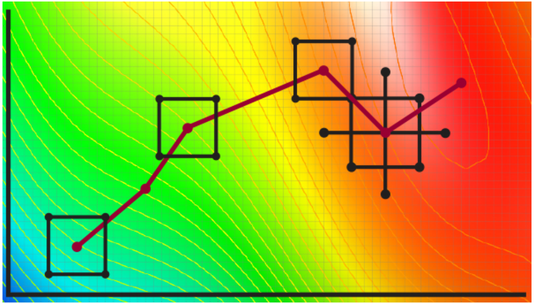
Experimentation for Improvement



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Design and Analysis of Experiments

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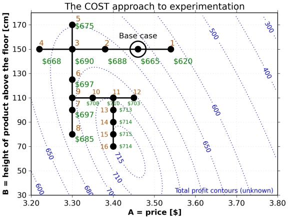
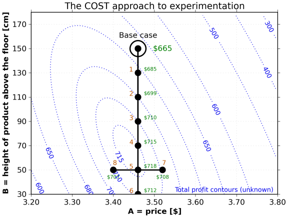
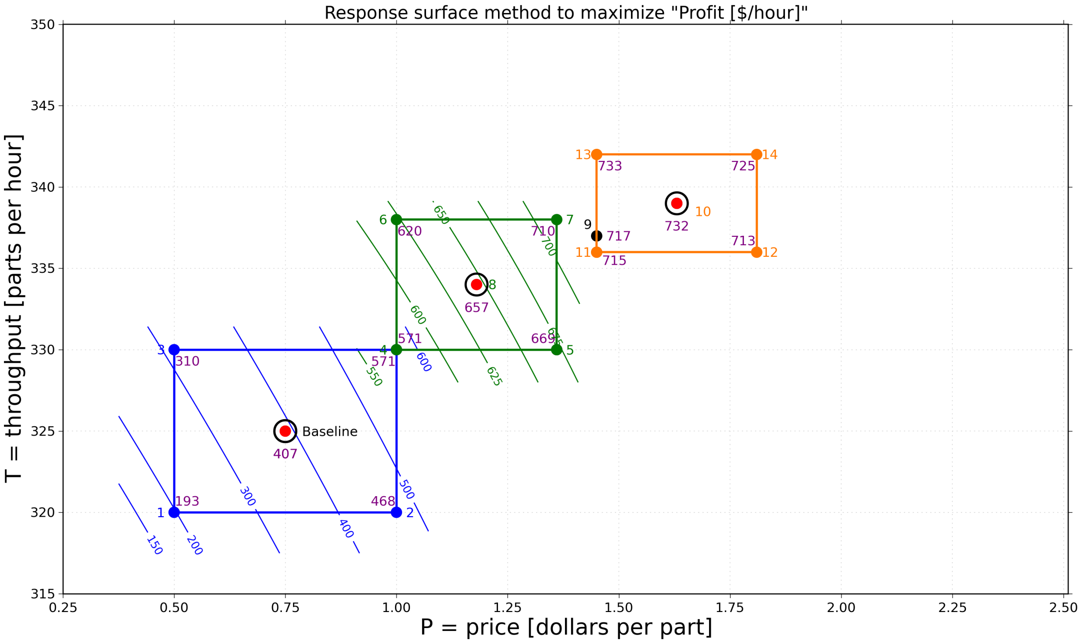
Before we get started: a quick look back at OFAT (COST)

OFAT is not recommended for a variety of reasons:

- With OFAT you are never really sure you are at the peak.
  - you keep iterating through all the factors
  - no conclusive indication that you converged
- OFAT is order-dependent (it is a lottery*!*)
- OFAT does not scale well
  - For 3 or more factors: we often end up using more runs
  - For 2 factors: I find OFAT and RSM (response surface methods) use about the same number of runs
- We don't learn about interactions; only about main effects







Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
Baseline
407
193
468
310
571
620
710
669
657
733
725
732
717
713
715
1
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700
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0.50
0.75
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1.50
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2.50
315
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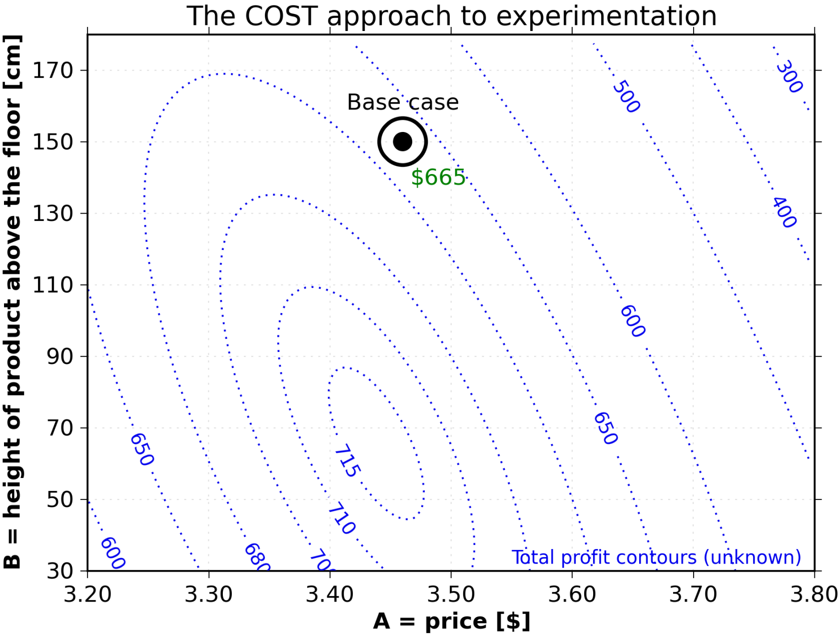
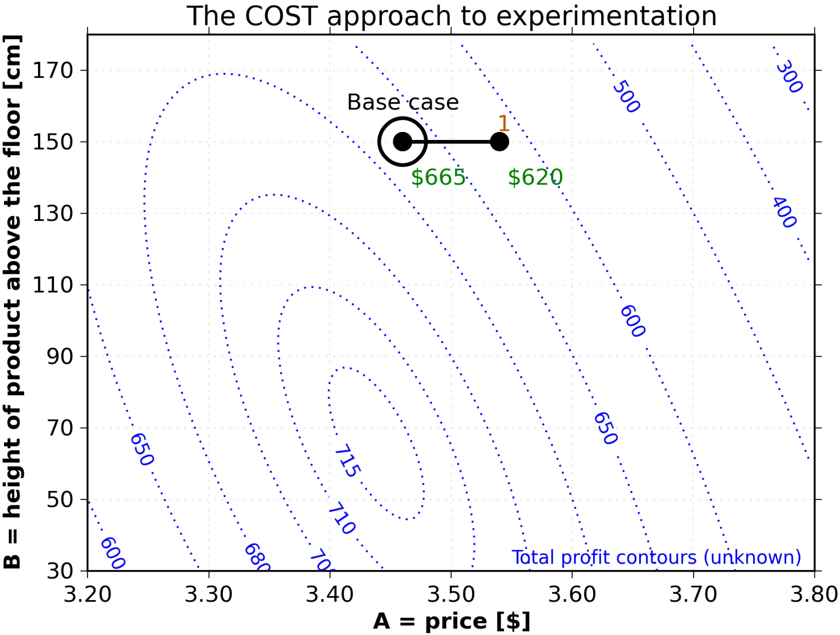
345
350



The COST approach to experimentation
Base case
$665
Total profit contours (unknown)
300
400
500
600
650
680
700
710
715
B = height of product above the floor [cm]
A = price [$]
170
150
130
110
90
70
50
30
3.20
3.30
3.40
3.50
3.60
3.70
3.80


The COST approach to experimentation
Base case
1
$665
$620
500
300
400
600
650
650
600
680
70
715
710
Total profit contours (unknown)
B = height of product above the floor [cm]
A = price [$]
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150
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110
90
70
50
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3.20
3.30
3.40
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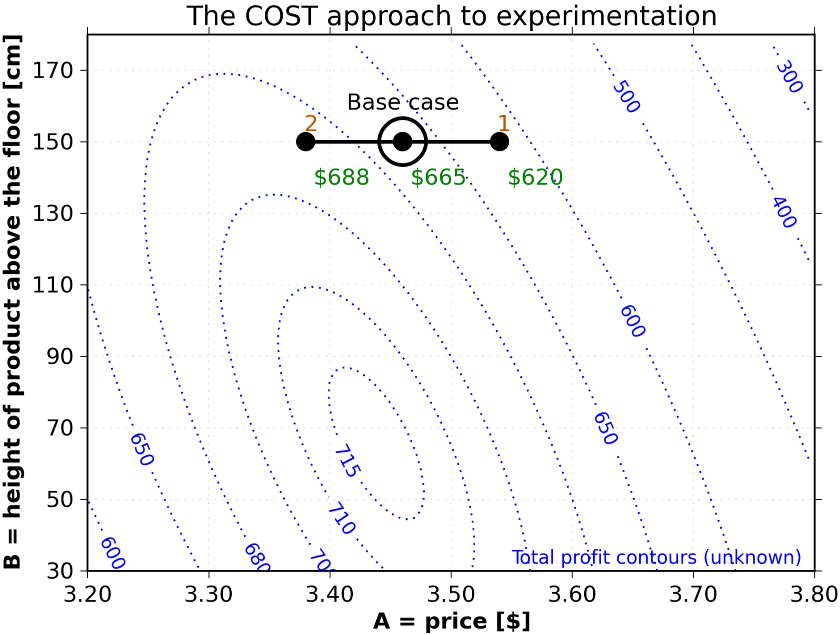
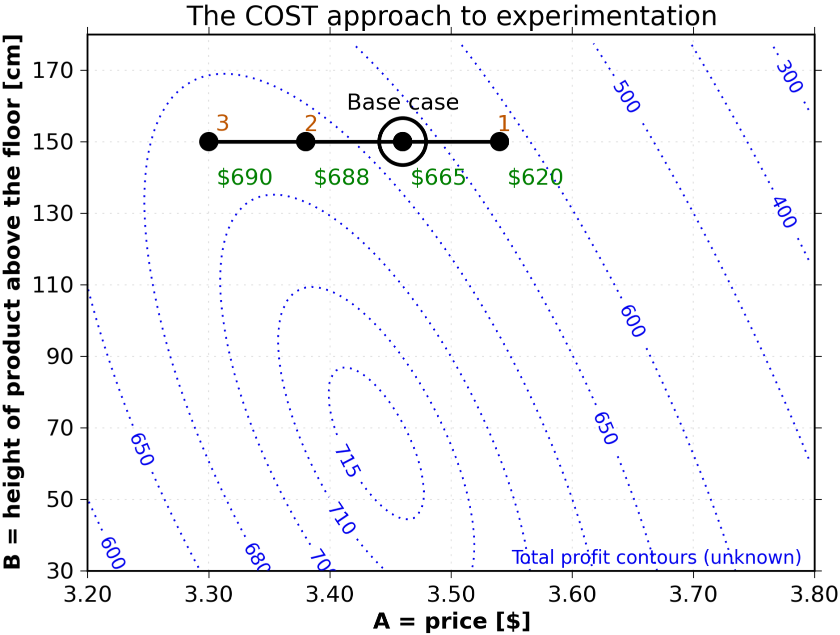
3.60
3.70
3.80



The COST approach to experimentation
Base case
2
1
$688
$665
$620
300
400
500
600
650
715
710
680
700
650
600
Total profit contours (unknown)
170
150
130
110
90
70
50
30
3.20
3.30
3.40
3.50
3.60
3.70
3.80
A = price [$]
B = height of product above the floor [cm]


The COST approach to experimentation
Base case
3
2
1
$690
$688
$665
$620
300
400
500
600
650
600
650
680
700
710
715
Total profit contours (unknown)
170
150
130
110
90
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3.80
A = price [$]
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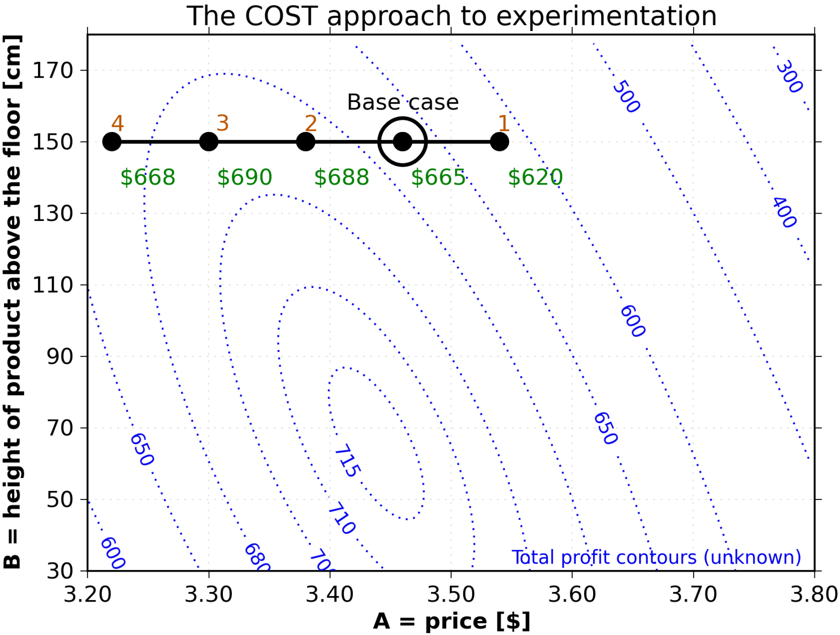
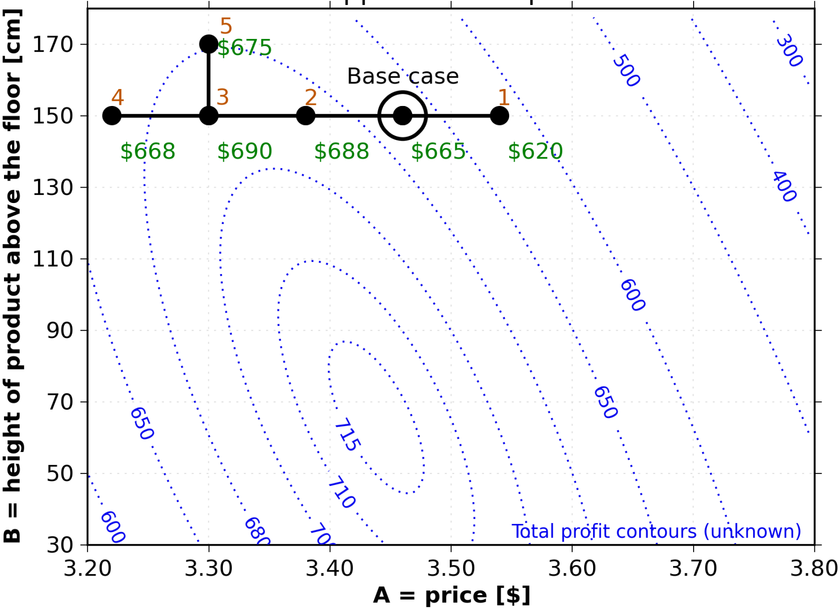
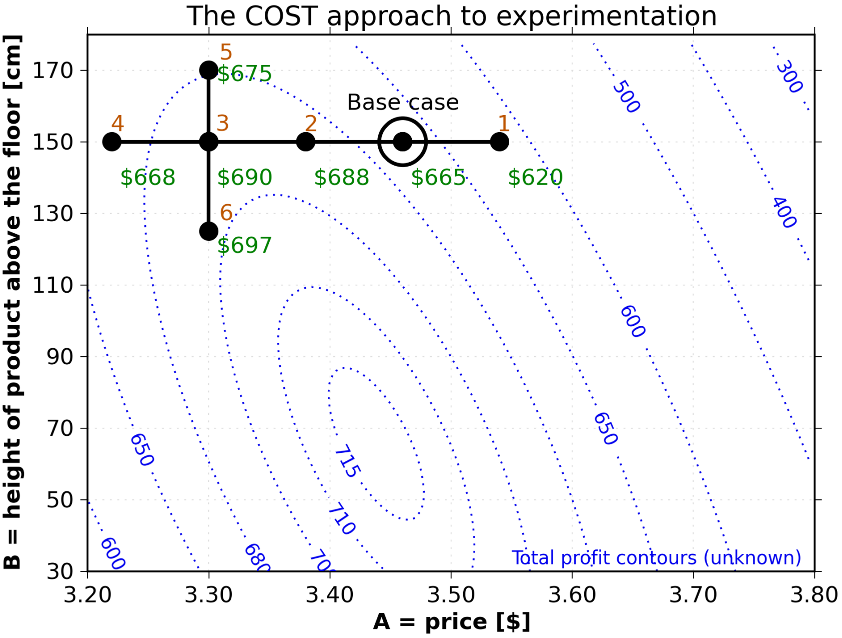


The COST approach to experimentation
Base case
4
3
2
1
$668
$690
$688
$665
$620
300
400
500
600
650
680
700
710
715
Total profit contours (unknown)
170
150
130
110
90
70
50
30
3.20
3.30
3.40
3.50
3.60
3.70
3.80
A = price [$]
B = height of product above the floor [cm]

The COST approach to experimentation





The COST approach to experimentation
Base case
1
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$675
$668
$690
$688
$665
$620
$697
300
400
500
600
650
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700
710
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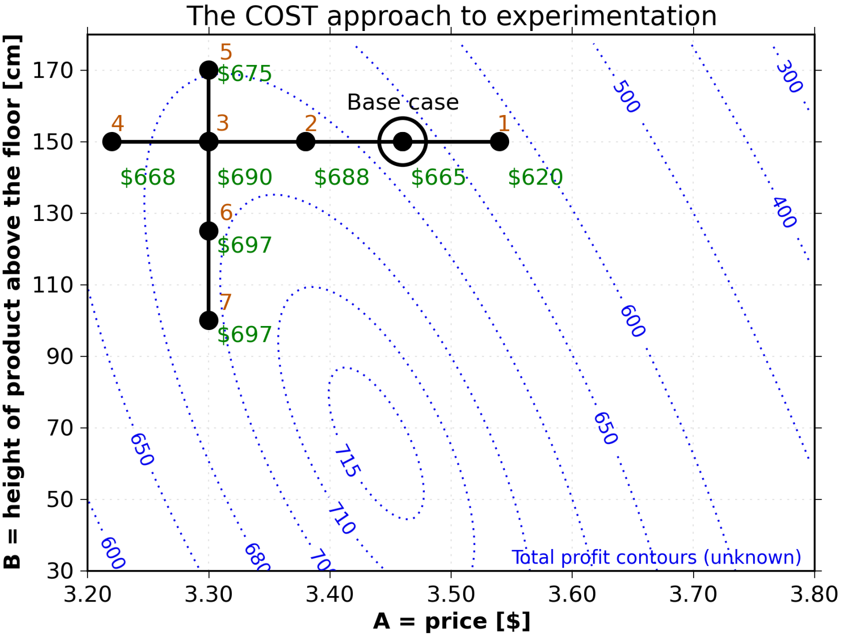
The COST approach to experimentation
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A = price [$]
170
150
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3.70
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$675
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Base case
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$668
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$688
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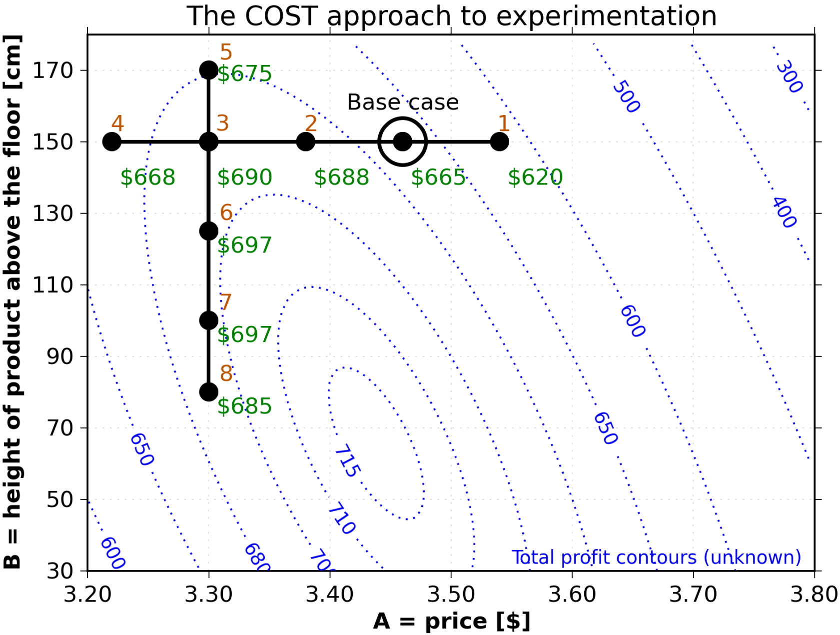
600
Total profit contours (unknown)



The COST approach to experimentation
Base case
1
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$675
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$620
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300
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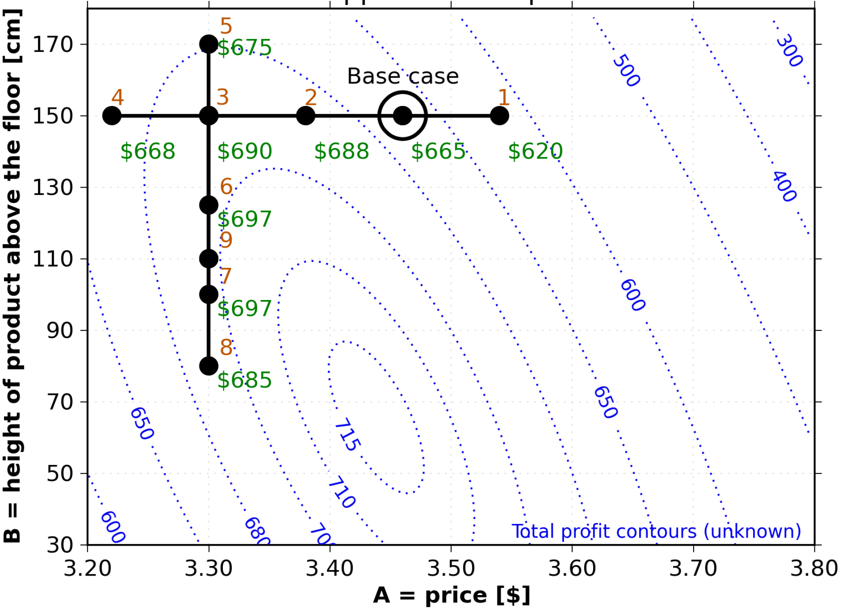
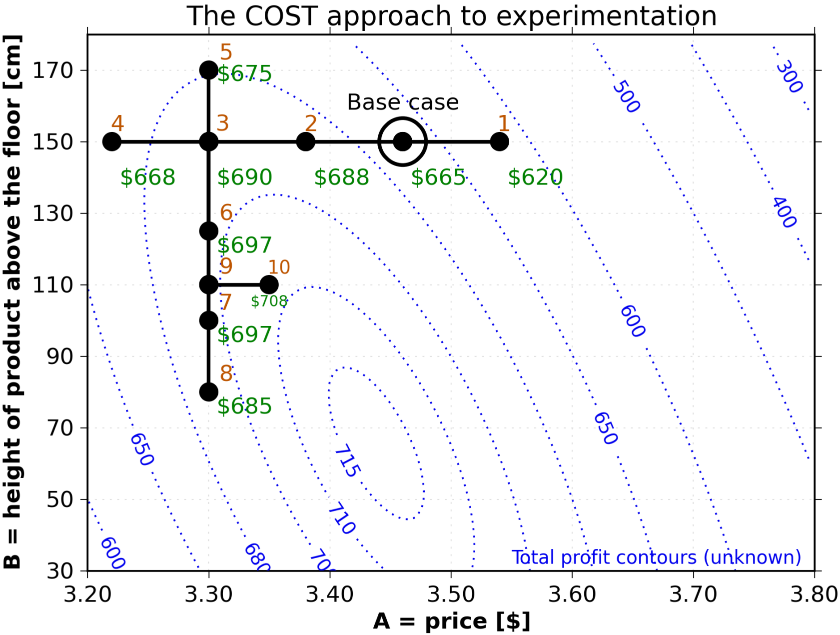
The COST approach to experimentation





The COST approach to experimentation
B = height of product above the floor [cm]
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Base case
1
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$675
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Total profit contours (unknown)
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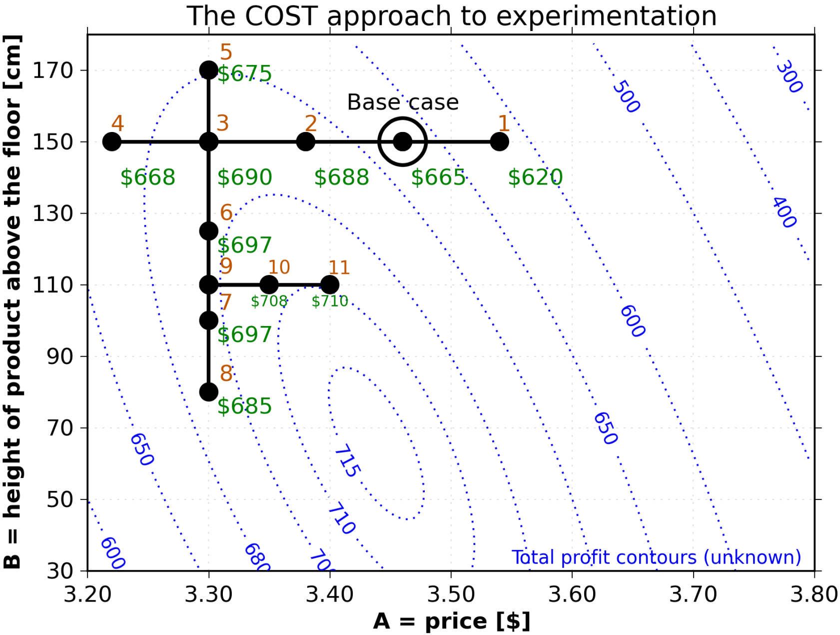
The COST approach to experimentation
Base case
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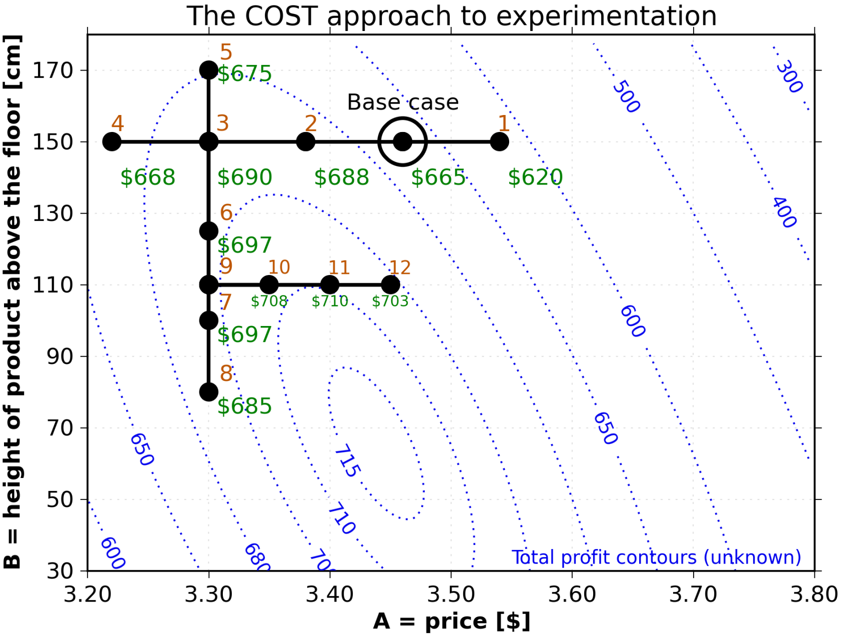

The COST approach to experimentation
Base case
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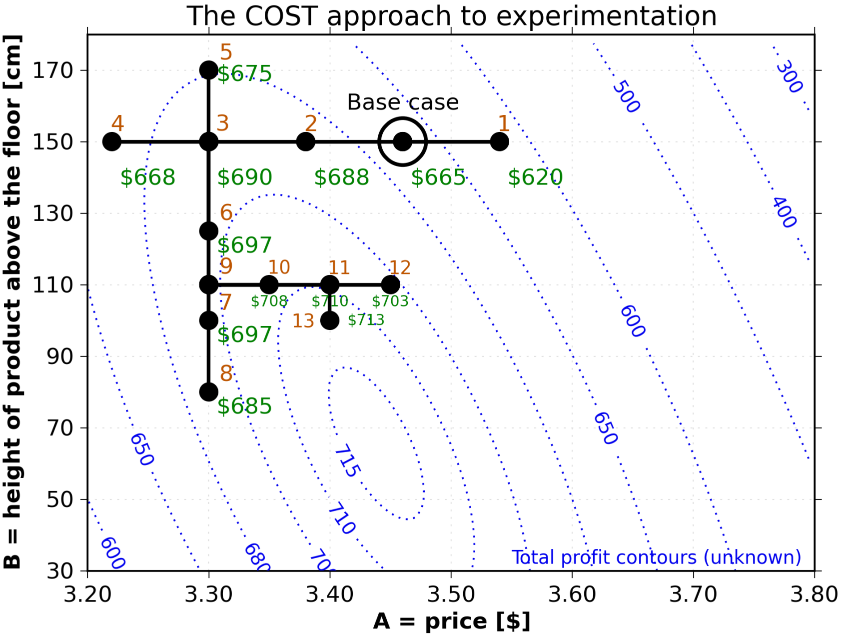
$685
Total profit contours (unknown)
A = price [$]
B = height of product above the floor [cm]


The COST approach to experimentation
Base case
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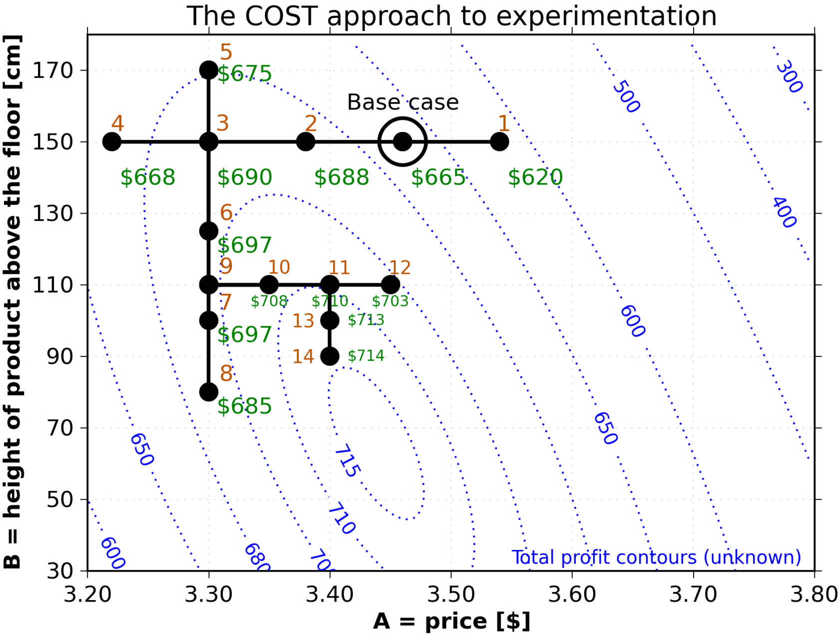
The COST approach to experimentation
Base case
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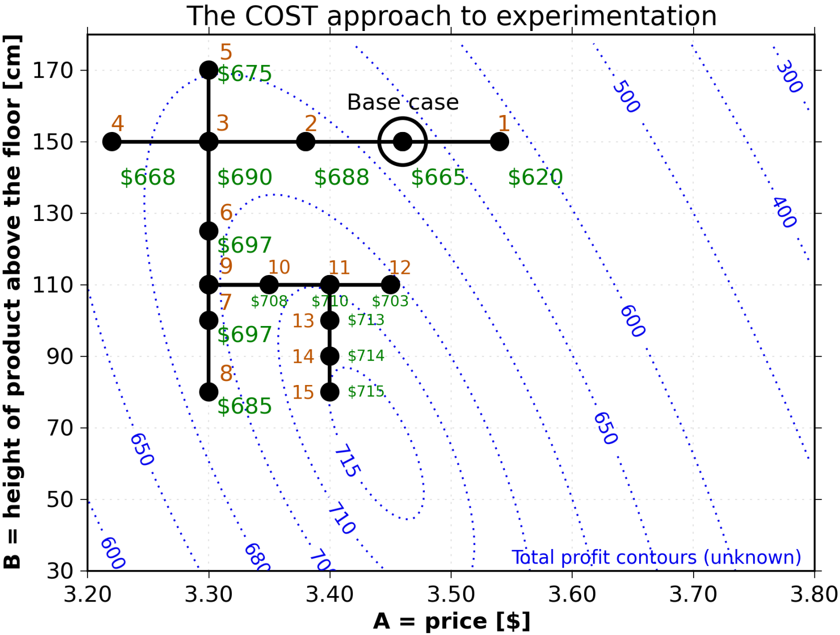
The COST approach to experimentation
Base case
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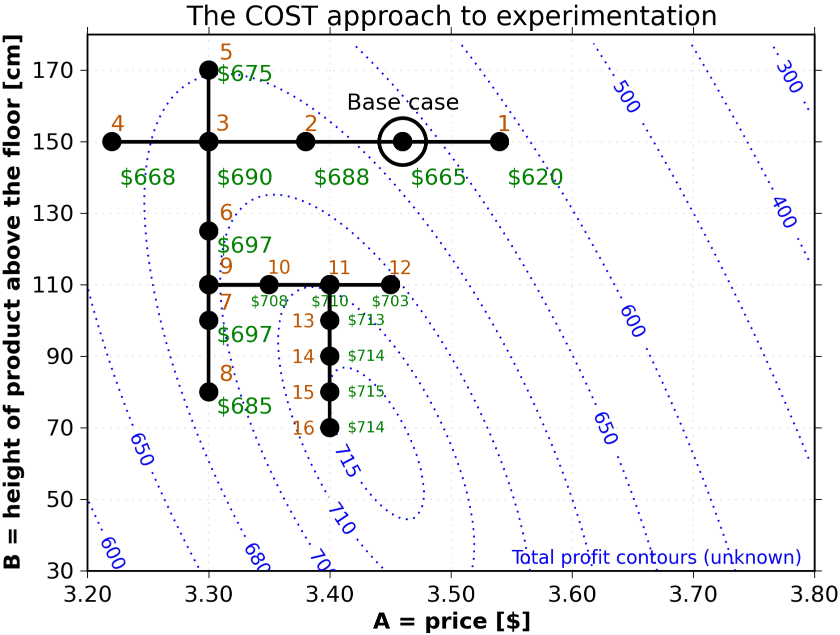

The COST approach to experimentation
Base case
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$690
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$620
$697
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$714
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$685
$714
Total profit contours (unknown)
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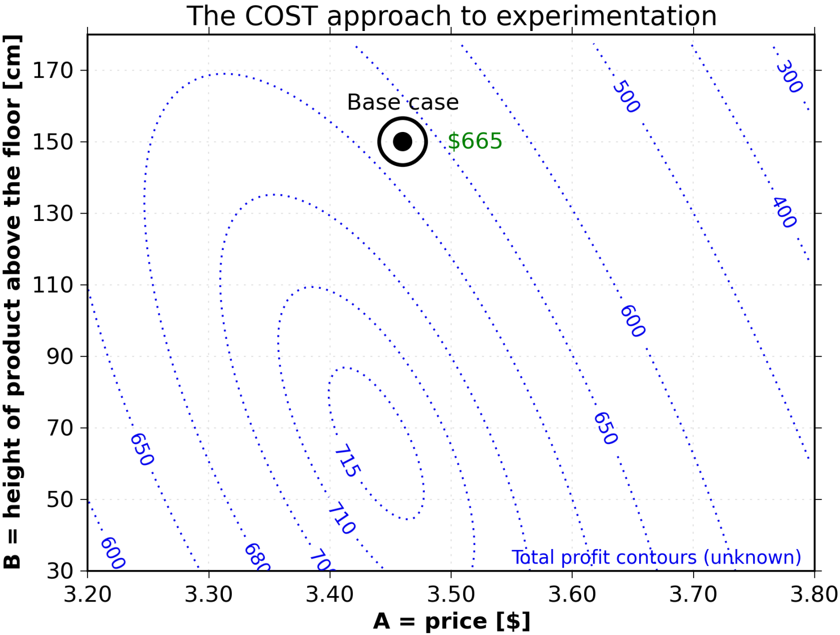

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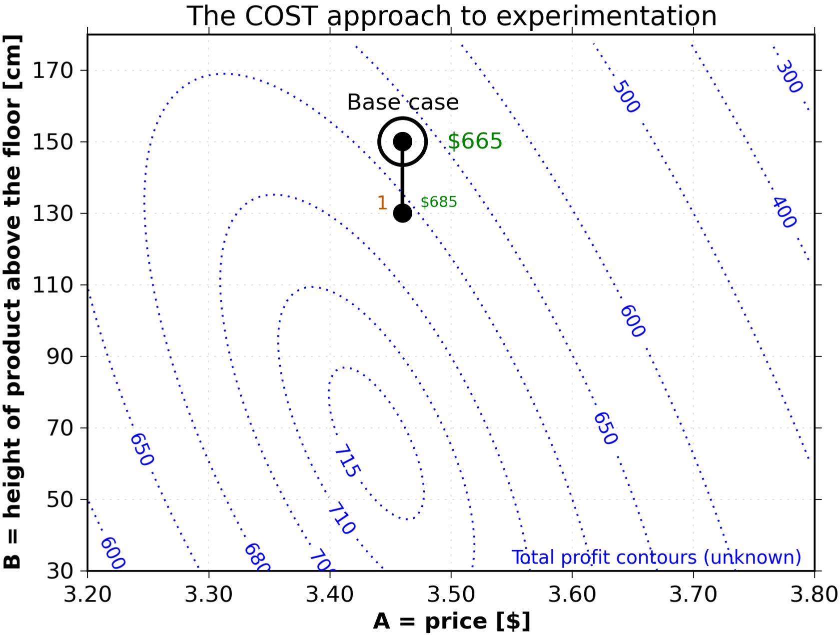
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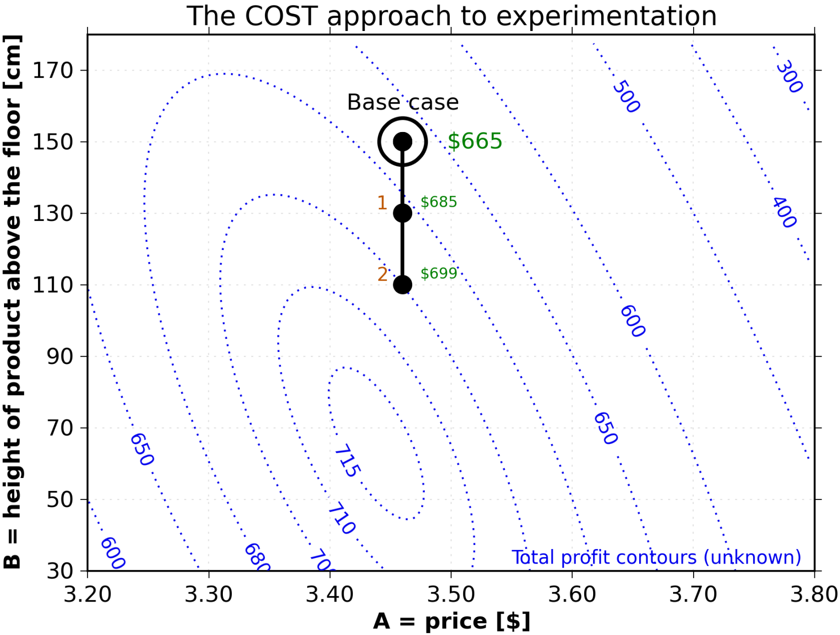

The COST approach to experimentation
Base case
$665
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$685
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$699
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400
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600
650
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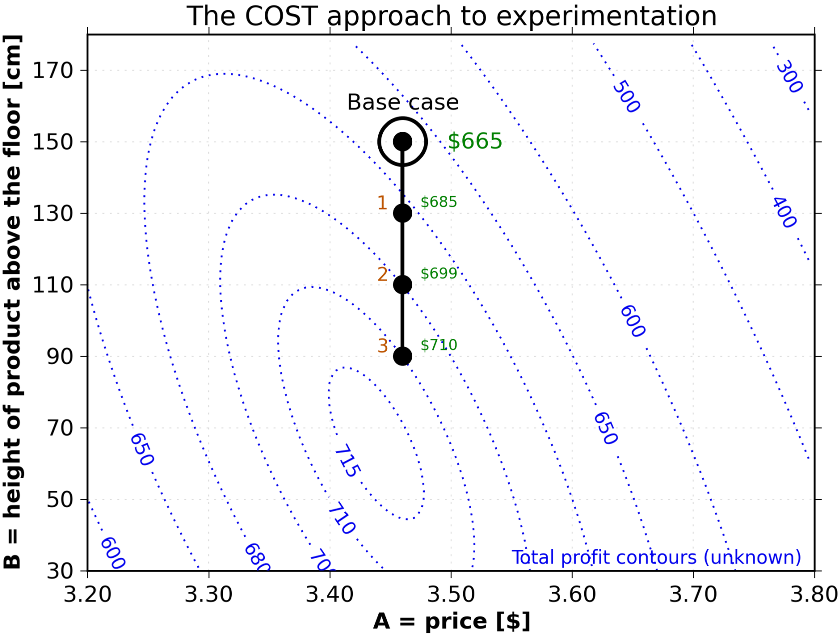
The COST approach to experimentation
Base case
$665
1
$685
2
$699
3
$710
300
400
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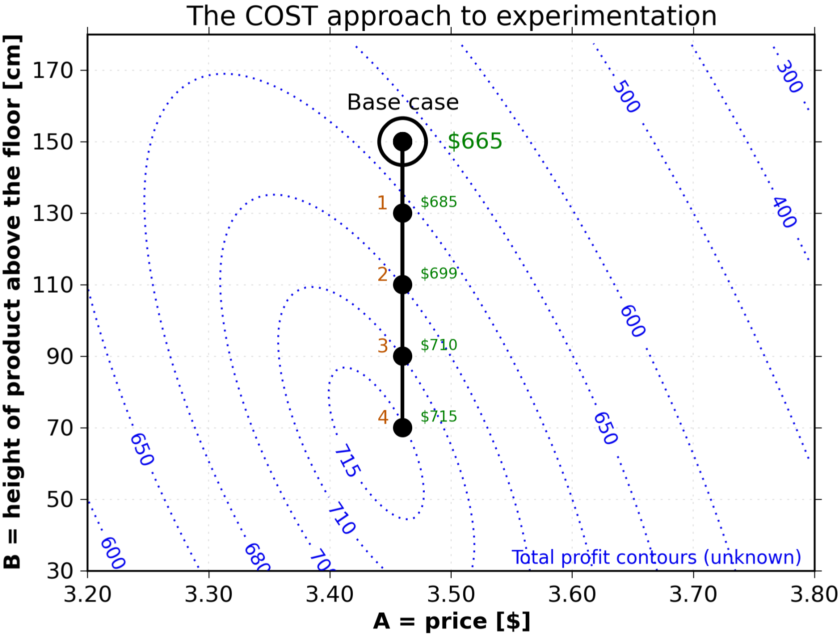
The COST approach to experimentation
Base case
$665
1
$685
2
$699
3
$710
4
$715
500
300
400
600
650
715
710
650
600
680
70
Total profit contours (unknown)
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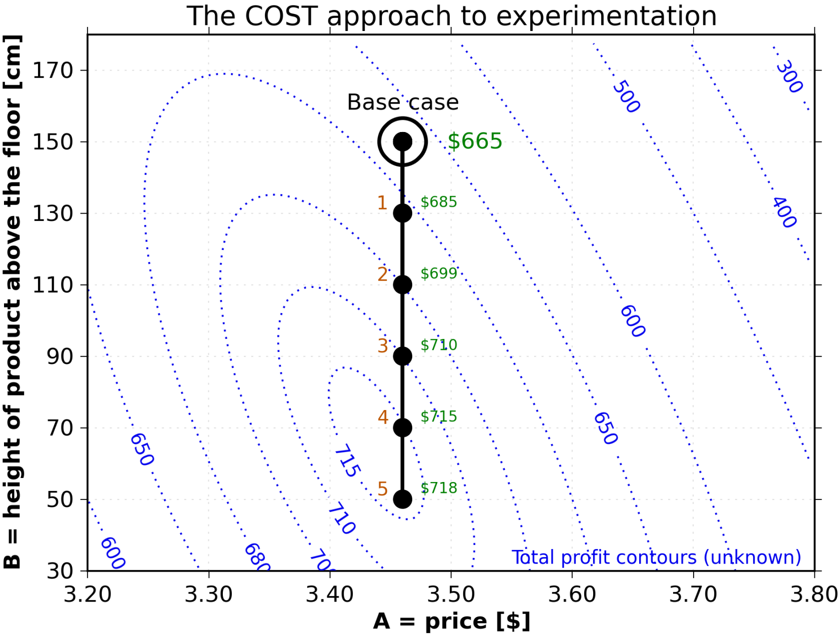
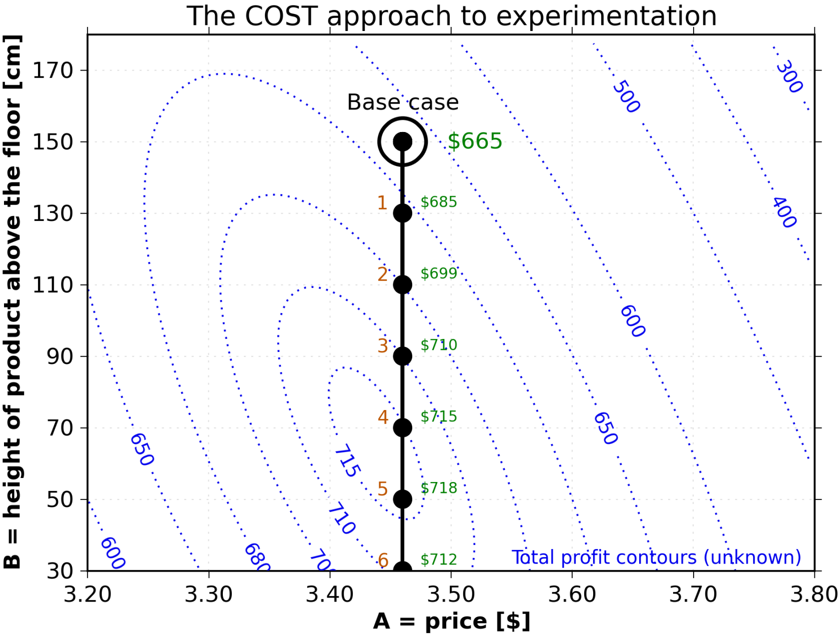


The COST approach to experimentation
Base case
$665
1
$685
2
$699
3
$710
4
$715
5
$718
Total profit contours (unknown)
300
400
500
600
650
680
700
710
715
B = height of product above the floor [cm]
A = price [$]
30
50
70
90
110
130
150
170
3.20
3.30
3.40
3.50
3.60
3.70
3.80



The COST approach to experimentation
Base case
$665
1
$685
2
$699
3
$710
4
$715
5
$718
6
$712
300
400
500
600
650
680
710
715
Total profit contours (unknown)
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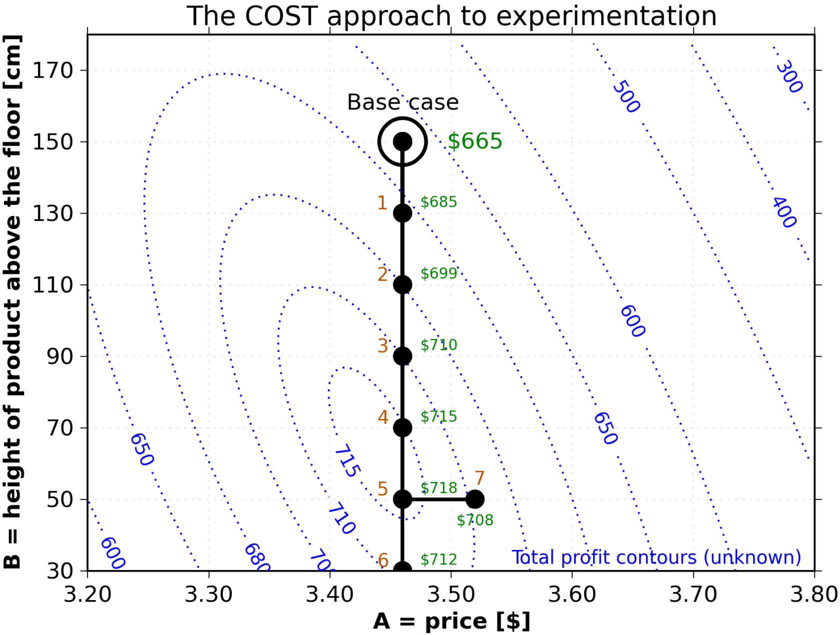

The COST approach to experimentation
Base case
$665
1
$685
2
$699
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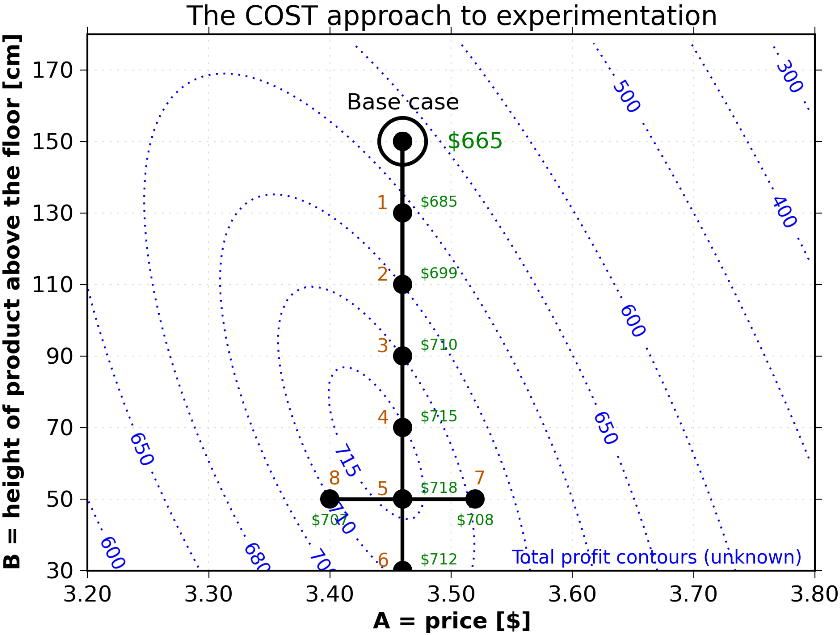

The COST approach to experimentation
Base case
$665
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$685
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$699
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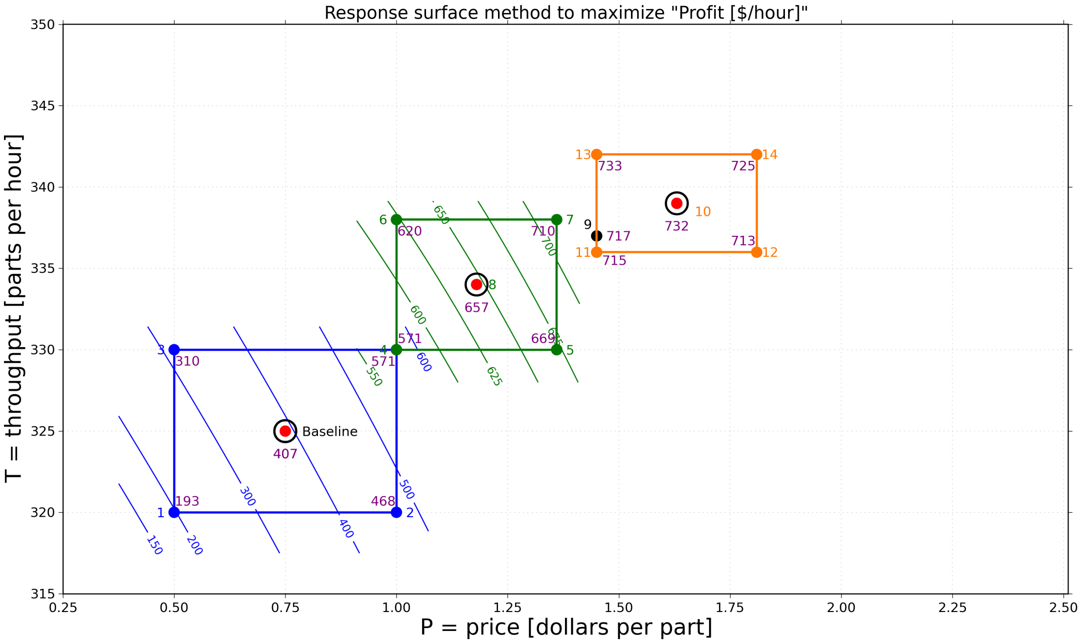

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T = throughput [parts per hour]
P = price [dollars per part]
Baseline
407
193
468
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571
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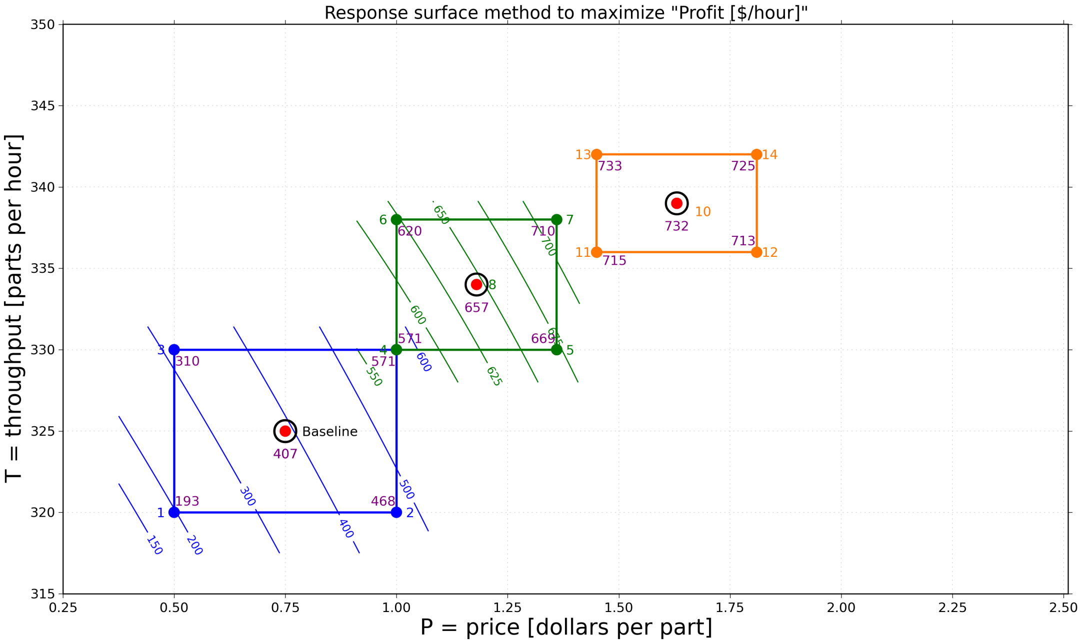

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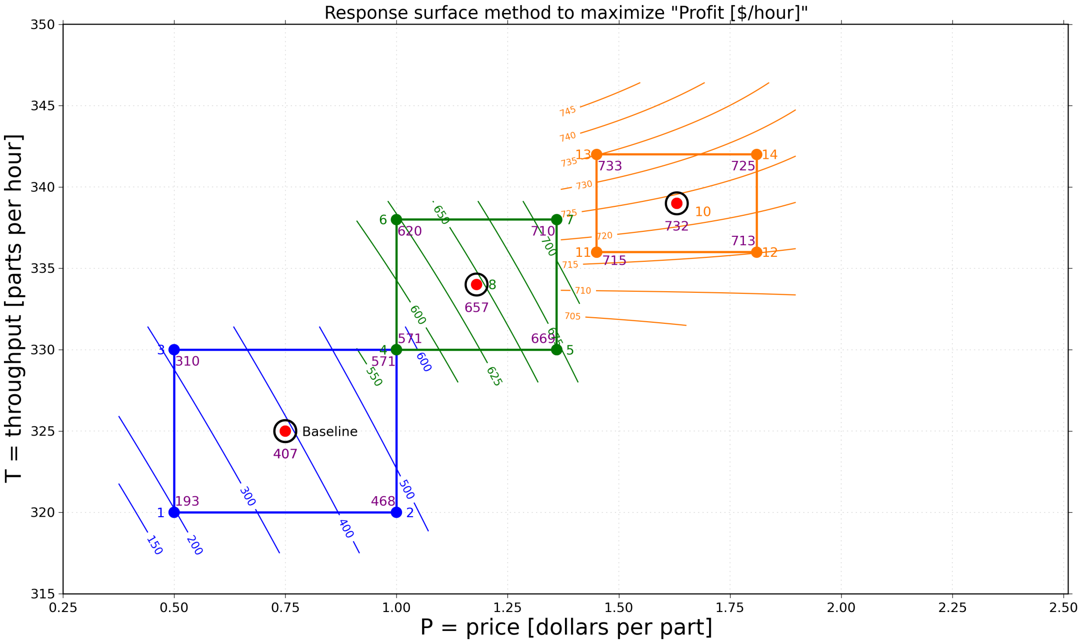

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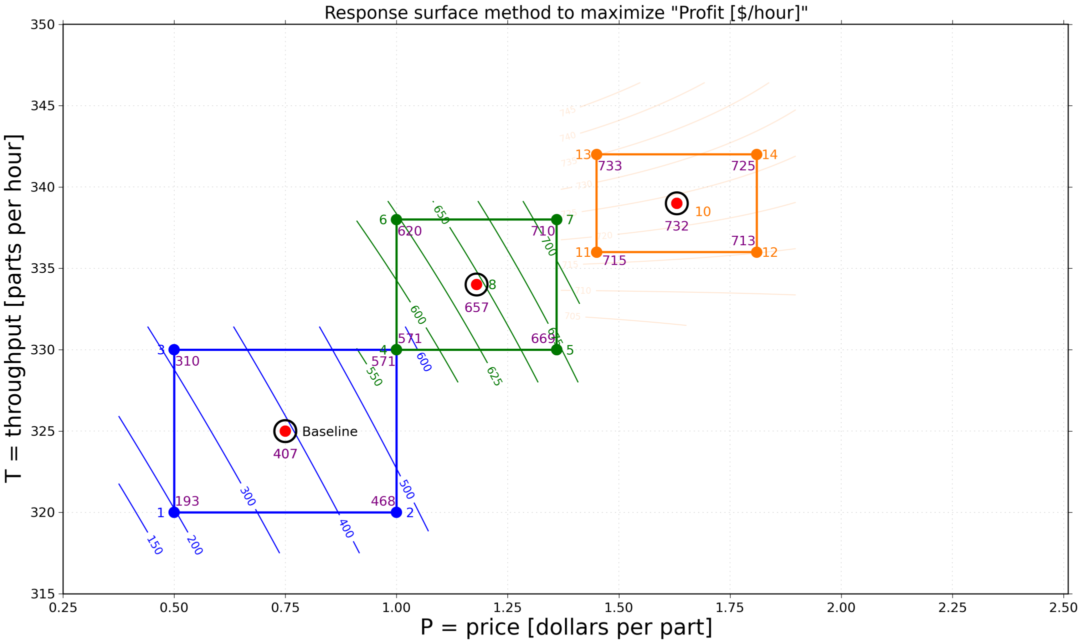
Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
Baseline
407
193
468
310
571
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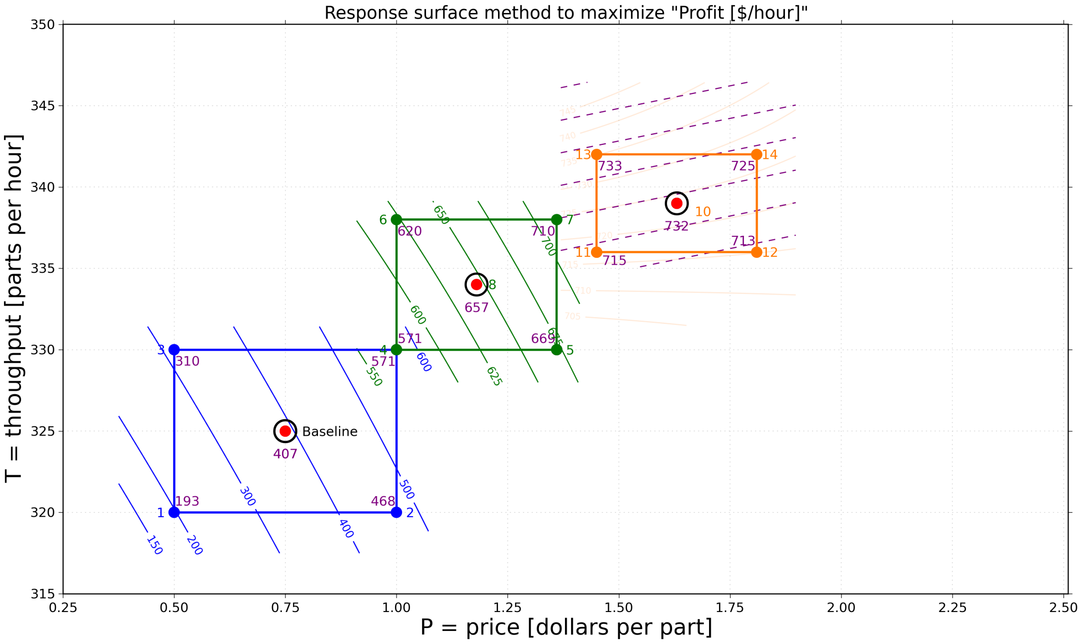
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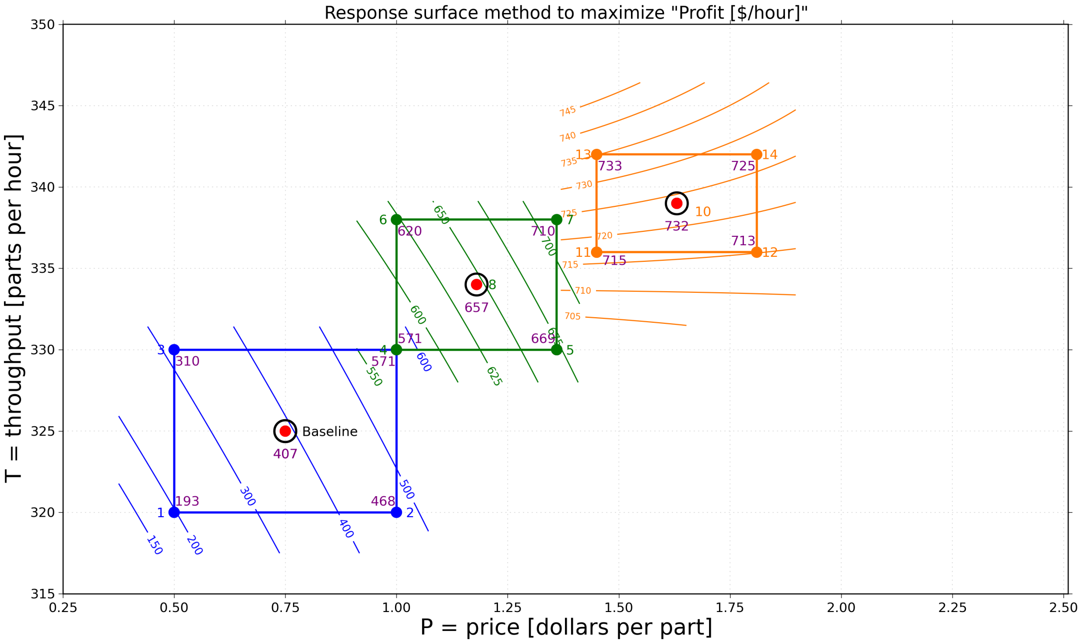

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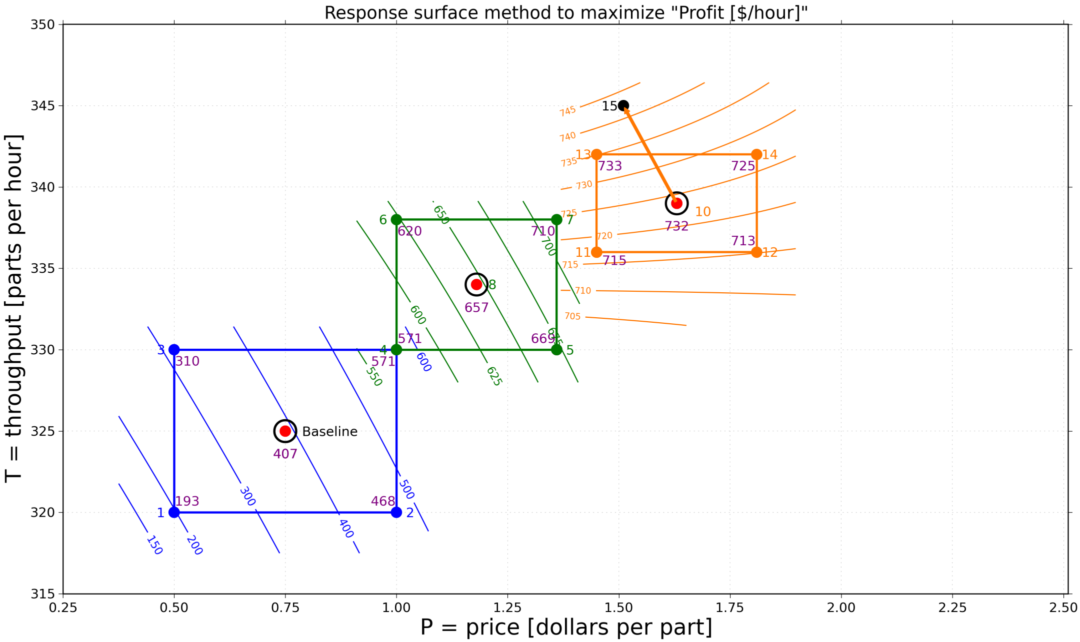

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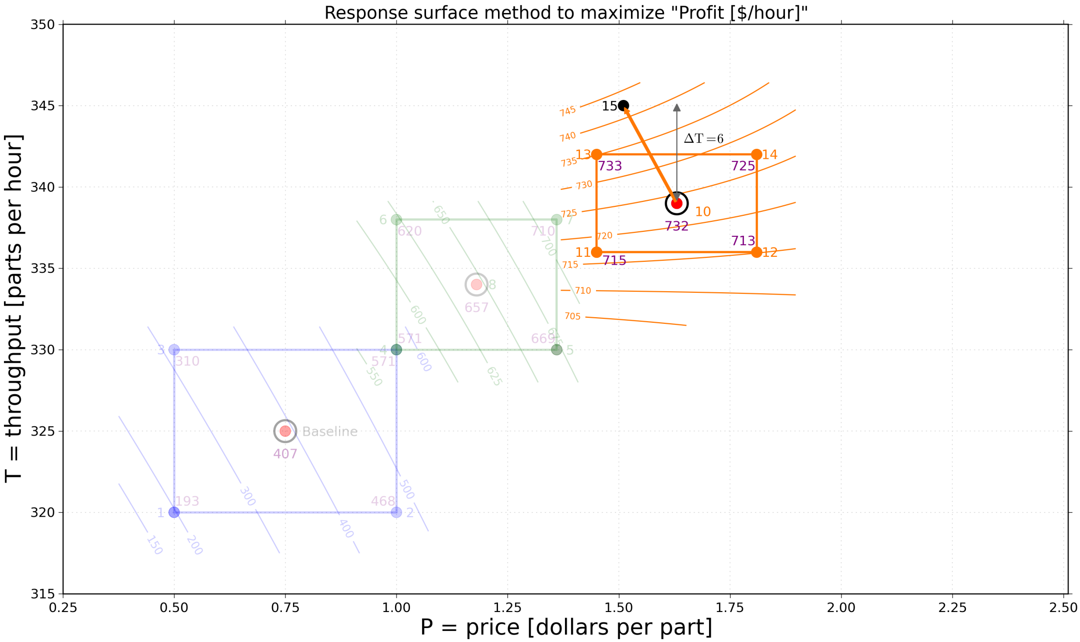

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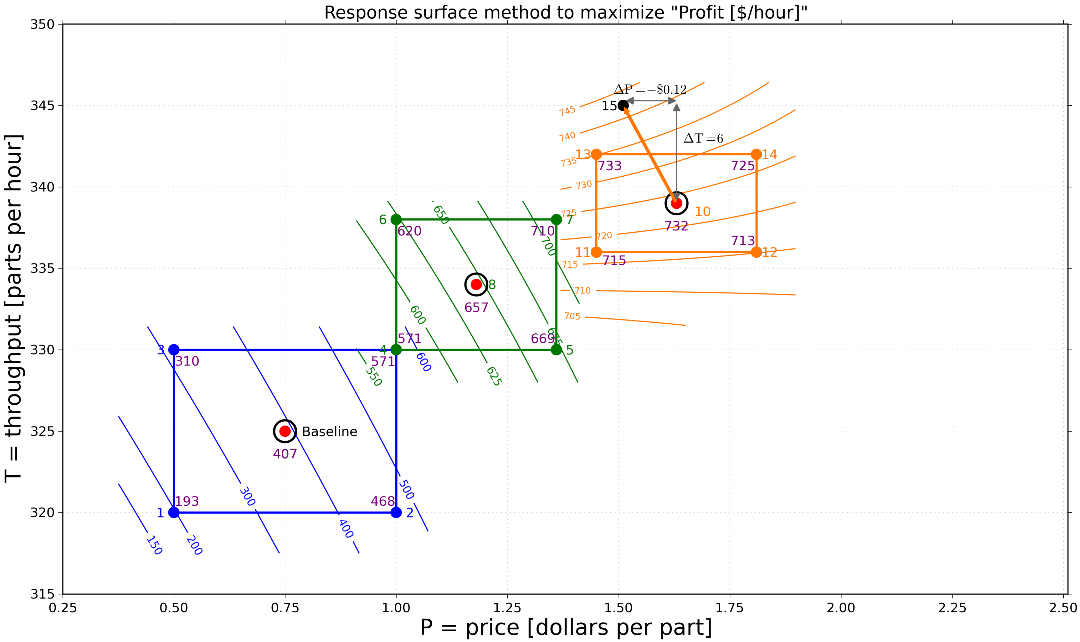
$\Delta T = 6$



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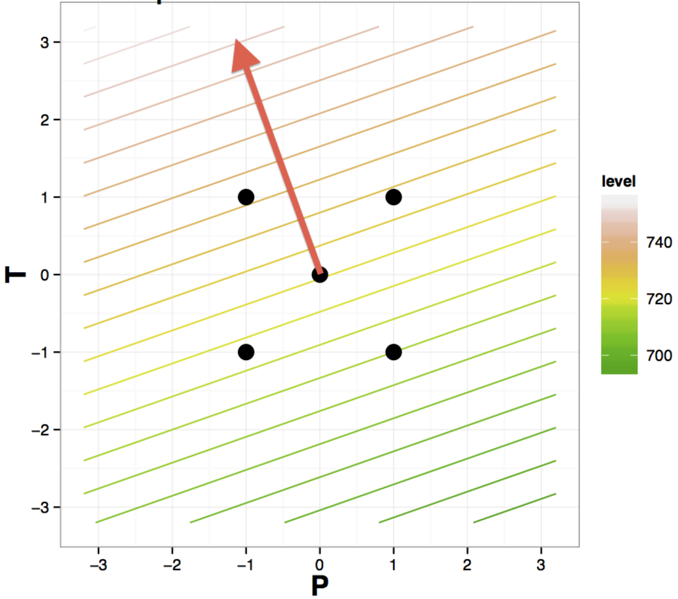
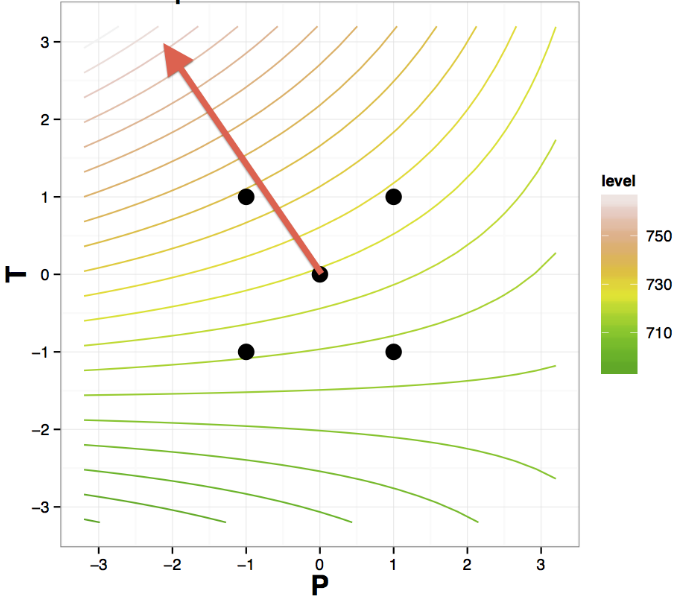
$\Delta P = -\$0.12$
$\Delta T = 6$

Contour plot with NO interaction term



Contour plot WITH interaction term



The 3 models we have built so far

Factorial 1:

$$\hat{y} = 389.8 + 134x_{\mathsf{P}} + 55x_{\mathsf{T}} - 3.5x_{\mathsf{P}}x_{\mathsf{T}}$$

Factorial 2:

$$\hat{y} = 645.4 + 47x_{\mathsf{P}} + 22.500x_{\mathsf{T}} - 2.0x_{\mathsf{P}}x_{\mathsf{T}}$$

Factorial 3:

$$\hat{y} = 723.6 - 2.5x_{\mathsf{P}} + 7.500x_{\mathsf{T}} - 1.5x_{\mathsf{P}}x_{\mathsf{T}}$$

Taking the next step for experiment 15: solution

1. Pick change in coded units in one factor.
2. Find the ratios for the other factor(s).
3. Calculate step size in coded units.
4. Convert these to real-world *changes*.
5. Get the real-world location of the next experiment.
6. Convert these back to coded-units.
7. Predict the next experiment's outcome.
8. Now run the next experiment, and record the values

Price

$$\Delta x_{\mathsf{P}} = = \frac{b_{\mathsf{P}}}{b_{\mathsf{T}}} \times \Delta x_{\mathsf{T}}$$

$$\Delta x_{\mathsf{P}} = \frac{-2.5}{7.5} \times 2 = -\frac{2}{3}$$

$$\Delta \mathsf{P} = -\frac{2}{3} \cdot \frac{1}{2}(0.36) = -\$0.12$$

$$\mathsf{P}^{(15)} = \mathsf{P}^{(10)} + \Delta \mathsf{P} = \$1.63 - 0.12 = \$1.51$$

$$x_{\mathsf{P}}^{(15)} = \frac{1.51 - 1.63}{\frac{1}{2} \cdot (0.36)} = -\frac{2}{3}$$

Throughput

$\Delta x_{\mathsf{T}} = 2$ (this was chosen)

$$\Delta x_{\mathsf{T}} = 2$$

$$\Delta \mathsf{T} = 6$$

$$\mathsf{T}^{(15)} = \mathsf{T}^{(10)} + \Delta \mathsf{T}$$

$$\mathsf{T}^{(15)} = 339 + 6 = 345$$

$$x_{\mathsf{T}}^{(15)} = 2$$

$$\hat{y} = 723.6 - 2.5 x_{\mathsf{P}} + 7.5 x_{\mathsf{T}} - 1.5 x_{\mathsf{P}} x_{\mathsf{T}}$$

$$\hat{y}^{(15)} \approx \text{\$742 profit per hour}$$

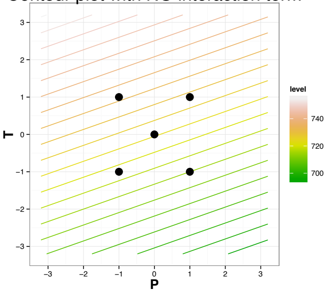
$$y^{(15)} = \text{\$735 profit per hour}$$

Dealing with curvature when we suspect we are approaching an optimum

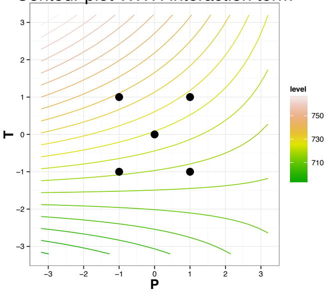
We detect curvature in several ways (in practice, you will detect one or more of these)

1. interaction terms are comparable to main effects

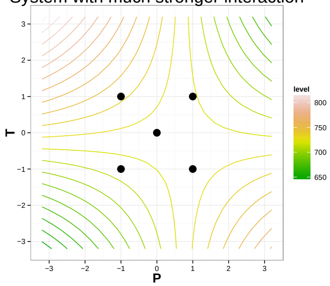
Contour plot with NO interaction term



Contour plot WITH interaction term



System with much stronger interaction



$$\hat{y} = -2.5x_{\mathsf{P}} + 7.5x_{\mathsf{T}} + \cancel{0 \cdot x_{\mathsf{P}} x_{\mathsf{T}}} \quad 0$$

$$\hat{y} = -2.5x_{\mathsf{P}} + 7.5x_{\mathsf{T}} - 1.5x_{\mathsf{P}} x_{\mathsf{T}}$$

$$\hat{y} = -2.5x_{\mathsf{P}} + 5.35x_{\mathsf{T}} - 7.5x_{\mathsf{P}} x_{\mathsf{T}}$$

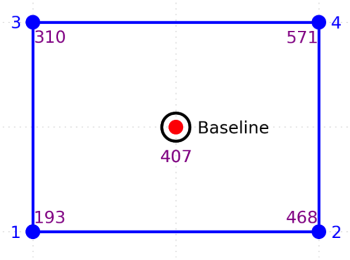
Dealing with curvature when we suspect we are approaching an optimum

We detect curvature in several ways (in practice, you will detect one or more of these)

2. differences in the spread* are becoming smaller and smaller

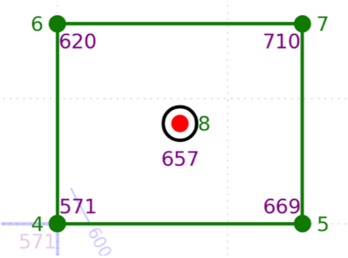
*can be crudely quantified as (highest outcome) − (lowest outcome)

Factorial 1



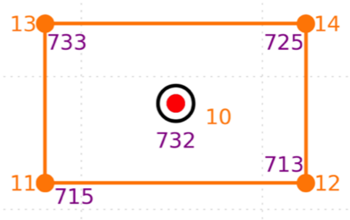
spread = \$ 378

Factorial 2



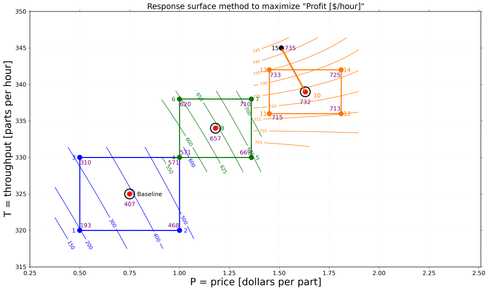
spread = \$ 139

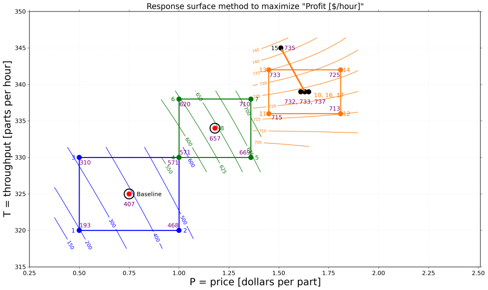
Factorial 3



spread = \$ 20

Aside: If we are judging differences, we need an estimate of "noise"





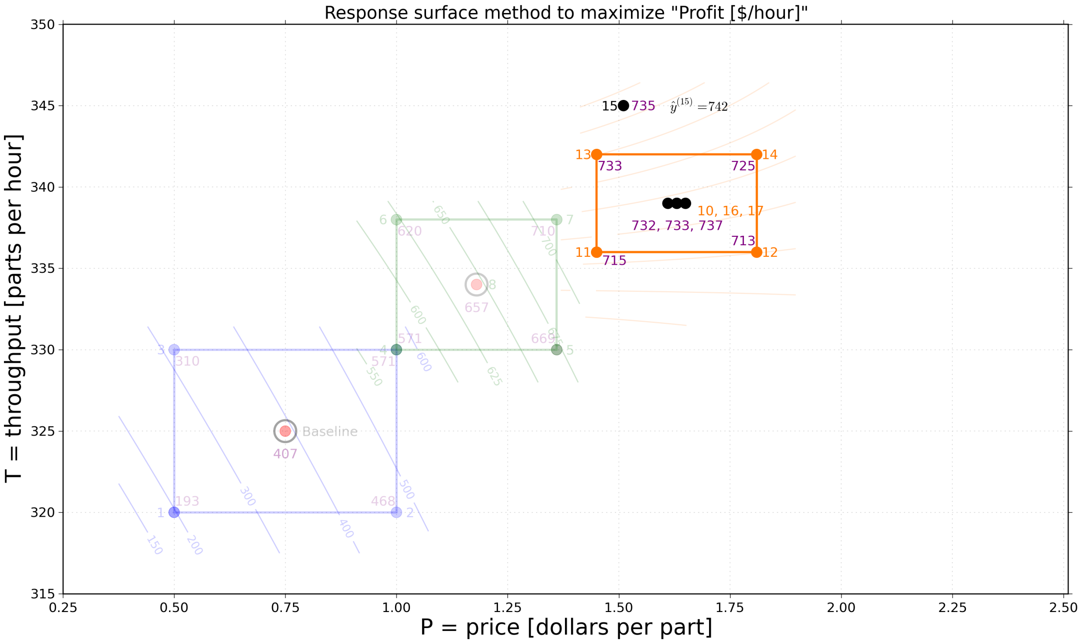
Dealing with curvature when we suspect we are approaching an optimum

We detect curvature in several ways (in practice, you will detect one or more of these)

3. we notice prediction errors with our model that indicate the surface is changing

[Flickr: rmkoske]

[Flickr: Digital Temi]

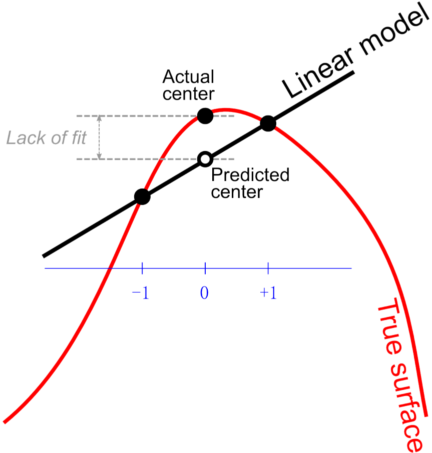


Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
15 735 $\hat{y}^{(15)} = 742$
13 733
14 725
10, 16, 17
732, 733, 737
11 715
12 713
6 620
7 710
8 657
4 571
5 669
3 310
1 193
2 468
Baseline 407

Dealing with curvature when we suspect we are approaching an optimum

We detect curvature in several ways (in practice, you will detect one or more of these)

4. we detect "lack of fit" in our empirical model



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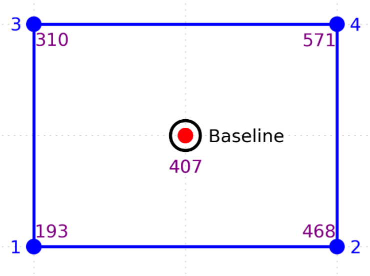
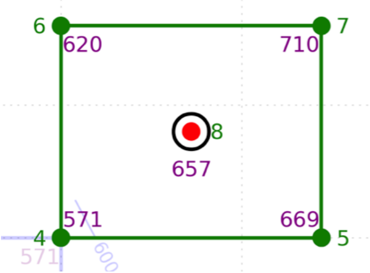


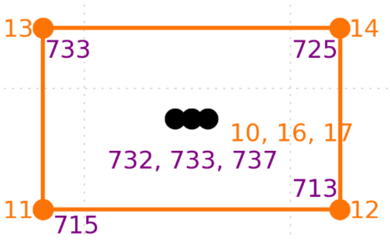
$\hat{y} = 390 + 134x_{\text{P}} + 55x_{\text{T}} - 3.5x_{\text{P}}x_{\text{T}}$

$$\left.\begin{aligned} y^{(0)} &= \$407 \\ \hat{y}^{(0)} &= \$390 \end{aligned}\right\}$$ difference = $17



$\hat{y} = 645 + 47x_{\text{P}} + 22.5x_{\text{T}} - 2.0x_{\text{P}}x_{\text{T}}$

$$\left.\begin{aligned} y^{(8)} &= \$657 \\ \hat{y}^{(8)} &= \$645 \end{aligned}\right\}$$ difference = $12



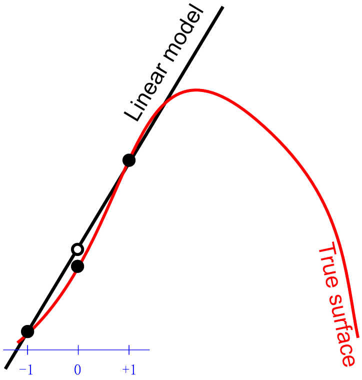
$\hat{y} = 724 - 2.5x_{\text{P}} + 7.5x_{\text{T}} - 1.5x_{\text{P}}x_{\text{T}}$

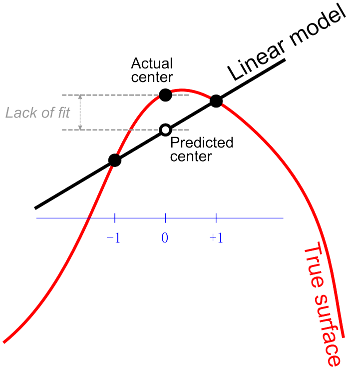
$$\left.\begin{aligned} y^{(\text{mid})} &= \$734 \\ \hat{y}^{(\text{mid})} &= \$724 \end{aligned}\right\}$$ difference = $10

The one-dimensional equivalent of what we are seeing in two dimensions

We detect curvature in several ways (in practice, you will detect one or more of these)

4. we detect "lack of fit" in our empirical model





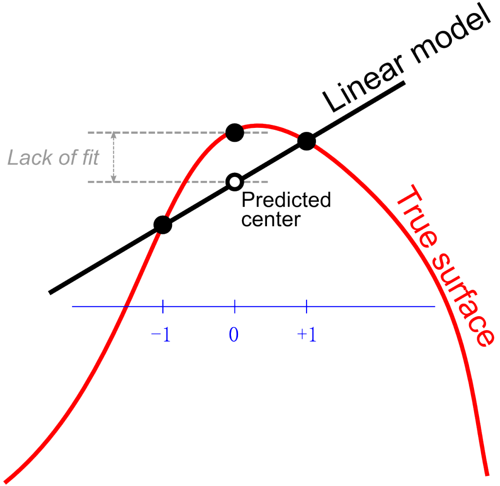
Dealing with curvature when we suspect we are approaching an optimum

We detect curvature in several ways (in practice, you will detect one or more of these)

1. interaction terms are comparable to main effects
2. differences in the spread are becoming smaller and smaller
3. we notice prediction errors with our model that indicate the surface is changing
4. we detect "lack of fit"
5. confidence intervals show some model coefficients are not significant

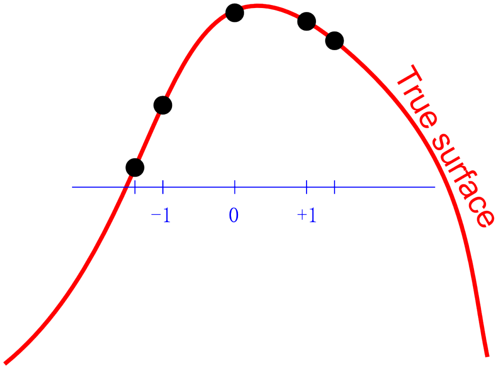
Improving the model's prediction ability by adding quadratic terms

Current situation, displaying lack of fit:



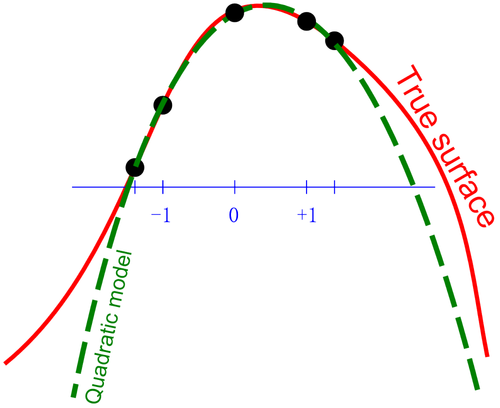
Improving the model's prediction ability by adding quadratic terms

Add specially placed points:

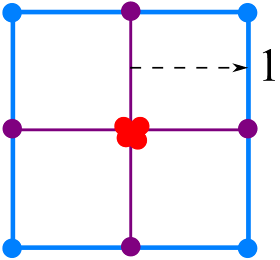


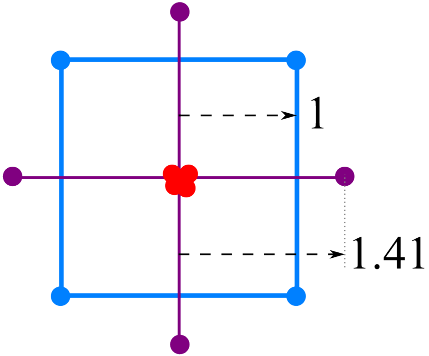
Improving the model's prediction ability by adding quadratic terms

And fit a quadratic model now:



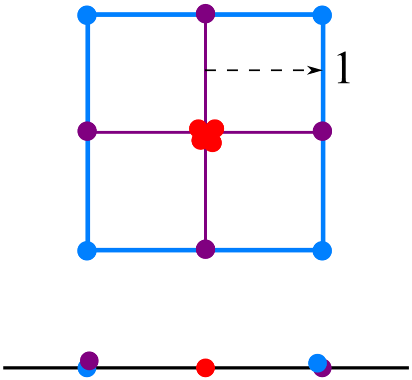
Adding experiments so we can fit quadratic terms: where do we put them?

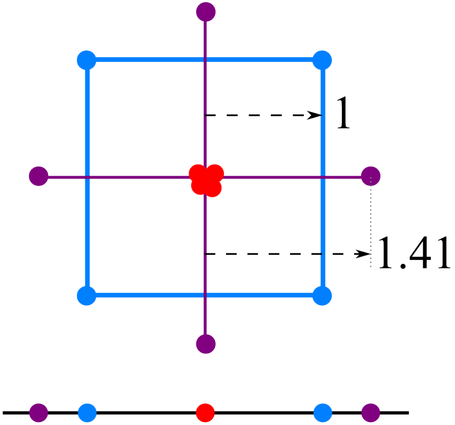




$\alpha = 1.41$

Adding experiments so we can fit quadratic terms: where do we put them?





$\alpha = 1.41$

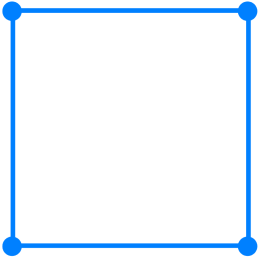
Adding experiments so we can fit quadratic terms: where do we put them?

- Run the factorial points first
- Then run the star (axial) points after*

$$\alpha = \left(2^k\right)^{0.25}$$

  - $k = 2$; then $\alpha = \sqrt{2} \approx 1.41$
  - $k = 3$; then $\alpha = 2^{\frac{3}{4}} \approx 1.68$
- Run several center points randomly, during the above two stages

* Astute viewers will wonder about confounding disturbances, since these runs are not randomized.
You can show that these **two groups** of experiments are blocked.



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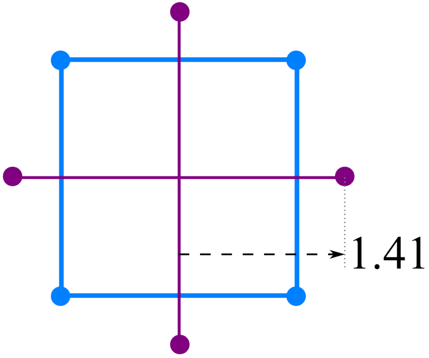

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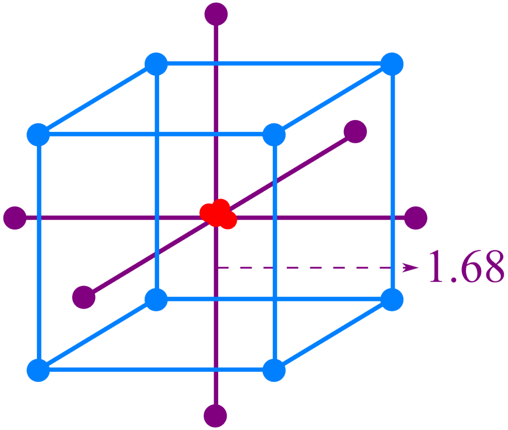

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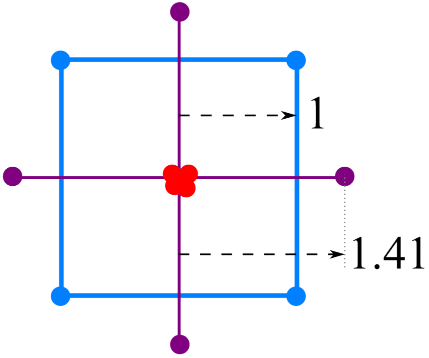

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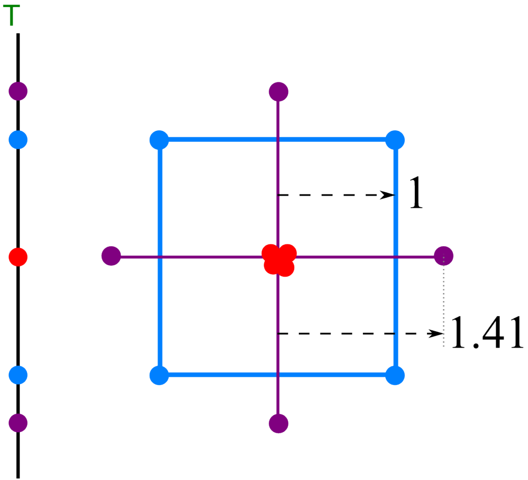

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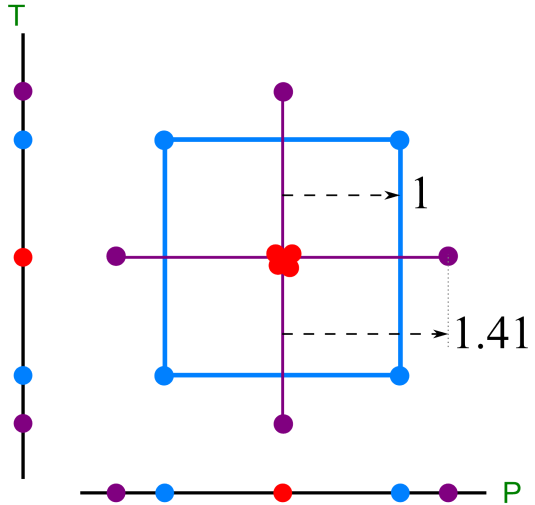

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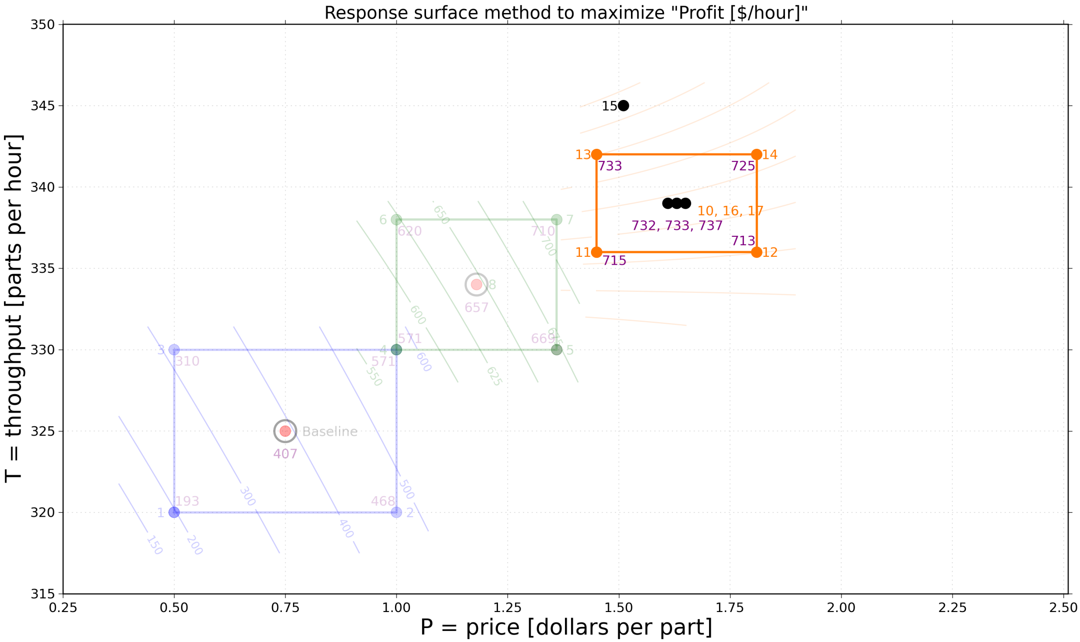

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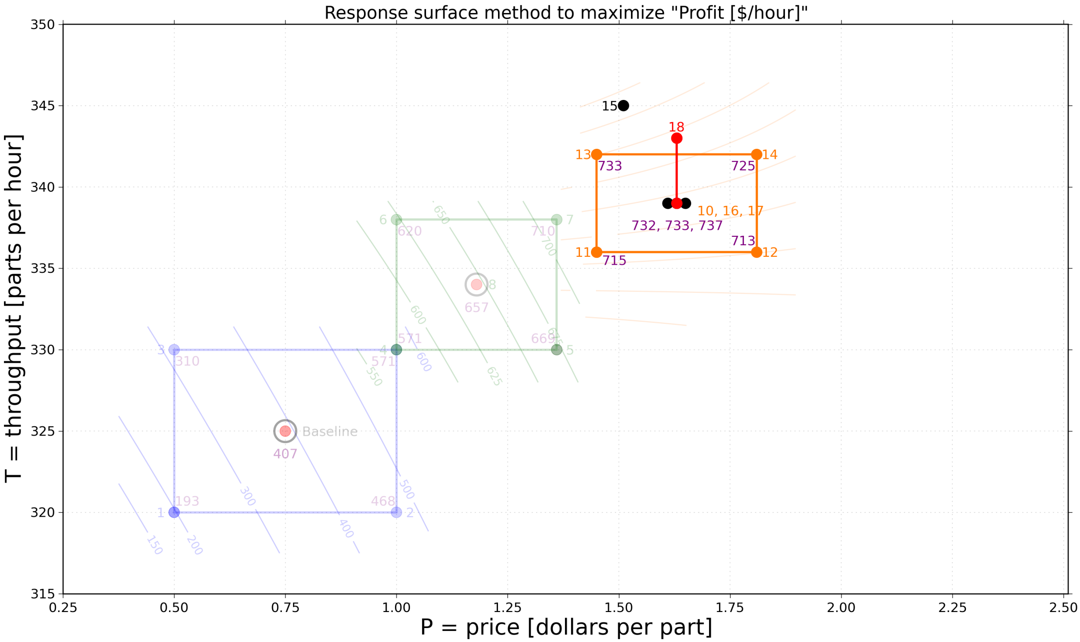

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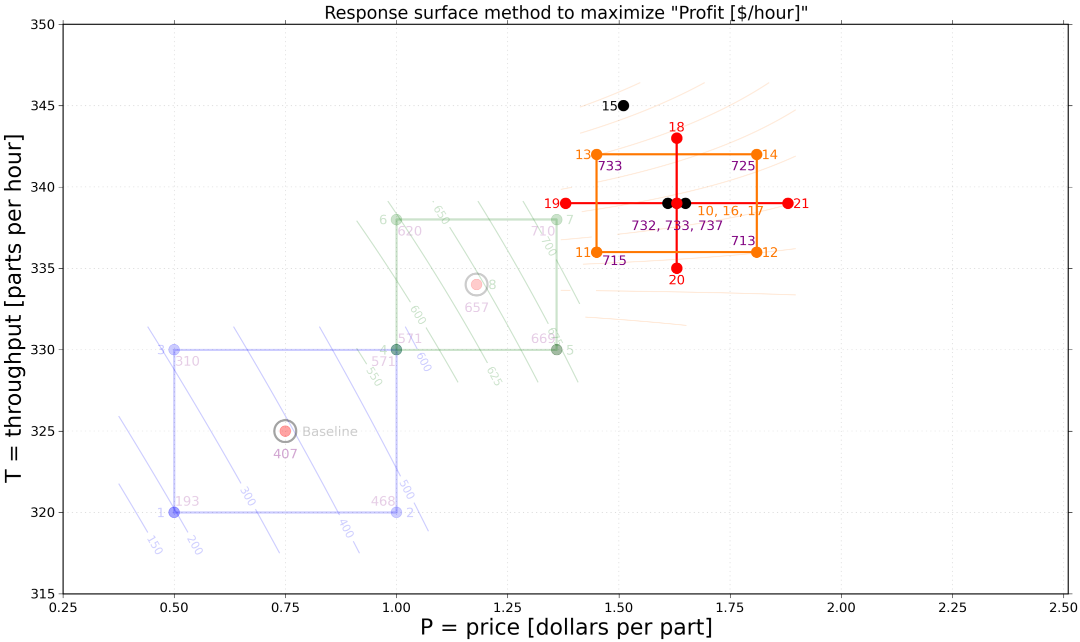

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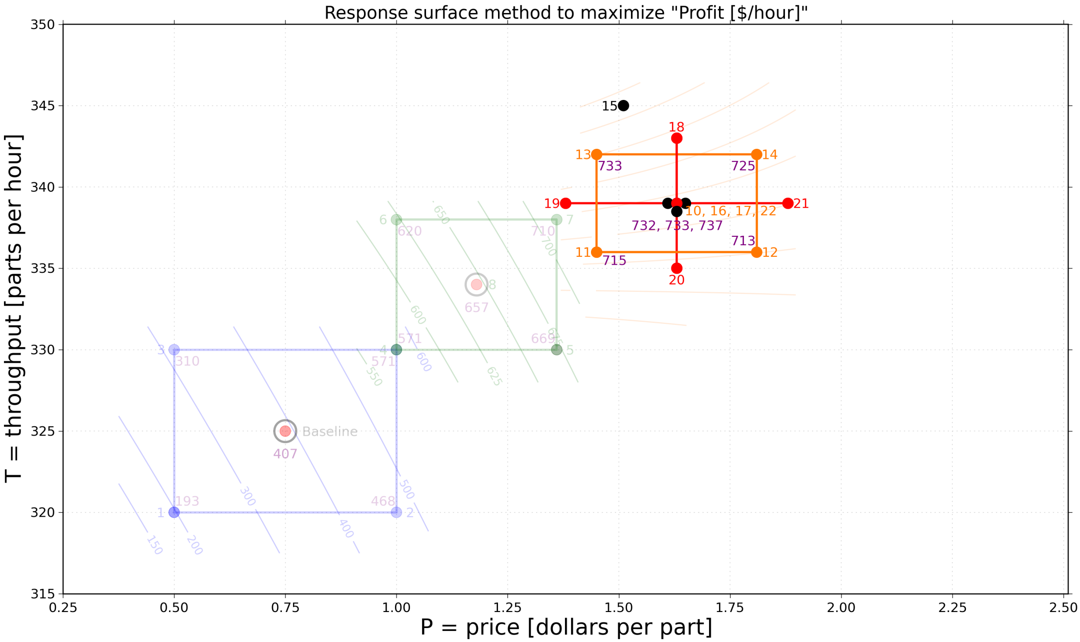
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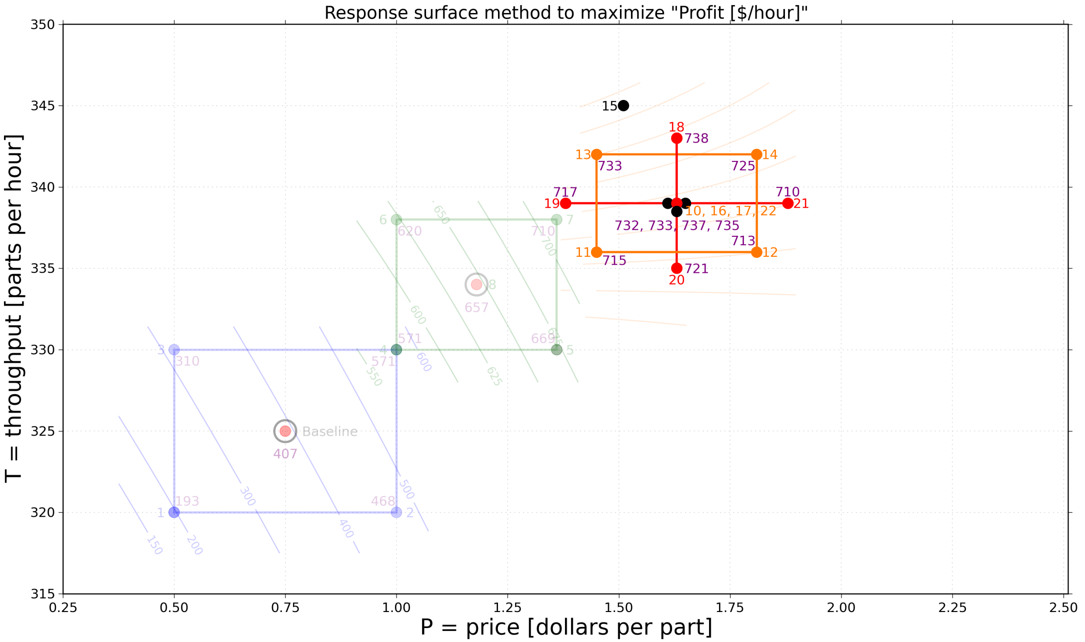
10, 16, 17, 22



Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
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732, 733, 737, 735
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721
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List of all experiments

Experiment	P	T	x_{P}	x_{T}	$\hat{y}$	Actual $= y$	Type
10	1.63	339	0	0		\$ 732	Center
11	1.45	336	-1	-1		715	Factorial
12	1.81	336	$+1$	-1		713	Factorial
13	1.45	342	-1	$+1$		733	Factorial
14	1.81	342	$+1$	$+1$		725	Factorial
15	1.51	345	$-\frac{2}{3}$	$+2$	\$ 742	735	Steepest path
16	1.63	339	0	0		\$ 733	Center
17	1.63	339	0	0		737	Center
18	1.63	343	0	$+1.41$		\$ 738	Star
19	1.38	339	-1.41	0		717	Star
20	1.63	335	0	-1.41		721	Star
21	1.88	339	$+1.41$	0		710	Star
22	1.63	339	0	0		\$ 735	Center
23							Optimum?

Results of the 12 runs: (factorial + star + center). Are we nearly there?

$$\hat{y} = 734.23 - 2.5x_{\mathsf{P}} + 6.97x_{\mathsf{T}} - 10.6x_{\mathsf{P}}^2 - 2.5x_{\mathsf{T}}^2 - 1.5x_{\mathsf{P}}x_{\mathsf{T}}$$

1. We get good predictions at the center, indicating a "small lack of fit"
   - $\hat{y}^{(\text{center})} = \$734$
   - $y_{\text{actual}}^{(\text{center})} = \$734.25$

2. We have good predictions of other points in the region

```
# Predict value of point 15 (actual value was $735)
predict(model.7, data.frame(P=-2/3, T=2))
```

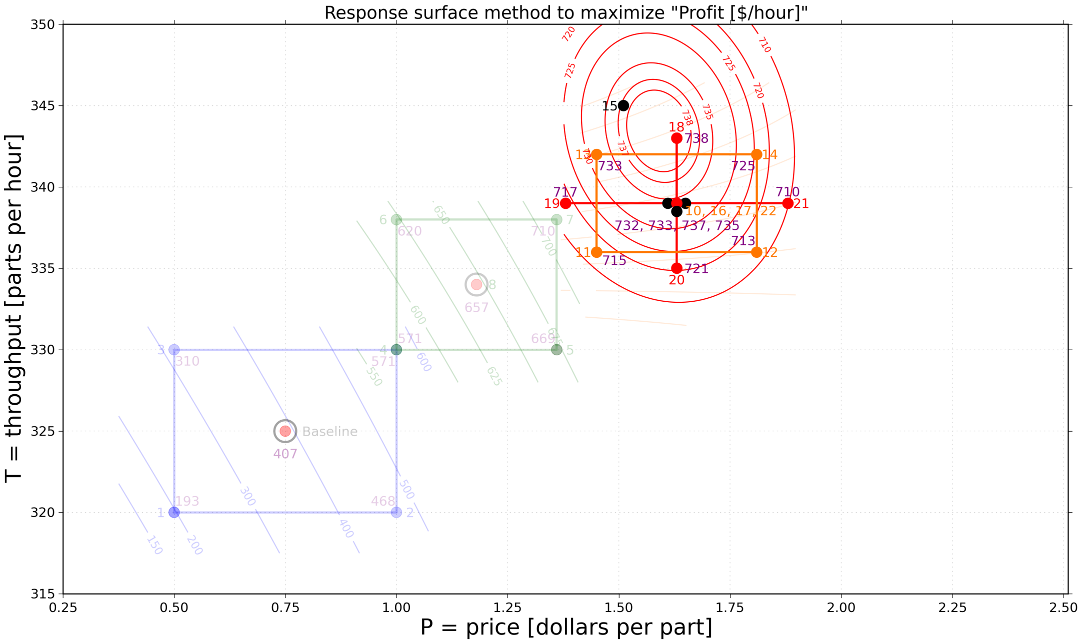
   - $\hat{y}^{(15)} = \$737$
   - $y_{\text{actual}}^{(15)} = \$735$

Output from the R software:

```
Residuals:
    Min      1Q  Median      3Q     Max 
-2.2248 -0.5393 -0.2176  0.7893  2.7752 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 734.2248     0.8642 849.571  < 2e-16 ***
P            -2.5098     0.6175  -4.064  0.00662 ** 
T             6.9706     0.6298  11.069 3.24e-05 ***
I(P^2)      -10.5762     0.6989 -15.133 5.25e-06 ***
I(T^2)       -2.4604     0.7435  -3.309  0.01622 *  
P:T          -1.5000     0.8655  -1.733  0.13379    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.731 on 6 degrees of freedom
Multiple R-squared:  0.9841,	Adjusted R-squared:  0.9709 
F-statistic: 74.37 on 5 and 6 DF,  p-value: 2.581e-05
```



Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
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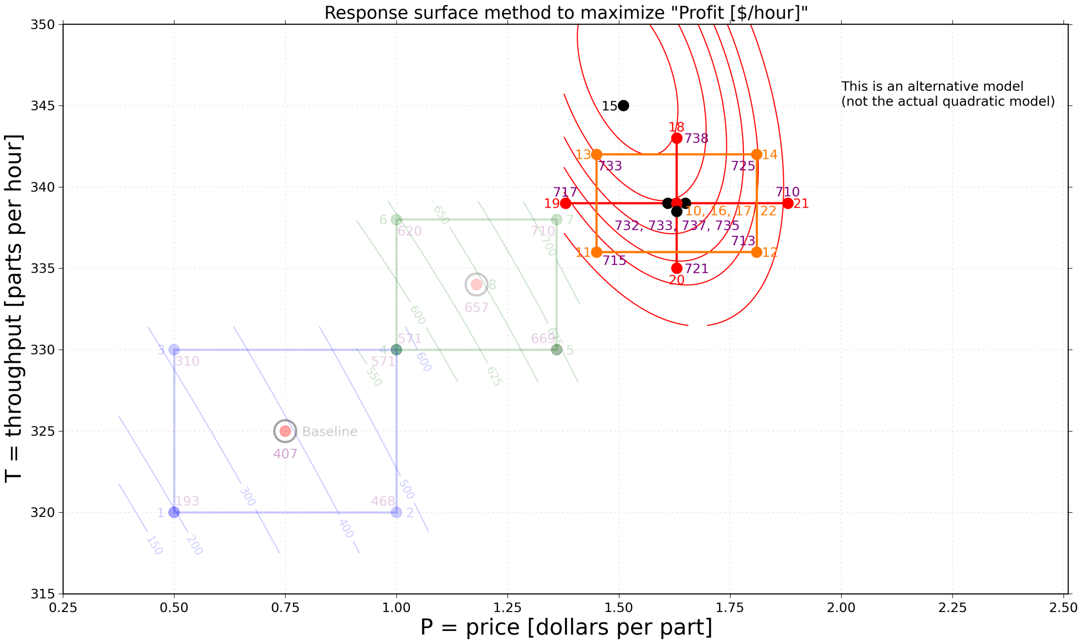

Response surface method to maximize "Profit [$/hour]"
This is an alternative model
(not the actual quadratic model)
T = throughput [parts per hour]
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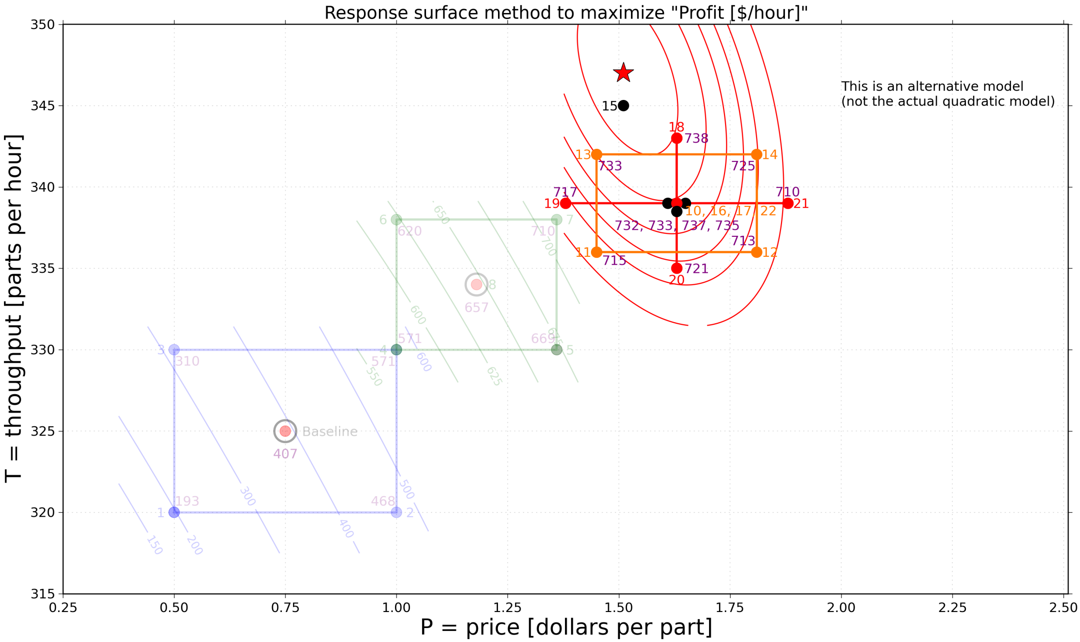

Response surface method to maximize "Profit [$/hour]"
This is an alternative model
(not the actual quadratic model)
Baseline
10, 16, 17, 22
732, 733, 737, 735
P = price [dollars per part]
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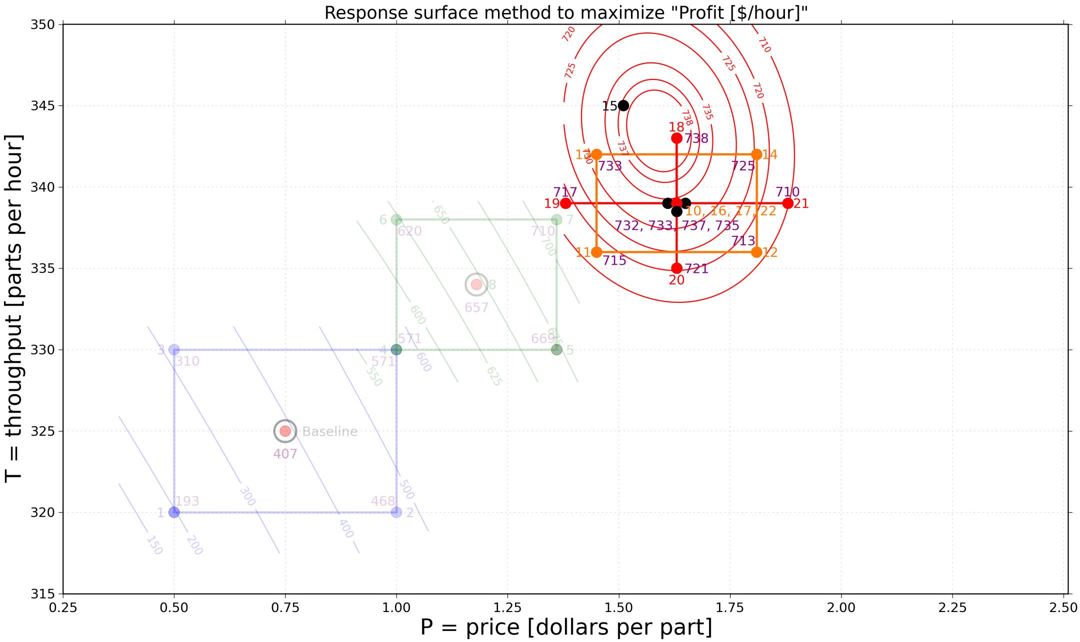

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Finding the optimum analytically (with differentiation)

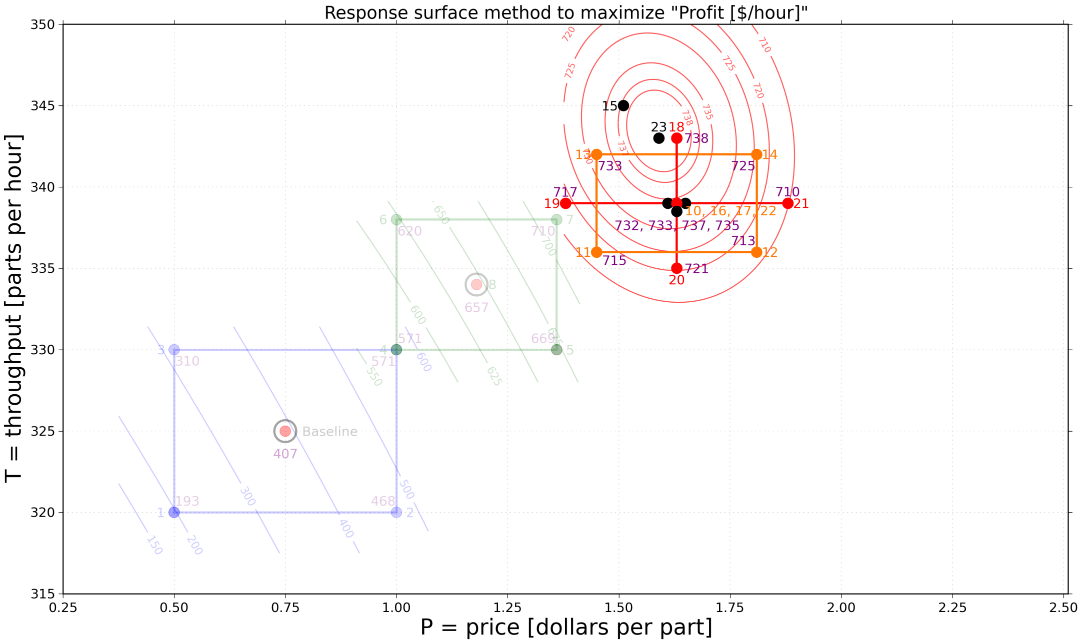
$$\hat{y} = 734 - 2.5x_{\mathsf{P}} + 6.97x_{\mathsf{T}} - 10.6x_{\mathsf{P}}^2 - 2.5x_{\mathsf{T}}^2 - 1.5x_{\mathsf{P}}x_{\mathsf{T}}$$

$$\begin{aligned} \frac{\partial \hat{y}}{\partial x_{\mathsf{P}}} &= -2.5 - 21.2x_{\mathsf{P}} - 1.5x_{\mathsf{T}} = 0 \\ \frac{\partial \hat{y}}{\partial x_{\mathsf{T}}} &= 6.97 - 1.5x_{\mathsf{P}} - 5.0x_{\mathsf{T}} = 0 \end{aligned}$$

$$\begin{pmatrix} -21.2 & -1.5 \\ -1.5 & -5.0 \end{pmatrix} \begin{pmatrix} x_{\mathsf{P}} \\ x_{\mathsf{T}} \end{pmatrix} = \begin{pmatrix} 2.5 \\ -6.97 \end{pmatrix}$$

$$x_{\mathsf{P}}^{(23)} = -0.22 \qquad \mathsf{P}^{(23)} = -0.22 \cdot \tfrac{1}{2}(0.36) + 1.63 = \$1.59$$

$$x_{\mathsf{T}}^{(23)} = 1.46 \qquad \mathsf{T}^{(23)} = 1.46 \cdot \tfrac{1}{2}(6) + 339 \approx 343 \text{ parts per hour}$$



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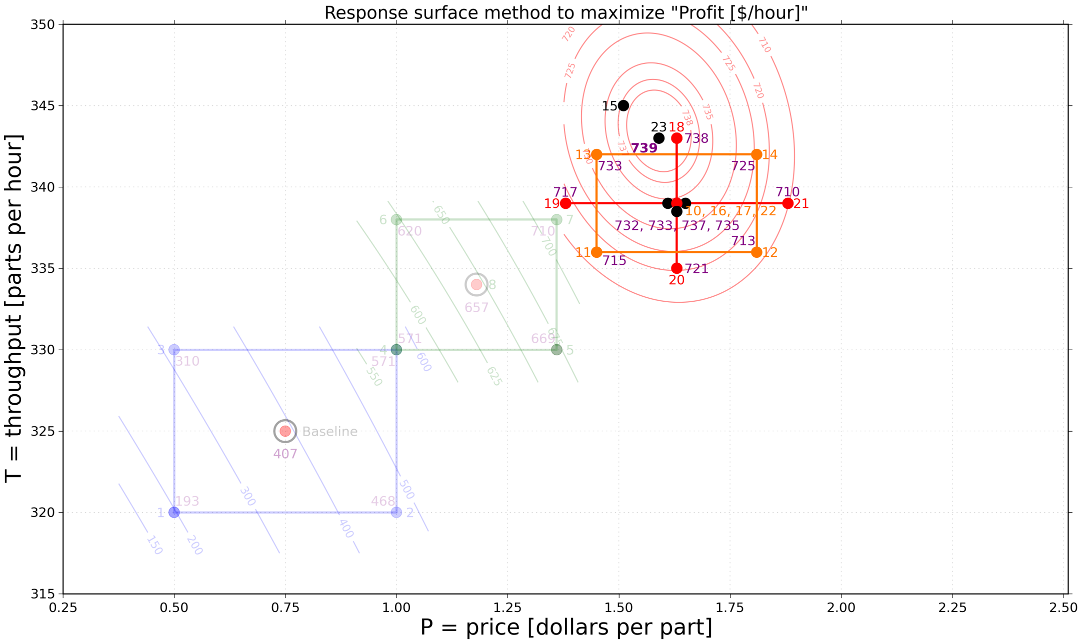

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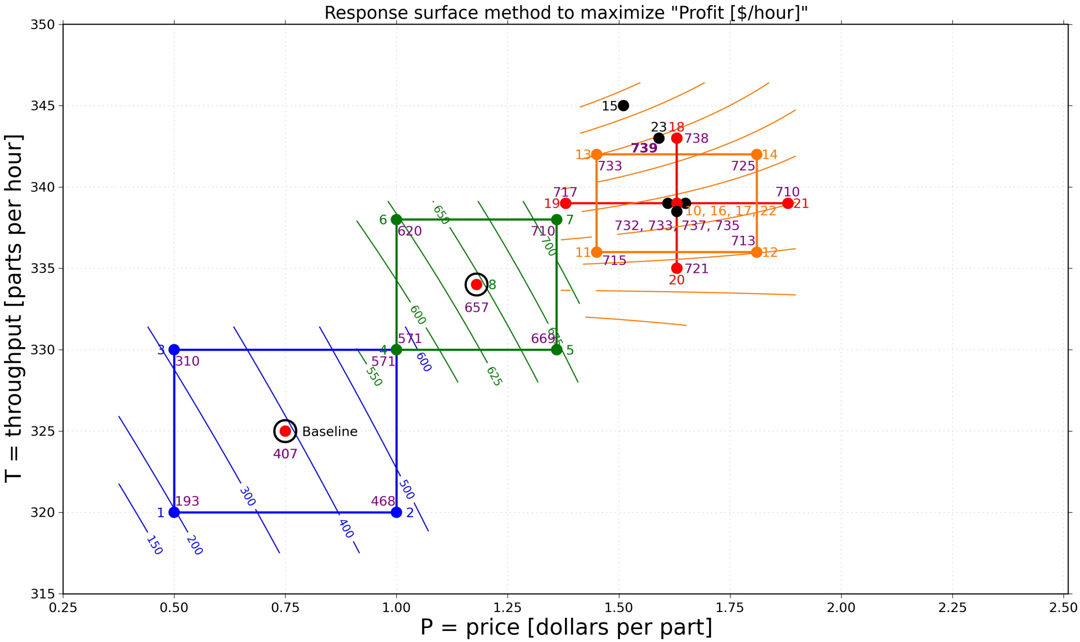

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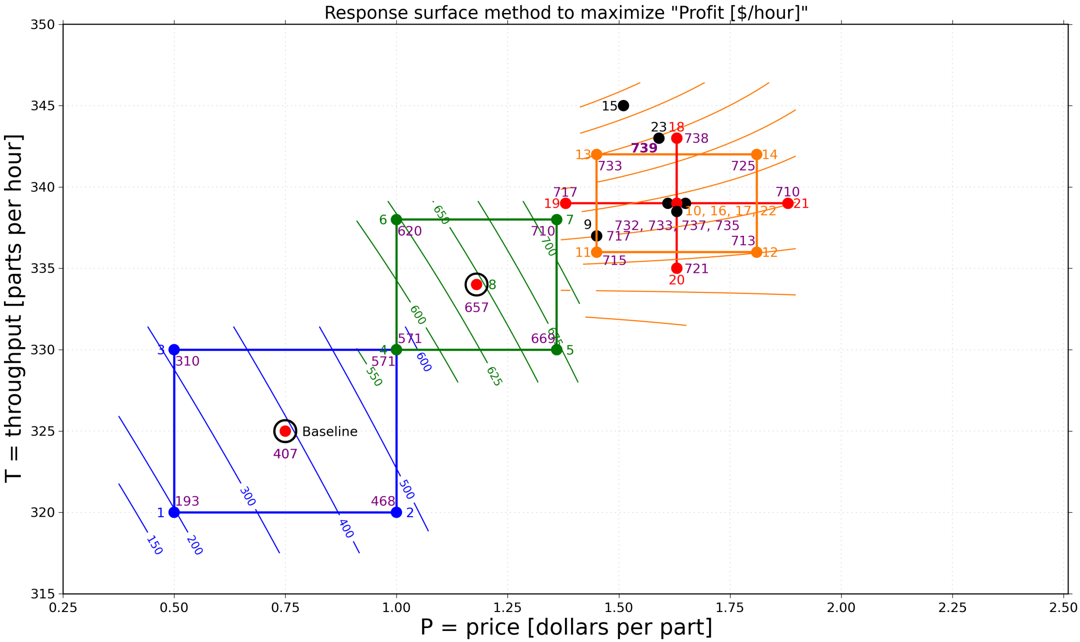

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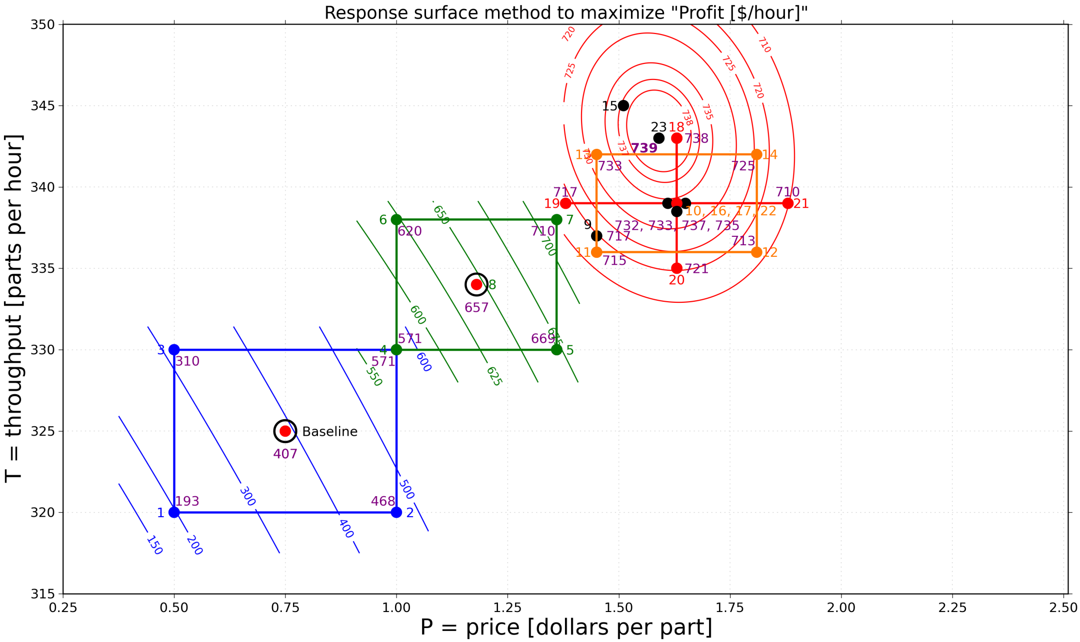

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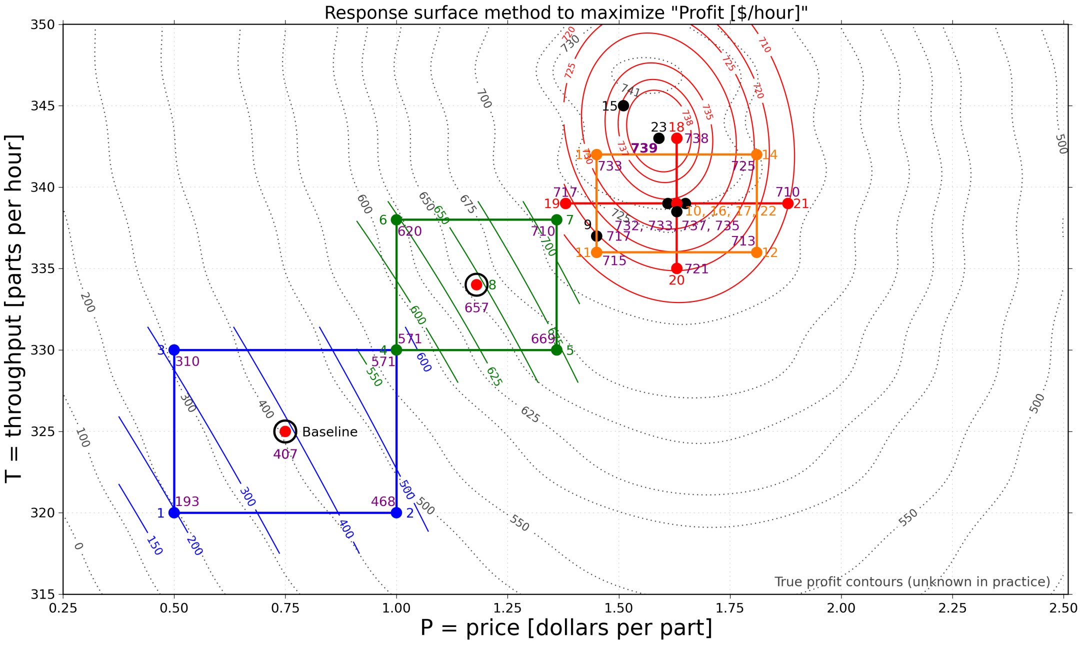
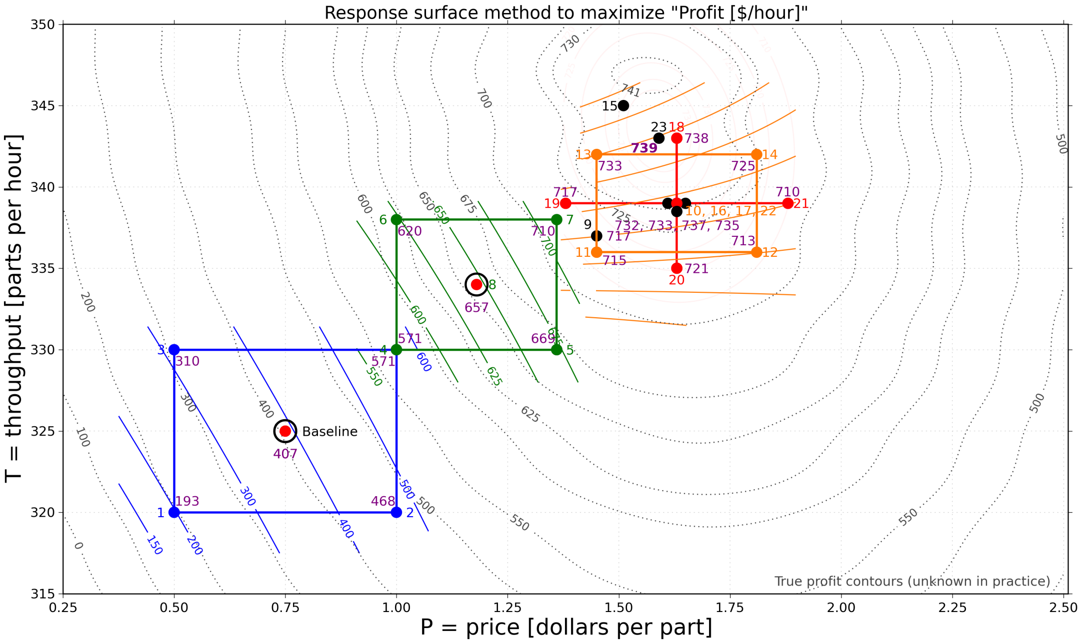


Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
Baseline
True profit contours (unknown in practice)



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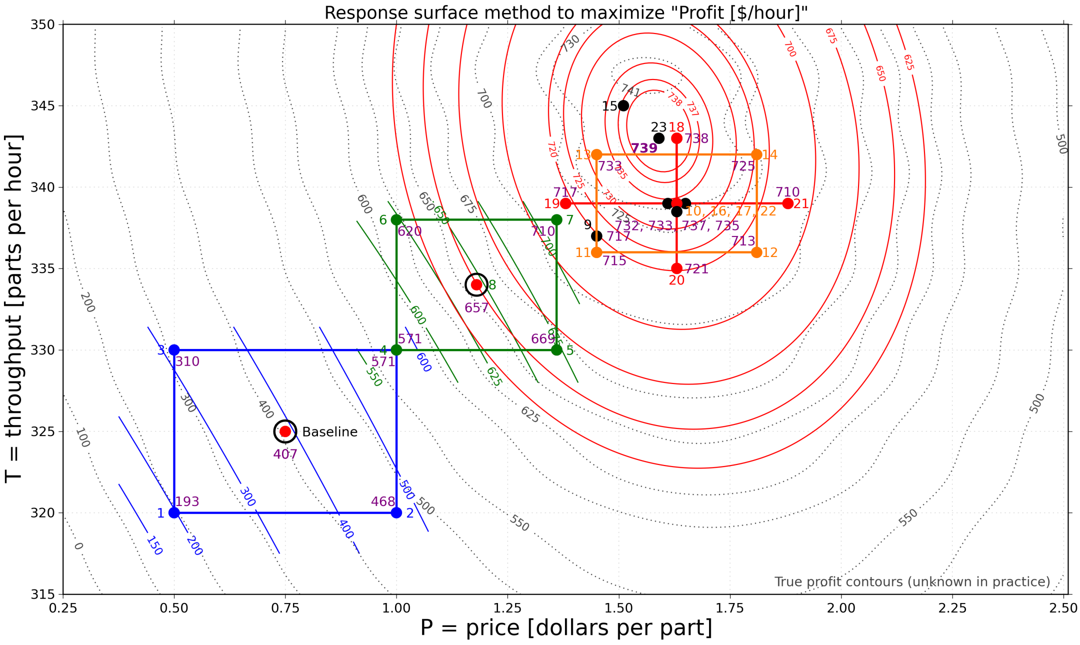

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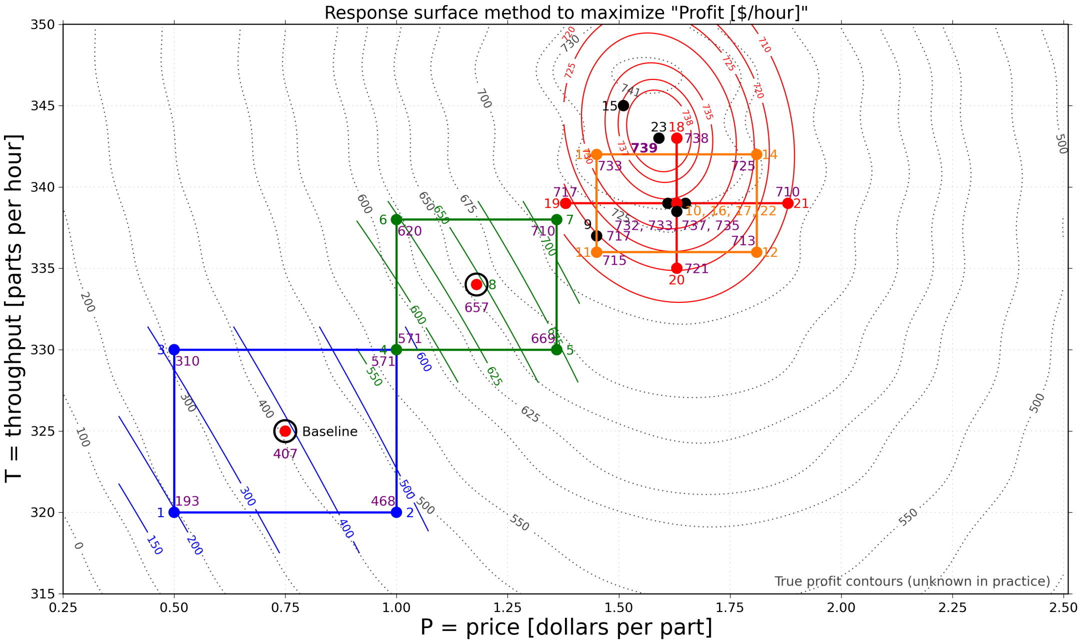

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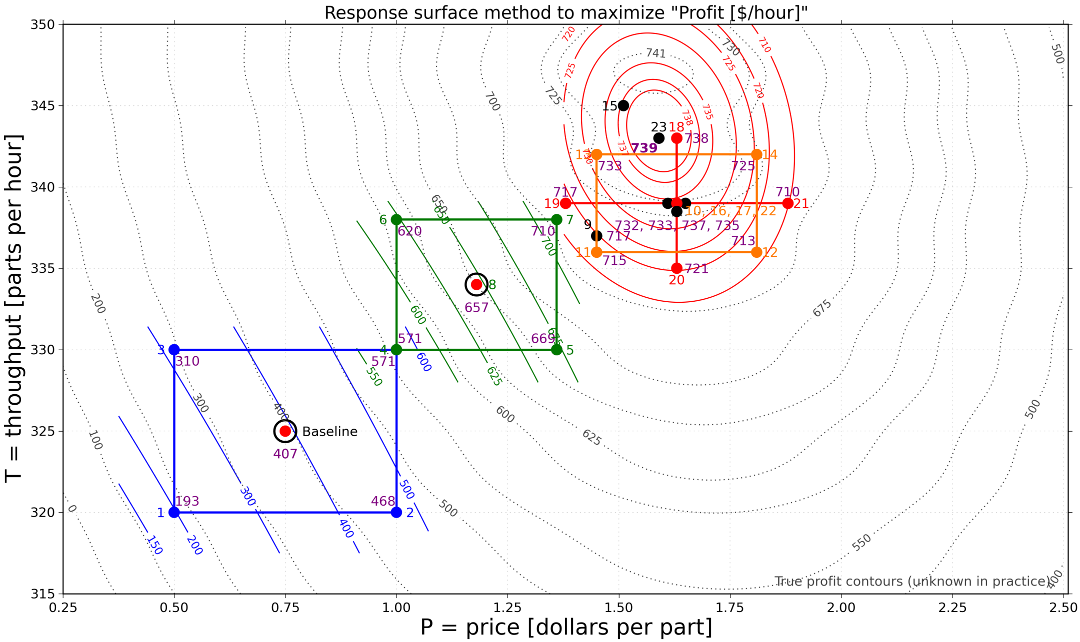
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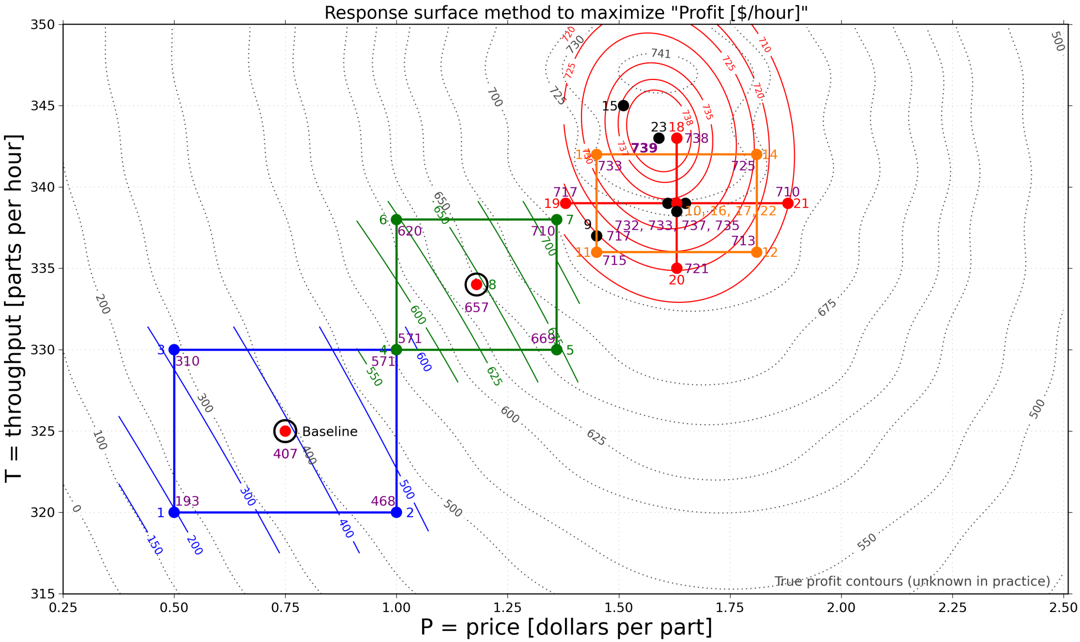

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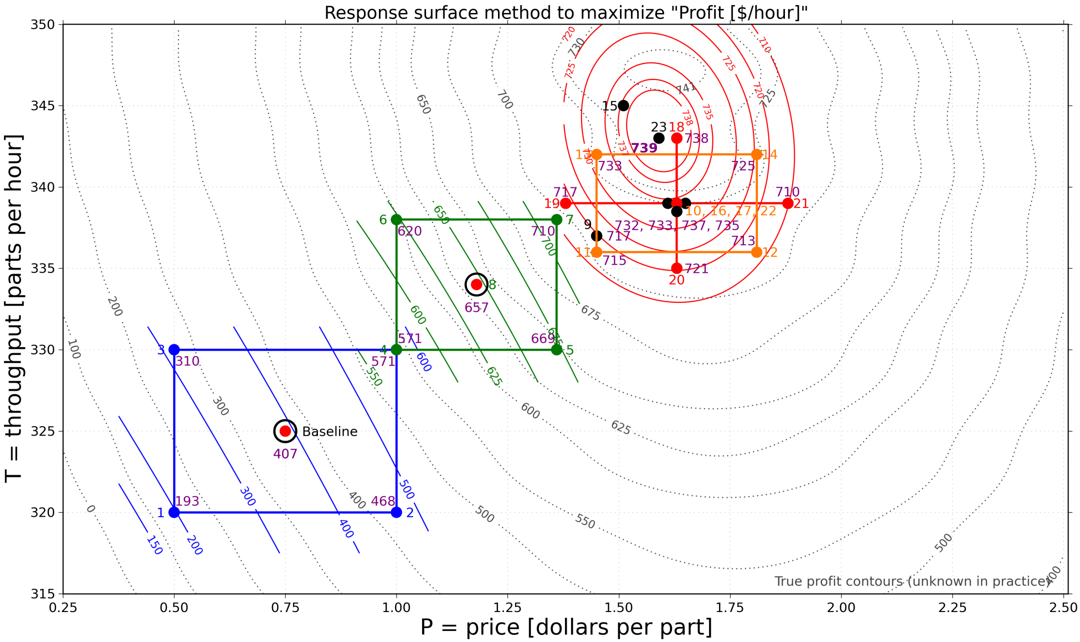

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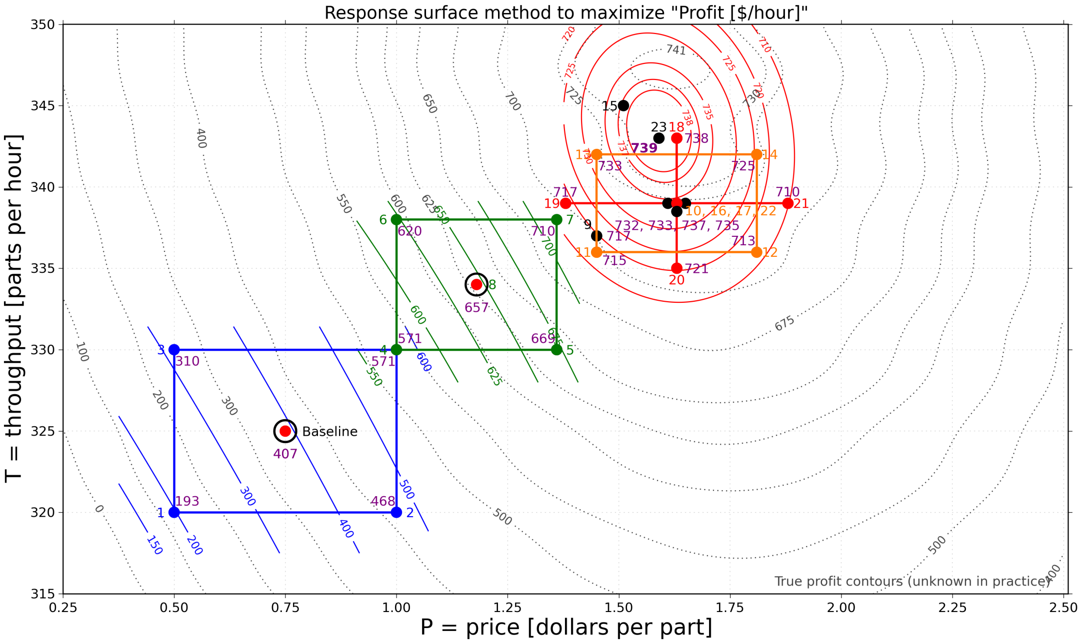

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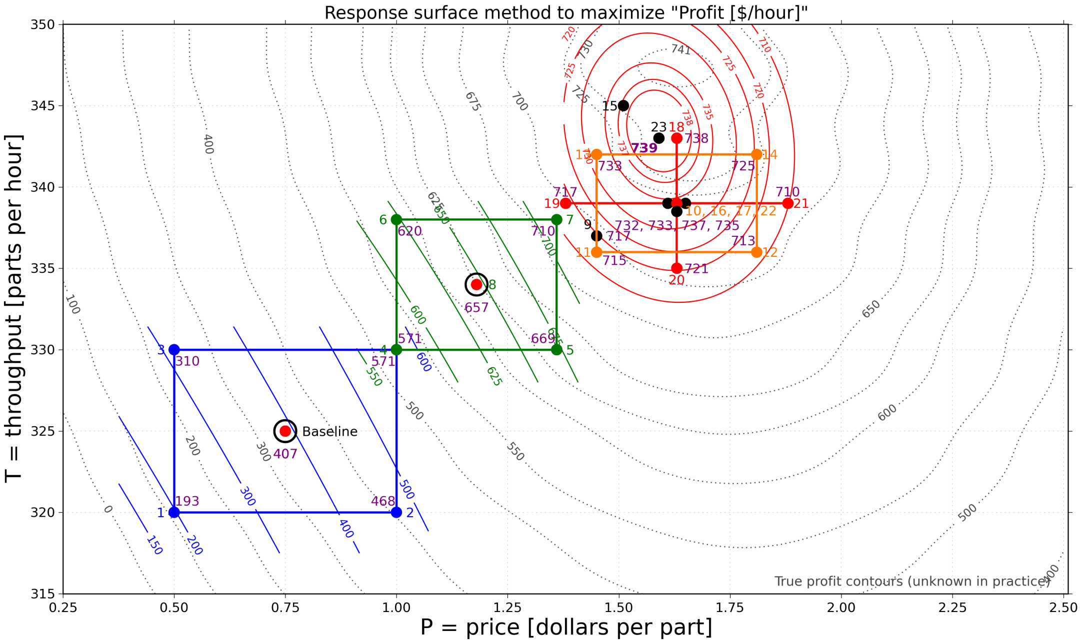

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