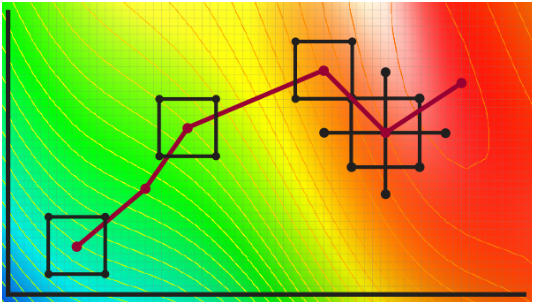
Experimentation for Improvement



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Design and Analysis of Experiments

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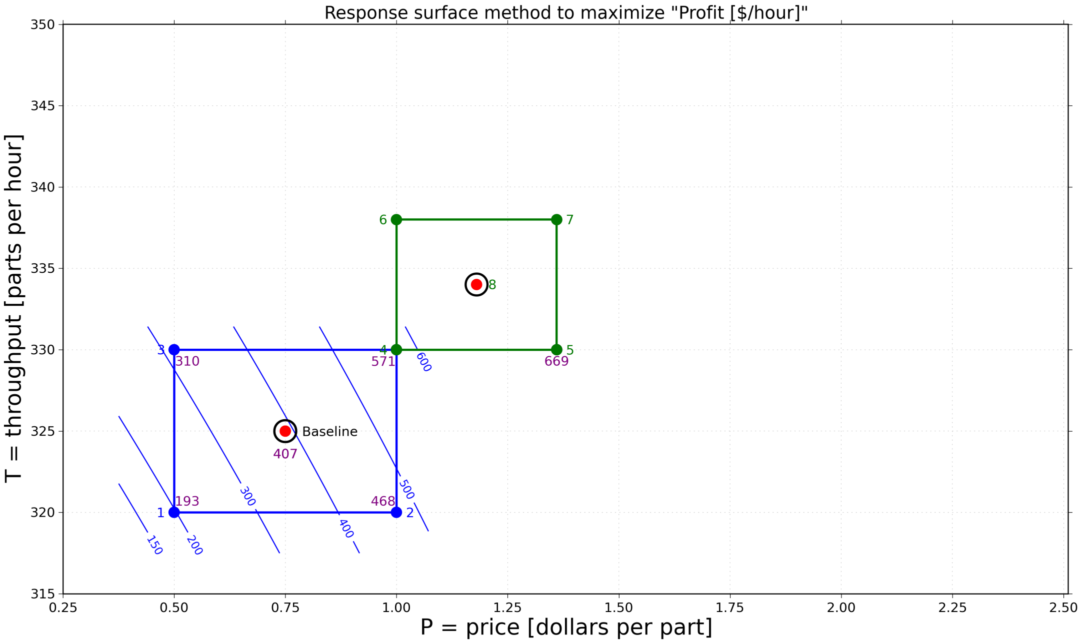
Outline of topics in this video

1. We keep climbing up the mountain ...
2. but, what happens if we encounter a constraint?
3. and, what happens when mistakes happen?

$$\text{coded value} = \frac{(\text{real value}) - (\text{center point})}{\frac{1}{2}\text{range}}$$

$$x_{\mathsf{T}} = \frac{337 - 339}{\frac{1}{2}(6)} = -\frac{2}{3}$$


Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
Baseline
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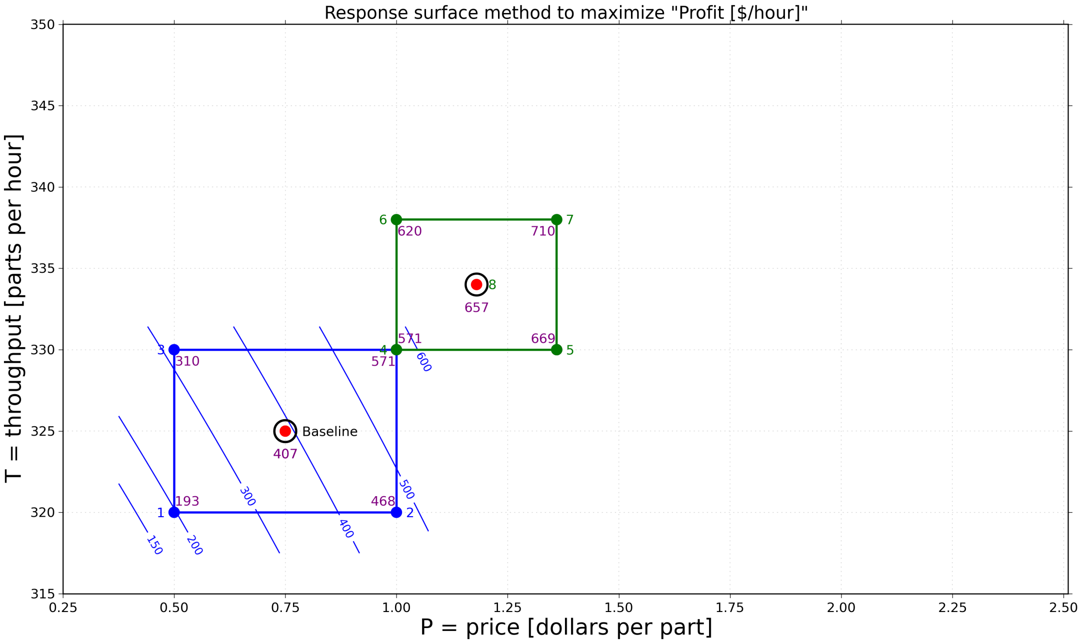

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T = throughput [parts per hour]
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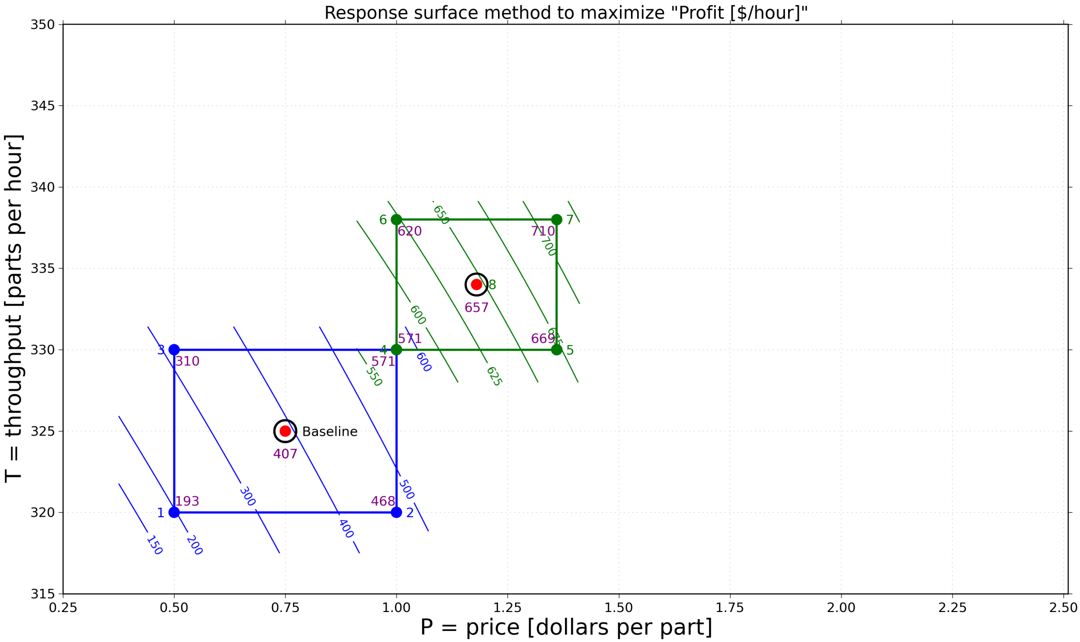
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Taking the next step for experiment 9: fill in the blanks

1. Pick change in coded units in one factor.
2. Find the ratios for the other factor(s).
3. Calculate step size in coded units.
4. Convert these to real-world *changes*.
5. Get the real-world location of the next experiment.
6. Convert these back to coded-units.
7. Predict the next experiment's outcome.
8. Now run the next experiment, and record the values

Price	Throughput
$\Delta x_{\mathsf{P}} = 1.5$ (this was chosen)	
$\Delta x_{\mathsf{P}} = 1.5$	$\Delta x_{\mathsf{T}} =$ ____________
$\Delta \mathsf{P} =$ ____________	$\Delta \mathsf{T} =$ ____________ parts per hour
$\mathsf{P}^{(9)} =$ ____________	$\mathsf{T}^{(9)} =$ ____________ parts per hour
$x_{\mathsf{P}}^{(9)} =$ ____________	$x_{\mathsf{T}}^{(9)} =$ ____________

$\hat{y} =$ ____________ profit per hour

$y^{(9)} =$ ____________ profit per hour

Taking the next step for experiment 9

	Price	Throughput
1. Pick change in coded units in one factor.	$\Delta x_{\mathsf{P}} = 1.5$ (this was chosen)	
2. Find the ratios for the other factor(s).		$\Delta x_{\mathsf{T}} = \dfrac{b_{\mathsf{T}}}{b_{\mathsf{P}}} \cdot \Delta x_{\mathsf{P}}$ $\Delta x_{\mathsf{T}} = \dfrac{22.5}{47} \cdot 1.5$
3. Calculate step size in coded units.	$\Delta x_{\mathsf{P}} = 1.5$	$\Delta x_{\mathsf{T}} = 0.718$
4. Convert these to real-world *changes*.	$\Delta \mathsf{P} = \Delta x_{\mathsf{P}} \cdot \frac{1}{2}(0.36)$ $\Delta \mathsf{P} = \$0.27$	$\Delta \mathsf{T} = \Delta x_{\mathsf{T}} \cdot \frac{1}{2}(8)$ $\Delta \mathsf{T} = 2.87 \approx 3$ parts per hour
5. Finally, take a step from the baseline! Get the real-world location of the next experiment.	$\mathsf{P}^{(9)} = \mathsf{P}^{(8)} + \Delta \mathsf{P}$ $\mathsf{P}^{(9)} = \$1.18 + 0.27$ $\mathsf{P}^{(9)} = \$1.45$	$\mathsf{T}^{(9)} = \mathsf{T}^{(8)} + \Delta \mathsf{P}$ $\mathsf{T}^{(9)} = 334 + 3$ $\mathsf{T}^{(9)} = 337$ parts per hour

Taking the next step for experiment 9

5. Get the real-world location of the next experiment.
6. Convert these back to coded-units.
7. Predict the next experiment's outcome.
8. Now run the next experiment, and record the values

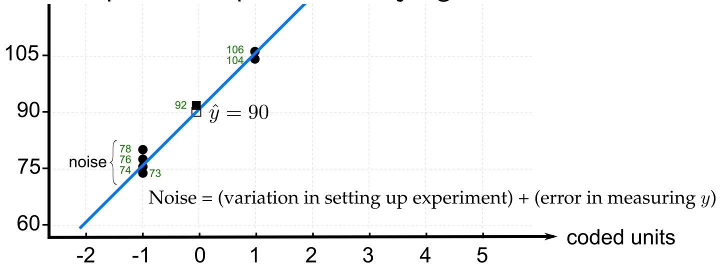
Price	**Throughput**
$\mathsf{P}^{(9)} = \$1.45$	$\mathsf{T}^{(9)} = 337$ parts per hour
$x_{\mathsf{P}}^{(9)} = 1.5$	$x_{\mathsf{T}}^{(9)} = \dfrac{337-334}{\frac{1}{2}(8)} = 0.75$ (not 0.718)

$$
\begin{aligned}
\hat{y} &= 645 + 47x_{\mathsf{P}} + 22.5x_{\mathsf{T}} - 2x_{\mathsf{P}}x_{\mathsf{T}}\\
\hat{y}^{(5)} &= 645 + 47(1.5) + 22.5(0.75) - 2(1.5)(0.75)\\
\hat{y}^{(5)} &\approx 645 + 71 + 17 - 2\\
\hat{y}^{(5)} &\approx 731 \text{ profit per hour}
\end{aligned}
$$

$$y^{(5)} = \$717 \text{ profit per hour}$$

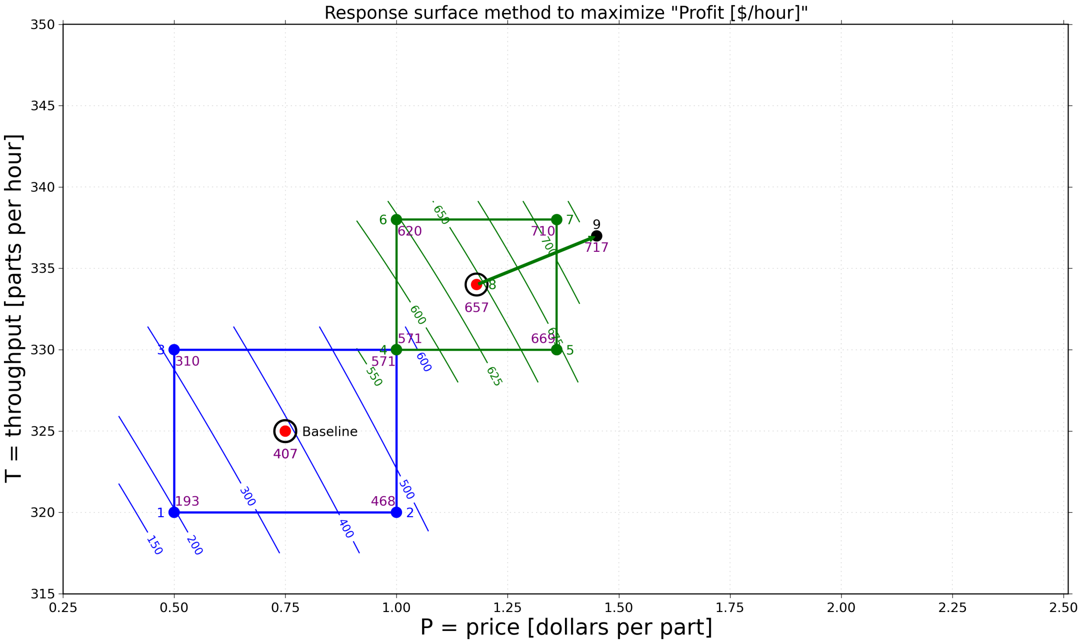
Judging the prediction error

1. We should have replicated experiments to judge the noise level



We will see more on replicated experiments in the next video

2. Contrast prediction error with the model's coefficients:
   - $\hat{y}^{(9)} = 645 + 47x_{\text{P}} + 22.5x_{\text{T}} - 2x_{\text{P}}x_{\text{T}} = \$731$
   - $y^{(9)}_{\text{actual}} = \$717$
   - error $= \hat{y} - y \approx \$13$ (ignoring the decimals)



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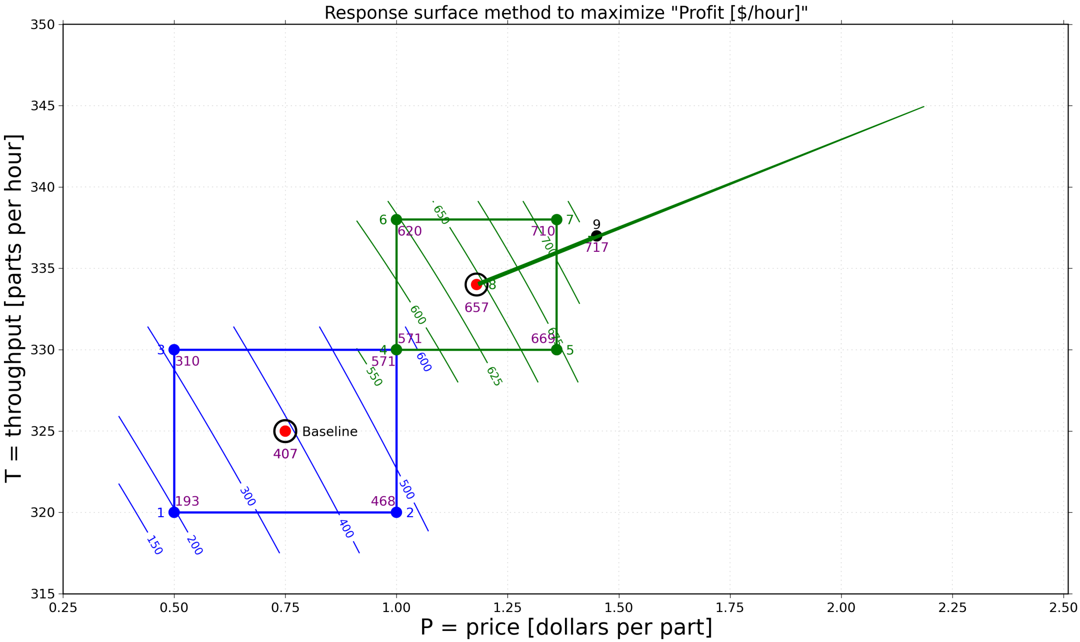

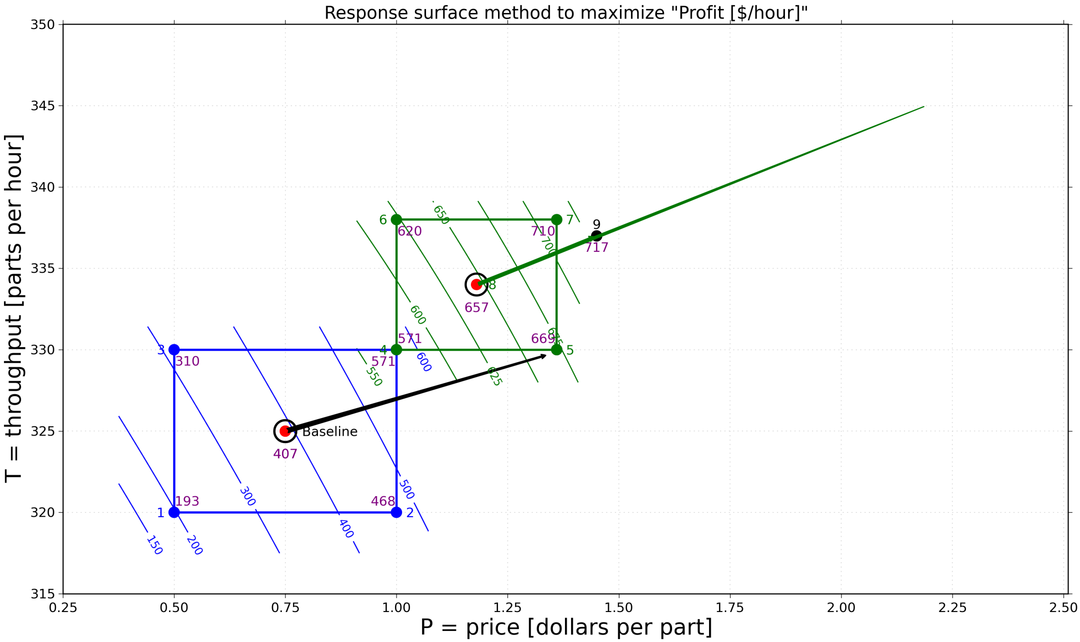

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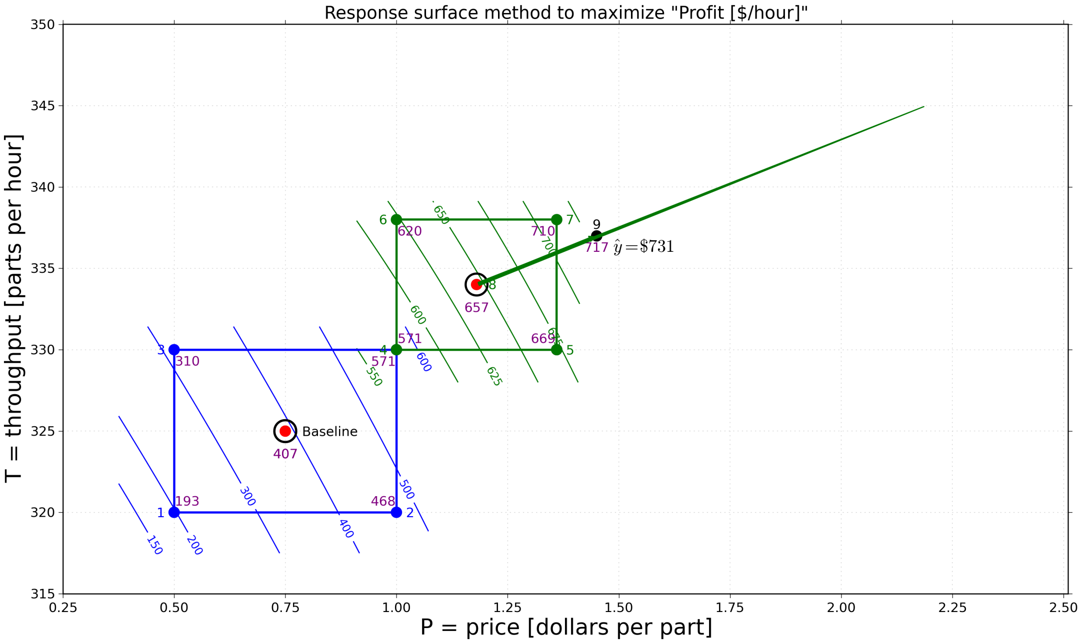

Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
Baseline
407
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657
717
$\hat{y} = \$731$

Taking the next step for experiment 10: fill in the blanks

1. Pick change in coded units in one factor.
2. Find the ratios for the other factor(s).
3. Calculate step size in coded units.
4. Convert these to real-world *changes*.
5. Get the real-world location of the next experiment.
6. Convert these back to coded-units.
7. Predict the next experiment's outcome.
8. Now run the next experiment, and record the values

Price	**Throughput**
$\Delta x_{\text{P}} = 2.5$ (this was chosen)	
$\Delta x_{\text{P}} = 2.5$	$\Delta x_{\text{T}} =$ __________
$\Delta \text{P} =$ __________	$\Delta \text{T} =$ __________ parts per hour
$\text{P}^{(10)} =$ __________	$\text{T}^{(10)} =$ __________ parts per hour
$x_{\text{P}}^{(10)} =$ __________	$x_{\text{T}}^{(10)} =$ __________

$\hat{y}^{(10)} =$ __________ profit per hour

$y^{(10)} =$ __________ profit per hour

Taking the next step for experiment 10: solution

	Price	Throughput
1. Pick change in coded units in one factor.	$\Delta x_{\mathsf{P}} = 2.5$ (this was chosen)	
2. Find the ratios for the other factor(s).		
3. Calculate step size in coded units.	$\Delta x_{\mathsf{P}} = 2.5$	$\Delta x_{\mathsf{T}} = 1.2 = \dfrac{b_{\mathsf{T}}}{b_{\mathsf{P}}} \times \Delta x_{\mathsf{P}} = \dfrac{22.5}{47} \times 2.5$
4. Convert these to real-world *changes*.	$\Delta \mathsf{P} = \$0.45$	$\Delta \mathsf{T} = 4.8 \approx 5$ parts per hour
5. Get the real-world location of the next experiment.	$\mathsf{P}^{(10)} = \$1.63$	$\mathsf{T}^{(10)} = 334 + 5 = 339$ parts per hour
6. Convert these back to coded-units.	$x_{\mathsf{P}}^{(10)} = 2.5$	$x_{\mathsf{T}}^{(10)} = \dfrac{339 - 334}{\frac{1}{2} \cdot (8)} = 1.25$

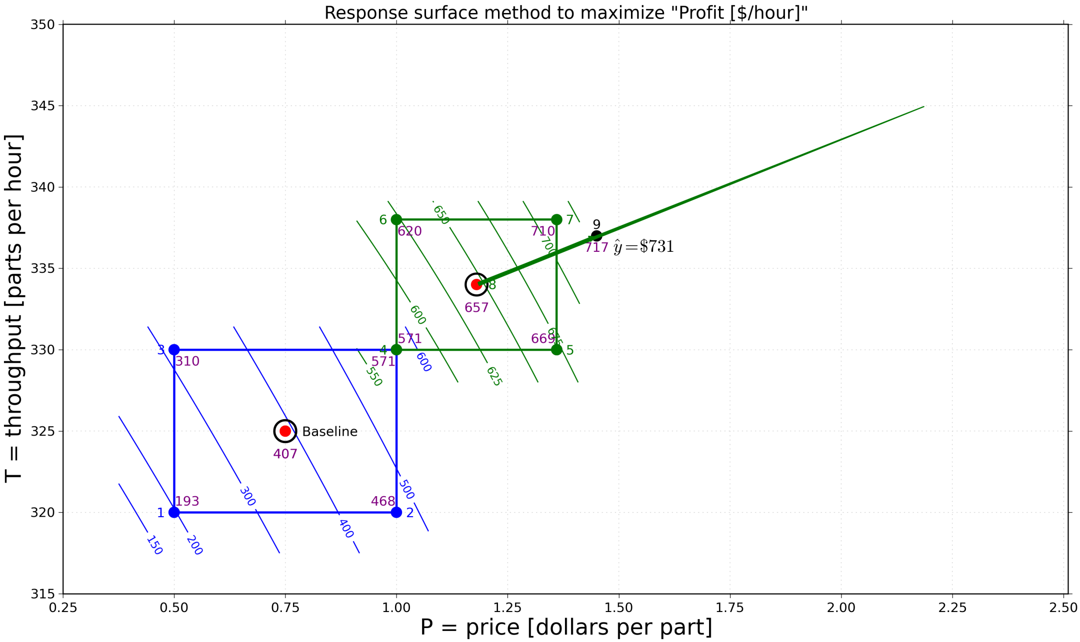
7. Predict the next experiment's outcome.

$$\hat{y} = 645 + 47x_{\mathsf{P}} + 22.5x_{\mathsf{T}} - 2x_{\mathsf{P}}x_{\mathsf{T}}$$

$$\hat{y}^{(10)} \approx \$785 \text{ profit per hour}$$

8. Now run the next experiment, and record the values

$$y^{(10)} = \$732 \text{ profit per hour}$$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$\hat{y} = \$731$
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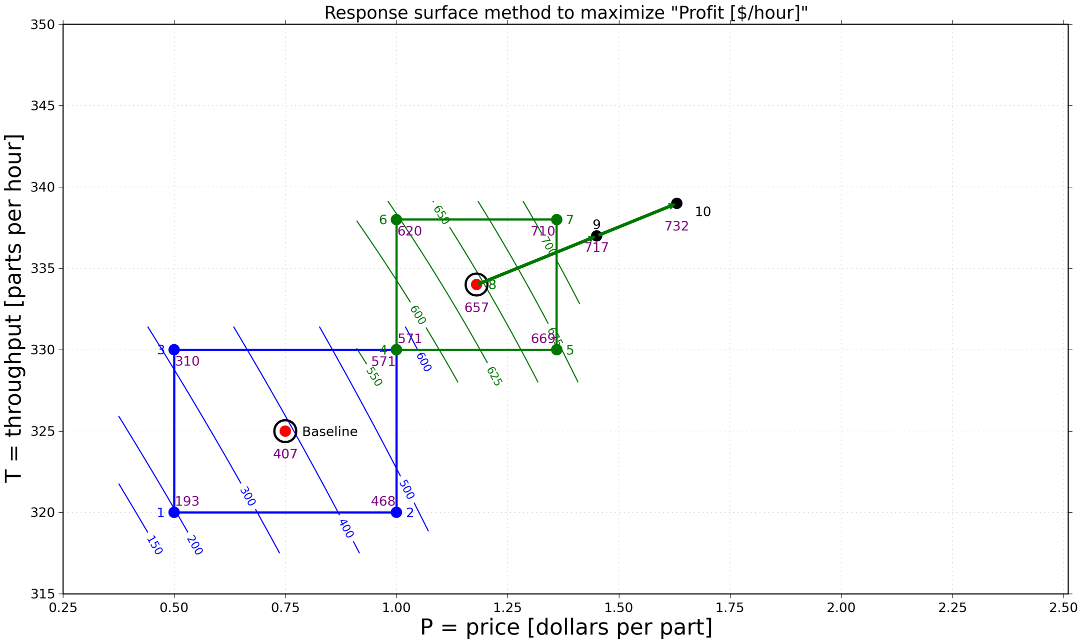
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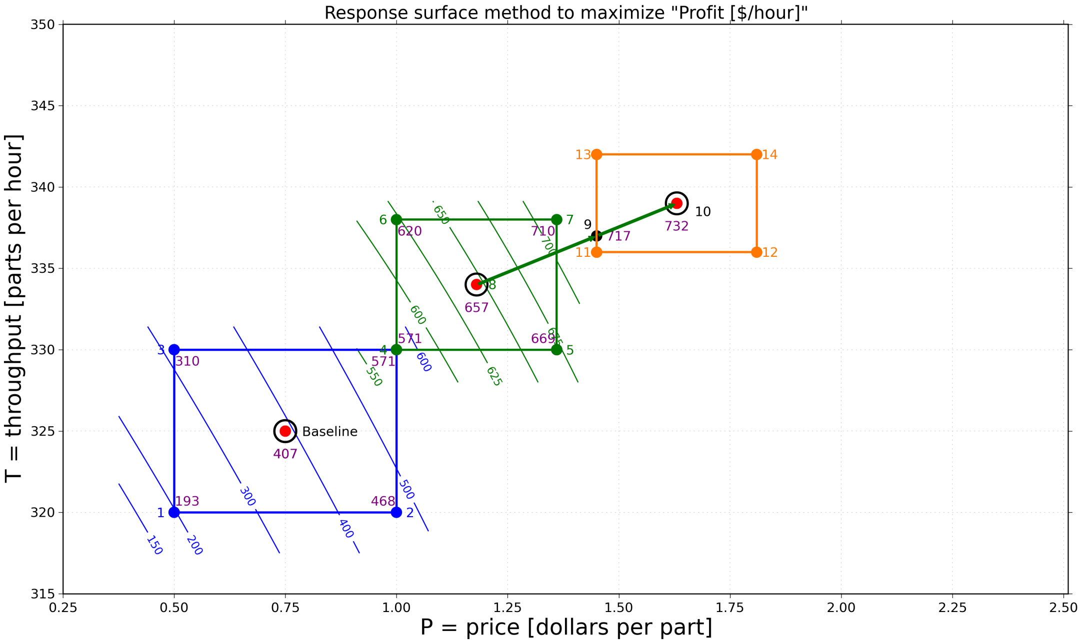
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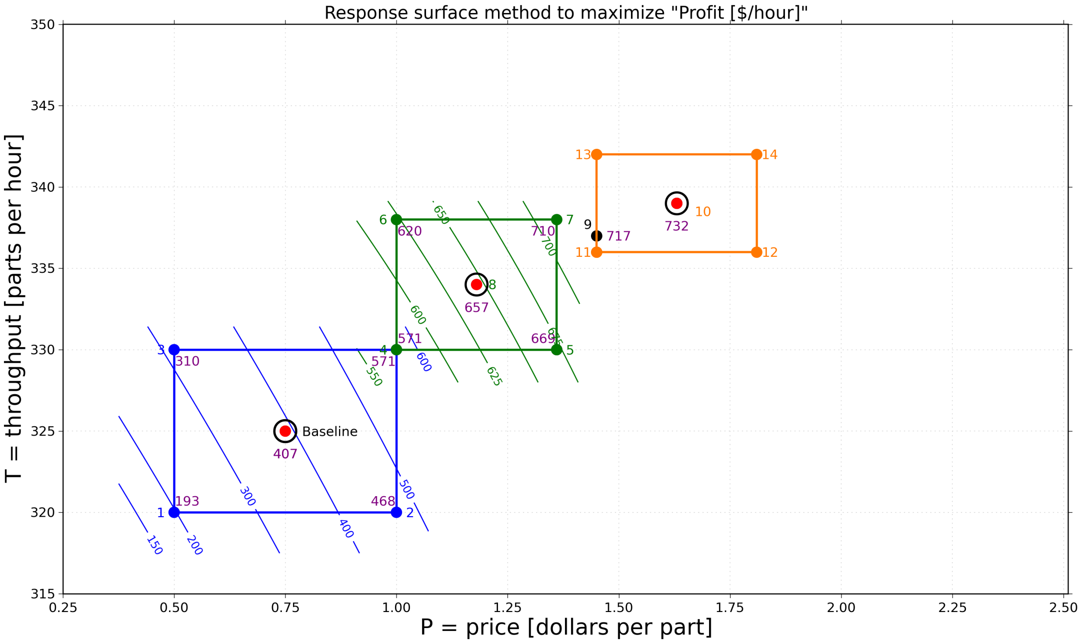

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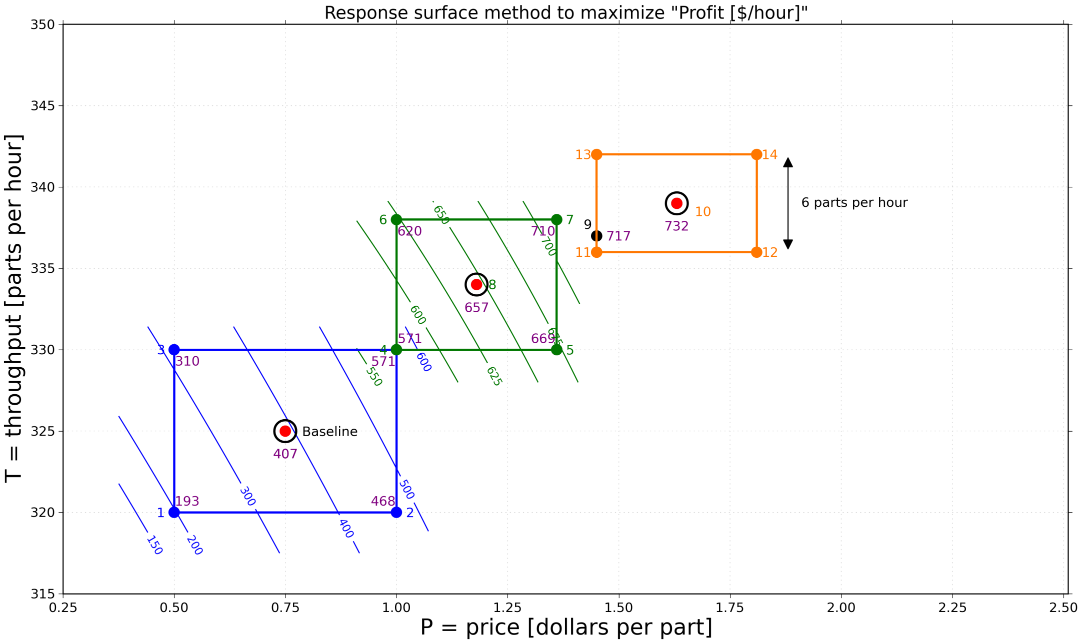

Response surface method to maximize "Profit [$/hour]"
T = throughput [parts per hour]
P = price [dollars per part]
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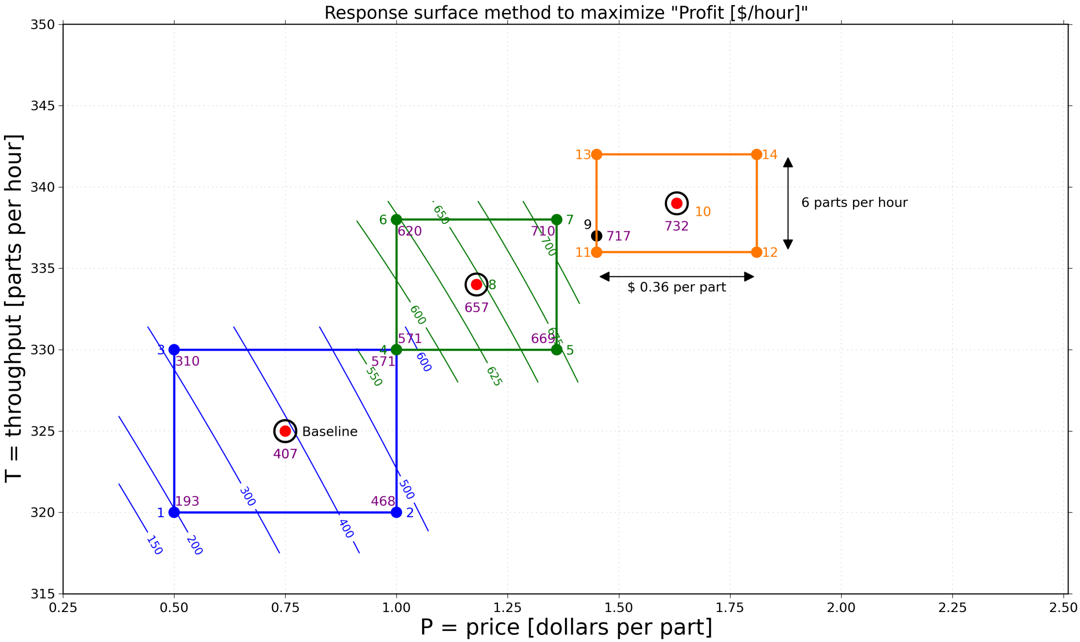

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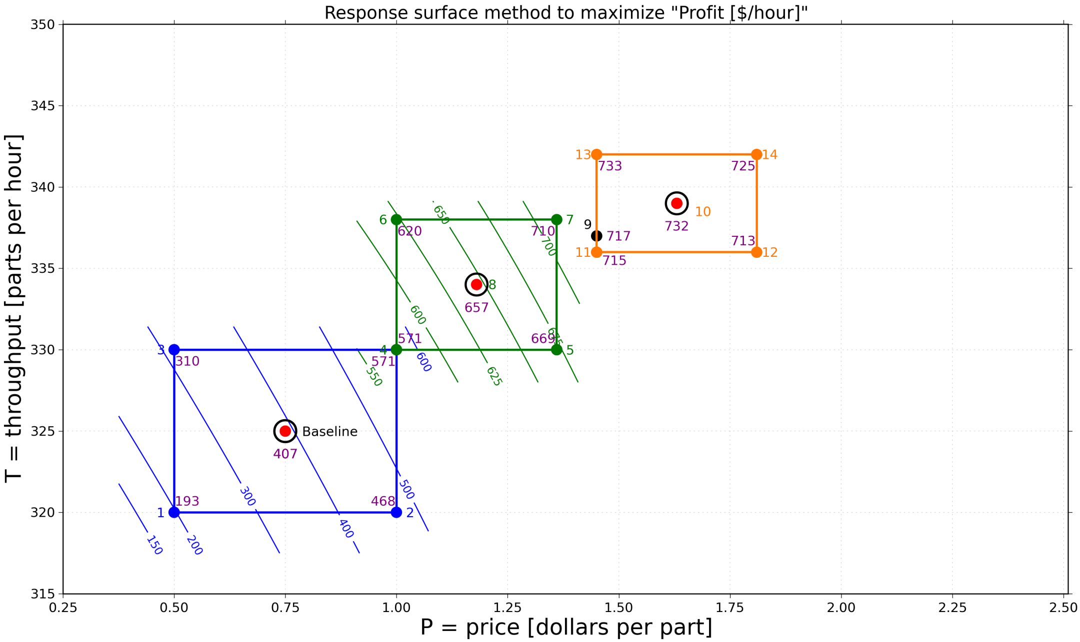
675
700
6 parts per hour
$ 0.36 per part

Step size is a function of the factorial range

Let's work backwards:

- You choose the step size, e.g. $\Delta x_{\mathsf{P}} = 2.5$
- Real-world step size $= \Delta \mathsf{P} = \Delta x_{\mathsf{P}} \cdot \frac{1}{2}\left(\text{range}_{\mathsf{P}}\right)$
- where the "range$_{\mathsf{P}}$" is picked by the experimenter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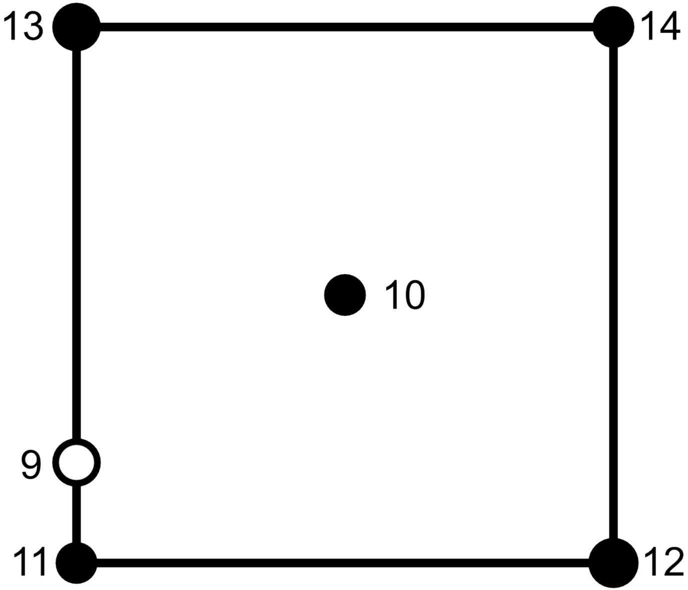
List of all experiments

Experiment	P	T	$x_{\mathbf{P}}$	$x_{\mathbf{T}}$	Prediction $= \hat{y}$	Actual $= y$	Model
9	1.45	337	1.5	0.75		\$ 717	2
10	1.63	339	0	0		\$ 732	3
11	1.45	336	-1	-1		715	3
12	1.81	336	$+1$	-1		713	3
13	1.45	342	-1	$+1$		733	3
14	1.81	342	$+1$	$+1$		725	3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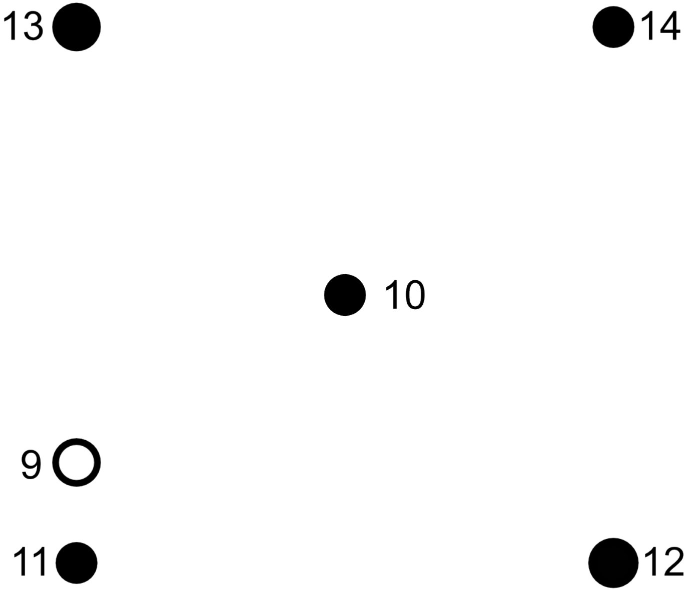
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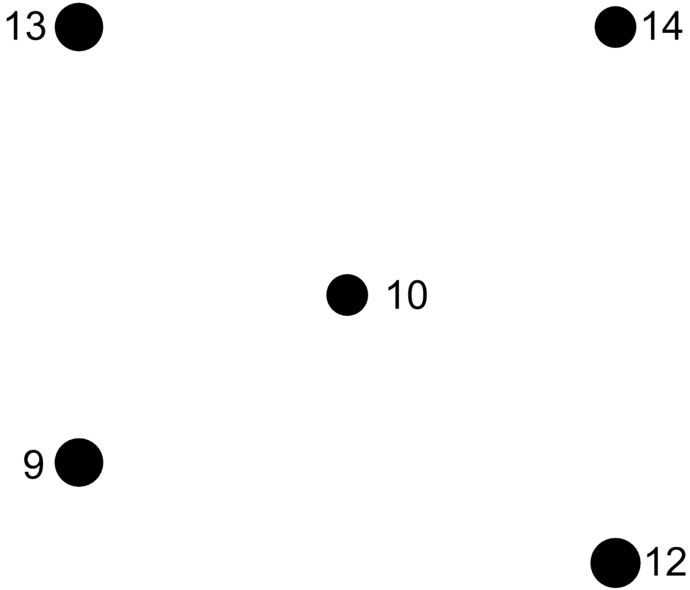
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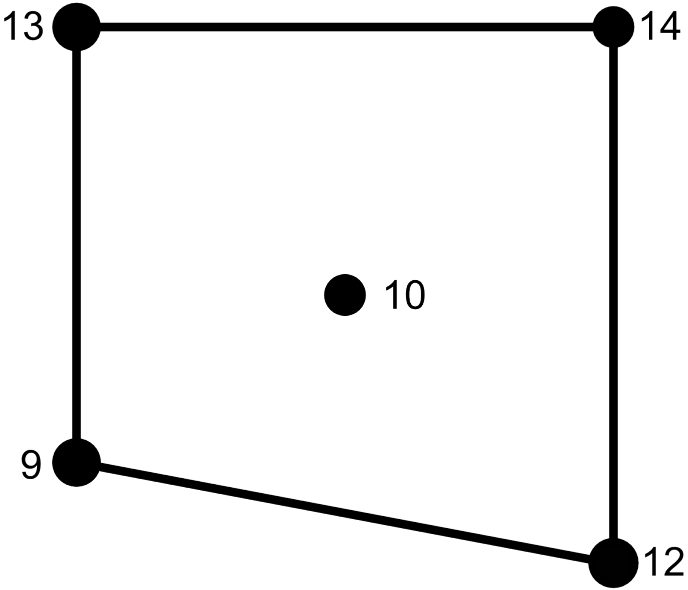
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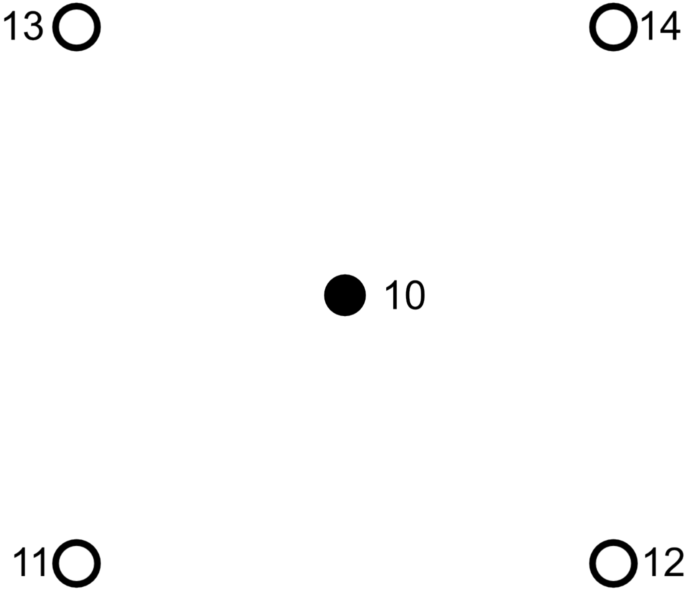
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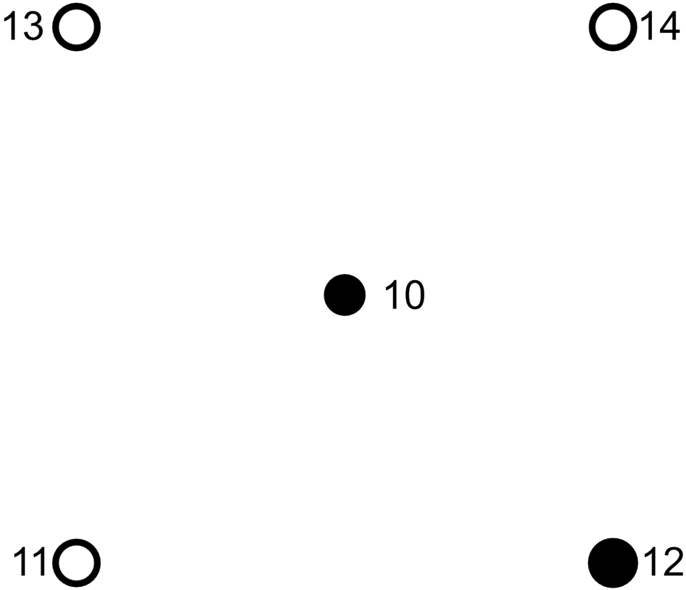
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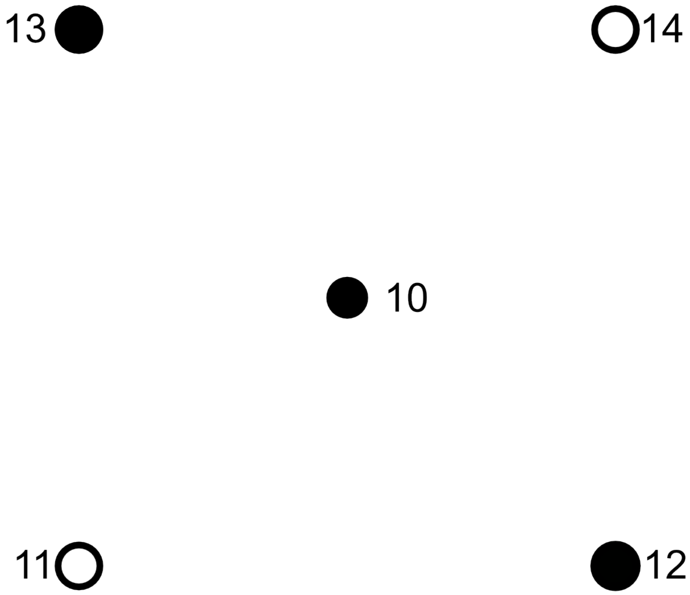
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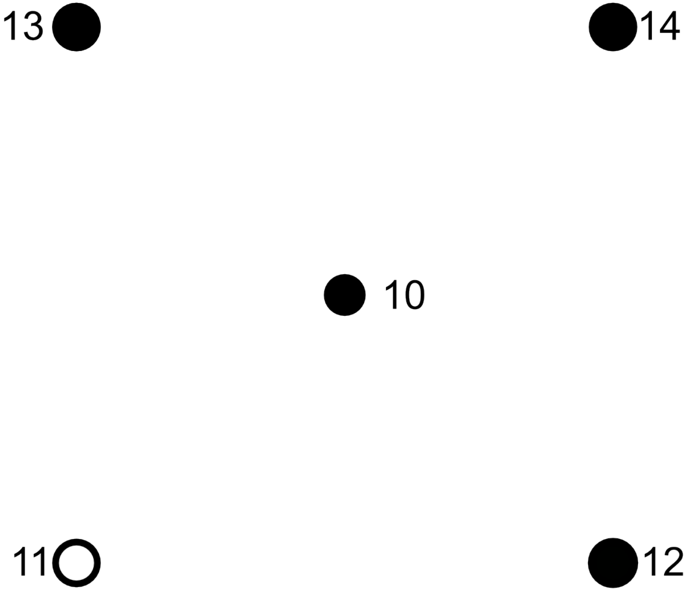
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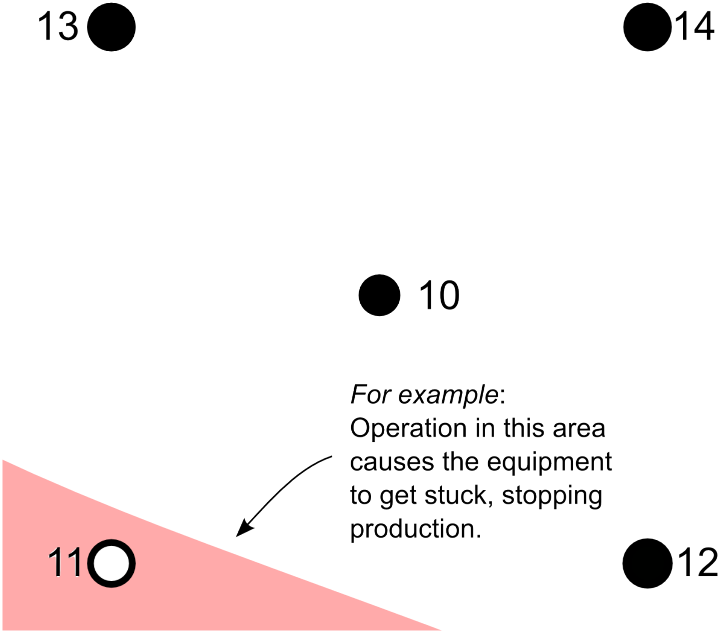
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For example:
Operation in this area causes the equipment to get stuck, stopping production.
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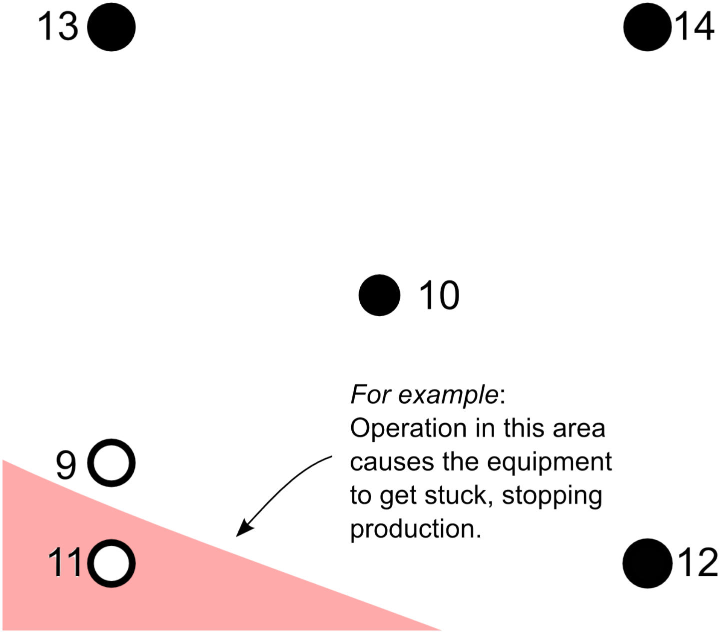
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Operation in this area causes the equipment to get stuck, stopping production.
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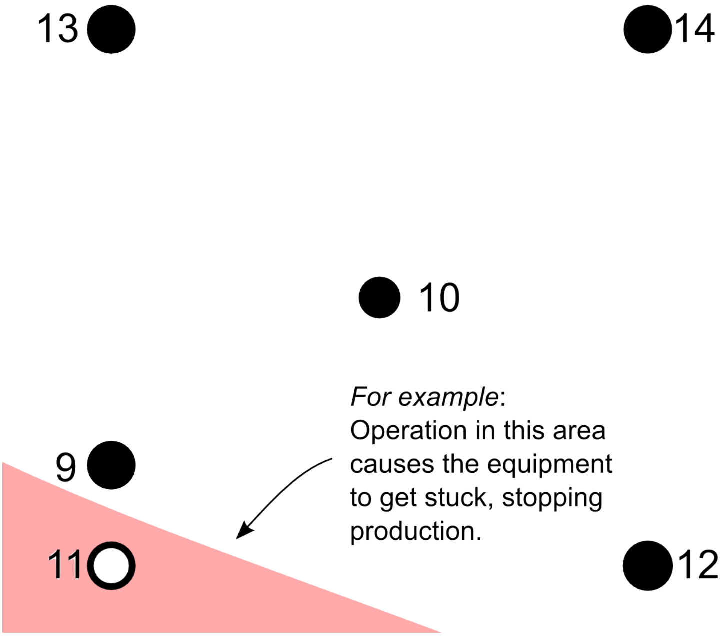
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For example:
Operation in this area
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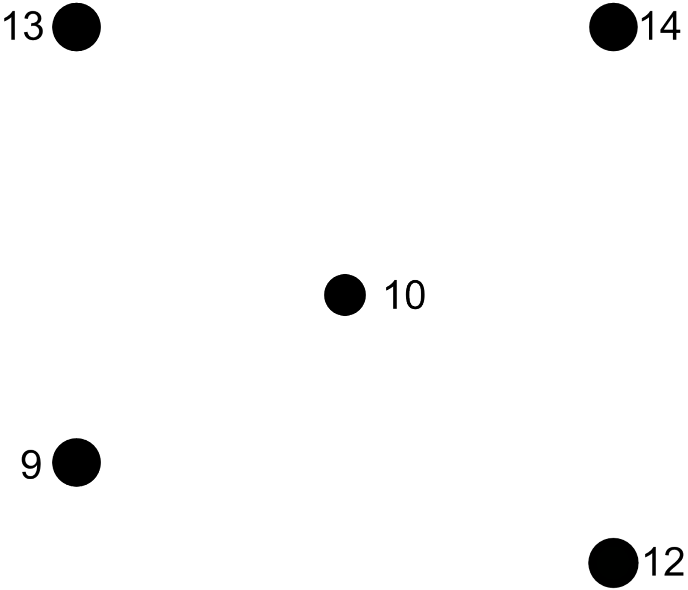
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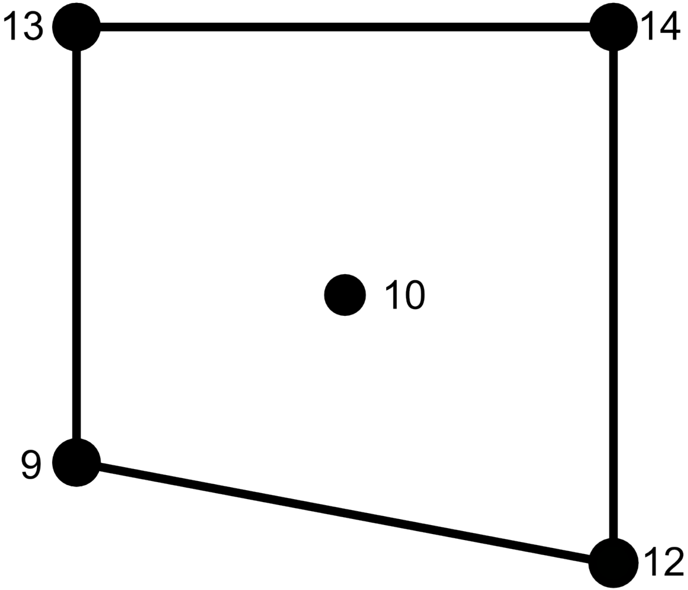
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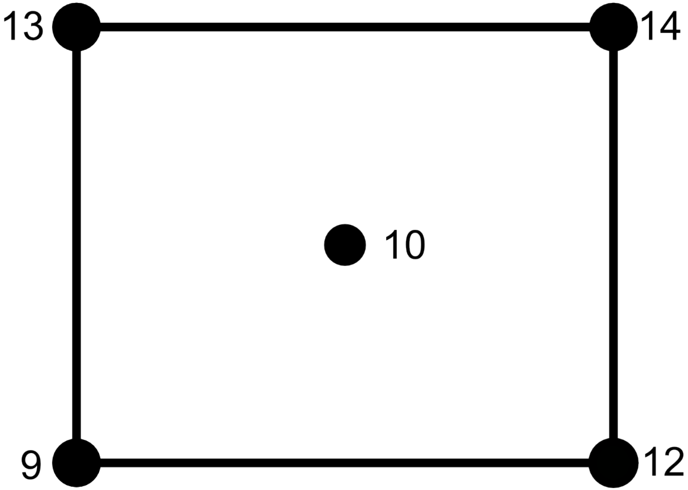
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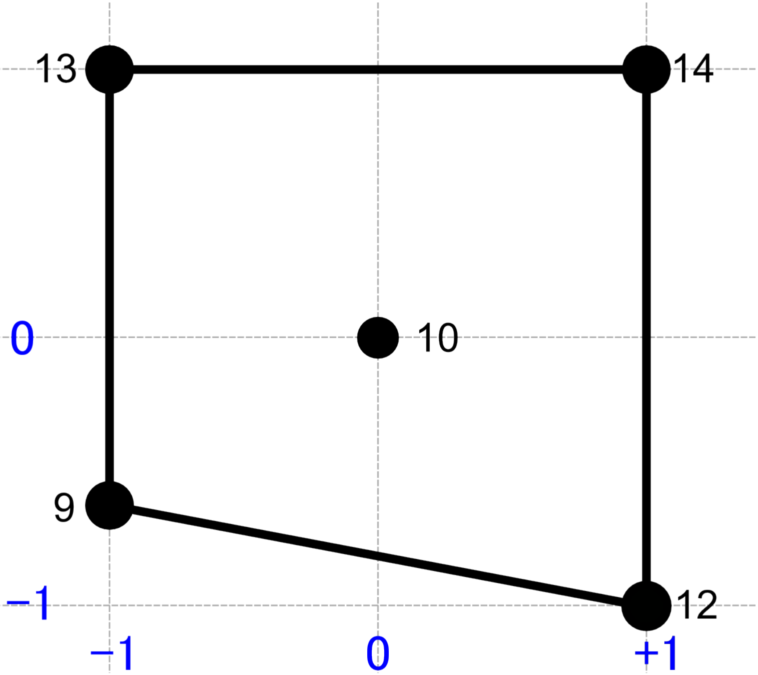
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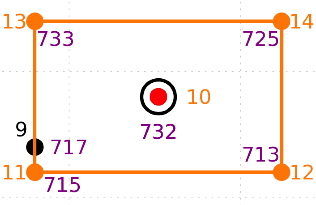
−1
0
+1

Botched design

```r
P <- c(  0,   -1,  +1,  -1,  +1)
T <- c(  0, -2/3,  -1,  +1,  +1)
y <- c(732,  717, 713, 733, 725)
mod.base.4 <- lm(y ~ P*T)
summary(mod.base.4)
contourPlot(mod.base.4, P, T,
            title="Botched design")
```



Regular design

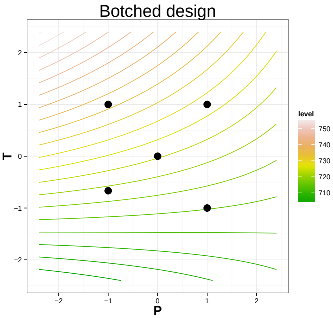
```r
P <- c(  0,  -1,  +1,  -1,  +1)
T <- c(  0,  -1,  -1,  +1,  +1)
y <- c(732, 715, 713, 733, 725)
mod.base.3 <- lm(y ~ P*T)
summary(mod.base.3)
contourPlot(mod.base.3, P, T,
            title="Regular design")
```

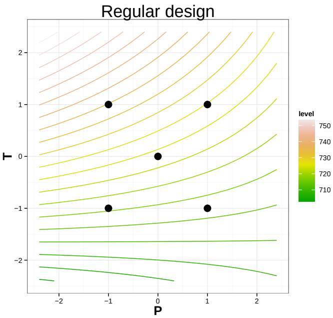
Data points used: 10, **9**, 12, 13, 14

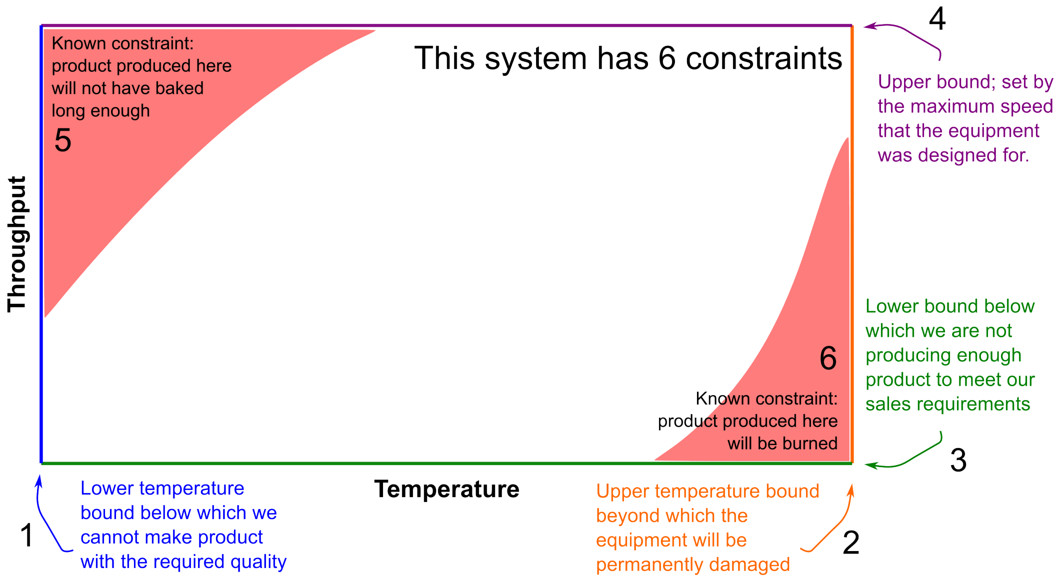
Data points used: 10, **11**, 12, 13, 14

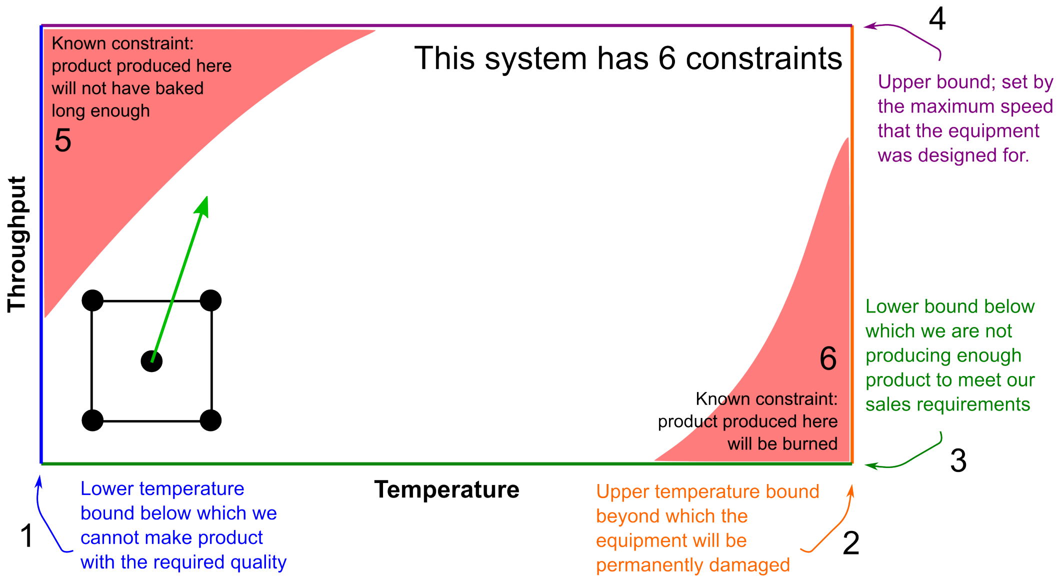
$$\hat{y} = 723.4 - 2.243 x_{\mathsf{P}} + 7.542 x_{\mathsf{T}} - 1.542 x_{\mathsf{P}} x_{\mathsf{T}}$$

$$\hat{y} = 723.6 - 2.500 x_{\mathsf{P}} + 7.500 x_{\mathsf{T}} - 1.500 x_{\mathsf{P}} x_{\mathsf{T}}$$

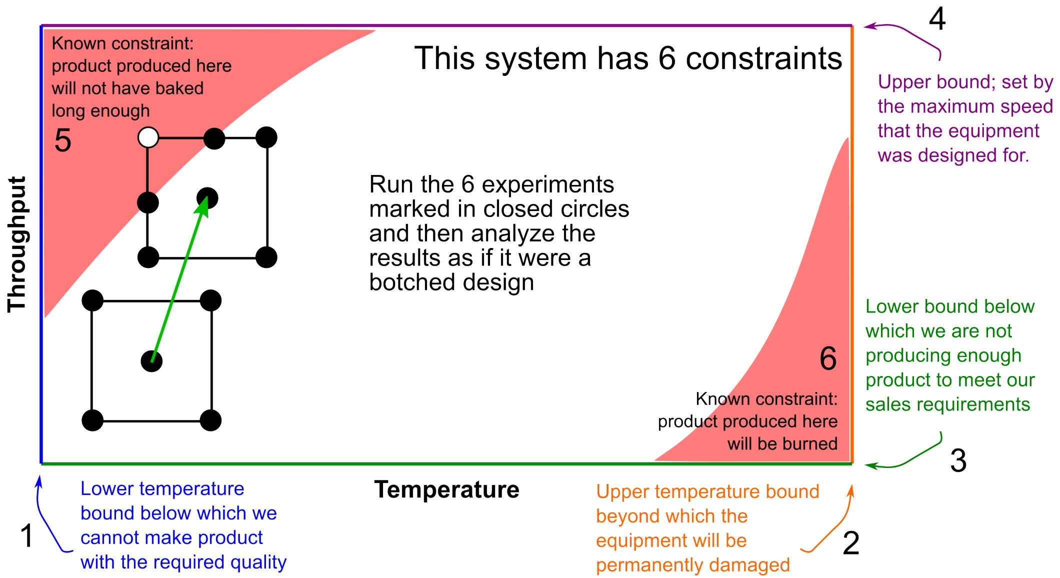




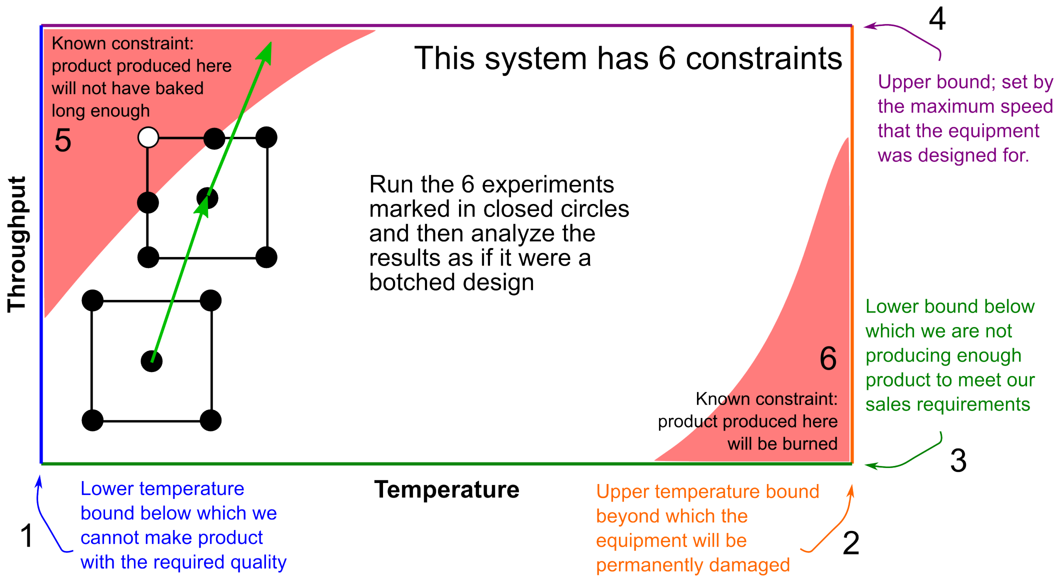

This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
6
Known constraint: product produced here will be burned
Throughput
Temperature
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2

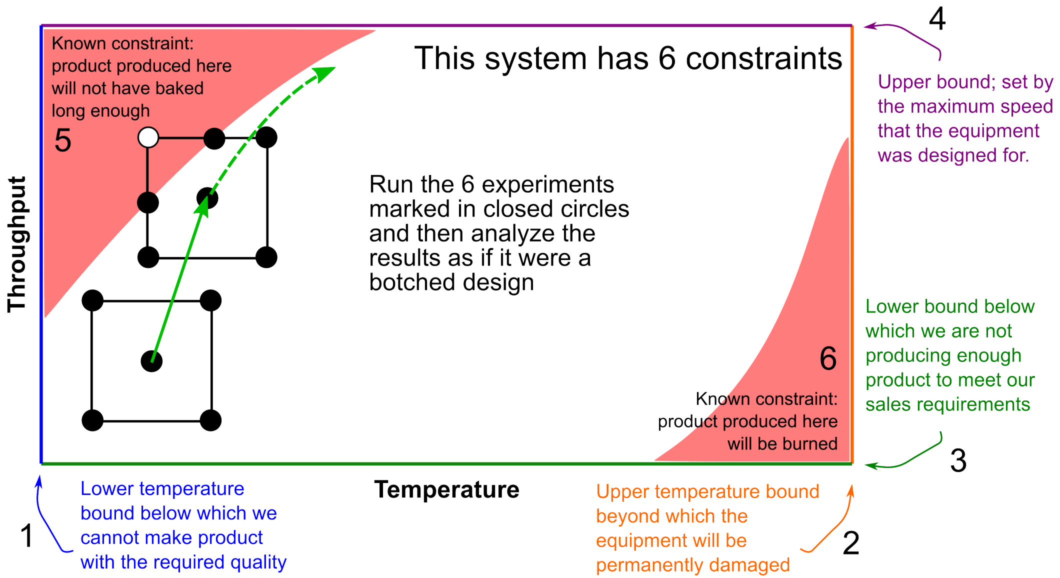


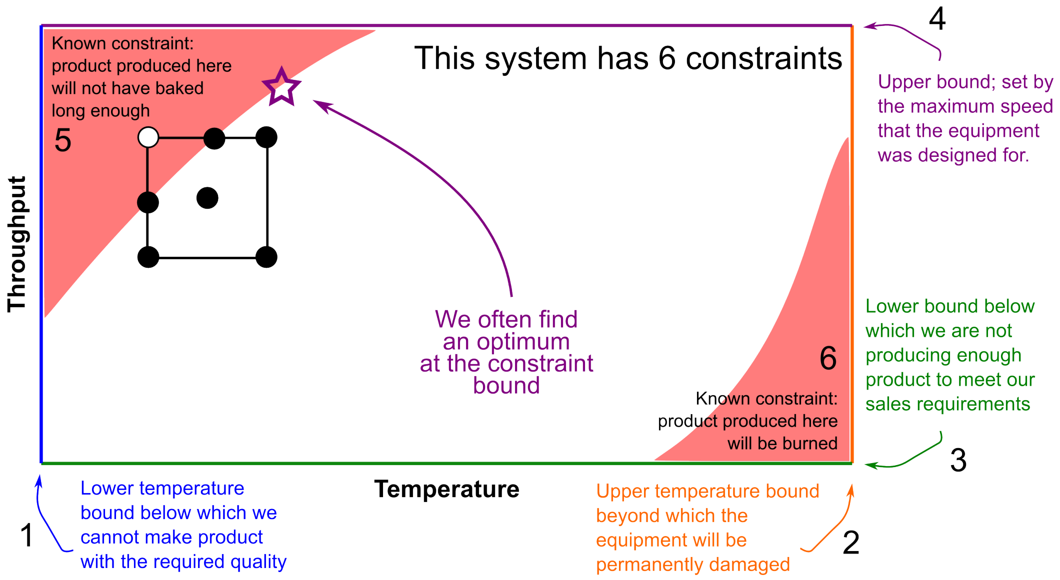
This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
Known constraint: product produced here will be burned
6
Throughput
Temperature
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2



This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
Run the 6 experiments marked in closed circles and then analyze the results as if it were a botched design
6
Known constraint: product produced here will be burned
Throughput
Temperature
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2


This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
Run the 6 experiments marked in closed circles and then analyze the results as if it were a botched design
6
Known constraint: product produced here will be burned
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
Throughput
Temperature
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2


This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
Run the 6 experiments marked in closed circles and then analyze the results as if it were a botched design
6
Known constraint: product produced here will be burned
Throughput
Temperature
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2


This system has 6 constraints
Known constraint: product produced here will not have baked long enough
5
Throughput
We often find an optimum at the constraint bound
6
Known constraint: product produced here will be burned
4
Upper bound; set by the maximum speed that the equipment was designed for.
Lower bound below which we are not producing enough product to meet our sales requirements
3
Temperature
1
Lower temperature bound below which we cannot make product with the required quality
Upper temperature bound beyond which the equipment will be permanently damaged
2