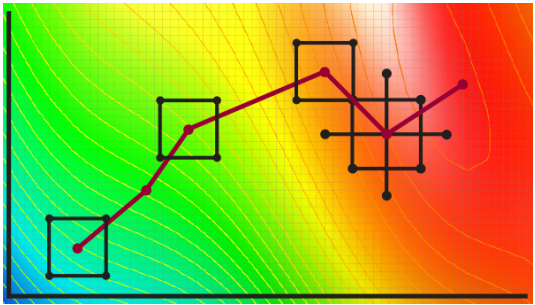
Experimentation for Improvement



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Design and Analysis of Experiments

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Now that we understand aliasing; how can we work with it in our system?

We'd like to take advantage of doing half the work, but still get the most benefit. But, recall we showed that this will lead to a loss of some accuracy:

Full factorial model

$$\begin{aligned}\hat{y} &= 11.25 \\ &+ 6.25\, x_{\mathsf{A}} \\ &+ 0.75\, x_{\mathsf{B}} \\ &- 7.25\, x_{\mathsf{C}} \\ &+ 0.25\, x_{\mathsf{A}} x_{\mathsf{B}} \\ &- 6.75\, x_{\mathsf{A}} x_{\mathsf{C}} \\ &- 0.25\, x_{\mathsf{B}} x_{\mathsf{C}} \\ &- 0.25\, x_{\mathsf{A}} x_{\mathsf{B}} x_{\mathsf{C}}\end{aligned}$$

Fractional factorial model

$$\begin{aligned}\hat{y} &= 11.0 \\ &+ 6.0\, x_{\mathsf{A}} \\ &- 6.0\, x_{\mathsf{B}} \\ &- 7.0\, x_{\mathsf{C}} \\ &+ \cancel{b_{\mathsf{AB}}\, x_{\mathsf{A}} x_{\mathsf{B}}} \\ &+ \cancel{b_{\mathsf{AC}}\, x_{\mathsf{A}} x_{\mathsf{C}}} \\ &+ \cancel{b_{\mathsf{BC}}\, x_{\mathsf{B}} x_{\mathsf{C}}} \\ &+ \cancel{b_{\mathsf{ABC}}\, x_{\mathsf{A}} x_{\mathsf{B}} x_{\mathsf{C}}}\end{aligned}$$

Example: treating water (again)

We have identified 3 factors to investigate

1. Water treatment chemical used
2. Temperature of the treatment
3. Stirring speed

Budget: for 4, maybe 5, experiments

Other criteria:

- We don't expect interactions between temperature, and stirring speed
- We want a good (i.e. unbiased) estimate of the chemical effect

Example: treating water (again)

Now we choose to assign the factor letters: **A**, **B**, and **C**

- **A**: Temperature of the treatment
- **B**: Stirring speed
- **C**: Water treatment chemical used

Reasons for this choice of letters (assignment):

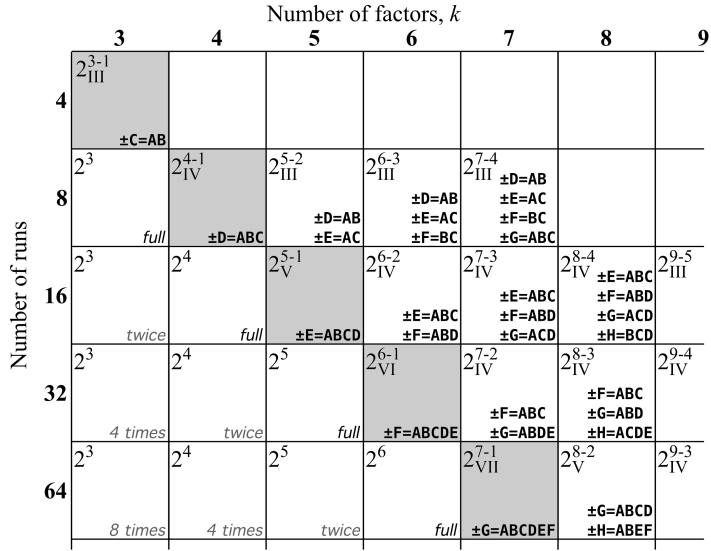
- We don't expect interaction between temperature (**A**), and stirring speed (**B**). That implies $b_{\mathsf{AB}} \approx 0$
- We want a clear, unbiased estimate of the chemical effect (**C**). So an unbiased b_{C} is desired.
- We know that $\hat{b}_{\mathsf{C}}$ will be the estimated chemical effect in a half-fraction. It will be confounded:
  - $\hat{b}_{\mathsf{C}} = b_{\mathsf{C}} + b_{\mathsf{AB}} = b_{\mathsf{C}} + \cancel{b_{\mathsf{AB}}}^{\approx 0}$ so then $\hat{b}_C \approx b_{\mathsf{C}}$

plan your experiments carefully ahead of time

Experiments are expensive to run!

- so plan not only the experiments, but also **how you will analyze** the results
- you don't need outcome values (y-values) to do this

- Re-allocate your letters if you don't like the confounding



Notice where all the half fractions lie in the table.
Do you observe the recurring pattern?

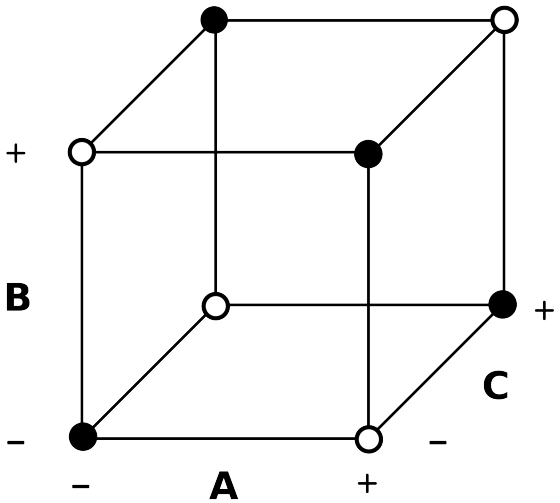
the utility of half-fractions

Four factors: requires 16 runs for a full factorial. Why not start with a half fraction?

- Start with 8 runs
- Then come back and do the other 8 later on

Coming up later in this module: we will see quarter, or even $\frac{1}{8}^{\text{th}}$ fractions, applied in the same way.

Demonstration on an example we can visualize: a 3-factor system



Start with a half fraction *

Experiment	A	B	C = AB
1	−	−	+
2	+	−	−
3	−	+	−
4	+	+	+

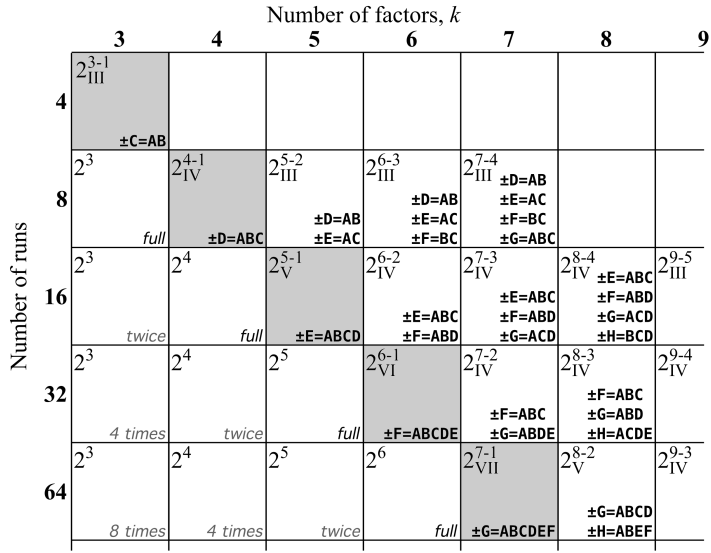
These are the 4 runs with the open circles.

Later on ... come back and complete the complementary half-fraction.*

Experiment	A	B	C = −AB
5	−	−	−
6	+	−	+
7	−	+	+
8	+	+	−

These are the 4 runs with the closed circles.

*Always randomize within the group of 4!



We will see how entries in this table are good building blocks.
Consider how you might use it for your experiments.

Half fractions are one of the experimental building blocks available to you

Initial groups of experiments can be build on, and extended later.

[Flickr: rahego]