressed against the benchmarks: letting  $R_i$  be the excess return of asset  $i$  and  $\mathbf{R}_b$  the  $K \times 1$  vector of excess returns to the benchmarks, we have the decomposition

$$R_i = \alpha_i + \beta'_i \mathbf{R}_b + \varepsilon_i.$$

From this equation, we have the usual decomposition of the covariance matrix of assets into a term that arises from exposures to the benchmarks and an idiosyncratic term:

$$\mu = \alpha + \beta' \mu_b,$$

where  $\beta$  is the  $K \times N$  matrix obtained by putting the column vectors  $\beta_1, \dots, \beta_N$  next to each other, and  $\Sigma_\varepsilon$  is the covariance

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

matrix of the residuals. (In general, residuals are not uncorrelated across assets.) For expected excess returns, we have

$$\mu = \alpha + \beta' \mu_b,$$

where  $\alpha$  is the vector of alphas. Here, alphas are not zero in general, except when the benchmarks happen to be pricing factors, in the sense of Section 2.1. This precisely corresponds to the optimal grouping, as we shall see in a moment.

The "decentralised manager" takes the benchmarks as building blocks and chooses how much to allocate to them: the vector of weights allocated to the benchmarks is  $\mathbf{x}_b$ , with possibly a positive or a negative position in the risk-free asset. This portfolio can also be regarded as a portfolio of the individual securities, so its Sharpe ratio is by construction lower than that of the maximum Sharpe ratio attainable by optimally combining the securities. This inequality holds even if the decentralised manager forms the maximum Sharpe ratio (MSR) allocation to the benchmarks. If the MSR portfolio of benchmarks has strictly lower a Sharpe ratio than the MSR portfolio of individual securities, then the two-step process causes an efficiency loss with respect to the one-step process. The following proposition, which is a reformulation of a result established in Martellini and Milhau (2015), gives a necessary and sufficient condition for the efficiency loss to be zero.

**Proposition A** *Consider an investment universe made of  $N$  risky securities and one risk-free asset, and consider a two-step allocation process in which the  $N + 1$  assets are first grouped into  $K$  benchmarks, and a decentralised manager*

*chooses an allocation to the benchmarks in the second step. The portfolio of benchmarks has the maximum Sharpe ratio attainable with the  $N$  risky securities if, and only if, the following two conditions are met:*

- *The excess returns to the  $K$  benchmarks are pricing factors for the  $N$  securities (i.e., all securities have zero alpha);*
- *The decentralised manager implements the maximum Sharpe ratio allocation to the benchmarks.*

The proof is given in Section F of the Technical Appendix. This result implies that forming pricing factors is the optimal way of grouping securities in the first step. It also says that the optimal number of groups equals the number of pricing factors. For instance, if there is only one factor, then there is no loss of efficiency from having access only to one portfolio of securities and the risk-free asset, as opposed to having access to all individual securities, provided the portfolio in question prices all of them. If there is a portfolio that prices all assets, then it is necessarily the MSR portfolio of these assets: indeed, any portfolio with a non-zero specific risk has a lower Sharpe ratio than the benchmark, because specific risk is not compensated by higher expected return. A mathematical proof of this property is given in Section G of the Technical Appendix. Hence, in the one-factor case, Proposition 1 implies the well-known two-fund separation theorem: the portfolios that achieve the maximum Sharpe ratio (in other words, the efficient frontier) can be obtained by combining the maximum Sharpe ratio portfolio (fully invested in risky assets by definition) and the risk-free asset.

When there is more than one pricing factor, Proposition 1 implies that one can

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

reach the best possible Sharpe ratio by combining  $K$  portfolios that replicate the pricing factors. This represents a dramatic reduction in the number of parameters to estimate if  $K$  is much less than  $N$ . For instance, if the market factor and the long-short size, value and momentum factors of Carhart (1997) are pricing factors for all US individual stocks, then it suffices to efficiently combine these four portfolios to have as high a Sharpe ratio as by optimally combining stocks. (In fact, it is known that these four factors do not fully explain the cross-section of expected returns, since they do not, for instance, account for the low volatility effect.) Any other method to group securities in the first stage, such as forming sector or industry portfolios in stocks, before combining the benchmarks, implies an efficiency loss with respect to the one-step process.

### 4.1.3 Risk Factors and the Curse of Dimensionality

Proposition A shows that pricing factors are useful for the purpose of constructing the MSR portfolio. To put this result in practice, one needs to know a complete set of pricing factors and the risk premia on these factors. The first requirement is already hard to satisfy because it means that one would be able to explain any pattern in expected returns. Second, even if pricing factors are known, their premia are not easier to estimate than risk premia on assets, even though there are fewer factors than assets. In case one does not want to take any stand on premia, there still is the option to choose a weighting scheme independent of expected returns, like the global minimum variance or the risk parity portfolios. Both portfolios are functions of the covariance matrix only. But overall, Proposition A has limited applicability in practice.

Nevertheless, a factor approach is helpful for the construction of a well-diversified portfolio because it alleviates the curse of dimensionality involved in the estimation of the covariance matrix. In Section 2.3, we have shown that if one postulates the existence of risk factors such that idiosyncratic returns are cross-sectionally uncorrelated, the number of parameters to estimate to construct the full covariance matrix of assets dramatically decreases with respect to a situation where no factor structure is assumed: outside a factor model, all covariances are treated as independent parameters, and their number is a quadratic function of the number of assets. In contrast, when returns are generated by a set of risk factors, the number of independent parameters to estimate becomes a linear function of the universe size, and therefore grows much more slowly.

## 4.2 Factor Investing and Benchmarking: What?

In this section, we review the most commonly accepted risk premia starting with equity markets, and then turn to bond markets. Extending beyond traditional equity and bond factors, we also discuss alternative risk premia that can be collected across asset classes such as the value and momentum everywhere factors.

### 4.2.1 Risk Premia in Equity Markets

This section provides a concise review of the literature on equity factors, and more detailed presentations with additional references can be found in Amenc et al. (2015) and Martellini and Milhau (2015).

#### *The Market Factor*

The CAPM of Sharpe (1964) and Lintner (1965b) predicts that the market beta



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

– the beta with respect to the market portfolio – is the sole determinant of expected excess returns on stocks, and more generally on any capital asset. While early empirical studies have tended to confirm this prediction (Fama and MacBeth 1973), it has been rapidly recognised that it is an inaccurate description of observed returns: the model underestimates the returns to low-beta assets and overstates the returns to high-beta ones (Miller and Scholes 1972), which forces a close-to-flat adjustment line when risk premia are fitted to betas (Black, Jensen, and Scholes 1972). Furthermore, the study of Black, Jensen, and Scholes (1972) shows that the relation between the risk premium and the beta is decreasing in some periods. These findings have led to the recent introduction of the “betting-against-beta factor” by Frazzini and Pedersen (2014): this factor represents the excess return of a portfolio of low-beta securities over a portfolio of high-beta securities, the long leg being leveraged and the short leg being de-leveraged so that both sides have a beta of one. In contrast with the CAPM prediction, which implies that the expected return on this long-short portfolio should be zero, a positive average excess return is empirically observed, which cannot be explained by the exposure to other standard equity factors like size, value, momentum and liquidity. Interestingly, the low-beta effect is not restricted to stocks and is also observed in other asset classes, including notably Treasuries and credits: roughly speaking, it means that low-beta securities have higher Treynor ratios than high-beta ones.<sup>11</sup>

The second set of empirical facts at odds with the CAPM is that some stock characteristics are associated with higher average returns and that this

outperformance cannot be attributed to higher betas: as shown by Fama and French (1992), stocks with high-book-to-market ratio outperform those with low ratio despite having almost the same beta, and small stocks outperform large ones. It is true that small stocks have higher betas than large ones (Table II of Fama and French (1992)), but once size is controlled for, average returns are no longer positively related to the beta (their Table I). Numerous such patterns have been reported in the literature since early work on the price-earnings ratio (Basu 1977) and the size as measured by the market capitalisation (Banz 1981). The recent survey by Harvey, Liu, and Zhu (2016) counts over 300 of empirical regularities, which are “anomalies” in that they cannot be explained by the CAPM, and they conclude that with so many patterns at hand, severe statistical tests should be applied to identify the robust ones.

Several explanations have been put forward to account for the lack of empirical success of the CAPM. The market portfolio is not well proxied by a broad cap-weighted index (Roll 1977) and nonmarketable assets such as the human capital should be included (Mayers 1973; Jagannathan and Wang 1996); the beta is not constant over time, so the CAPM can hold conditionally without being true unconditionally (Jagannathan and Wang 1996; Ang and Chen 2007); investors do not have access to a risk-free asset, or are at least limited in their ability to borrow at the risk-free rate (Black 1972). Addressing these issues can improve the model's empirical performance: in particular, conditional versions of the CAPM, in which the linear relationship between the expected excess return and the market beta holds conditionally

11 – The Treynor ratio is a measure of the risk-adjusted return equal to the expected excess return over the risk-free rate divided by the market beta.



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

rather than unconditionally, have shown some ability to explain, or at least reduce the magnitude of, the size effect (Jagannathan and Wang 1996), and the alpha of a strategy that trades value stocks against growth stocks (Ang and Chen 2007).<sup>12</sup> However, the consensus view today is that equity risk is multi-dimensional and is not summarised by the market beta alone. Debates focus on which patterns in expected returns exist in the first place, which are the important risk factors beyond the market, and which anomalies are due to risk exposure (risk-based explanation) versus mispricing (behavioural explanation).

Although the market factor alone is unable to describe the cross-section of expected stock returns, it remains necessary to explain why equities earn a premium over cash. Fama and French (1993) test a two-factor model with the size and the value factors, which are shown to be important to explain differences in average returns across stocks, but the residual alphas remain statistically significant (their Table 9a). They conclude (p. 35) that "size and book-to-market factors explain the strong differences in average returns across stocks. But the large intercepts also say that [they] do not explain the average premium of stock returns over one-month bill returns." Hence, the market factor must be included in an equity factor model in order to explain the level of the broad equity premium.

### Size

Small cap stocks tend to outperform their large counterparts. Early empirical evidence was provided by Banz (1981), and has been subsequently confirmed – over a longer sample – by Fama and French (1992). A comprehensive survey of empirical evidence, including

international equity markets, and the related explanations is provided by Van Dijk (2011).

The risk-based explanation for the outperformance of small stocks is that it comes as a compensation for bearing higher exposure to a rewarded factor. Fama and French (1993) introduce a long-short "small-minus-big" (SMB) factor, to which small stocks are more exposed than large ones almost by construction, but this procedure, which has become standard for this and other factors, does not reveal the economic nature of the source of risk that is of concern to investors. To fit in one of the theoretical models described in Section 2, a rational explanation for the size effect has to relate it to the marginal utility of consumption, innovations in the opportunity set (ICAPM) or a risk factor that impacts all stocks (APT). Chan and Chen (1991) invoke microeconomic characteristics of small firms to justify their higher sensitivity to adverse economic shocks, arguing that many of them are "marginal firms", with inefficient production processes and difficult access to funding. Fama and French (1993) echo this idea by noting that small firms have historically exhibited commonalities in their reaction to recessions and expansions: these arguments are reminiscent of APT. Petkova (2006) proposes an ICAPM story, by showing that Fama and French's SMB and HML factors are proxies for the innovations in state variables like the T-bill rate, the dividend yield, the term spread and the default spread, which have ability to predict future returns.

Another class of explanations involve investors' behaviour or market frictions. Daniel and Titman (1997) reject the existence of a risk factor associated to size,

12 - Unlike the traditional CAPM, which states that unconditional expected excess returns (in practice, long-term average excess returns) are linearly related to the market beta, the conditional CAPM focuses on conditional expected returns and betas, explicitly treats them as time-varying quantities and postulates that the CAPM equation holds conditionally. The conditional CAPM can be obtained as a special case of Merton's ICAPM (Merton 1973) when the opportunity set is constant, but when this condition is satisfied, Jagannathan and Wang (1996) show that the conditional CAPM implies the unconditional CAPM. More generally, when the market premium is not constant, the conditional CAPM cannot be regarded as a special case of Merton's ICAPM.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

arguing that it is the size characteristic, not the exposure to a hypothetical factor, that determines higher expected returns. As noted by Van Dijk (2011), the overreaction argument that has been put forward for the value effect can apply in principle for the size effect too, if small firms are indeed firms that have done poorly in the past. Gompers and Metrick (2001) point clientele effects arising from the increasing share of large institutions among equity investors and their preference for large and liquid stocks, which has driven up prices of large stocks. Finally, smaller stocks tend to be less liquid (Acharya and Pedersen 2005), so illiquidity could at least partially account for the size premium.

### Value

Broadly speaking, a stock is said to be "value" if it is inexpensive according to some valuation ratio. Stattman (1980) and Rosenberg, Reid, and Lanstein (1985) report that stocks with higher book-to-market ratio have higher average returns, and international evidence across developed equity markets is provided by Fama and French (2012). While the book-to-market ratio (B/M) has become a standard measure of cheapness since the work of Fama and French (1992) and Fama and French (1993), other indicators have been proposed. For instance, the negative relationship between the price-earnings ratio and expected returns documented by Basu (1977) is also a manifestation of a value effect. Empirically, the B/M effect also appears to be related to the long-term reversal effect in stock returns (Fama and French 1996): stocks that have best performed over the past three to five years tend to underperform past losers over a similar period. This makes intuitive sense, as stocks that have been decreasing for a long time are likely to be relatively cheap.

Like they do for the size effect, Fama and French (1993) relate the value effect to a common risk factor that they proxy as the "high-minus-low" (HML) factor, defined as the excess return of a portfolio of value stocks over a portfolio of growth stocks. They note that high B/M is associated with durably low earnings, an interpretation pursued by Fama and French (1995), who refer to these firms possessing this characteristic as "distressed firms". However, the distress explanation is questioned by Campbell, Hilscher, and Szilagyi (2008), who compute a probability of failure as a function of accounting and market variables and show that average returns and alphas are almost monotonically *decreasing* in the likelihood of default (their Table VI). Other studies relate the value effect to Merton's ICAPM. In this perspective, Liew and Vassalou (2000) and Petkova (2006) provide evidence that the HML factor has some ability to predict changes in the opportunity set, which makes it a state variable in the Mertonian sense and contributes to justify its positive expected excess return. In the former study, the authors show that in most countries, high current HML tends to be associated with high future GDP growth. Campbell and Vuolteenaho (2004) show that growth stocks covary more with negative news about discount rates than value stocks do (their Tables 4 and 5): hence, growth stocks are better hedges against unfavourable changes in the opportunity set, which justifies lower expected returns than for value stocks.

In an effort to find a risk-based explanation for the value premium, another strand of research has studied it in the context of the conditional CAPM, which allows for time variation in the market betas of value and growth stocks. Petkova and Zhang

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

(2005) show that value stocks are indeed more exposed to the market in times the market premium is high (yet another possible notion of "bad times", distinct from the notion of bad times defined with respect to marginal utility), while the opposite is true for growth stocks, which justifies higher expected returns for the former. Unfortunately, this effect does not account for the magnitude of the observed premium. Zhang (2005) proposes an economic justification for this finding, based on the fact that value firms have more assets in place than growth options, and that this capital puts a weight on them in bad times. Consistent with these findings, Ang and Chen (2007) show that the conditional CAPM can rationalise the value premium, but this conclusion is disputed by Lewellen and Nagel (2006), who use a different measure of conditional betas.

There remains the possibility that the value premium is due to behavioural biases. Overreaction is often at the heart of such explanations, as in DeBondt and Thaler (1985), who focus on long-term reversal, and in Lakonishok, Shleifer, and Vishny (1994), who argue that investors are prone to extrapolate past good news about growth stocks and overreact to past bad news about value stocks, which leads them to overbid the former and to neglect the latter.

### *Momentum*

The first systematic study of the short-term continuation in stock returns was published by Jegadeesh and Titman (1993): stock returns tend to be positively autocorrelated at horizons from one to four quarter(s), so portfolios of winner stocks formed over such a period continue to outperform portfolios of loser stocks during the same period. Thus, there is an

interesting pattern when stock returns are examined in the time series: a short-term reversal, at the one-month horizon (Jegadeesh 1990), then continuation, at horizons of one to four quarter(s), and a long-term reversal, over three to five years. Subsequent studies have shown that the momentum effect is robust across countries (Rouwenhorst 1998) and exists in other asset classes, like futures contracts on commodities, equity indices, bonds and currencies (Moskowitz, Ooi, and Pedersen 2012; Asness, Moskowitz, and Pedersen 2013). Since some of these papers were published about 20 years after Jegadeesh's seminal paper, it is remarkable that momentum has stood the test of time after its first publication. It is a major reason why the momentum factor has become the most commonly adopted equity factor after size and value.

Behavioural explanations have often been put forward to explain momentum profits, following the work of Jegadeesh and Titman (1993), who invoke short-term underreaction to firm-specific news as a possible cause, and the recognition by Fama and French (1996) that the momentum effect is the "main embarrassment" for the three-factor model, given that it is past losers that are more exposed than past winners to the rewarded SMB and HML factors. Hong, Lim, and Stein (2000) find empirical support for the behavioural model developed by Hong and Stein (1999), where public or initially private information gets only gradually incorporated in prices. For instance, momentum strategies are more profitable among stocks with low analysts' coverage, for which information diffuses even more slowly (their Table IV).

Another body of research has tried to relate the momentum effect to systematic

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

risk, starting with Johnson (2002), who argues that price returns are increasing in the dividend growth rate, so a sort on past returns is tantamount to a sort on the past growth rate, and that higher growth rate determines higher exposure to growth rate risk: to the extent that this risk must be compensated by additional expected return, past winners have higher expected returns than past losers. Liu and Zhang (2008) propose the growth rate in industrial production as a common risk factor in the firm-level dividend growth rate, and they empirically show that past winners are more exposed to this variable in a way that explains more than half of momentum profits. Their introduction also reviews several articles that have proposed risk-based explanations for the profitability of these strategies.

There has been a fierce debate on whether the profitability of the "relative strength strategies" of Jegadeesh and Titman (1993), which sell past losers and buy past winners, survives transaction costs, given the large amount of turnover that they involve. This question is of obvious practical interest, but it can also shed light on the debate on the behavioural or fundamental origins of momentum, because sufficiently high trading costs represent a limit to arbitrage, that is an obstacle that prevents traders from drying up the profits arising from temporary mispricing. A related question is whether momentum profits are more due to the short side than to the long side, because shorting stocks is more costly. Lesmond, Schill, and Zhou (2004) argue that the short side is more important and that both sides of momentum – and especially the short one – consist mainly of small stocks traded outside the NYSE, for which trading costs are higher than average. The final impact of transaction costs on

the significance of momentum profits appears to depend strongly on how these costs are modelled, and evidence is mixed: Lesmond, Schill, and Zhou (2004) find that they turn the significantly positive returns of paper strategies into insignificant or even negative returns (their Table 3), while Korajczyk and Sadka (2004) reach an opposite conclusion (their Table IV). More recent research shows that rebalancing rules can be adapted in such a way as to limit transaction costs and to preserve at least some of the profitability of paper strategies (Frazzini, Israel, and Moskowitz 2012; Novy-Marx and Velikov 2016).

### *Low Volatility*

Common sense suggests that expected return above the risk-free rate is a compensation for risk, but the notion of risk is in fact ambiguous. Factor models in asset pricing theory do predict a positive risk-return relationship, but risk is a multi-dimensional concept summarised by the betas with respect to pricing factors: a higher exposure to a positively rewarded factor (a factor that tends to be high in "good times" and low in "bad times" defined by marginal utility) implies a higher risk premium. Idiosyncratic risk is irrelevant for the risk premium. Merton (1987) shows that this is no longer the case in the presence of frictions such as incomplete information about security characteristics for some investors: if not all investors are informed about a security, the market alpha of this security is increasing in the idiosyncratic risk. But these theories do not make explicit predictions as to the existence and the direction of a relation between expected return and volatility.<sup>13</sup>

Empirical evidence barely confirms, or even contradicts, the intuition of a positive link between risk and return. Haugen and

13 – In asset pricing theory, a positive relation between expected return and volatility appears only for efficient portfolios in the presence of a risk-free asset: any efficient portfolio is a combination of the maximum Sharpe ratio portfolio and the risk-free asset, and its expected excess return is a linear function of its volatility.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

Baker (1996) find that stocks with the highest expected returns (as estimated by an implicit factor model similar to the Barra model) tend to have lower volatilities than the worst performers: this is an instance of the "tv (total volatility) puzzle", which is also documented by Blitz and Van Vliet (2007) and Baker, Bradley, and Wurgler (2011). It is Ang et al. (2006) who conduct the first direct study of the relation between idiosyncratic volatility (with respect to the Fama-French model) and expected returns, and they uncover the "iv (idiosyncratic volatility) puzzle": stocks with higher specific volatilities earn significantly lower average returns. These results, obtained with US data, are extended by Ang et al. (2009) to other developed equity markets. Recently, Blitz (2016) has re-affirmed the independence of the low volatility effect with respect to the value effect, by showing that it is only among large stocks, and during a 20-year period, that exposures to the Fama-French value factor can explain the outperformance of the less risky stocks.

Robustness to measurement issues has been debated for the volatility puzzles more than for any other effect. Bali and Cakici (2008) point that the low idiosyncratic volatility effect reverses if stocks are equally weighted as opposed to being value-weighted in the test portfolios, that it is highly sensitive to the definition of volatility quintiles, and that it becomes insignificant if volatility is estimated from monthly instead of daily returns, or if the stocks with the lowest prices and the smallest liquidity are removed from the analysis. Other papers have focused on the relation between expected returns and the conditional idiosyncratic volatility, using dynamic models to estimate this time-varying parameter: Fu (2009) reports a positive

relation, but Fink, Fink, and He (2012) and Guo, Kassa, and Ferguson (2014) find that it disappears when a look-ahead bias in Fu's estimation procedure is eliminated.

Explanations for the volatility effects have mostly focused on the role of behavioural biases and of certain aspects of market functioning. High-volatility stocks are likely to attract more attention, so they are more appealing to individual investors, who are prone to buy stocks with high media coverage (Barber and Odean 2013; Blitz, Falkenstein, and Vliet 2014). A similar effect is at work in investment management companies, as Baker and Haugen (2012) note, since analysts tend to recommend buying stocks with higher presence in the media. High risk is also associated with the potential for very high returns, although chances are admittedly low, so that investors seeking "lottery tickets" find them attractive (Kumar 2009). This explanation is related to the preference for skewness, which is not taken into account in the CAPM's mean-variance framework (Baker, Bradley, and Wurgler 2011). Mitton and Vorkink (2007) note that higher volatility implies higher skewness, because stock prices are bounded from below, and Barberis and Huang (2008) show that skewness can be priced in the cross section of stock returns, with more skewed securities having lower average returns. Lotteries are especially important to investors who operate mental separation between different accounts, such as one where they behave roughly like mean-variance optimisers and another one where they seek opportunities to reach high wealth levels through highly risk investments (Shefrin and Statman 2000; Blitz and Van Vliet 2007). On the institutional side, fund managers have also incentives to purchase high-risk stocks because the bonus for high performance



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

is similar to an option-like contract, the value of which rises with the volatility of the underlying asset (Baker and Haugen 2012). Blitz, Falkenstein, and Vliet (2014) also point out other features of investors' behaviour, like the fact that they tend to focus on a few "success stories", in which some risky stocks have enjoyed impressive returns. Overconfidence in their own stock selection skills is yet another bias, which leads them to take large amounts of risk.

### *Investment and Profitability*

The investment and the profitability effects are relatively more recent to the literature than the classical size, value and momentum effects, but they are backed by straightforward microeconomic explanations, so the corresponding investment and profitability factors can be regarded as strategies with reliable long-term positive performance.

Unlike the size, value, momentum and volatility scores, which use market prices at some stage of their construction, profitability measures are based solely on accounting data, but they vary across studies. Haugen and Baker (1996), who are perhaps the first to report a positive cross-sectional relation between expected return and profitability, use the return on equity, defined as the ratio of earnings to book value. These results are confirmed by Fama and French (2006), who also show that replacing past profitability by a forward-looking measure of expected profitability does not result in a stronger relation with expected returns. Novy-Marx (2013) makes the case for another profitability measure, defined as gross profits scaled by book assets, and they also find a significant positive relation with expected returns, even after controlling for the other characteristics known to

impact average performance (size, book-to-market, short-term reversal and mid-term momentum).

The investment effect refers to the finding that firms with higher investment display lower stock returns. Again, different measures have been tested, but the findings concur. Titman, Wei, and Xie (2004) focus on "abnormal capital investment", by looking at the distance between the level of capital expenditures and its past three-year moving average and variants of this definition, Xing (2008) has a closely related measure with the growth in capital expenditures, and Fama and French (2006) and Hou, Xue, and Zhang (2015) consider the growth in total assets. It is systematically found that lower investment is associated with higher expected returns.

As explained by Fama and French (2006), a simple dividend-discount model can be invoked to rationalise these effects. A stock price is the sum of expected future dividends discounted at the expected stock return,  $\rho$ :

$$S_t = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t[D_{t+h}]}{(1 + \rho)^h},$$

where  $D_{t+h}$  is the dividend in period  $t + h$ , and the summation index,  $h$ , is the horizon of dividends. The sum goes to infinity because a stock is similar to a perpetual claim on a fraction of dividends. An accounting identity states that the dividend equals earnings ( $Y_{t+h}$ ) minus the change in book value of equity per share ( $\Delta B_{t+h}$ ). Replacing dividends with their expression and dividing by the book value, we obtain the market-to-book ratio:

$$\frac{S_t}{B_t} = \sum_{h=1}^{\infty} \frac{\mathbb{E}_t[Y_{t+h} - \Delta B_{t+h}]}{B_t(1 + \rho)^h}$$

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

Holding the book-to-market ratio and the expected change in book value fixed, a higher ratio of expected earnings to book value implies a higher expected return: this is the profitability effect. Holding the book-to-market ratio and the expected earnings fixed, a higher expected growth in book value results in a lower expected return: this is the investment effect.

Hou, Xue, and Zhang (2015) propose a similar pricing framework, with a few modifications: they consider a two-period model for simplicity, and they explicitly model the firm's decision of how much to allocate to production. The optimality condition for the firm implies the same predictions as Fama and French's dividend-discount model: the expected return for future cash flows is decreasing in the ratio of investments to assets, and increasing in the ratio of profits to assets. As they note, this justification of the investment and profitability effects is different from the traditional approach in asset pricing, which relies on a model of the consumption side: the economic origin of risk premia lies in consumption betas, like in the consumption-based asset pricing model of Breeden (1979) (see also Section 2.1). In investment-based asset pricing, the focus is set on the production side of the economy. In the case of the investment and profitability effects, an economic justification for the factor premia can be easily provided, based on an analysis of the producer's decision, but as Hou, Xue, and Zhang (2015) explain, this does not suffice to claim that the premia arise from exposures to consumption risk, as would be required in a risk-based model.

### 4.2.2 Factor Investing Beyond Equities

It is fair to say that the smart beta approach is now firmly grounded in equity

investment practices, and the key question for an increasing majority of institutional investors is not whether one should use smart beta, but instead which and how much smart beta to use. In parallel, interest in smart beta equity products is fast increasing in retail and private wealth management. In contrast, the concept of smart beta in the fixed-income space, not to mention other asset classes, is still relatively less mature, despite the obvious importance and relevance of the subject. Over the recent years, a number of concerns have been expressed, however, about the (ir)relevance of existing forms of corporate and sovereign bond indices offered by index providers. One of the major problems with bond indices which simply weight the debt issues by their market value is the already mentioned "bums' problems" highlighted by Siegel (2003): given the large share of the total debt market accounted for by issuers with large amounts of outstanding debt, market value-weighted corporate bond indices will have a tendency to overweight bonds with large amounts of outstanding debt. It is often argued that such indices will thus give too much weight to riskier assets. While it is debatable whether debt-weighting really leads to an overweight in the most risky securities, it is clear that market value-weighted indices lead to concentrated portfolios which are in opposition with investors' needs for efficient risk premia harvesting which involves holding well-diversified portfolios. In a nutshell, a good case can be made that existing bond indices, just like cap-weighted equity indices, tend to be poorly diversified portfolios, regardless of whether or not the overweighting applies to the wrong constituents.

In addition to the problem of concentration, fluctuations in risks'



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

exposure such as duration or credit risk in existing indices are another source of concern. Such uncontrolled time variation in risk exposures is incompatible with investors' requirements that these risks exposures be relatively stable so that allocation decisions are not compromised by implicit choices made by an unstable index. For example, an asset-liability mismatch would be generated by changes in the duration of the bond index if the bond index is used as a benchmark for a pension fund bond portfolio. More generally, it appears that existing bond indices can be regarded as more "issuer-friendly" than "investor-friendly", in the sense that these bond indices passively reflect the collective decisions of issuers regarding the maturity and size of bond issues, with no control over the risk factor exposures associated with such choices or over the reward that investors should deserve from holding a well-diversified portfolio of such factor exposures.

The following paragraphs give a rapid overview of the factors identified in the fixed-income class and in other classes. More detailed reviews and discussions can be found in Martellini and Milhau (2015) for fixed-income and commodities (see also Rebonato (2017) for fixed-income factors).

### *Level, Slope and Curvature*

The names of these classical factors refer to the location and the shape of the term structure of interest rates. They are traditionally obtained as the first three principal factors in a principal component analysis of panel data on interest rates, but they also show up in a principal component analysis of bond returns (Deguest et al. 2013): all bonds are positively exposed to the level factor (up to a change of sign of the factor), with long-term bonds having larger exposures,

while long-term and short-term bonds have diverging sensitivities to the slope factor. By nature, they are risk factors, that collectively explain more than 90% of the common variance of the returns to bonds from the same issuer, but they are not "priced factors" in the sense of profitable strategies, except for the level factor, which is strongly related to the term factor. Bonds with long maturities are more exposed to the level factor than the short-maturity ones. However, the unconditional term premium is low (see Fama and French (1993)).

The slope factor is also a conditioning variable, because it is closely related to the slope of the term structure and to forward rates, which have shown ability to predict excess bond returns (Fama and Bliss 1987; Cochrane and Piazzesi 2005). Evidence for predictability is also evidence that the expected excess return on long-term bonds (the "conditional term premium") varies over time, and research has focused on estimating this time-varying term premium (see Fama and French (1989) in addition to the aforementioned references) as opposed to estimating the unconditional term premium. In fact, it is because the conditional premium changes sign over time that the unconditional premium is close to zero. In the spirit of their long-short equity factors, they also introduce a term factor defined as the excess return of a portfolio of corporate bonds over Treasuries: given that a sort on maturities replicates a sort on sensitivities to the level of interest rates, this construction is yet another possible definition for the level factor.

### *Credit*

It is straightforward to observe that defaultable bonds have higher discount

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

rates than bonds credited to be default-free (e.g., bonds issued by developed and politically stable countries). More difficult is the task of separating out the various components of the credit spread, defined as the excess discount rate: in the absence of frictions, the spread is a function of the likelihood of default and the recovery rate in the event of default (Duffie and Singleton 1999), but real-world spread also include contributions from liquidity and from differences in tax treatment (Longstaff, Mithal, and Neis 2005; Driessen 2005).

In any case, credit spreads are not good estimates of the expected excess returns to defaultable bonds over default-free ones, if only because the former bonds can default: Yu (2002) and Driessen (2005) provide mathematical decompositions of the credit risk premium. Empirically, credit spreads tend to overestimate the premium, which is low: Fama and French (1993) report a value of 2 basis points per month for a mixture of corporate bonds ranging from low grade to Aaa over the period from 1963 to 1991. But as highlighted by Hallerbach and Houweling (2013) and Asvanunt and Richardson (2016), taking the average excess return of corporate bonds over Government bonds ignores differences in the durations of the bonds and leads to underestimating the true credit risk premium. This effect is sizable, as shown by Asvanunt and Richardson (2016), who find a premium of 1.80% per year in investment grade bonds with the correction, versus 0.34% with the classical approach.

In addition to being a priced factor, the credit factor is also a common risk factor for bonds, as appears from the regressions of Fama and French (1993), since all bond portfolios have significant exposure to it.

### *Value and Momentum*

Value and momentum are twin effects that represent competing forces, in that value investing involves buying "cheap" assets while momentum trading is a form of trend following that favours past winners over past losers. Of course, they are not mutually inconsistent because returns exhibit mid-term continuation exploited in momentum and long-term reversal, from which value strategies benefit. The negative correlations reported by Asness, Moskowitz, and Pedersen (2013) between value and momentum strategies in various zones and asset classes (their Table II) reinforce this view of value and momentum as opposite effects.

Value and momentum effects have been extensively documented in stock returns, so it is natural to ask whether they also exist in bond returns. All empirical studies so far tend to show that momentum profits are low and insignificant among sovereign and investment-grade bonds (Gebhardt, Hvidkjaer, and Swaminathan 2005; Israel and Moskowitz 2013; Jostova et al. 2013), and are stronger in non-investment grade bonds (Pospisil and Zhang 2010; Jostova et al. 2013). Like in equities, the momentum effect refers to the continuation of returns computed over periods ranging from 3 to 12 months. Jostova et al. reject the explanation of bond momentum as a duplicate of stock momentum, but they propose to attribute it to the slow diffusion of information in bond prices, as in the behavioural model of Hong and Stein (1999).

To detect a value effect in the cross section of bonds, one needs a measure of value, which is not obvious to reach because there is no notion of book value for bonds. But empirically, the book-to-market effect in stocks appears to be related to

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

the long-term reversal: Fama and French (1996) report a 97% correlation between the portfolio of stocks with high book-to-market ratio and the portfolio of stocks with the worst returns over the past five years (their Table X).<sup>14</sup> This finding suggests that a value score for bonds can be defined by looking at the past long-term return, as Asness, Moskowitz, and Pedersen (2013) do. Asness et al. (2015) suggest another metrics, based on the ex-ante real rate, to have a measure of the "cheapness" of a bond.

Beyond the well-documented equity class and the fixed-income universe, Asness, Moskowitz, and Pedersen (2013) show that other currency and commodity futures also exhibit a tendency to mid-term continuation of returns (momentum) and to long-term reversal (value). Empirical evidence for commodity momentum is also given by Miffre and Rallis (2007), who note that momentum strategies share some common features with strategies exploiting the shape of the term structure: specifically, they tend to go long backwardated contracts (with a positively sloped term structure and positive roll return) and short contangoed contracts (with the opposite characteristics), although exceptions exist and sorting on past returns is not equivalent to sorting on the slope of the term structure of futures prices.

The presence of value and momentum patterns in multiple asset classes, as well as the positive correlations between the value or the momentum strategies of various classes, motivate the definition by Asness, Moskowitz, and Pedersen (2013) of "value everywhere" and "momentum everywhere" factors as cross-class averages of the single-class factors. They test the ability of these multi-class

factors to play the role of asset pricing factors for a universe made of portfolios from multiple classes. They conclude that higher exposure to these factors is indeed rewarded by higher expected return, that is, the factors are priced in the cross section (their Table V), and they also report that a model that includes these two factors plus a broad market factor leaves a smaller fraction of expected returns unexplained than competing models, including in particular the Fama-French factors (their Table VI). Hence, the value and momentum everywhere factors prove that research can be successful at uncovering factors that statistically explain differences in average returns across multiple asset classes. Further work is needed to refine the models with additional meaningful factors and to relate them to more fundamental sources of risk, as has been done in the equity class.

### Carry

Carry is another example of a multi-class pattern, which exploits observable characteristics endowed with predicting ability and allows profitable strategies to be constructed. Kojien et al. (2013) conduct a detailed theoretical and empirical study of carry in several major asset classes like US equities and Treasuries, corporate bonds, commodity and currency futures and options. The general definition of carry is as the excess return to a strategy rolling over futures contracts under the assumption that the spot price of the underlying is constant. Kojien et al. (2013) show that the corresponding general formula is

$$C_t = \frac{S_t - F_t}{F_t},$$

where  $S_t$  is the spot price and  $F_t$  is the futures price. With this definition, the

14 - When computing the long-term reversal score, Fama and French use the last five years of data, skipping the last year to avoid muddling the measure with the momentum effect. In their Table X, many correlations are greater than 80%, even between past losers and past winners (84%), between past losers and growth stocks (86%) and between past winners and value stocks (88%), but a correlation of 97% is higher.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

expected excess return on the roll-over strategy is the sum of the carry and the expected change in spot price divided by the futures price.

The practical way to compute the carry depends on the asset class considered. The previous formula is directly applicable when good spot price data is available, like for equities and currencies, but the spot price is replaced by the price of the nearest-to-expiration contract in equity index futures, currencies and commodities. In this case,  $F_t$  is the price of the second-nearest-to-expiration contract. For country bond indices, the same formula serves as a basis for the definition, but in the absence of data on liquid futures contracts, synthetic futures prices derived from the yield curve are computed. For individual bonds, the carry can be computed from the yield-to-maturity and the duration, as Kojien et al. explain.

The authors show that the carry score can in general be regarded as an increasing function of the expected return, so carry trade strategies that purchase high carry securities and sell low carry ones tend to go long assets with the highest expected returns and short the least attractive ones. As a matter of fact, these strategies post positive long-term returns and Sharpe ratios on a long period.<sup>15</sup> These results are confirmed by Asness et al. (2015), who also define multi-class carry portfolios by mixing the single-class strategies.

### *Low Risk (Beta or Volatility)*

Various low-risk effects have been documented in the returns to stocks, bonds or futures contracts, so it is important to specify the reference risk measure: it can be a measure of systematic risk (the beta with respect to

a market factor appropriate to the asset class), a measure of specific risk (the volatility of the residuals from a factor model), or a measure of risk specific to the asset class considered (e.g., the duration-based or spread-based measures used in de Carvalho et al. (2014) for bonds). These patterns give rise to "factors" understood as strategies that have proved profitable in the past, and whose future profitability can be justified by economic explanations.

Together with value, momentum and carry, low risk (or defensive) is the last investment style that Asness et al. (2015) admit as a source of positive long-term returns in multiple classes with robust historical evidence and sound economic justification. It is rooted in the "low beta anomaly", which was first uncovered by Black, Jensen, and Scholes (1972): low-beta assets earn too high average returns compared to the CAPM's prediction of a risk premium equal to the beta times the market premium, and conversely, high-beta assets earn too low premia (see their Table 2). They find that the empirical relation between expected returns and the betas is better described by the equation

$$\mathbb{E}[R_i] = \mathbb{E}[R_z] + \beta_i [\mathbb{E}[R_m] - \mathbb{E}[R_z]], \quad (4.1)$$

where  $R_m$  is the market excess return,  $\beta_i$  is the standard market beta and  $R_z$  is the excess return to a zero-beta portfolio. In the standard CAPM, the zero-beta expected return equals the risk-free rate, but Black, Jensen, and Scholes (1972) find it to be greater, so the slope of the empirical capital market line is lower than the market premium.

The work of Black, Jensen, and Scholes (1972) focuses on US stocks, but the

15 - The extent of the period depends on the asset class: it ranges from 1983 to 2012 for currencies and country bond indices, from 1980 to 2012 for commodities, from 1971 to 2012 for US Treasuries and from 1988 to 2012 for equity index futures.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

low-beta anomaly is present in many asset classes. Pilotte and Sterbenz (2006) document that the Sharpe and the Treynor ratios of bonds are decreasing in the maturity, so a portfolio of short-term bonds levered to have the same volatility or the same beta as a portfolio of long-term bonds has better performance. Similarly, Frazzini and Pedersen (2014) show that this pattern is encountered in equities, equity indices, US Treasuries, credit indices, country bond indices, commodities and currencies. In Frazzini and Pedersen's work, the market factor against which the beta is measured is specific to each class: for commodities, contracts are weighted by the reciprocal of their volatility to ensure equal risk contribution, while for equity indices, country bond indices and currencies, they are weighted by the gross domestic product to reflect economic size. As the authors show in an Appendix to their paper, "betting-against-beta" strategies that go long low-beta assets and short high-beta ones remain profitable if the beta is taken with respect to a global market factor similar to the one of Asness, Frazzini, and Pedersen (2012), which is an equal risk contribution portfolio of the stock, bond and commodity market factors. Asness et al. (2015) provide concurring evidence for the profitability of these strategies. It should be noted that the low-beta anomalies documented by Black, Jensen, and Scholes (1972) and Frazzini and Pedersen (2014) do not imply that low-beta securities have systematically higher average returns than high-beta ones: expected returns sometimes align well with betas (see Table 2 of Black, Jensen and Schole's paper, and Table 3 of Frazzini and Pedersen's paper). What is decreasing in the beta is the residual alpha.

An economic justification for the low-beta effect is the existence of frictions in capital markets. Indeed, an empirical relation of the form (4.1) can be rationalised in an economy where agents have no access to riskless borrowing (Black 1972), or face restrictions on leverage (Frazzini and Pedersen 2014). The intuition is that investors chasing high returns through high exposure have no choice but to purchase high-beta assets, because they cannot lever low-beta assets as much as would be necessary to reach the same level of expected return. As a result, they overbid high-beta securities, thereby depressing their returns. Blitz, Falkenstein, and Vliet (2014) point the role of regulatory capital requirements, which discourage large holdings in low-beta stocks. More generally, they provide a detailed review of the violations of the CAPM hypothesis that can justify the low-beta effect.

Systematic risk (the beta) is not the only risk measure giving rise to a low-risk anomaly. Fuertes, Miffre, and Fernandez-Perez (2015) document an idiosyncratic volatility effect in commodity futures similar to the one studied by Ang et al. (2006) in individual stocks: strategies that go long contracts with low past idiosyncratic volatility and short those with high past signal earn a positive return over the period from 1985 through 2011. Here, the specific volatility is the volatility of residuals in a regression on the S&P GSCI. In investment-grade corporate bonds from US, European, UK or Japan issuers, de Carvalho et al. (2014) find that average returns (measured over the period from 1997 through 2012) are decreasing in several risk measures: the modified duration, the duration multiplied by the option-adjusted spread and the duration times the yield-to-

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

maturity. These results are established for the individual corporate bonds that make up the universe of the BofA Merrill Lynch indices.

### 4.3 Factor Investing and Benchmarking: How?

Once a proper understanding of the main sources of rewarded risk in an asset class has been obtained, the outstanding question from an investor perspective is to determine the best way to harvest such multiple risk premia. This decision is in fact embedded within the choice of a benchmark, which is then used as a reference portfolio for passive or active mandates and is intended as a better choice than the cap-weighted or debt-weighted index which is often used in the equity and bond classes. After choosing the factor(s) they would like to be exposed to, investors should decide on which methodology they want to rely to achieve exposure. In this section, we divide the problem in two steps: first, achieve exposure to a single factor by constructing *factor strategies*; then, combine the strategies to gain exposure to multiple factors.

#### 4.3.1 Exposure to a Single Factor: Constructing Smart Factor Strategies

*Factor strategies* aim to provide exposure to a single factor in an asset class. *Smart factor strategies* are engineered to provide not only a pronounced factor tilt emanating from the stock selection procedure (right risk exposure), but also a high Sharpe ratio following from the efficient diversification of unrewarded risks related to individual stocks (fair reward for the risk exposure). This two-step procedure was introduced by Amenc, Goltz, and Lodh (2012), as a means to allow investors to make conscious choices of factor exposures, as opposed to dealing

with factor biases implied by construction methodologies. As emphasised by Amenc et al. (2014b), it differs from the early conceptions of smart beta, which focused on diversification by deviating from cap weighting, and from traditional factor indices, which pursue the objective of maximising the factor exposure, be it at the cost of concentration.

The difference between smart factor strategies and smart-weighted strategies lies in the fact that the latter are subject to factor biases with respect to a cap-weighted benchmark that result from the employed methodology but are not made explicit at any point of the construction process. For instance, an equally-weighted portfolio will mechanically overweight small stocks with respect to a cap-weighted one, and will thus have a small size bias. It will also have a bias towards past losers, since equal weighting is a countercyclical rule. While equal weighting requires no input parameters and no optimisation, many weighting schemes are derived from mathematical algorithms. Minimum variance and risk parity are two popular examples of such rules: the former minimises the ex-ante variance, and the risk parity rule equates the contributions of constituents to portfolio volatility. Both have by construction a strong bias towards low volatility stocks. The selection-weighting method of Amenc, Goltz, and Lodh (2012) is intended to be transparent as to the choice of factor exposures, by applying the smart weighting scheme to a subset of stocks, as opposed to the full universe. The selection step leads to a sub-universe of stocks with a pronounced tilt towards an attribute associated with a rewarded factor: for instance, selecting stocks with the highest book-to-market ratio leads implies an exposure to the value factor,



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

while selecting stocks with the highest past one-year return results in an exposure to the momentum factor. The second step is the weighting of selected stocks: a "smart weighting scheme" is applied to ensure diversify away unrewarded risks. The final result is a "smart factor" or "smart beta" index or strategy, which combines a factor tilt with a diversified weighting scheme.

The second difference is between smart factor strategies and factor strategies that target "purity" in terms of factor exposure, but do not attempt to be well-diversified. This construction method implies that a substantial amount of idiosyncratic risk can remain in the portfolio, while this risk is not rewarded by additional expected return. But in theory, a pure factor-replicating portfolio should have both a beta of one with respect to the factor and zero exposures with respect to the other factors, but it should also have no residual risk. In factor strategies that do not control the amount of unrewarded risk, stocks are generally weighted by their capitalisation or by a score that represents their factor exposure, the objective being to maximise the index exposure. The practice of weighting by score is also present in the construction of "fundamental indices", where the weight of each constituent is meant to reflect its economic size as measured by a range of fundamental measures. It is argued by Arnott, Hsu, and Moore (2005) that such indices provide a reliable access to the value factor.

The two-step process involves several degrees of freedom, related to the ways the selection and the weighting are performed. The following list gives examples for the value factor, but it could be adapted to other equity factors:

- The characteristic used to separate growth and value stocks. book-to-market

ratio has become a consensual variable since the work of Fama and French (1993), which introduces the high-minus-low factor, but other valuation ratios can play that role, and some index providers even use more sophisticated, proprietary, methodologies to define a value score;

- Measurement details in the characteristic: as emphasised by Asness and Frazzini (2013), there are different ways of measuring the book-to-market ratio. They show that even if the book value is lagged by 6 months to ensure observability at the stock selection date (as is the current practice), using the current market value as opposed to the lagged value produces a better approximation of the actual (but unobservable) book-to-market ratio;

- Intensity of selection: the "value portfolio" is made of the stocks with the highest book-to-market ratio, but one must decide how many are these "top stocks", and the same remark applies to the growth portfolio. Fama and French (1993) retain 30% of the universe on both sides, but there is a trade-off between a stricter selection, which in principle improves the factor exposure, and a larger universe, which leaves more room for diversification;

- Weighting scheme: cap-weighting has long been the default option, but it is well known that alternative weighting schemes exist, which are based on scientific or heuristic diversification techniques and lead to a better riskreturn profile (see Amenc, Goltz, and Lodh (2012)).

Table 4 illustrates the two-step process with analytics on equity indices constructed from the universe of the 500 largest US stocks. The six sub-universes correspond to the six traditional factors equity factors listed in Section 4.2.1: size, value, momentum, volatility, investment



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

and profitability. Each of these subsets is made up of stocks with a characteristic associated with higher expected returns. The definition of characteristics is taken to be as simple as possible, and the intensity of the selection is set to 50% of the universe, to ensure a balance between purity and diversification. The mid-cap universe consists of the 50% stocks with the smallest capitalisation, the value universe contains the 50% stocks with the highest book-to-market ratio, the high momentum universe the 50% with the highest past one-year return, the low volatility universe the 50% with the lowest two-year volatility, the high profitability universe the 50% with the highest gross profit-to-asset ratio, and the low investment universe the 50% with the lowest total asset growth. In each sub-universe, we consider three weighting schemes. To limit the effects of methodological choices in parameter estimation, we restrict to schemes that do not require the estimation of the entire covariance matrix: cap-weighting, equal weighting and weighting by the reciprocal of volatility.<sup>16</sup>

The sample period in Table 4 goes from 1970 to 2015, and thus encompasses a variety of market conditions, which allows "long-term" conclusions to be drawn. These conclusions are in line with the findings of previous literature (Amenc, Goltz, and Lodh 2012, 2016; Amenc et al. 2014b). First, the selection step always improves performance with respect to the broad cap-weighted index: indeed, all cap-weighted portfolios from sub-universes have a higher annualised return and a higher Sharpe ratio than the broad benchmark. This indicates that a long-only version of the six factors delivered better returns and better risk-adjusted returns than the standard index over

this period. Second, the weighting step further improves both analytics, which confirms that the smart weighting scheme adds value. Third, the factor tilt and the weighting scheme imply short-term deviations from the broad cap-weighted index: tracking errors often exceed 5%, and peak within the mid cap selection (at around 6%), which restrict to the stocks that are the least represented in the benchmark. Short-term risk also results in a positive relative maximum drawdown: this indicator is the maximum drawdown of the series of relative values, where the relative value of an index is simply its value divided by the value of the benchmark on the same date. In all universes, the cap-weighted portfolio has the lowest tracking error, and in most of them except for the High Profitability selection, it has also the lowest maximum relative drawdown. But in absolute terms, smart factor indices do not systematically have a higher maximum drawdown than the broad cap-weighted index, so they do not appear to have much more extreme risk.

Selection and weighting have impacts on the factor exposures. To evaluate them, we use the standard Fama-French-Carhart model, in which the four factors are the market, size, value and momentum. The series are downloaded from Ken French's data library.<sup>17</sup> The market factor is the excess return of a cap-weighted portfolio of the stocks listed on the NYSE, AMEX or NASDAQ, so its universe extends beyond that of the ERI Scientific Beta cap-weighted index, which restricts to the largest 500 US stocks. As a result, the two series of excess returns are not equal, though they are close to each other, with a R-square of 99.4% and a market beta of 1 for the index. Due to its weighting scheme, the index displays a bias towards

16 - Volatilities of individual stocks are estimated as the sample volatilities of weekly returns over a two-year rolling window (source: ERI Scientific Beta glossary).

17 - We are grateful to Ken French for making the series of the market, the small-minus-big, the high-minus-low and the winners-minus-losers factors, as well as the series of the risk-free rate, available on his website (see [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)).

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Table 4. Effects of selection and weighting in equity benchmarks; 1970-2015.

	All stocks			Mid Cap			Value		
	CW	EW	IV	CW	EW	IV	CW	EW	IV
Ann. ret. (%)	10.80	12.76	12.83	13.42	14.13	14.10	12.29	14.54	14.48
Ann. vol. (%)	16.90	16.76	16.12	17.12	17.06	16.29	17.19	16.72	16.16
Sharpe ratio	0.35	0.47	0.49	0.50	0.54	0.56	0.43	0.58	0.59
Tracking err. (%)	0.00	3.99	3.98	5.71	6.36	6.19	4.41	5.50	5.48
Info. ratio	-	0.49	0.51	0.46	0.52	0.53	0.34	0.68	0.67
Max. DD (%)	54.63	56.43	54.39	57.09	54.81	54.77	60.01	56.80	55.59
Max. rel. DD (%)	0.00	30.07	34.10	35.94	43.71	45.73	20.31	32.80	36.86
$\beta$ MKT	1.00	1.01	0.96	1.03	1.01	0.97	1.03	1.01	0.97
$\beta$ SIZ	-0.21	0.02	-0.03	0.14	0.18	0.15	-0.16	0.04	0.00
$\beta$ VAL	0.00	0.17	0.17	0.23	0.23	0.26	0.38	0.40	0.39
$\beta$ MOM	0.00	-0.03	-0.02	0.05	-0.02	0.01	0.03	0.00	0.01
R-square (%)	99.4	96.2	95.6	92.7	90.8	90.8	96.5	94.2	93.6

	High Momentum			Low Volatility			High Profitability		
	CW	EW	IV	CW	EW	IV	CW	EW	IV
Ann. ret. (%)	11.68	13.63	13.63	11.12	13.34	13.26	11.12	13.27	13.34
Ann. vol. (%)	17.28	17.05	16.45	15.41	14.54	14.34	17.03	16.79	16.13
Sharpe ratio	0.39	0.51	0.53	0.40	0.58	0.58	0.37	0.50	0.52
Tracking err. (%)	3.47	4.73	4.60	4.24	5.53	5.66	3.21	4.33	4.16
Info. ratio	0.25	0.60	0.61	0.07	0.46	0.43	0.10	0.57	0.61
Max. DD (%)	50.81	55.75	54.11	51.10	52.49	50.89	52.29	50.52	50.18
Max. rel. DD (%)	14.44	19.40	19.98	33.82	42.52	43.81	24.52	23.35	24.05
$\beta$ MKT	1.02	1.03	0.99	0.87	0.84	0.83	0.96	0.98	0.95
$\beta$ SIZ	-0.20	0.01	-0.03	-0.35	-0.14	-0.16	-0.25	0.04	-0.01
$\beta$ VAL	-0.03	0.11	0.13	0.09	0.23	0.22	-0.26	-0.06	-0.04
$\beta$ MOM	0.19	0.18	0.18	0.03	0.01	0.01	0.01	-0.04	-0.03
R-square (%)	97.2	94.8	94.6	93.4	90.7	90.1	97.0	95.0	94.6

	Low Investment		
	CW	EW	IV
Ann. ret. (%)	12.72	14.20	13.94
Ann. vol. (%)	15.85	15.98	15.38
Sharpe ratio	0.49	0.58	0.59
Tracking err. (%)	3.78	5.10	5.20
Info. ratio	0.51	0.66	0.60
Max. DD (%)	51.12	54.33	52.58
Max. rel. DD (%)	26.47	38.03	41.11
$\beta$ MKT	0.93	0.96	0.92
$\beta$ SIZ	-0.20	0.03	-0.02
$\beta$ VAL	0.16	0.27	0.26
$\beta$ MOM	0.06	0.03	0.05
R-square (%)	94.7	93.3	92.4

Statistics are computed from the ERI Scientific Beta database for long-term US indices. The data is daily and spans the period from Jun. 19, 1970 through Dec. 31, 2015. CW denotes a cap-weighted portfolio, EW an equally-weighted portfolio and IV a portfolio in which the stocks are weighted by the reciprocal of their volatility. The tracking error, the information ratio and the maximum relative drawdown are calculated with respect to the broad cap-weighted index. The market, size, value and momentum factors are borrowed from Ken French's data library. The betas are obtained by regressing the excess returns of the stock indices on the four factors.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

large firms, with a negative size beta: this bias arises in all selections, except for the mid-cap one. In all universes, it is the equally-weighted portfolio that has the largest size exposure, because small stocks are relatively overrepresented. It can also be verified that the selection step achieves its objective: portfolios based on the mid-cap, the value or the high momentum selections have respectively the highest size, value and momentum betas.

Interestingly, some selections appear to induce biases with respect to the uncontrolled factors. The high profitability portfolios have negative value betas, perhaps because stocks with high profits are more likely to be growth than value: this echoes the characterisation of value firms as firms with durably low earnings in Fama and French (1995). Similarly, the low volatility portfolios have a large cap bias, as illustrated by their negative size betas. Investors confident in the value or in the size premium and holding a portfolio tilted towards high profitability or low volatility stocks may be worried that they bear factor exposures that go against their convictions, but this happens because none of the smart factor indices aims to simultaneously control several factor exposures. To obtain exposure to several factors, one can use a combination of smart factor strategies, as we argue in the next section.

### 4.3.2 Exposure to Multiple Factors: Allocating to Smart Factor Strategies

The theoretical result on the equivalence between the MSR portfolio of individual securities and the MSR portfolio of pricing factors suggests that it is interesting to combine factors. The last empirical illustration that we present takes as building blocks the six equity factors already used in the previous section,

namely size, value, momentum, volatility, profitability and investment. Table 5 shows performance and risk indicators for these constituents, and Tables 6 and 7 display out-of-sample statistics for multi-factor portfolios. The main purpose of the backtests is to show that multi-factor portfolios display in general risk and return properties substantially better compared to the average of the single factors. In this context where all long-only equity factors are highly correlated and relatively similar in terms of risk characteristics, we do not expect strong value to be added by the use of competing portfolio construction methodologies, but we do report the performance of various weighting schemes for comparison purposes.

### *Backtest Methodology*

The six factors are considered in their long-only version and are represented by six ERI Scientific Beta indices, all of which are made with US stocks and are cap-weighted: the Mid-Cap index, the Value index, the High-Momentum index, the Low-Volatility index, the Low-Investment index and the High-Profitability index. These indices have the advantage that they can be regarded as investable building blocks because their construction procedure involves a control for turnover and capacity. The series are available on a daily basis for the period from June 19, 1970 through December 31, 2015.

Except for the equally-weighted scheme, all weighting schemes require a covariance matrix for the factors as an input. At each rebalancing date, this parameter is estimated from two years of weekly returns, an approach that is similar to that employed for individual stocks in the calculation of ERI Scientific Beta indices. All portfolios are rebalanced quarterly and are left buy-and-hold

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

within a quarter. The first two years of the dataset are used to estimate the first covariance matrix, so the backtests actually start on July 3, 1972.

Together with the covariance matrix, implicit risk factors are extracted by principal component analysis and by minimum linear torsion. The extraction method is irrelevant for the construction of equally-weighted, minimum variance and risk parity portfolios, but it matters wherever the effective number of bets is involved. The ENUB of any portfolio depends on how the implicit factors have been estimated, and the composition of a FRP portfolio as well as that of any portfolio subject to a minimum ENUB constraint is also contingent to the choice of the method. For this reason, we report results pertaining to PCA and MLT factors in separate tables.

The first four methods (equal weighting, variance minimisation with short sales allowed, variance minimisation without short sales and risk parity) do not make any use of implicit factors. In the factor risk parity method, it should be noted that two notions of factors are involved: on the one hand, constituents are rewarded equity factors, and on the other hand, the weighting scheme targets diversification across risk factors. Since there are  $2^6 - 1 = 32$  FRP portfolios, we select one by minimising leverage, defined as the sum of absolute values of negative weights. This portfolio is the most easily "investable" of all FRP portfolios in that it minimises the magnitude of short positions, but it can still involve a substantial amount of residual leverage. In what follows, we refer to it as "the" FRP portfolio. Except for the long-short global minimum variance (GMV) and the FRP portfolios, all tested portfolios are long-only in terms

of their constituents, hence in terms of the individual stocks that constitute the factors.

In addition to the FRP portfolio based on the factors extracted from asset returns, we also compute a "relative FRP portfolio", which is constructed exactly like the FRP portfolio, except for the noticeable difference that it is based on implicit factors extracted from excess returns over the cap-weighted benchmark, as opposed to being extracted from raw returns. To estimate the factors in this case, principal component analysis and minimum linear torsion are applied to the covariance matrix of excess returns as opposed to the standard covariance matrix. The resulting factors span the relative risk of the various factors, and the 32 relative FRP portfolios equate the contributions of these factors to the volatility of the portfolio excess returns with respect to the benchmark, that is its tracking error. We select the relative FRP portfolio that has the lowest volatility among those that have the minimum leverage. Indeed, with relative factors, it happens in some calibration windows that more than one relative FRP portfolio is long-only, so there are multiple portfolios with zero leverage. In this context, the volatility minimisation criterion allows one of them to be selected, and we refer to it as "the" relative FRP portfolio.

Because we now have two sets of implicit factors for each calibration window, we calculate two ENUB measures: the "absolute ENUB" measures diversification in terms of the absolute factors – extracted from plain constituent returns – while the "relative ENUB" has a focus on the relative factors – extracted from excess returns.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

In the presence of a long-only constraint, the maximum attainable absolute ENUB may be less than six, which is the value attained by FRP portfolios. To find this maximum, we compute a maximum (absolute) ENUB portfolio subject to a long-only constraint. The last three weighting schemes that we consider are portfolios subject to a long-only and a minimum ENUB constraint, where the lower bound on the ENUB is taken to be 75% of the maximum, and they respectively minimise volatility, maximise the ENC and maximise the ENCB. We end up with a total of nine portfolios.

The leverage, the ENC, the ENCB and the ENUB of each portfolio are recorded at each rebalancing date and averaged across dates, to give a single number reported in the tables. We also calculate the quarterly one-way turnover, defined at each rebalancing date as

$$\frac{1}{2} \sum_{i=1}^6 |w_{it} - w_{it-}|,$$

where  $w_{it}$  is the target weight of factor  $i$  and  $w_{it-}$  is the weight just before rebalancing. This figure gives a sense of the amount of trading involved in the rebalancing process, and the coefficient  $1/2$  is included to avoid double-counting a buy or sell operation in a security that is compensated by an opposite operation in another security. It should be noted that this turnover number measures the change in the composition in terms of the factors, not in terms of the underlying stocks.

We also report a series of standard risk and return indicators including annualised (geometric) return, volatility, Sharpe ratio and maximum drawdown. The risk-free rate in the Sharpe ratio is taken to be

the annualised return to a strategy that rolls over short-term bonds every day. The daily rate of return of this money market account is the secondary market rate on US Treasury bills with three-month maturity on that day. The tables also display relative indicators, namely the tracking error and the information ratio, both of which are computed with respect to the ERI Scientific Beta US cap-weighted index, which is a good proxy for the S&P 500 index. The maximum relative drawdown measures the worst relative loss of a portfolio with respect to the broad index, and is computed as the maximum drawdown of the time series of portfolio values divided by the benchmark values, a series that represents the value of the portfolio expressed in the benchmark numeraire as opposed to the dollar numeraire.

### *Single-Factor versus Multi-Factor Portfolios*

Table 5 shows standard performance and risk analytics on the six factors. They differ from those shown in Table 4 because the sample period here starts in 1972 as opposed to 1970, but all factors outperform the broad cap-weighted index, as can be seen from the positive information ratios, as in Table 4. Size is still the factor that has performed best in the sample, while it has the second worst maximum drawdown, at 57.09%, consistent with the interpretation of the size premium as a risk premium.

As expected, these long-only equity factors are highly correlated, with correlation coefficients all greater than 90%. This is due to the presence of a common factor, namely the market factor, which explains a large part of the time series variation in each factor. By subtracting market returns from the returns to each factor,

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Table 5. Performance and risk analysis of equity factors; 1972–2015.

	Annualised return (%)	Annualised volatility (%)	Sharpe ratio	Maximum drawdown (%)	Tracking error (%)	Information ratio	Max. rel. drawdown (%)
Size	12.89	17.34	0.46	57.09	5.77	0.45	35.94
Value	12.15	17.46	0.41	60.01	4.45	0.41	20.31
Momentum	11.26	17.53	0.36	50.81	3.52	0.27	14.44
Volatility	10.65	15.63	0.37	51.10	4.33	0.08	33.82
Profitability	10.40	17.26	0.32	52.29	3.25	0.03	24.52
Investment	12.37	16.07	0.46	51.12	3.82	0.54	26.47
<b>Average</b>	<b>11.62</b>	<b>16.88</b>	<b>0.40</b>	<b>53.74</b>	<b>4.19</b>	<b>0.29</b>	<b>25.92</b>

	Correlations and relative correlations					
	Size	Value	Momentum	Volatility	Profitability	Investment
Size	-	0.94	0.92	0.91	0.91	0.95
Value	0.38	-	0.93	0.94	0.91	0.96
Momentum	-0.01	-0.12	-	0.94	0.97	0.95
Volatility	-0.03	0.13	-0.10	-	0.94	0.97
Profitability	-0.31	-0.77	0.17	-0.12	-	0.94
Investment	0.31	0.38	0.03	0.55	-0.28	-

Each equity factor is long-only and is represented by an ERI Scientific Beta index. Performance and risk indicators are computed from daily returns over the period from July 3, 1972 through December 31, 2015. Tracking errors and information ratios are computed with respect to the broad US cap-weighted index. Row "Average" displays metrics averaged across the six factors. The upper part (above diagonal) of the correlation matrix contains correlations between factor returns over the sample period, and the lower part contains the correlations between excess returns over the broad cap-weighted index.

18 - To completely eliminate exposure to the market factor, market returns should be multiplied by the market beta to form a long-short beta-neutral portfolio. Here, we consider long-short zero-dollar portfolios.

the influence of this factor is strongly reduced, since the market betas of all factors cluster around 1 (see Table 4).<sup>18</sup> Correlations between excess returns are much lower, and in fact often negative.

Table 6 shows the average performance and risk analytics of the six factors, so the "average properties" of the factors can be compared with those of an equally-weighted portfolio of all factors, reported in Table 6. Absolute indicators are improved by mixing the factors equally, but to a moderate extent. Because of the presence of an exceedingly high correlation level, the portfolio average return, volatility and Sharpe ratio are close to the average indicators for the constituents. The maximum drawdown decreases slightly, from 53.74% on average to 52.3%. On the other hand, the portfolio differs significantly from the average of its

constituents when risk and performance are measured in relative terms. Thus the tracking error of the EW portfolio of the six constituents is less than half the average tracking error (2.0% versus 4.19%), and its information ratio is much higher than the average of the six constituents, at 0.95 versus 0.72. These results suggest that mixing highly correlated factors with equal weights hardly brings an improvement over the average absolute metrics of the constituents, while the relative metrics are improved as long because the excess returns to the factors display much lower correlation levels.

### Back to Eggs and Baskets: Diversification Across Assets or Risk Factors

With multiple analytics on weights, it is possible to gauge the diversification of each portfolio from different perspectives.



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Table 6. Properties of multi-factor equity portfolios; principal factors used for ENUB; 1972-2015.

	Analytics on weights (averaged)					
	Leverage (%)	ENC	ENCB	ENUB	Rel. ENUB	Quarterly turnover (%)
Equal Weight	0.0	6.0	6.0	1.0	2.1	0.8
GMV (L/S)	275.7	0.2	0.2	2.3	2.4	84.4
GMV (L/O)	0.0	1.4	1.4	1.1	2.0	13.4
Risk Parity	0.0	6.0	6.0	1.0	2.1	1.1
Factor Risk Parity	583.4	0.0	0.1	6.0	3.7	327.3
Relative Factor Risk Parity	3.5	3.6	3.6	1.0	6.0	20.1
Max ENUB (L/O)	0.0	1.0	1.0	1.2	2.3	20.2
Max Rel. ENUB (L/O)	0.0	3.6	3.5	1.0	5.8	21.6
Max ENC (L/O + ENUB cst.)	0.0	5.8	5.8	1.0	2.1	2.5
Min Vol (L/O + ENUB cst.)	0.0	1.4	1.4	1.1	2.0	13.6
Max ENCB (L/O + ENUB cst.)	0.0	5.8	5.8	1.0	2.1	2.6
Max ENC (L/O + Rel. ENUB cst.)	0.0	5.1	5.0	1.0	4.4	9.1
Min Vol (L/O + Rel. ENUB cst.)	0.0	2.7	2.7	1.0	4.4	26.6
Max ENCB (L/O + Rel. ENUB cst.)	0.0	5.1	5.1	1.0	4.4	7.9

	Performance and risk analysis						
	Ann. return (%)	Ann. volatility (%)	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
Equal Weight	11.74	16.45	0.41	52.30	1.97	0.72	13.32
GMV (L/S)	9.04	14.15	0.29	43.57	11.10	-0.12	74.85
GMV (L/O)	11.25	15.46	0.41	47.84	3.98	0.23	36.90
Risk Parity	11.76	16.36	0.42	51.96	2.09	0.69	14.73
Factor Risk Parity	5.59	18.61	0.04	63.57	18.39	-0.26	94.04
Relative Factor Risk Parity	11.23	16.77	0.38	52.66	1.30	0.70	5.34
Max ENUB (L/O)	9.92	16.26	0.31	51.67	3.85	-0.10	34.13
Max Rel. ENUB (L/O)	11.10	16.80	0.37	54.17	1.30	0.60	3.73
Max ENC (L/O + ENUB cst.)	11.94	16.40	0.43	52.30	2.35	0.69	13.78
Min Vol (L/O + ENUB cst.)	11.25	15.46	0.41	47.82	4.03	0.23	36.89
Max ENCB (L/O + ENUB cst.)	11.91	16.31	0.43	51.96	2.41	0.66	14.96
Max ENC (L/O + Rel. ENUB cst.)	11.43	16.68	0.39	52.29	1.45	0.77	5.28
Min Vol (L/O + Rel. ENUB cst.)	11.10	16.32	0.38	53.79	1.99	0.40	11.38
Max ENCB (L/O + Rel. ENUB cst.)	11.38	16.61	0.39	52.60	1.48	0.72	5.68

Multi-factor portfolios are made with the six equity factors and are rebalanced every quarter from July 1972 through December 2015. The covariance matrix and the relative covariance matrix (i.e., the matrix of covariances between excess returns with respect to the broad cap-weighted index) are estimated over a two-year rolling window of weekly observations. Two sets of six implicit factors are extracted respectively from the covariance matrix and the relative covariance matrix by principal component analysis. They are used to construct factor risk parity (FRP) and relative factor risk parity portfolios. We retain the FRP portfolio with the lowest leverage and the relative FRP portfolio with the lowest leverage and the lowest volatility. The ENUB is computed with respect to the implicit factors from the covariance matrix, while the relative ENUB is based on the factors from the relative covariance matrix. When an ENUB or relative ENUB constraint is applied, the minimum ENUB or relative ENUB required is equal to 75% of the maximum ENUB or relative ENUB attainable subject to the long-only constraint. The tracking error, the information ratio and the maximum relative drawdown are taken with respect to the broad cap-weighted index.

In particular, the ENC, ENCB and ENUB scores provide measures of diversification pertaining to the various definitions of

eggs and baskets introduced in Section 3. It first turns out that the equally-weighted and the risk parity portfolios are close to



## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

each other in this particular situation, with both portfolios displaying an average ENC and an average ENCB of 6. There are still small differences between the two portfolios. For instance, the risk parity portfolio has a slightly higher turnover, at 1.1% per quarter instead of 0.8%, and the performance metrics are also slightly different. Again, this similarity is due to the large correlations across the long-only factors, which are greater than 80% in 86% of the calibration periods, and to the rather limited dispersion of volatilities. While the low-volatility factor has the lowest volatility, as expected, the difference between the top and the bottom volatilities is never more than 9.4%, a much narrower range compared to a stock-bond allocation exercise. As a result, contributions to risk are almost as well spread in the equally-weighted portfolio as in the risk parity portfolio. More generally, the ENC and the ENCB measures are close to each other for all portfolios, but this would not be the case in portfolios of constituents exhibiting more dispersion of volatilities and correlations, such as multi-class portfolios.

On the other hand, the FRP portfolio is very different from these two portfolios, if only because it features a substantial amount of leverage. This is especially true with PCA factors, because any long-only portfolio is by construction dominated by the first factor, which is similar to the market factor. For the contributions of the other factors to be as large as that of the first factor, a contrived allocation is needed, with extremely large short positions in some assets. Unsurprisingly, the turnover of this highly unreasonable long-short portfolio is huge, at 327.3% per quarter on average. MLT factors are by definition approximations to the constituents, so contributions of

factors in long-only portfolios are not as dispersed as with PCA factors, and it is possible to find a FRP portfolio with a moderate degree of leverage, at 18.3% on average. Turnover, at 12.0%, is also more reasonable. These findings confirm that the MLT approach is a more reasonable method for extracting factors in the context of constructing a FRP portfolio compared to the PCA method.

Another way to analyse these results is to note that long-only portfolios have low ENUBs – no greater than 1.2 out of a total of 6 – with respect to PCA factors, while ENUBs with respect to MLT factors are much higher – no less than 5.4. Moreover, in Table 6, no portfolio simultaneously achieves a “high” ENC (or ENCB, since these two measures are very similar here) and a “high” ENUB, where “high” means at least greater than 3. With MLT factors, the picture is different with many portfolios achieving high scores in the three dimensions. For instance, the equally-weighted and the risk parity portfolios have respective ENUBs of 5.5 and 5.6. Overall, there is a clear conflict between diversification in terms of assets and diversification in terms of PCA factors, which is largely resolved when MLT factors are employed.

For each extraction method, there are in fact two sets of implicit factors, those that explain the covariances of assets, and those that explain their relative covariances, i.e. the covariances between their excess returns over the cap-weighted index. Using these relative factors is another way to mitigate the conflict between diversification across assets and across factors. With the PCA method, relative ENUBs are greater than ENUBs for all portfolios, except of course for the FRP portfolio, which means that

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Table 7. Properties of multi-factor equity portfolios; minimum torsion factors used for ENUB; 1972-2015.

	Analytics on weights (averaged)					
	Leverage (%)	ENC	ENCB	ENUB	Rel. ENUB	Quarterly turnover (%)
Equal Weight	0.0	6.0	6.0	5.5	3.6	0.8
GMV (L/S)	275.7	0.2	0.2	3.2	3.2	84.4
GMV (L/O)	0.0	1.4	1.4	5.6	2.0	13.4
Risk Parity	0.0	6.0	6.0	5.6	3.6	1.1
Factor Risk Parity	18.3	2.3	2.2	6.0	3.0	12.0
Relative Factor Risk Parity	0.1	4.6	4.5	5.4	6.0	3.6
Max ENUB (L/O)	0.0	2.8	2.9	5.9	2.6	8.7
Max Rel. ENUB (L/O)	0.0	4.5	4.5	5.4	6.0	4.7
Max ENC (L/O + ENUB cst.)	0.0	6.0	6.0	5.5	3.6	0.8
Min Vol (L/O + ENUB cst.)	0.0	1.4	1.4	5.6	2.0	13.3
Max ENCB (L/O + ENUB cst.)	0.0	6.0	6.0	5.6	3.6	1.1
Max ENC (L/O + Rel. ENUB cst.)	0.0	5.7	5.6	5.5	4.5	3.5
Min Vol (L/O + Rel. ENUB cst.)	0.0	3.4	3.4	5.6	4.5	17.0
Max ENCB (L/O + Rel. ENUB cst.)	0.0	5.7	5.7	5.5	4.5	2.3

	Performance and risk analysis						
	Ann. return (%)	Ann. Volatility	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
Equal Weight	11.74	16.45	0.41	52.30	1.97	0.72	13.32
GMV (L/S)	9.04	14.15	0.29	43.57	11.10	-0.12	74.85
GMV (L/O)	11.25	15.46	0.41	47.84	3.98	0.23	36.90
Risk Parity	11.76	16.36	0.42	51.96	2.09	0.69	14.73
Factor Risk Parity	11.26	15.28	0.42	46.99	3.90	0.24	33.91
Relative Factor Risk Parity	11.09	16.78	0.37	53.18	1.13	0.69	4.46
Max ENUB (L/O)	11.53	15.56	0.42	48.30	3.31	0.37	30.80
Max Rel. ENUB (L/O)	11.09	16.81	0.37	53.18	1.12	0.69	5.18
Max ENC (L/O + ENUB cst.)	11.74	16.45	0.41	52.30	1.97	0.72	13.32
Min Vol (L/O + ENUB cst.)	11.25	15.46	0.41	47.82	3.99	0.23	36.89
Max ENCB (L/O + ENUB cst.)	11.76	16.36	0.42	51.96	2.09	0.69	14.73
Max ENC (L/O + Rel. ENUB cst.)	11.24	16.56	0.38	52.49	1.68	0.55	13.27
Min Vol (L/O + Rel. ENUB cst.)	11.24	16.13	0.39	50.15	2.41	0.39	18.94
Max ENCB (L/O + Rel. ENUB cst.)	11.35	16.54	0.39	52.28	1.58	0.66	8.77

Multi-factor portfolios are made with the six equity factors and are rebalanced every quarter from July 1972 through December 2015. The covariance matrix and the relative covariance matrix (i.e., the matrix of covariances between excess returns with respect to the broad cap-weighted index) are estimated over a two-year rolling window of weekly observations. Two sets of six implicit factors are extracted respectively from the covariance matrix and the relative covariance matrix by minimum linear torsion. They are used to construct factor risk parity (FRP) and relative factor risk parity portfolios. We retain the FRP portfolio with the lowest leverage and the relative FRP portfolio with the lowest leverage and the lowest volatility. The ENUB is computed with respect to the implicit factors from the covariance matrix, while the relative ENUB is based on the factors from the relative covariance matrix. When an ENUB or relative ENUB constraint is applied, the minimum ENUB or relative ENUB required is equal to 75% of the maximum ENUB or relative ENUB attainable subject to the long-only constraint. The tracking error, the information ratio and the maximum relative drawdown are taken with respect to the broad cap-weighted index.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

contributions of factors to the portfolio tracking error are more balanced than the contributions to volatility. With the MLT approach, the opposite effect takes place: relative ENUBs are lower than ENUBs, except for the relative FRP portfolio that has a relative ENUB of 6, but they remain greater than 3 in most cases, the exceptions being two of the three minimum variance portfolios. When relative factor contributions are forced to be equal, the relative ENUB is maximal, and the leverage and the turnover are lower than for FRP portfolios. The difference is especially spectacular with PCA factors. Thus, despite the large correlations across constituents, it is possible to construct portfolios with reasonable asset weights and well-balanced factor contributions if one focuses on the factors that explain risk of factor indices relative to the market index.

### *Effects of Diversification on Performance and Risk*

A first observation is that the minimum variance portfolios achieve their objective out of sample since each of them has the lowest out-of-sample volatility in its category, a property that holds both in Tables 6 and 7. A second, and unsurprising, finding is that portfolios with extreme weights such as the long-short GMV and the FRP portfolios have disappointing returns, risk-adjusted returns and information ratios. For instance, the FRP portfolio based on PCA factors has an annualised return of 5.59% and a minuscule Sharpe ratio of 0.04, much less than that of any of its constituents, and it delivers a negative information ratio of -0.26. Thus, portfolios computed by quantitative diversification algorithms have poor out-of-sample properties if these methods lead to sizable short positions that are not corrected. This is

a standard result of the literature on the empirical properties of mean-variance efficient portfolios (Frost and Savarino 1988; Jagannathan and Ma 2003; DeMiguel, Garlappi, and Uppal 2009).

While introducing long-only constraints greatly improves the situation, it is not a sufficient condition to maximise the benefits of diversification. The long-only GMV has the same Sharpe ratio as the equally-weighted while having a lower information ratio, and the portfolio that maximises the ENUB of PCA factors still has a negative information ratio and underperforms the equally-weighted allocation. In both tables, the GMV portfolios that incorporate an ENUB or a relative ENUB constraint in addition to the long-only constraint exhibit higher returns, Sharpe ratio and information ratio than the long-only GMV, but they are still dominated by the simple equally-weighted portfolio. Interestingly, these long-only portfolios have lower ENCs and ENCBs than their competitors. This suggests that long-only constraints should be complemented by a sufficient level of "naïve" diversification across constituents. Indeed, they do not by themselves avoid concentration of dollar or risk contributions of assets, so introducing an ENC or ENCB constraint can prove useful. The weighting schemes that outperform the equally-weighted allocation in terms of both return and Sharpe ratio have ENCs and ENCBs greater than 3. There are actually few of them, namely the maximum ENC or ENCB with an ENUB-PCA constraint in Table 6, and the maximum ENCB with an ENUB-MLT constraint in Table 7. These portfolios have average ENCs and ENCBs of 5.8 at least.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Relative risk is a concern to many equity managers, and a key question is in particular whether deviations from the broad cap-weighted reference were rewarded by additional returns. The equally-weighted allocation of the six factors proves again to be a difficult-to-beat reference: it has lower tracking error and maximum relative drawdown, and a higher information ratio than the GMV, the risk parity and the FRP portfolios, as well as the three portfolios subject to an ENUB constraint. This result is observed whether implicit factors are obtained by PCA or by MLT. It is the the portfolios including a control of relative risk that best compete with the equally-weighted benchmark. For instance, it is the relative FRP portfolio and the maximum relative ENUB – its long-only best proxy – that have the lowest tracking errors, between 1.12% and 1.58% versus 1.97% for the EW benchmark. The maximum ENC, the minimum volatility and the maximum ENCB portfolios have lower relative drawdowns when they are forced to respect a minimum relative ENUB than when they are subject to an ENUB constraint. With the PCA method in Table 6, they also have higher information ratios, but this is no longer true with the MLT approach in Table 7. Although they bring an improvement in relative terms, portfolios with a focus on or a control of relative risk do not outperform the equally-weighted allocation in terms of performance and Sharpe ratio.

As a conclusion, combining equity factors even by the simple equal weighting rule improves the risk and return characteristics, both in absolute and in relative terms, which makes a clear case for multi-factor allocations. Other attractive multi-factor portfolios can be designed by ensuring a correct level of

diversification in terms of assets and/or risk contributions, but outperforming the equally-weighted allocation, whether in performance, Sharpe ratio or information ratio, is by no means an easy task. This is not impossible, however, and some weighting schemes based on quantitative allocation techniques improve at least one indicator. For instance, portfolios with a control of relative risk are likely to outperform this benchmark in terms of tracking error, information ratio and maximum relative drawdown. These findings confirm that it is in terms of relative returns, which are not as highly correlated as the long-only raw returns, that diversification benefits can be best harvested.

### *Robustness Checks in Sub-Periods*

To test the robustness of the previous conclusions to the choice of the sample period, we repeat the backtests over four non-overlapping decades that span the period from January 1975 to January 2015. To save space, we only report the performance and risk metrics, and we focus on minimum linear torsion as the method for extracting implicit factors, because this approach allows for an easier reconciliation between the objectives to diversify across assets and to diversify across sources of risk. Table 8 shows the results per sub-period.

The benefits of diversifying across across the six equity factors are remarkably consistent over time. In each subperiod, a simple equally-weighted portfolio has greater performance and Sharpe ratio, and lower volatility and maximum drawdown than the average of the constituents, even though the difference is more sizable in some sample periods than in others.<sup>19</sup> Again, the benefits are larger when it comes to relative indicators,

19 - The fact that the average return of the equally-weighted portfolio is larger than the equally-weighted average of the constituents' returns over a sufficiently long period is a manifestation of the rebalancing bonus, which indicates that a rebalanced portfolio tends to outperform a buy-and-hold portfolio.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

---

and we find that the tracking error is divided by a factor between 1.8 and 2.9, the information ratio is multiplied by more than 2, and the maximum relative drawdown is very substantially reduced. As highlighted previously, the bigger improvement in the relative indicators can be attributed to the fact that relative returns are much less correlated than raw returns.

Just as in the full 45-year period, it is hard to consistently achieve a higher out-of-sample Sharpe ratio than the equally-weighted benchmark. For instance, the long-only GMV and the minimum volatility portfolio with an ENUB constraint are successful between 1975 and 1985 and then between 2005 and 2015, but they lag behind in the other decades. A possible explanation for the good Sharpe ratios of minimum variance allocations during the 2005-2015 sub-period is the severe bear market of 2009, which favoured more defensive strategies, but it is only partial because a bear market occurred at the turn of the millennium too, without this resulting in a higher Sharpe ratio for minimum variance portfolios between 1995 and 2005.

Turning to tracking error and maximum relative drawdown, we find that portfolios with a control of relative ENUB dominate their counterparts that focus on the absolute ENUB. The relative FRP displays lower values in terms of these relative risk indicators compared to the FRP portfolio, an observation that can be repeated for the maximum ENC, the maximum ENCB and the minimum volatility portfolios with or without a relative ENUB constraint. The effect of switching from an absolute to a relative risk control on information ratio is more erratic. While the relative FRP portfolio always has a higher information

ratio than the FRP portfolio, this is not always the case for portfolios with a relative ENUB constraint compared to those with an ENUB constraint. In any case, the information ratio of the equally-weighted portfolio is hard to improve, since it has the highest value across all tested portfolios, except in the 1985-1995 window, where the maximum ENC portfolio with a relative ENUB constraint exhibits a better score, at 1.09 versus 0.99.

## 4. Factor Investing and Benchmarking Decisions: A More Efficient Harvesting of Risk Premia

Table 8: Performance and risk indicators of multi-factor equity portfolios in sub-periods; minimum torsion factors used for ENUB.

	1975–1985						
	Ann. return (%)	Ann. Volatility	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
<b>Average</b>	<b>17.41</b>	<b>13.34</b>	<b>0.65</b>	<b>19.64</b>	<b>3.36</b>	<b>0.62</b>	<b>11.04</b>
Equal Weight	17.48	12.99	0.67	17.39	1.83	1.49	3.28
GMV (L/S)	14.66	11.25	0.52	17.70	9.15	-0.01	43.97
GMV (L/O)	17.57	12.62	0.70	14.83	2.82	0.99	8.98
Risk Parity	17.48	12.97	0.67	17.23	1.85	1.47	3.36
Factor Risk Parity	17.04	12.49	0.66	14.93	2.64	0.86	9.89
Relative Factor Risk Parity	16.10	13.22	0.55	16.39	1.22	1.09	2.88
Max ENUB (L/O)	17.11	12.65	0.66	15.41	2.32	1.01	6.48
Max Rel. ENUB (L/O)	16.20	13.20	0.56	16.39	1.26	1.15	3.45
Max ENC (L/O + ENUB cst.)	17.48	12.99	0.67	17.39	1.83	1.49	3.28
Min Vol (L/O + ENUB cst.)	17.57	12.62	0.70	14.83	2.84	0.99	9.03
Max ENCB (L/O + ENUB cst.)	17.48	12.97	0.67	17.23	1.85	1.47	3.36
Max ENC (L/O + Rel. ENUB cst.)	16.46	13.09	0.59	16.83	1.55	1.10	3.05
Min Vol (L/O + Rel. ENUB cst.)	16.56	12.73	0.61	15.79	2.01	0.89	4.15
Max ENCB (L/O + Rel. ENUB cst.)	16.48	13.07	0.59	16.82	1.55	1.11	3.09

	1985–1995						
	Ann. return (%)	Ann. Volatility	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
<b>Average</b>	<b>15.15</b>	<b>16.04</b>	<b>0.59</b>	<b>33.05</b>	<b>2.83</b>	<b>0.37</b>	<b>11.50</b>
Equal Weight	15.23	15.80	0.60	32.58	1.03	0.99	1.44
GMV (L/S)	13.19	14.34	0.52	28.03	8.61	-0.12	33.36
GMV (L/O)	13.90	15.21	0.54	30.35	2.58	-0.12	9.74
Risk Parity	15.19	15.77	0.60	32.48	1.02	0.97	1.56
Factor Risk Parity	14.73	15.28	0.59	30.81	1.86	0.28	7.64
Relative Factor Risk Parity	15.07	15.87	0.59	32.82	0.74	1.17	0.72
Max ENUB (L/O)	14.66	15.36	0.58	30.91	1.72	0.27	7.14
Max Rel. ENUB (L/O)	15.28	15.88	0.60	32.82	0.73	1.46	0.69
Max ENC (L/O + ENUB cst.)	15.23	15.80	0.60	32.58	1.03	0.99	1.44
Min Vol (L/O + ENUB cst.)	13.88	15.21	0.54	30.35	2.59	-0.12	9.83
Max ENCB (L/O + ENUB cst.)	15.19	15.77	0.60	32.48	1.02	0.97	1.56
Max ENC (L/O + Rel. ENUB cst.)	15.22	15.82	0.60	32.61	0.94	1.09	1.01
Min Vol (L/O + Rel. ENUB cst.)	15.30	15.69	0.61	31.98	1.18	0.93	2.78
Max ENCB (L/O + Rel. ENUB cst.)	15.21	15.81	0.60	32.59	0.93	1.07	1.05

See the legend of Table 7. Backtests are conducted over the specified period, from the first of January to the first of January, and calibration of the covariance matrix starts two years before the first date. Row "Average" contains the average performance and risk statistics for the six equity factors.

## 4. Factor Investing and Benchmarking Decisions. A More Efficient Harvesting of Risk Premia

Table 8 (cont.): Performance and risk indicators of multi-factor equity portfolios in sub-periods; minimum torsion factors used for ENUB.

	1995-2005						
	Ann. return (%)	Ann. Volatility	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
Average	13.29	17.27	0.55	39.40	5.76	0.17	23.12
Equal Weight	13.46	16.54	0.58	34.76	3.04	0.42	12.55
GMV (L/S)	6.72	15.56	0.18	43.57	12.04	-0.45	65.15
GMV (L/O)	11.39	15.64	0.48	29.41	5.92	-0.13	31.63
Risk Parity	13.51	16.41	0.59	33.20	3.32	0.40	14.07
Factor Risk Parity	12.12	15.51	0.53	31.24	5.30	-0.01	30.92
Relative Factor Risk Parity	12.62	17.32	0.50	43.96	1.54	0.28	4.46
Max ENUB (L/O)	12.44	15.69	0.55	31.14	4.98	0.05	27.79
Max Rel. ENUB (L/O)	12.48	17.40	0.49	44.49	1.49	0.20	5.18
Max ENC (L/O + ENUB cst.)	13.46	16.54	0.58	34.76	3.04	0.42	12.55
Min Vol (L/O + ENUB cst.)	11.39	15.64	0.48	29.40	5.93	-0.13	31.58
Max ENCB (L/O + ENUB cst.)	13.51	16.41	0.59	33.20	3.32	0.40	14.07
Max ENC (L/O + Rel. ENUB cst.)	12.54	16.87	0.51	40.38	2.64	0.14	12.97
Min Vol (L/O + Rel. ENUB cst.)	11.82	16.51	0.48	38.43	3.59	-0.10	18.85
Max ENCB (L/O + Rel. ENUB cst.)	12.91	16.94	0.53	40.46	2.27	0.32	8.44

	2005-2015						
	Ann. return (%)	Ann. Volatility	Sharpe ratio	Max. draw. (%)	Tracking error (%)	Info. ratio	Max. rel. draw. (%)
Average	8.91	20.12	0.38	52.48	4.00	0.23	12.45
Equal Weight	8.99	19.82	0.38	52.30	1.42	0.70	4.12
GMV (L/S)	6.71	15.04	0.35	34.98	14.54	-0.09	41.41
GMV (L/O)	9.04	17.85	0.43	47.84	4.05	0.26	8.45
Risk Parity	8.94	19.63	0.38	51.96	1.50	0.63	4.58
Factor Risk Parity	8.05	17.28	0.38	46.99	5.07	0.01	16.67
Relative Factor Risk Parity	8.62	20.04	0.36	53.18	0.94	0.66	3.00
Max ENUB (L/O)	9.14	17.97	0.43	48.30	3.61	0.32	8.09
Max Rel. ENUB (L/O)	8.62	20.04	0.36	53.18	0.94	0.66	3.00
Max ENC (L/O + ENUB cst.)	8.99	19.82	0.38	52.30	1.42	0.70	4.12
Min Vol (L/O + ENUB cst.)	9.05	17.85	0.43	47.82	4.06	0.26	8.45
Max ENCB (L/O + ENUB cst.)	8.94	19.63	0.38	51.96	1.50	0.63	4.58
Max ENC (L/O + Rel. ENUB cst.)	8.86	19.76	0.38	52.49	1.32	0.65	4.13
Min Vol (L/O + Rel. ENUB cst.)	8.85	18.86	0.39	50.15	2.56	0.33	8.30
Max ENCB (L/O + Rel. ENUB cst.)	8.89	19.68	0.38	52.28	1.40	0.63	4.41

See the legend of Table 7. Backtests are conducted over the specified period, from the first of January to the first of January, and calibration of the covariance matrix starts two years before the first date. Row "Average" contains the average performance and risk statistics for the six equity factors.



## 5. Conclusions and Perspectives



## 5. Conclusions and Perspectives

---

Asset pricing theory has proposed an unambiguous definition for meaningful asset pricing factors as factors that explain differences in expected returns across assets. In investment practice, the notion of factor is more polysemic and the relevance of a given definition appears to depend on the targeted application. Factors are often defined as factor strategies that deliver a long-term premium over a benchmark. The most popular examples can be found in equity markets, where size, value, momentum, volatility, investment and profitability are widely documented sources of profitability with sound economic rationale. Capturing these premia subject to investability conditions is the goal of the "factor indices" offered by index providers. In addition to delivering a factor exposure, "smart factor indices" reduce unrewarded risk and improve risk-adjusted performance by deviating from the usual capitalisation weighting scheme. The results presented in this paper confirm the well-known fact that adopting a smart weighting scheme improves the long-term Sharpe ratio and information ratio with respect to a cap-weighted index. Our understanding of risk premia in non-equity asset classes is comparatively more limited, although recent research has identified effects such as value, momentum or carry in multiple classes. It is of critical importance for future research to continue searching for robust and economically justified rewarded factors outside equities, and for the asset management industry to provide cost-efficient vehicles to access the corresponding premia. A particularly relevant effort relates to a detailed analysis of the existence and persistence of rewarded risk factors in fixed income markets.

Other definitions of factors are practically relevant. In particular, measuring exposures to risk factors is useful to explain the commonalities, or the absence thereof, between various securities or asset classes. Risk factors are naturally involved in the construction of liability-hedging portfolios by techniques such as duration or duration-convexity matching, but they also appear whenever one wants to decompose the risk of an equity, bond or multi-class portfolio. The recognition that seemingly disparate asset classes bear exposures to common risk factors leads to questioning the notion of "well-diversified portfolio", which is traditionally regarded as a synonym for "equally-weighted portfolio". Equal weighting leaves the risk of a portfolio concentrated among the most volatile constituents, and even a risk parity portfolio can still have very unbalanced contributions of uncorrelated risk factors, even though it seeks to equalise risk contributions across correlated assets. This paper provides a detailed discussion of how to measure diversification in terms of underlying risk factors with two competing methods for extracting factors from the data, namely principal component analysis and minimum linear torsion analysis. We recommend the use of the second method, because it finds the uncorrelated factors that are the closest to the original assets, thereby enhancing interpretability and robustness.

A third definition of factors is as *state variables* that explain time variation in risk premia, volatilities and correlations of various assets. It turns out that this notion is mostly used in an asset allocation context, and this paper provides a simple illustration with the average return and the risk of equities, sovereign bonds, credits, commodities and real estate in

## 5. Conclusions and Perspectives

---

various regimes of economic growth and inflation. State variables like the dividend yield or term-structure-related variables are also commonly used to predict equity and bond returns.

The three notions of factors described in this paper need not be mutually exclusive and can co-exist within a comprehensive framework for factor allocation. The last section of this paper presents various allocation methods to six explicit equity factors, some of which control the concentration of the portfolio across risk factors. In particular, portfolios with a control of the relative ENUB, an indicator that measures how well the tracking error is split across risk factors, are shown to have lower tracking error and maximum relative maximum drawdown than portfolios that focus on the decomposition of absolute risk (volatility). Another example of a situation in which different notions of factors are involved is the prediction of the returns to factor strategies with suitably chosen state variables. While research has mostly focused on finding predictors of the equity market or the bond market as a whole, having predictors for various factors within equity and bond markets would be extremely useful for the purpose of constructing performance-seeking portfolios that react to market conditions without resorting to subjective views on future factor returns. This provides another important avenue for future research.

## 5. Conclusions and Perspectives

---

# Technical Appendix



# Technical Appendix

## A Factor Premia and Stochastic Discount Factor

In asset pricing theory, factor models postulate a linear relationship between the stochastic discount factor (SDF)  $m$  and a set of factors. In mathematical notation, we have

$$m = a + \mathbf{b}'\mathbf{F}, \quad (\text{A.1})$$

where  $a$  is a scalar constant,  $\mathbf{b}$  is a constant vector and  $\mathbf{F}$  is the  $K \times 1$  vector of factor values. Cochrane (2005) shows (his Theorem 6.3) that the SDF has the representation (A.1) if, and only if, the expected excess return of any asset can be decomposed as

$$\mu_i = \sum_{k=1}^K \beta_{ki} \Lambda_k.$$

This equation can be rewritten in vector form, by introducing the vector  $\boldsymbol{\mu}$  of the expected excess returns on the  $N$  assets, the  $K \times N$  matrix of factor exposures  $\boldsymbol{\beta}$  and the  $K \times 1$  vector of factor premia:

$$\boldsymbol{\mu} = \boldsymbol{\beta}'\boldsymbol{\Lambda}.$$

Let  $\mathbf{R}$  be the  $N \times 1$  vector of excess returns on assets. By definition of the SDF, the expected excess returns satisfy

$$\boldsymbol{\mu} = -\frac{1}{\mathbb{E}[m]} \mathbb{Cov}[\mathbf{R}, m],$$

hence

$$\mu_i = -\frac{1}{\mathbb{E}[m]} \mathbf{C}'\mathbf{b}.$$

Now, let  $\boldsymbol{\beta}$  be the  $K \times N$  matrix of the asset betas with respect to factors and  $\mathbf{C}$  be the  $K \times N$  matrix of covariances between the factors and the assets:

$$\boldsymbol{\beta} = \boldsymbol{\Sigma}_F^{-1} \mathbf{C}.$$

We have

$$\begin{aligned} \mu_i &= -\frac{1}{\mathbb{E}[m]} \boldsymbol{\beta}' \boldsymbol{\Sigma}_F \mathbf{b} \\ &= -\frac{1}{\mathbb{E}[m]} \boldsymbol{\beta}' \mathbb{Cov}[m, \mathbf{F}]. \end{aligned}$$

Hence, the vector of factor premia is

$$\boldsymbol{\Lambda} = -\frac{1}{\mathbb{E}[m]} \mathbb{Cov}[m, \mathbf{F}].$$

In a consumption-based model,  $m$  equals the marginal utility of future consumption,  $u'(c)$ , times a positive scaling constant (Cochrane (2005), Chap. 1.2). Hence, the factor premia can also be written as

$$\boldsymbol{\Lambda} = -\frac{1}{\mathbb{E}[u'(c)]} \mathbb{Cov}[u'(c), \mathbf{F}].$$

## B Linear Transformation of Pricing Factors

Consider a set of  $K$  random variables  $\mathbf{F}$  (not necessarily pricing factors), and a linear transformation of them, written as

$$\tilde{\mathbf{F}} = \mathbf{A}'\mathbf{F}.$$

$\mathbf{A}$  is a non-singular matrix, so it is possible to go back and forth between the original variables and the transformed variables. If  $m$  is a stochastic discount factor, it is clear that  $m$  is a linear combination of the original variables of the form

$$m = a + \mathbf{b}'\tilde{\mathbf{F}}$$

if, and only if, it is also a combination of the transformed variables:

$$m = a + [\mathbf{A}^{-1}\mathbf{b}]' \tilde{\mathbf{F}}.$$

In other words, the  $\tilde{\mathbf{F}}$  are pricing factors if, and only if, the  $\mathbf{F}$  are pricing factors. In particular, given a set  $\mathbf{F}$  of  $K$  pricing factors, one can construct infinitely many other sets of pricing factors by applying invertible linear transformations to them.

Now, consider the decomposition of asset excess returns on the factors:

$$R_i = c_i + \boldsymbol{\beta}_i' \mathbf{F} + \varepsilon_i.$$

The residual term  $\varepsilon_i$  is the idiosyncratic return, and its standard deviation is

## Technical Appendix

the idiosyncratic risk of the asset. The regression equation can equivalently be written as

$$R_i = c_i + [\mathbf{A}^{-1}\boldsymbol{\beta}_i]' \tilde{\mathbf{F}} + \varepsilon_i.$$

The residual  $\varepsilon_i$  is by construction orthogonal to the factors  $\mathbf{F}$  (in the sense that the expectation of the vector  $\varepsilon_i \mathbf{F}$  is the null vector), hence it is orthogonal to  $\tilde{\mathbf{F}}$  too. This shows that  $\varepsilon_i$  is also the residual in the regression of  $R_i$  on  $\tilde{\mathbf{F}}$ , and that idiosyncratic is invariant under a linear transformation of the factors.

**Proposition B** *Let  $\mathbf{F}$  be a set of  $K$  pricing factors and  $\tilde{\mathbf{F}}$  be a set of  $K$  factors obtained by applying a linear invertible transformation to the original  $K$  factors. Then,  $\tilde{\mathbf{F}}$  is another set of  $K$  pricing factors, and the idiosyncratic returns of assets with respect to either set of factors are identical.*

### C Extracting Factors by Minimum Linear Torsion

Factor values are given by  $\mathbf{F} = \mathbf{A}'\mathbf{R}$ , so their covariance matrix is  $\mathbf{A}'\boldsymbol{\Sigma}\mathbf{A}$ , and the squared tracking error of  $F_i$  with respect to  $R_i$  is

$$\mathbb{V}[F_i - R_i] = [\mathbf{A}_{:,i} - \mathbf{e}_i]' \boldsymbol{\Sigma} [\mathbf{A}_{:,i} - \mathbf{e}_i],$$

where  $\mathbf{A}_{:,i}$  is the  $i^{\text{th}}$  column of  $\mathbf{A}$  and  $\mathbf{e}_i$  is the vector filled with zeros, except for the  $i^{\text{th}}$  element, equal to 1. Hence, the sum of squared tracking errors of factors with respect to assets is

$$\sum_{i=1}^N \mathbb{V}[F_i - R_i] = \text{tr}[(\mathbf{A} - \mathbf{I})' \boldsymbol{\Sigma} (\mathbf{A} - \mathbf{I})], \quad (\text{C.1})$$

where  $\mathbf{I}$  is the identity matrix of size  $N$ , and  $\text{tr}$  denotes the trace operator.

Factors must be orthogonal and must have the same variances as the original

assets, so we have

$$\mathbf{A}'\boldsymbol{\Sigma}\mathbf{A} = \mathbf{V}^2,$$

where  $\mathbf{V}$  is the diagonal matrix of standard deviations. By expanding the right-hand side of (C.1), we get

$$\begin{aligned} & \text{tr}[(\mathbf{A} - \mathbf{I})' \boldsymbol{\Sigma} (\mathbf{A} - \mathbf{I})] \\ &= \text{tr}[\mathbf{A}'\boldsymbol{\Sigma}\mathbf{A}] - 2\text{tr}[\boldsymbol{\Sigma}\mathbf{A}] + \text{tr}[\boldsymbol{\Sigma}], \end{aligned}$$

so the minimum linear torsion problem is equivalent to

$$\max_{\mathbf{A}} \text{tr}[\boldsymbol{\Sigma}\mathbf{A}] \quad \text{subject to } \mathbf{A}'\boldsymbol{\Sigma}\mathbf{A} = \mathbf{V}^2. \quad (\text{C.2})$$

Consider the spectral decomposition of  $\boldsymbol{\Sigma}$ :

$$\boldsymbol{\Sigma} = \mathbf{P}\mathbf{L}^2\mathbf{P}',$$

where  $\mathbf{P}$  is orthogonal and  $\mathbf{L}$  is the diagonal matrix whose diagonal coefficients are the square roots of the eigenvalues of  $\boldsymbol{\Sigma}$ . Let  $\mathbf{B} = \mathbf{L}\mathbf{P}'\mathbf{A}\mathbf{V}^{-1}$ , so problem (C.2) is equivalent to

$$\max_{\mathbf{B}} \text{tr}[\mathbf{P}\mathbf{L}\mathbf{B}\mathbf{V}] \quad \text{subject to } \mathbf{B}'\mathbf{B} = \mathbf{I}. \quad (\text{C.3})$$

The objective function to be maximised can be rewritten as  $\text{tr}[\mathbf{B}\mathbf{V}\mathbf{P}\mathbf{L}]$ . Consider the singular value decomposition of  $\mathbf{V}\mathbf{P}\mathbf{L}$ :

$$\mathbf{V}\mathbf{P}\mathbf{L} = \mathbf{U}\mathbf{S}\mathbf{W}',$$

where  $\mathbf{U}$  and  $\mathbf{W}$  are two orthogonal matrices and  $\mathbf{S}$  is the diagonal matrix of singular values, all of which are positive since the matrices  $\mathbf{V}$ ,  $\mathbf{P}$  and  $\mathbf{L}$  are non-singular. Then, let  $\mathbf{Z} = \mathbf{W}'\mathbf{B}\mathbf{U}$ . Problem (C.3) is equivalent to

$$\max_{\mathbf{Z}} \text{tr}[\mathbf{Z}\mathbf{S}] \quad \text{subject to } \mathbf{Z}'\mathbf{Z} = \mathbf{I}. \quad (\text{C.4})$$

Consider a matrix  $\mathbf{Z}$  such that  $\mathbf{Z}'\mathbf{Z} = \mathbf{I}$ . The only singular value of  $\mathbf{Z}$  is 1, so we have

$$\mathbf{Z} = \mathbf{X}\mathbf{Y}',$$

where  $\mathbf{X}$  and  $\mathbf{Y}$  are two orthogonal matrices. Let  $\sqrt{\mathbf{S}}$  be the matrix with the squared roots of the diagonal elements



# Technical Appendix

of  $\mathbf{S}$  along the diagonal and zeros off diagonal. We have:

$$tr[\mathbf{ZS}] = tr[\mathbf{Y}'\sqrt{\mathbf{S}}\sqrt{\mathbf{S}}\mathbf{X}],$$

hence, by the matrix version of Cauchy-Schwartz inequality:<sup>20</sup>

$$\begin{aligned} tr[\mathbf{ZS}] &\leq \sqrt{tr[\mathbf{Y}'\sqrt{\mathbf{S}}\sqrt{\mathbf{S}}\mathbf{Y}]} \times \sqrt{tr[\mathbf{X}'\sqrt{\mathbf{S}}\sqrt{\mathbf{S}}\mathbf{X}]} \\ &= tr[\mathbf{S}]. \end{aligned}$$

Thus, the maximum possible value for  $tr[\mathbf{ZS}]$  subject to the constraint  $\mathbf{Z}'\mathbf{Z} = \mathbf{I}$  is  $tr[\mathbf{S}]$ .

If  $\mathbf{Z}$  is a matrix that achieves this maximum, then, we have  $\sqrt{\mathbf{S}}\mathbf{Y} = t\sqrt{\mathbf{S}}\mathbf{X}$  for some  $t$ , so

$$\mathbf{Z} = t\mathbf{X}\mathbf{X}' = t\mathbf{I}.$$

Since  $\mathbf{Z}'\mathbf{Z} = \mathbf{I}$ ,  $t$  must be -1 or +1. To maximise  $tr[\mathbf{ZS}]$ , we must take  $t = +1$ . Hence,  $\mathbf{Z} = \mathbf{I}$  is the unique solution to problem (C.4).

The corresponding matrix  $\mathbf{A}$  is

$$\mathbf{A} = \mathbf{P}\mathbf{L}^{-1}\mathbf{W}\mathbf{U}'\mathbf{V}.$$

By using the equality  $\mathbf{VPL} = \mathbf{USW}'$ , we can rewrite  $\mathbf{A}$  as

$$\mathbf{A} = \mathbf{V}\mathbf{U}\mathbf{S}^{-1}\mathbf{U}'\mathbf{V},$$

which highlights the symmetric nature of  $\mathbf{A}$ .

## D Minimum Variance Portfolios in Fama-MacBeth Procedure

Consider a panel dataset consisting of  $T$  observations for the excess returns to  $N$  securities and for the values of  $K$  factors (not necessarily defined as excess returns). The Fama-MacBeth procedure (Fama and MacBeth 1973) produces estimates of betas by running cross section regressions: let  $\hat{\beta}_i$  be the  $K \times 1$  vector of estimated betas for security  $i$ , and  $\hat{\beta}$  be the  $KN$  matrix of

estimates. Returns at date  $t$  are stacked in the vector  $\mathbf{R}_t$ . In the second step, the cross section of returns is regressed on the exposures at each date, to generate a time series of estimates for the factor premia:

$$R_{it} = d_t + \hat{\beta}_i' \lambda_t + u_{it}.$$

Note that the residuals are not homoscedastic and cross-sectionally uncorrelated, as is often assumed in linear regressions: because the terms  $d_t$  and  $\hat{\beta}_i' \lambda_t$  in the right-hand side are treated as non-random quantities, the covariance matrix of residuals is the covariance matrix of returns,  $\Sigma$ . Let  $\hat{\Sigma}$  be a consistent estimator for  $\Sigma$ .

In the presence of heteroscedasticity (unequal variances) and correlations, the best estimator for the premia at date  $t$  is the Generalised Least Squares estimator, obtained by minimising

$$[\mathbf{R}_t - \hat{\beta}' \lambda_t]' \hat{\Sigma}^{-1} [\mathbf{R}_t - \hat{\beta}' \lambda_t]$$

with respect to  $\lambda_t$ . The GLS estimators for factor premia are

$$\hat{\lambda}_t = [\hat{\beta}' \hat{\Sigma} \hat{\beta}]^{-1} \hat{\beta}' \Sigma^{-1} \mathbf{R}_t.$$

The factor premia estimates are the time series averages of these quantities.

Consider now the problem of minimising the variance of a portfolio subject to the constraint of having a beta of one with respect to factor  $k$  and zero betas with respect to the others. Letting  $\mathbf{e}_k$  be the  $k^{\text{th}}$  column of the  $K \times K$  identity matrix, we write this program as

$$\min_{\mathbf{w}} \mathbf{w}' \hat{\Sigma} \mathbf{w}, \quad \text{subject to } \hat{\beta} \mathbf{w} = \mathbf{e}_k.$$

20 - The Cauchy-Schwartz inequality for matrices states that for any two matrices  $\mathbf{A}$  and  $\mathbf{B}$ , we have

$$|tr[\mathbf{A}'\mathbf{B}]| \leq \sqrt{tr[\mathbf{A}'\mathbf{A}]} \sqrt{tr[\mathbf{B}'\mathbf{B}]},$$

and equality holds if, and only if,  $\mathbf{B}$  is the null matrix, or  $\mathbf{B}$  is non-zero and there exists a  $t$  such that  $\mathbf{A} = t\mathbf{B}$ .

## Technical Appendix

The first-order condition in this program is

$$\hat{\Sigma} \mathbf{w} = \hat{\beta}' \mathbf{v},$$

where  $\mathbf{v}$  is the vector of Lagrange multipliers. Multiplying both sides of this equation by  $\hat{\Sigma}^{-1}$  and using the constraint, we obtain

$$\mathbf{w} = \hat{\Sigma}^{-1} \hat{\beta}' [\hat{\beta} \hat{\Sigma}^{-1} \hat{\beta}']^{-1} \mathbf{e}_k.$$

The return to this portfolio between dates  $t-1$  and  $t$  is

$$\begin{aligned} R_{p,k,t} &= \mathbf{w}' \mathbf{R}_t \\ &= \mathbf{e}_k' [\hat{\beta} \hat{\Sigma}^{-1} \hat{\beta}']^{-1} \hat{\beta} \hat{\Sigma}^{-1} \mathbf{R}_t \\ &= \mathbf{e}_k' \hat{\lambda}_t. \end{aligned}$$

This is the estimate for the  $k^{\text{th}}$  factor premium at date  $t$ .

### E Factor Risk Parity Portfolios

In this section, we give analytical expressions for factor risk parity (FRP) portfolios and we count them, as is done in DMM13.

Factor values are  $\mathbf{F} = \mathbf{A}' \mathbf{R}_t$ , and the portfolio return is the weighted sum of constituents' returns:

$$R_p = \mathbf{w}' \mathbf{R}_t.$$

It can be rewritten as

$$R_p = \beta_p' \mathbf{F},$$

where  $\beta_p$  is the vector of factor exposures, equal to  $\mathbf{A}^{-1} \mathbf{w}$ . Hence, the portfolio variance is

$$\sigma_p^2 = \sum_{k=1}^K \beta_{p,k}^2 \sigma_{F,k}^2,$$

and the contribution of factor  $k$  is

$$c_{F,k} = \frac{\beta_{p,k}^2 \sigma_{F,k}^2}{\sigma_p^2}.$$

The contributions sum up to the portfolio volatility.

A FRP portfolio is defined by the condition that all contributions to volatility are equal:

$$c_{F,1}(\mathbf{w}) = \dots = c_{F,K}(\mathbf{w}).$$

This condition is equivalent to having the following equality, for all  $k$ :

$$\beta_{p,k} = \frac{\sigma_p}{\sqrt{K}} \times \frac{\varepsilon_k}{\sigma_{F,k}}$$

where  $\varepsilon_k$  is a number equal to  $-1$  or  $+1$ .  $\varepsilon_k$  can be negative because the factor contribution can be negative. In vector form, we have

$$\beta_p = \frac{\sigma_p}{\sqrt{K}} [\boldsymbol{\varepsilon} \odot \boldsymbol{\sigma}_F^{-1}],$$

where  $\boldsymbol{\sigma}_F^{-1}$  is a shortcut notation for the vector of the reciprocals of factor volatilities and  $\odot$  denotes element-by-element product between two vectors.

Asset weights are recovered from factor exposures as

$$\mathbf{w} = \mathbf{A} \beta_p.$$

The unknown portfolio volatility can be found by using the condition that asset weights should sum up to one. The composition of FRP portfolios is given by

$$\mathbf{w} = \frac{1}{\mathbf{1}' [\mathbf{A} [\boldsymbol{\varepsilon} \odot \boldsymbol{\sigma}_F^{-1}]]} \mathbf{A} [\boldsymbol{\varepsilon} \odot \boldsymbol{\sigma}_F^{-1}].$$

There are  $2^N$  possible choices for the vector  $\boldsymbol{\varepsilon}$  (2 choices for each of the  $N$  entries), but two vectors that are opposite from each other ( $\boldsymbol{\varepsilon}$  and  $-\boldsymbol{\varepsilon}$ ) lead to the same vector of asset weights. Hence, the number of possible FRP portfolios is  $2^N/2 = 2^{N-1}$ .

# Technical Appendix

## F Optimal Grouping of Securities

Notations are defined as follows:

- $\mathbf{W}$  is the  $N \times K$  matrix that gives the weights of the benchmarks in the risky assets;
- $\Sigma$  and  $\mu$  are the  $N \times N$  covariance matrix of the risky assets and the  $N \times 1$  vector of their expected excess returns;
- $\Sigma_b$  and  $\mu_b$  are the  $K \times K$  covariance matrix of the benchmarks and the  $K \times 1$  vector of their expected excess returns;
- $\mathbf{C}$  is the  $K \times N$  matrix of covariances between the benchmarks and the risky assets, equal to  $\mathbf{W}'\Sigma$ .

We have the following relations between the expected returns and the covariance matrix of the benchmarks and those of the assets:

$$\begin{aligned}\mu_b &= \mathbf{W}'\mu, \\ \Sigma_b &= \mathbf{W}'\Sigma\mathbf{W}.\end{aligned}$$

Let  $\beta$  be the  $K \times N$  matrix of asset betas with respect to the benchmarks, obtained by running linear regressions of asset excess returns on benchmark excess returns. It is given by

$$\beta = \Sigma_b^{-1}\mathbf{C} = \Sigma_b^{-1}\mathbf{W}'\Sigma.$$

Consider now the output of the second step, which is a portfolio of benchmarks with weights stacked in the  $K \times 1$  vector  $\mathbf{x}_b$ . Its Sharpe ratio is

$$\lambda = \frac{\mathbf{x}_b'\mu_b}{\sqrt{\mathbf{x}_b'\Sigma_b\mathbf{x}_b}} = \frac{[\mathbf{W}\mathbf{x}_b]'\mu}{\sqrt{[\mathbf{W}\mathbf{x}_b]'\Sigma[\mathbf{W}\mathbf{x}_b]}}.$$

This is the Sharpe ratio of a portfolio of assets with the weights  $\mathbf{W}\mathbf{x}_b$ . It is equal to the maximum Sharpe ratio that can be achieved with the  $N$  assets if, and only if, the vector  $\mathbf{W}\mathbf{x}_b$  is proportional to the vector  $\Sigma^{-1}\mu$ . There must exist a positive constant  $v$  such that

$$\mathbf{W}\mathbf{x}_b = v\Sigma^{-1}\mu.$$

This condition implies that

$$\mu = \frac{1}{v}\Sigma\mathbf{W}\mathbf{x}_b = \frac{1}{v}\beta'\Sigma_b\mathbf{x}_b.$$

We also have

$$\begin{aligned}\mu_b &= \mathbf{W}'\tilde{\mu} = \frac{1}{v}\mathbf{W}'\beta'\Sigma_b\mathbf{x}_b \\ &= \frac{1}{v}\mathbf{W}'\Sigma\mathbf{W}\Sigma_b^{-1}\Sigma_b\mathbf{x}_b = \frac{1}{v}\Sigma_b\mathbf{x}_b.\end{aligned}$$

Hence:

$$\mu = \beta'\mu_b. \quad (\text{F.1})$$

This means that if the two-step portfolio is efficient, then the output of the first step must be a group of asset pricing factors.

Moreover, the equality  $\mathbf{W}\mathbf{x}_b = v\Sigma^{-1}\mu$  implies that:

$$\mathbf{W}'\Sigma\mathbf{W}\mathbf{x}_b = v\mathbf{W}'\mu = v\mu_b,$$

hence:

$$\mathbf{x}_b = v\Sigma_b^{-1}\mu_b. \quad (\text{F.2})$$

Hence, the output of the second step must be a combination of the MSR portfolio of benchmarks and cash (possibly leveraged).

Conversely, assume that the  $K$  benchmarks are pricing factors and that the portfolio constructed in the second step has weights proportional to those of the MSR portfolio: this means that Equations (F.1) and (F.2) are satisfied for some positive constant  $v$ . By (F.2), we have

$$\Sigma_b^{-1}\mu_b = \frac{1}{v}\mathbf{x}_b,$$

hence, by substituting in (F.1):

$$\mu = \frac{1}{v}\Sigma\mathbf{W}\mathbf{x}_b,$$

so

$$\mathbf{W}\mathbf{x}_b = v\Sigma^{-1}\mu.$$

Hence, the portfolio of benchmarks achieves the maximum Sharpe ratio of a portfolio of risky assets.

## Technical Appendix

### G Efficiency of Pricing Portfolio

Consider the case where the excess return of one portfolio prices all assets. The expected excess returns of risky assets are thus proportional to their betas with respect to the pricing portfolio:

$$\boldsymbol{\mu} = \mu_b \boldsymbol{\beta}',$$

and their covariance matrix can be decomposed as

$$\boldsymbol{\Sigma} = \sigma_b^2 \boldsymbol{\beta}' \boldsymbol{\beta} + \boldsymbol{\Sigma}_\varepsilon.$$

Let  $\mathbf{w}$  be a portfolio fully invested in risky assets, and assume that its expected excess return is positive. Then, the expected return of this portfolio is

$$\mu_p = \mathbf{w}' \boldsymbol{\mu} = \mu_b \boldsymbol{\beta}' \mathbf{w},$$

and the scalar  $\boldsymbol{\beta}' \mathbf{w}$  is positive. Hence, the Sharpe ratio of this portfolio can be written as

$$\begin{aligned} \lambda &= \frac{\mathbf{w}' \boldsymbol{\mu}}{\sqrt{\mathbf{w}' \boldsymbol{\Sigma} \mathbf{w}}} \\ &= \frac{\mu_b \times [\boldsymbol{\beta}' \mathbf{w}]}{\sqrt{\sigma_b^2 [\boldsymbol{\beta}' \mathbf{w}]^2 + \mathbf{w}' \boldsymbol{\Sigma}_\varepsilon \mathbf{w}}} \\ &= \frac{\mu_b}{\sqrt{\sigma_b^2 + \mathbf{v}' \boldsymbol{\Sigma}_\varepsilon \mathbf{v}}}, \end{aligned}$$

where  $\mathbf{v} = \mathbf{w} / \boldsymbol{\beta}' \mathbf{w}$ . In any case, the Sharpe ratio is less than or equal to  $\mu_b / \sigma_b$ , the Sharpe ratio of the benchmark. This shows that the benchmark itself achieves the maximum Sharpe ratio.

Are there other portfolios that do as well as the benchmark?  $\lambda$  equals  $\mu_b / \sigma_b$  if, and only if  $\boldsymbol{\Sigma}_\varepsilon \mathbf{v}$  is zero, that is if  $\mathbf{w}$  is such that

$$\boldsymbol{\Sigma}_\varepsilon \mathbf{w} = \mathbf{0}. \quad (\text{G.1})$$

Excess returns of assets can be decomposed as

$$\mathbf{R} = R_b \boldsymbol{\beta}' + \boldsymbol{\varepsilon}, \quad (\text{G.2})$$

where  $R_b$  is the benchmark excess return, equal to  $\mathbf{w}'_b \mathbf{R}$ . Multiply both sides of (G.2) by  $\mathbf{w}'$  on the left, and use (G.1) to obtain

$$\begin{aligned} \mathbf{w}' \mathbf{R} &= R_b \boldsymbol{\beta}' \mathbf{w} \\ &= [\boldsymbol{\beta}' \mathbf{w}] \times [\mathbf{w}'_b \mathbf{R}]. \end{aligned}$$

Hence

$$[\mathbf{w} - [\boldsymbol{\beta}' \mathbf{w}] \mathbf{w}_b]' \mathbf{R} = \mathbf{0}.$$

Because risky assets are not redundant, no linear combination is risk-free unless it has zero weights, so we have

$$\mathbf{w} = [\boldsymbol{\beta}' \mathbf{w}] \mathbf{w}_b.$$

The full investment constraint for the portfolio and the benchmark implies that  $\mathbf{w} = \mathbf{w}_b$ . Hence, the benchmark is the only portfolio fully invested in risky assets that achieves the maximum Sharpe ratio.

# Technical Appendix

---

# References



## References

---

- Acharya, V. V., and L. H. Pedersen. 2005. Asset Pricing with Liquidity Risk. *Journal of Financial Economics* 77(2): 375–410.
- Ahn, S. C., and A. R. Horenstein. 2013. Eigenvalue Ratio Test for the Number of Factors. *Econometrica* 81(3): 1203–1227.
- Amenc, N., G. Coqueret, and L. Martellini. 2015. Active Allocation to Smart Factor Indices. ERI Scientific Beta Publication.
- Amenc, N., R. Deguest, F. Goltz, A. Lodh, and E. Shirbini. 2014a. Risk Allocation, Factor Investing and Smart Beta: Reconciling Innovations in Equity Portfolio Construction. EDHEC-Risk Institute Publication.
- Amenc, N., F. Ducoulombier, M. Esakia, F. Goltz, and S. Sivasubramanian. 2017. Accounting for Cross-Factor Interactions in Multifactor Portfolios without Sacrificing Diversification and Risk Control. *The Journal of Portfolio Management* 43(5): 99–114.
- Amenc, N., and F. Goltz. 2016. Long-Term Rewarded Equity Factors: What Can Investors Learn from Academic Research? *Journal of Index Investing* 7(2): 39–56.
- Amenc, N., F. Goltz, V. Le Sourd, and A. Lodh. 2015. Alternative Equity Beta Investing: A Survey. EDHEC-Risk Institute Publication.
- Amenc, N., F. Goltz, and A. Lodh. 2012. Choose Your Betas: Benchmarking Alternative Equity Index Strategies. *Journal of Portfolio Management* 39(1): 88–111.
- —. 2016. Smart Beta Is Not Monkey Business. *The Journal of Index Investing* 6(4): 12–29.
- Amenc, N., F. Goltz, A. Lodh, and L. Martellini. 2014b. Towards Smart Equity Factor Indices: Harvesting Risk Premia Without Taking Unrewarded Risks. *The Journal of Portfolio Management* 40(4): 106–122.
- Ang, A. 2014. Asset Management: A Systematic Approach to Factor Investing. Oxford University Press.
- Ang, A., and G. Bekaert. 2002. International Asset Allocation with Regime Shifts. *Review of Financial Studies* 15(4): 1137–1187.
- Ang, A., M. Brière, and O. Signori. 2012. Inflation and Individual Equities. *Financial Analysts Journal* 68(4): 36–55.
- Ang, A., and J. Chen. 2007. CAPM Over the Long Run: 1926–2001. *Journal of Empirical Finance* 14(1): 1–40.
- Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2006. The Cross-Section of Volatility and Expected Returns. *Journal of Finance* 61(1): 259–299.
- —. 2009. High Idiosyncratic Volatility and Low Returns: International and Further US Evidence. *Journal of Financial Economics* 91(1): 1–23.
- Ang, A., and M. Piazzesi. 2003. A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables. *Journal of Monetary Economics* 50(4): 745–787.



## References

---

- Arnott, R. D., J. Hsu, and P. Moore. 2005. Fundamental Indexation. *Financial Analysts Journal* 61(2).
- Asness, C., and A. Frazzini. 2013. The Devil in HML's Details. *Journal of Portfolio Management* 39(4): 49–68.
- Asness, C., A. Iltanen, R. Israel, and T. Moskowitz. 2015. Investing with Style. *Journal of Alternative Investments* 13(1): 27–63.
- Asness, C. S., A. Frazzini, R. Israel, and T. J. Moskowitz. 2014. Fact, Fiction and Momentum Investing. *Journal of Portfolio Management*, 40(5): 75–92.
- Asness, C. S., A. Frazzini, and L. H. Pedersen. 2012. Leverage Aversion and Risk Parity. *Financial Analysts Journal* 68(1): 47–59.
- Asness, C. S., T. J. Moskowitz, and L. H. Pedersen. 2013. Value and Momentum Everywhere. *Journal of Finance* 68(3): 929–985.
- Asvanunt, A., and S. Richardson. 2016. The Credit Risk Premium. *The Journal of Fixed Income* 26(3): 6–24.
- Baele, L., G. Bekaert, and K. Inghelbrecht. 2010. The Determinants of Stock and Bond Return Comovements. *Review of Financial Studies* 23(6): 2374–2428.
- Bai, J., and S. Ng. 2002. Determining the Number of Factors in Approximate Factor Models. *Econometrica* 70(1): 191–221.
- Baker, M. P., B. Bradley, and J. Wurgler. 2011. Benchmarks as Limits to Arbitrage: Understanding the Low-Volatility Anomaly. *Financial Analysts Journal* 67(1): 40–54.
- Baker, N. L., and R. A. Haugen. 2012. Low Risk Stocks Outperform Within All Observable Markets of the World. Working Paper.
- Bali, T., and N. Cakici. 2008. Idiosyncratic Volatility and the Cross-Section of Expected Returns. *Journal of Financial and Quantitative Analysis* 43(1): 29–58.
- Banz, R. W. 1981. The Relationship Between Return and Market Value of Common Stocks. *Journal of Financial Economics* 9(1): 3–18.
- Barber, B., and T. Odean. 2013. The Behavior of Individual Investors, in *Handbook of the Economics of Finance*, chapter 22, 1533–1570. Elsevier.
- Barberis, N., and M. Huang. 2008. Stocks as Lotteries: The Implications of Probability Weighting for Security Prices. *The American Economic Review* 98(5): 2066–2100.
- Basu, S. 1977. Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis. *Journal of Finance* 32(3): 663–682.
- Black, F. 1972. Capital Market Equilibrium with Restricted Borrowing. *Journal of Business* 45(3): 444–455.
- Black, F., M. Jensen, and M. Scholes. 1972. The Capital Asset Pricing Model: Some Empirical Tests, in *Studies in the Theory of Capital Markets*. New York: Praeger.

## References

---

- Blitz, D. 2016. The Value of Low Volatility. *The Journal of Portfolio Management* 42(3): 94–100.
- Blitz, D., E. Falkenstein, and P. v. Vliet. 2014. Explanations for the Volatility Effect: An Overview Based on the CAPM Assumptions. *Journal of Portfolio Management* 40(3): 61–76.
- Blitz, D., and P. Van Vliet. 2007. The Volatility Effect: Lower Risk without Lower Return. *Journal of Portfolio Management* 34(1): 102–113.
- Bollerslev, T. 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* 31(3): 307–327.
- Breeden, D. 1979. An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. *Journal of Financial Economics* 7(3): 265–296.
- Brennan, M., and Y. Xia. 2002. Dynamic Asset Allocation under Inflation. *Journal of Finance* 57(3): 1201–1238.
- Brown, S. J. 1989. The Number of Factors in Security Returns. *Journal of Finance* 44(5): 1247–1262.
- Campbell, J., Y. Chan, and L. Viceira. 2003. A Multivariate Model of Strategic Asset Allocation. *Journal of Financial Economics* 67(1): 41–80.
- Campbell, J. Y., J. Hilscher, and J. Szilagyi. 2008. In Search of Distress Risk. *The Journal of Finance* 63(6): 2899–2939.
- Campbell, J. Y., and T. Vuolteenaho. 2004. Bad Beta, Good Beta. *American Economic Review* 94(5): 1249–1275.
- Carhart, M. M. 1997. On Persistence in Mutual Fund Performance. *Journal of Finance* 52(1): 57–82.
- Carli, T., R. Deguest, and L. Martellini. 2014. Improved Risk Reporting with Factor-Based Diversification Measures. EDHEC-Risk Institute Publication.
- Chamberlain, G., and M. Rothschild. 1983. Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets. *Econometrica* 51(5): 1281–1304.
- Chan, K., and N.-F. Chen. 1991. Structural and Return Characteristics of Small and Large Firms. *The Journal of Finance* 46(4): 1467–1484.
- Chen, N.-F., R. Roll, and S. A. Ross. 1986. Economic Forces and the Stock Market. *Journal of Business* 59(3): 383–403.
- Cieslak, A., and P. Povala. 2015. Expected Returns in Treasury Bonds. *The Review of Financial Studies* 28(10): 2859–2901.
- Cochrane, J. 2005. Asset Pricing Theory. Princeton University Press. Revised Edition.
- Cochrane, J. H., and M. Piazzesi. 2005. Bond Risk Premia. *The American Economic Review* 95(1): 138–160.
- Connor, G., and R. A. Korajczyk. 1993. A Test for the Number of Factors in an Approximate Factor Model. *Journal of Finance* 48(4): 1263–1291.

## References

---

- Dai, Q., K. Singleton, and W. Yang. 2004. Predictability of Bond Risk Premia and Affine Term Structure Models. Stanford University.
- Daniel, K., and S. Titman. 1997. Evidence on the Characteristics of Cross Sectional Variation in Stock Returns. *Journal of Finance* 52(1): 1–33.
- de Carvalho, R. L., P. Dugnolle, X. Lu, and P. Moulin. 2014. Low-Risk Anomalies in Global Fixed Income: Evidence from Major Broad Markets. *Journal of Fixed Income* 23(4): 51–70.
- DeBondt, W. F., and R. Thaler. 1985. Does the Stock Market Overreact? *Journal of Finance* 40(3): 793–805.
- Deguest, R., F. Fabozzi, L. Martellini, and V. Milhau. 2013. Bond Portfolio Optimization in the Presence of Duration Constraints. EDHEC-Risk Institute Publication.
- Deguest, R., L. Martellini, and A. Meucci. 2013. Risk Parity and Beyond – From Asset Allocation to Risk Allocation Decisions. Working Paper, Edhec-Risk Institute.
- DeMiguel, V., L. Garlappi, F. Nogales, and R. Uppal. 2009. A Generalized Approach to Portfolio Optimization: Improving Performance By Constraining Portfolio Norms. *Management Science* 55(5): 798–812.
- DeMiguel, V., L. Garlappi, and R. Uppal. 2009. Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? *Review of Financial Studies* 22(5): 1915–1953.
- Diebold, F. X., G. D. Rudebusch, and S. B. Aruoba. 2006. The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach. *Journal of Econometrics* 131(1): 309–338.
- Dimson, E., P. Marsh, and M. Staunton. 2008. Triumph of the Optimists: 101 Years of Global Investment Returns. Princeton University Press.
- Driessen, J. 2005. Is Default Event Risk Priced in Corporate Bonds? *Review of Financial Studies* 18(1): 165–195.
- Duffie, D., and K. J. Singleton. 1999. Modeling Term Structures of Defaultable Bonds. *Review of Financial Studies* 12(4): 87–720.
- Engle, R. 1982. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica* 55(2): 251–276.
- Fama, E., and K. French. 1989. Business Conditions and Expected Returns on Stocks and Bonds. *Journal of Financial Economics* 25(1): 23–49.
- —. 2014. A Five-Factor Asset Pricing Model. *Journal of Financial Economics* 116(1): 1–22.
- Fama, E. F., and R. R. Bliss. 1987. The Information in Long-Maturity Forward Rates. *The American Economic Review* 77(4): 680–692.
- Fama, E. F., and K. R. French. 1992. The Cross-Section of Expected Stock Returns. *Journal of Finance* 47(2): 427–465.
- —. 1993. Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics* 33(1): 3–56.

## References

---

- —. 1995. Size and Book-to-Market Factors in Earnings and Returns. *The Journal of Finance* 50(1): 131–155.
- —. 1996. Multifactor Explanations of Asset Pricing Anomalies. *Journal of Finance* 51(1): 55–84.
- —. 2006. Profitability, Investment and Average Returns. *Journal of Financial Economics* 82(3): 491–518.
- —. 2012. Size, Value, and Momentum in International Stock Returns. *Journal of Financial Economics* 105(3): 457–472.
- Fama, E. F., and J. D. MacBeth. 1973. Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 81(3): 607–636.
- Fink, J. D., K. E. Fink, and H. He. 2012. Expected Idiosyncratic Volatility Measures and Expected Returns. *Financial Management* 41(3): 519–553.
- Frazzini, A., R. Israel, and T. J. Moskowitz. 2012. Trading Costs of Asset Pricing Anomalies. Working Paper, AQR Capital Management and New York University.
- Frazzini, A., and L. H. Pedersen. 2014. Betting Against Beta. *Journal of Financial Economics* 111(1): 1–25.
- Frost, P. A., and J. Savarino. 1988. For Better Performance: Constrain Portfolio Weights. *Journal of Portfolio Management* 15(1): 29–34.
- Fu, F. 2009. Idiosyncratic Risk and the Cross-Section of Expected Stock Returns. *Journal of Financial Economics* 91(1): 24–37.
- Fuertes, A.-M., J. Miffre, and A. Fernandez-Perez. 2015. Commodity Strategies Based on Momentum, Term Structure, and Idiosyncratic Volatility. *Journal of Futures Markets* 35(3): 274–297.
- Gebhardt, W. R., S. Hvidkjaer, and B. Swaminathan. 2005. Stock and Bond Market Interaction: Does Momentum Spill Over? *Journal of Financial Economics* 75(3): 651–690.
- Gompers, P. A., and A. Metrick. 2001. Institutional Investors and Equity Prices. *Quarterly Journal of Economics* 116(1): 229–259.
- Graham, B. 1959. *The Intelligent Investor, A Book of Practical Counsel*. 3rd ed. New York: Harper & Brothers Publishers.
- Graham, B., and D. L. Dodd. 1934. *Security Analysis: Principles and Technique*. McGraw-Hill.
- Grinblatt, M., S. Titman, and R. Wermers. 1995. Momentum Investment Strategies, Portfolio Performance, and Herding: A Study of Mutual Fund Behavior. *The American Economic Review* 85(5): 1088–1105.
- Guo, H., H. Kassa, and M. F. Ferguson. 2014. On the Relation Between EGARCH Idiosyncratic Volatility and Expected Stock Returns. *Journal of Financial and Quantitative Analysis* 49(1): 271–296.
- Hallerbach, W. G., and P. Houweling. 2013. Ibbotson's Default Premium: Risky Data. *The Journal of Investing* 22(2): 95–105.

## References

---

- Harvey, C. R., Y. Liu, and H. Zhu. 2016. ... and the Cross-Section of Expected Returns. *Review of Financial Studies* 29(1): 5–68.
- Haugen, R. A., and N. L. Baker. 1996. Commonality in the Determinants of Expected Stock Returns. *Journal of Financial Economics* 41(3): 401–439.
- Hellerstein, R. 2011. Global Bond Risk Premiums. Working Paper.
- Hong, H., T. Lim, and J. C. Stein. 2000. Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies. *Journal of Finance* 55(1): 265–295.
- Hong, H., and J. C. Stein. 1999. A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets. *Journal of Finance* 54(6): 2143–2184.
- Hou, K., C. Xue, and L. Zhang. 2015. Digesting Anomalies: An Investment Approach. *Review of Financial Studies* 28(3): 650–705.
- Israel, R., and T. J. Moskowitz. 2013. The Role of Shorting, Firm Size, and Time on Market Anomalies. *Journal of Financial Economics* 108(2): 275–301.
- Jagannathan, R., and T. Ma. 2003. Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *Journal of Finance* 58(4): 1651–1683.
- Jagannathan, R., and Z. Wang. 1996. The Conditional CAPM and the Cross-Section of Expected Returns. *Journal of Finance* 51(1): 3–53.
- Jegadeesh, N. 1990. Evidence of Predictable Behavior of Security Returns. *Journal of Finance* 45(3): 881–898.
- Jegadeesh, N., and S. Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48(1): 65–91.
- Johansson, K. K., and T. Pekkala. 2013. Portfolio capacity. Working Paper. Available at [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=2200855](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2200855).
- Johnson, T. C. 2002. Rational Momentum Effects. *The Journal of Finance* 57(2): 585–608.
- Jostova, G., S. Nikolova, A. Philipov, and C. W. Stahel. 2013. Momentum in Corporate Bond Returns. *Review of Financial Studies* 26(7): 1649–1693.
- Kan, R., and G. Zhou. 2007. Optimal Portfolio Choice with Parameter Uncertainty. *Journal of Financial and Quantitative Analysis* 42(3): 621–655.
- King, B. F. 1966. Market and Industry Factors in Stock Price Behavior. *The Journal of Business* 39(1): 139–190.
- Koiien, R. S., T. J. Moskowitz, L. H. Pedersen, and E. B. Vrugt. 2013. Carry. Working Paper, National Bureau of Economic Research.
- Korajczyk, R. A., and R. Sadka. 2004. Are Momentum Profits Robust to Trading Costs? *The Journal of Finance* 59(3): 1039–1082.
- Kumar, A. 2009. Who Gambles in the Stock Market? *The Journal of Finance* 64(4): 1889–1933.
- Lakonishok, J., A. Shleifer, and R. W. Vishny. 1994. Contrarian Investment, Extrapolation, and Risk. *Journal of Finance* 49(5): 1541–1578.

## References

---

- Lesmond, D. A., M. J. Schill, and C. Zhou. 2004. The Illusory Nature of Momentum Profits. *Journal of Financial Economics* 71(2): 349–380.
- Lewellen, J., and S. Nagel. 2006. The Conditional CAPM Does Not Explain Asset-Pricing Anomalies. *Journal of Financial Economics* 82(2): 289–314.
- Liew, J., and M. Vassalou. 2000. Can Book-to-Market, Size and Momentum Be Risk Factors That Predict Economic Growth? *Journal of Financial Economics* 57(2): 221–245.
- Lintner, J. 1965a. Security Prices, Risk, and Maximal Gains from Diversification. *The Journal of Finance* 20(4): 587–615.
- —. 1965b. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics* 47(1): 13–37.
- Liu, L. X., and L. Zhang. 2008. Momentum Profits, Factor Pricing, and Macroeconomic Risk. *Review of Financial Studies* 21(6): 2417–2448.
- Long, J. B. 1974. Stock Prices, Inflation, and the Term Structure of Interest Rates. *Journal of Financial Economics* 1(2): 131–170.
- Longstaff, F. A., S. Mithal, and E. Neis. 2005. Corporate Yield Spreads: Default Risk or Liquidity? New Evidence From the Credit Default Swap Market. *Journal of Finance* 60(5): 2213–2253.
- Maillard, S., T. Roncalli, and J. Teiletche. 2010. The Properties of Equally Weighted Risk Contribution Portfolios. *Journal of Portfolio Management* 36(4): 60–70.
- Martellini, L., and V. Milhau. 2012. Dynamic Allocation Decisions in the Presence of Funding Ratio Constraints. *Journal of Pension Economics and Finance* 11(4): 549–580.
- —. 2015. Factor Investing: A Welfare-Improving New Investment Paradigm or Yet Another Marketing Fad? EDHEC-Risk Institute Publication.
- Mayers, D. 1973. Nonmarketable Assets and the Determination of Capital Asset Prices in the Absence of a Riskless Asset. *The Journal of Business* 46(2): 258–267.
- Merton, R. 1973. An Intertemporal Capital Asset Pricing Model. *Econometrica* 41(5): 867–887.
- —. 1980. On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics* 8(4): 323–361.
- —. 1992. An Intertemporal Capital Asset Pricing Model, in *Continuous-Time Finance*, ed. Merton, R., 475–523. Blackwell Publishers.
- Merton, R. C. 1987. A Simple Model of Capital Market Equilibrium with Incomplete Information. *Journal of Finance* 42(3): 483–510.
- Meucci, A. 2009. Managing Diversification. *Risk* 22(5): 74–79.
- Meucci, A., A. Santangelo, and R. Deguest. 2015. Measuring Portfolio Diversification Based on Optimized Uncorrelated Factors. *Risk* 11(29): 70–75.
- Miffre, J., and G. Rallis. 2007. Momentum Strategies in Commodity Futures Markets. *Journal of Banking & Finance* 31(6): 1863–1886.

## References

---

- Miller, M. H., and M. Scholes. 1972. Rates of Return in Relation to Risk: A Reexamination of Some Recent Findings. *Studies in the Theory of Capital Markets* 23.
- Mitton, T., and K. Vorkink. 2007. Equilibrium Underdiversification and the Preference for Skewness. *Review of Financial Studies* 20(4): 1255–1288.
- Moskowitz, T., Y. Ooi, and L. Pedersen. 2012. Time Series Momentum. *Journal of Financial Economics* 104: 228–250.
- MSCI. 2007. Barra Risk Model Handbook. MSCI.
- Mulvey, J. M., and H. Liu. 2016. Identifying Economic Regimes: Reducing Downside Risks for University Endowments and Foundations. *The Journal of Portfolio Management* 43(1): 100–108.
- Novy-Marx, R. 2013. The Other Side of Value: The Gross Profitability Premium. *Journal of Financial Economics* 108(1): 1–28.
- Novy-Marx, R., and M. Velikov. 2016. A Taxonomy of Anomalies and Their Trading Costs. *Review of Financial Studies* 29(1): 104–147.
- Petkova, R. 2006. Do the Fama-French Factors Proxy for Innovations in Predictive Variables? *Journal of Finance* 61(2): 581–612.
- Petkova, R., and L. Zhang. 2005. Is Value Riskier than Growth? *Journal of Financial Economics* 78(1): 187–202.
- Pilotte, E. A., and F. P. Sterbenz. 2006. Sharpe and Treynor Ratios on Treasury Bonds. *The Journal of Business* 79(1): 149–180.
- Pospisil, L., and J. Zhang. 2010. Momentum and Reversal Effects in Corporate Bond Prices and Credit Cycles. *Journal of Fixed Income* 20(2): 101–115.
- Qian, E. 2005. Risk Parity Portfolios: Efficient Portfolios Through True Diversification. PanAgora.
- Rebonato, R. 2017. Smart Beta Strategies in Fixed Income. Investment and Pensions Europe, EDHEC Research Insights Spring 2017 .
- Roll, R. 1977. A Critique of the Asset Pricing Theory's Tests. Part I: On Past and Potential Testability of the Theory. *Journal of Financial Economics* 4(2): 129–176.
- Roncalli, T., and G. Weisang. 2012. Risk Parity Portfolios with Risk Factors. Working Paper.
- Rosenberg, B. 1974. Extra-Market Components of Covariance in Security Returns. *Journal of Financial and Quantitative Analysis* 9(02): 263–274.
- Rosenberg, B., K. Reid, and R. Lanstein. 1985. Persuasive Evidence of Market Inefficiency. *Journal of Portfolio Management* 11(3): 9–16.
- Ross, S. 1976. The Arbitrage Theory of Capital Asset Pricing. *Journal of Economic Theory* 13(3): 341–360.
- Rouwenhorst, K. G. 1998. International Momentum Strategies. *Journal of Finance* 53(1): 267–284.



## References

---

- Sharpe, W. 1964. Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance* 19(3): 425–442.
- —. 1981. Decentralized Investment Management. *Journal of Finance* 36(2): 217–234.
- Shefrin, H., and M. Statman. 2000. Behavioral Portfolio Theory. *Journal of Financial and Quantitative Analysis* 35(2): 127–151.
- Siegel, L. B. 2003. Benchmarks and Investment Management. The Research Foundation of the Association for Investment Management and Research, Charlottesville, Virginia.
- Sørensen, C. 1999. Dynamic Asset Allocation and Fixed Income Management. *Journal of Financial and Quantitative Analysis* 34(4): 513–531.
- Spinu, F. 2013. An Algorithm for Computing Risk Parity Weights. Working Paper.
- Stattman, D. 1980. Book Values and Stock Returns. *The Chicago MBA: A Journal of Selected Papers* 4(1): 25–45.
- Teplá, L. 2001. Optimal Investment with Minimum Performance Constraints. *Journal of Economic Dynamics and Control* 25(10): 1629–1645.
- Titman, S., K. J. Wei, and F. Xie. 2004. Capital Investments and Stock Returns. *Journal of Financial and Quantitative Analysis* 39(4): 677–700.
- Trzcinka, C. 1986. On the Number of Factors in the Arbitrage Pricing Model. *Journal of Finance* 41(2): 347–368.
- van Binsbergen, J., M. Brandt, and R. Koijen. 2008. Optimal Decentralized Portfolio Management. *Journal of Finance* 63(4): 1849–1895.
- Van Dijk, M. A. 2011. Is Size Dead? A Review of the Size Effect in Equity Returns. *Journal of Banking & Finance* 35(12): 3263–3274.
- Xing, Y. 2008. Interpreting the Value Effect Through the Q-Theory: An Empirical Investigation. *The Review of Financial Studies* 21(4): 1767–1795.
- Yu, F. 2002. Modeling Expected Return on Defaultable Bonds. *Journal of Fixed Income* 12(2): 69–81.
- Zhang, L. 2005. The Value Premium. *Journal of Finance* 60(1): 67–103.

# About Amundi



## About Amundi

---

With more than €1,400 billion worldwide of assets under management<sup>1</sup>, Amundi is one of the world's leading Asset managers<sup>2</sup>. The ETF, Indexing and Smart Beta business line is one of the group's strategic business areas and totalizes €84 billion AuM<sup>1</sup>.

Built on strong commitments on cost-efficiency, innovation and transparency, the Amundi ETF platform is the 5<sup>th</sup> largest ETF provider in Europe<sup>3</sup> with 100 ETFs and >450 listings across Europe.

On Indexing and Smart Beta, innovation and customization are at the core of the client-oriented approach. The objective is to provide investors with robust, flexible and highly cost efficient solutions, leveraging on Amundi pricing power and extensive resources, including first class research capabilities in SRI and Factor investing.

1. Amundi figures as of 30 September 2017.

2. Source IPE "Top 400 asset managers" published in June 2017 and based on AUM as of end December 2016.

3. Source: DB-ETF Research ETF – as at 30 September 2017

# About EDHEC-Risk Institute



# About EDHEC-Risk Institute

---

Founded in 1906, EDHEC is one of the foremost international business schools. Accredited by the three main international academic organisations, EQUIS, AACSB, and Association of MBAs, EDHEC has for a number of years been pursuing a strategy of international excellence that led it to set up EDHEC-Risk Institute in 2001.

This institute now boasts a team of close to 50 permanent professors, engineers and support staff, as well as 39 research associates from the financial industry and affiliate professors.

## The Choice of Asset Allocation and Risk Management and the Need for Investment Solutions

EDHEC-Risk has structured all of its research work around asset allocation and risk management. This strategic choice is applied to all of the Institute's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing asset-liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds. EDHEC-Risk Institute has also developed an ambitious portfolio of research and educational initiatives in the domain of investment solutions for institutional and individual investors.

## Academic Excellence and Industry Relevance

In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by the Institute's advisory board. This board is made up of internationally recognised researchers, the Institute's business partners, and representatives of major international institutional investors. Management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Seven research programmes have been conducted by the centre to date:

- Investment Solutions in Institutional and Individual Money Management
- Equity Risk Premia in Investment Solutions
- Fixed-Income Risk Premia in Investment Solutions
- Alternative Risk Premia in Investment Solutions
- Multi-Asset Multi-Factor Investment Solutions
- Reporting and Regulation for Investment Solutions
- Technology, Big Data and Artificial Intelligence for Investment Solutions

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the EDHEC-Risk locations in the City of London in the United Kingdom; Nice and Paris in France.

EDHEC-Risk has developed a close partnership with a small number of sponsors within the framework of research chairs or major research projects:

- ETF, Indexing and Smart Beta Investment Strategies, in partnership with *Amundi*
- Regulation and Institutional Investment, in partnership with *AXA Investment Managers*
- Asset-Liability Management and Institutional Investment Management, in partnership with *BNP Paribas Investment Partners*
- New Frontiers in Risk Assessment and Performance Reporting, in partnership with *CACEIS*
- Exploring the Commodity Futures

## About EDHEC-Risk Institute

Risk Premium: Implications for Asset Allocation and Regulation, in partnership with *CME Group*

- Asset-Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with *Deutsche Bank*
- The Benefits of Volatility Derivatives in Equity Portfolio Management, in partnership with *Eurex*
- Innovations and Regulations in Investment Banking,

sponsored by *the French Banking Federation (FBF)*

- Maximizing and Harvesting the Rebalancing Premium in Equity Markets, in partnership with *the French Central Bank (BDF Gestion)*

- Risk Allocation Solutions, in partnership

with *Lyxor Asset Management*

- Infrastructure Equity Investment Management and Benchmarking, in partnership with *Meridiam and Campbell Lutyens*

- Risk Allocation Framework for Goal-Driven Investing Strategies, in partnership with *Merrill Lynch Wealth Management*

- Investment and Governance Characteristics of Infrastructure Debt Investments, in partnership with *Natixis*

- Advanced Modelling for Alternative Investments, in partnership with *Société Générale Prime Services (Newedge)*

- Advanced Investment Solutions for Liability Hedging for Inflation Risk, in partnership with *Ontario Teachers' Pension Plan*

- Cross-Sectional and Time-Series Estimates of Risk Premia in Bond Markets",

in partnership with *PIMCO*

- Active Allocation to Smart Factor Indices, in partnership with *Rothschild & Cie*
- Solvency II, in partnership with *Russell Investments*
- Structured Equity Investment Strategies for Long-Term Asian Investors, in partnership with *Société Générale Corporate & Investment Banking*

The philosophy of the Institute is to validate its work by publication in international academic journals, as well as to make it available to the sector through its position papers, published studies, and global conferences.

To ensure the distribution of its research to the industry, EDHEC-Risk also provides professionals with access to its website, [www.edhec-risk.com](http://www.edhec-risk.com), which is entirely devoted to international risk and asset management research. The website, which has more than 70,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC-Risk's analysis and expertise in the area of applied portfolio management research. Its quarterly newsletter is distributed to more than 200,000 readers.

### EDHEC-Risk Institute. Key Figures, 2014-2015

Number of permanent staff	48
Number of research associates & affiliate professors	36
Overall budget	€6,500,000
External financing	€7,025,695
Nbr of conference delegates	1,087
Nbr of participants at research seminars and executive education seminars	1,465

## About EDHEC-Risk Institute

---

### Research for Business

The Institute's activities have also given rise to executive education and research service offshoots. EDHEC-Risk's executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes. In partnership with CFA Institute, it has developed advanced seminars based on its research which are available to CFA charterholders and have been taking place since 2008 in New York, Singapore and London.

In 2012, EDHEC-Risk Institute signed two strategic partnership agreements with the Operations Research and Financial Engineering department of Princeton University to set up a joint research programme in the area of asset-liability management for institutions and individuals, and with Yale School of Management to set up joint certified executive training courses in North America and Europe in the area of risk and investment management.

As part of its policy of transferring know-how to the industry, in 2013 EDHEC-Risk Institute also set up ERI Scientific Beta. ERI Scientific Beta is an original initiative which aims to favour the adoption of the latest advances in smart beta design and implementation by the whole investment industry. Its academic origin provides the foundation for its strategy: offer, in the best economic conditions possible, the smart beta solutions that are most proven scientifically with full transparency in both the methods and the associated risks.



# EDHEC-Risk Institute Publications and Position Papers (2015–2018)



# EDHEC-Risk Institute Publications (2015–2018)

---

## 2017

- Amenc, N., F. Goltz, V. Le Sourd. EDHEC Survey on Equity Factor Investing (November).
- Amenc, N., F. Goltz, V. Le Sourd. The EDHEC European ETF and Smart Beta Survey 2016 (May).
- Martellini, L. and V. Milhau. Mass Customisation versus Mass Production in Retirement Investment Management. Addressing a "Tough Engineering Problem" (May).
- Esakia, M., F. Goltz, S. Sivasubramanian and J. Ulahel. Smart Beta Replication Costs (February).

## 2016

- Amenc, N., F. Goltz, V. Le Sourd. Investor Perceptions about Smart Beta ETFs (August).
- Giron, K., L. Martellini and V. Milhau Multi-Dimensional Risk and Performance Analysis for Equity Portfolios (July).
- Maeso, J.M., L. Martellini. Factor Investing and Risk Allocation. From Traditional to Alternative Risk Premia Harvesting (June).
- Amenc, N., F. Goltz, V. Le Sourd, A. Lodh and S. Sivasubramanian. The EDHEC European ETF Survey 2015 (February).
- Martellini, L. Mass Customisation versus Mass Production in Investment Management (January).

## 2015

- Blanc-Brude, F., M. Hasan and T. Whittaker. Cash Flow Dynamics of Private Infrastructure Project Debt (November).
- Amenc, N., G. Coqueret, and L. Martellini. Active Allocation to Smart Factor Indices (July).
- Martellini, L., and V. Milhau. Factor Investing. A Welfare Improving New Investment Paradigm or Yet Another Marketing Fad? (July).
- Goltz, F., and V. Le Sourd. Investor Interest in and Requirements for Smart Beta ETFs (April).
- Amenc, N., F. Goltz, V. Le Sourd and A. Lodh. Alternative Equity Beta Investing. A Survey (March).
- Amenc, N., K. Gautam, F. Goltz, N. Gonzalez, and J.P Schade. Accounting for Geographic Exposure in Performance and Risk Reporting for Equity Portfolios (March).
- Amenc, N., F. Ducoulombier, F. Goltz, V. Le Sourd, A. Lodh and E. Shirbini. The EDHEC European Survey 2014 (March).
- Deguest, R., L. Martellini, V. Milhau, A. Suri and H. Wang. Introducing a Comprehensive Risk Allocation Framework for Goals-Based Wealth Management (March).
- Blanc-Brude, F., and M. Hasan. The Valuation of Privately-Held Infrastructure Equity Investments (January).

# EDHEC-Risk Institute Position Papers (2015–2018)

---

## 2016

- Amenc, N., F. Ducoulombier, F. Goltz and J. Ulahel. Ten Misconceptions about Smart Beta (June).
- O'Kane, D. Initial Margin for Non-Centrally Cleared OTC Derivatives (June).



For more information, please contact:  
Maud Gauchon on +33 (0)4 93 18 78 87  
or by e-mail to: [maud.gauchon@edhec-risk.com](mailto:maud.gauchon@edhec-risk.com)

**EDHEC-Risk Institute**  
393 promenade des Anglais  
BP 3116 - 06202 Nice Cedex 3  
France  
Tel. +33 (0)4 93 18 78 87

**EDHEC Risk Institute—Europe**  
10 Fleet Place, Ludgate  
London EC4M 7RB  
United Kingdom  
Tel: + 44 207 332 5600

[www.edhec-risk.com](http://www.edhec-risk.com)