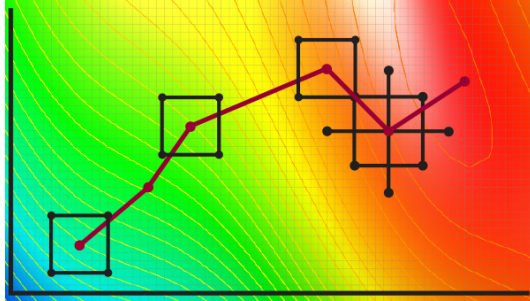


Experimentation for Improvement



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Design and Analysis of Experiments

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Number of factors	Total experiments	Cost of all experiments	Time to run experiments
2	4	\$300	1 day
3	8	\$600	2 days
4	16	\$1,200	4 days
5	32	\$2,400	8 days
6	64	\$4,800	16 days
7	128	\$9,600	32 days

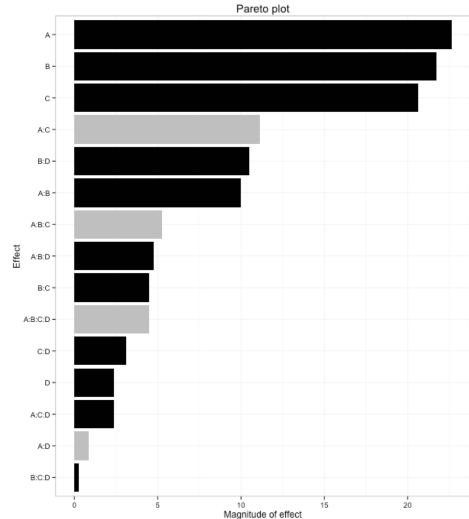
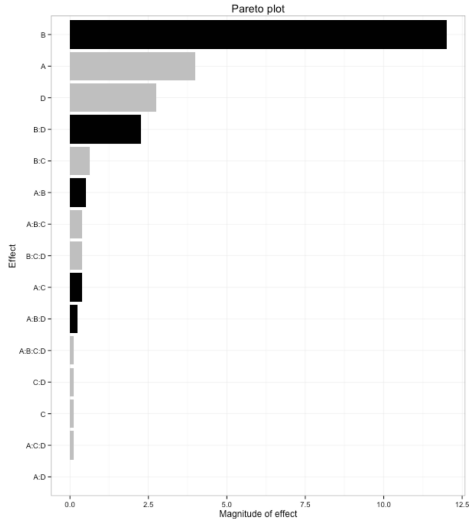
assuming 6 hours and \$150 per experiment

There are 2^k model parameters in a full-factorial: not all are meaningful!

For $k = 4$ factors:

$$\begin{aligned}\hat{y} = & b_0 \\ & + b_A x_A \\ & + b_B x_B \\ & + b_C x_C \\ & + b_D x_D \\ & + b_{AB} x_A x_B \\ & + b_{AC} x_A x_C \\ & + b_{BC} x_B x_C \\ & + b_{AD} x_A x_D \\ & + b_{BD} x_B x_D \\ & + b_{CD} x_C x_D \\ & + b_{ABC} x_A x_B x_C \\ & + b_{ABD} x_A x_B x_D \\ & + b_{ACD} x_A x_C x_D \\ & + b_{BCD} x_B x_C x_D \\ & + b_{ABCD} x_A x_B x_C x_D\end{aligned}$$

Most real systems exhibit minor interactions; main effects usually dominate



Core assumption regarding fractional factorials

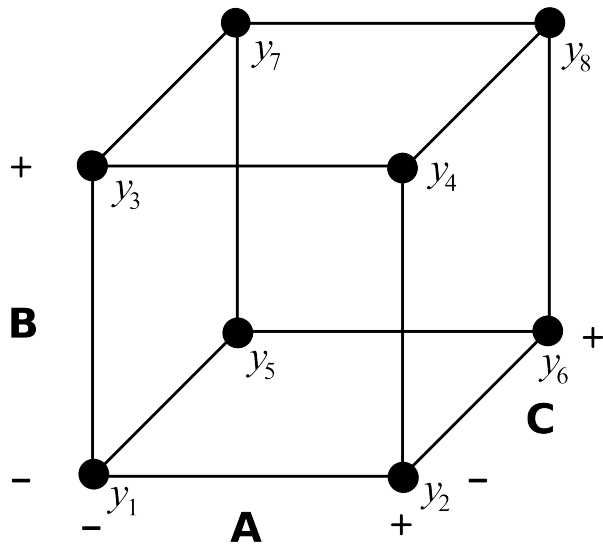
$$\begin{aligned}\hat{y} = & b_0 \\ & + b_A x_A \\ & + b_B x_B \\ & + b_C x_C \\ & + b_D x_D \\ & + b_{AB} x_A x_B \\ & + b_{AC} x_A x_C \\ & + b_{BC} x_B x_C \\ & + b_{AD} x_A x_D \\ & + b_{BD} x_B x_D \\ & + b_{CD} x_C x_D \\ & + b_{ABC} x_A x_B x_C \\ & + b_{ABD} x_A x_B x_D \\ & + b_{ACD} x_A x_C x_D \\ & + b_{BCD} x_B x_C x_D \\ & + b_{ABCD} x_A x_B x_C x_D\end{aligned}$$

The main effects and some two factor interactions are often the only parameters of interest

The higher order interactions can safely be ignored

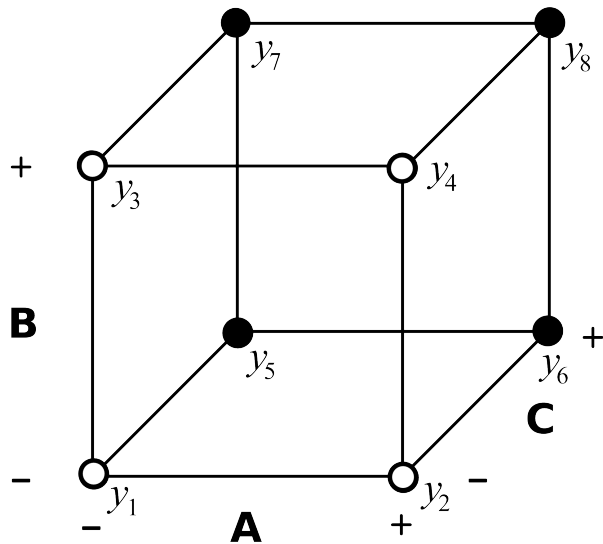
- ▶ Now it is an assumption, but it's reasonable in many cases
- ▶ The cost of obtaining them can be prohibitive

If you'd like to do half the work, which 4 experiments would you pick?



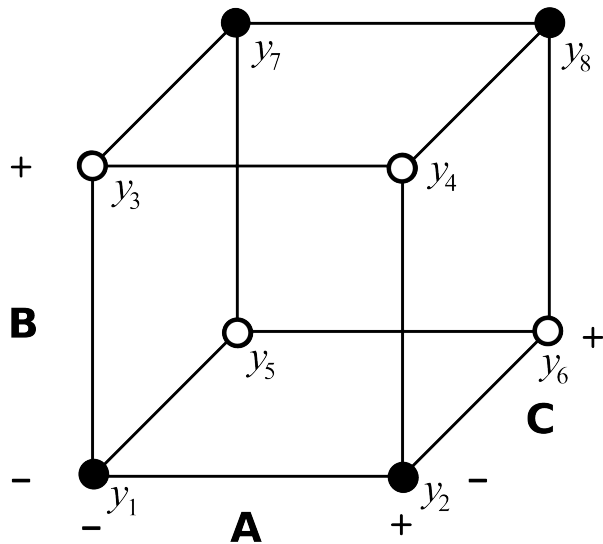
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



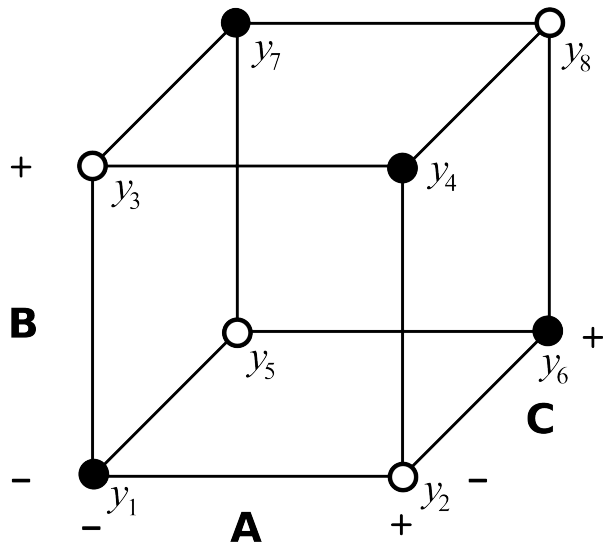
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



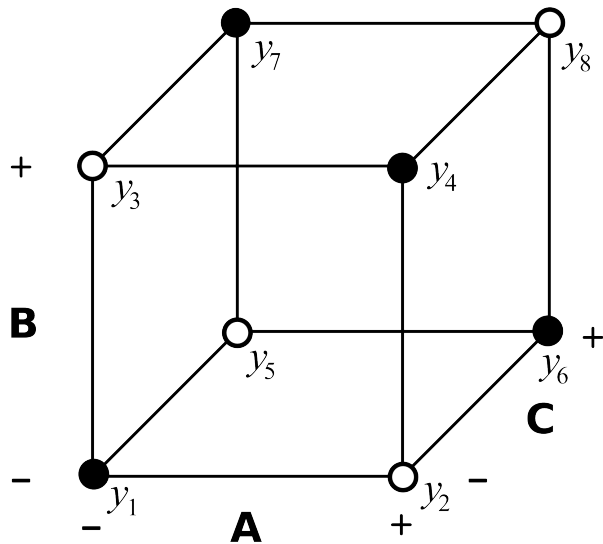
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



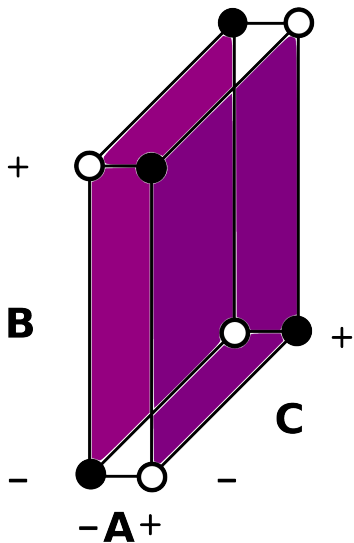
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



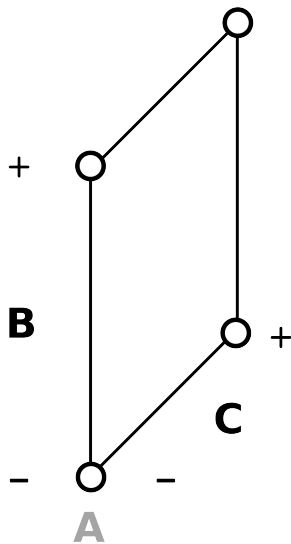
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



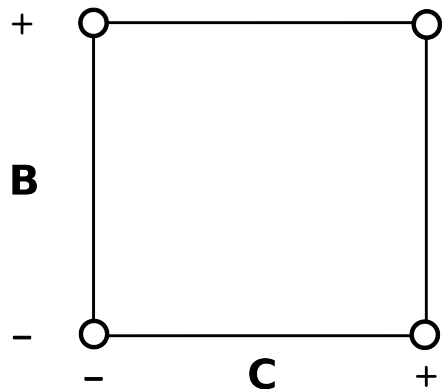
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



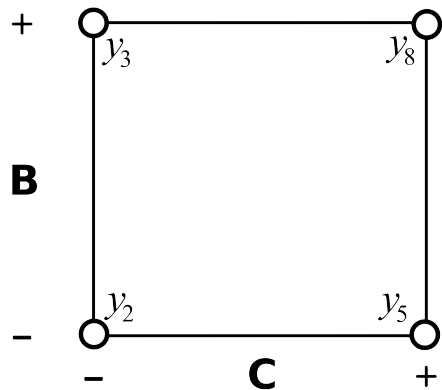
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



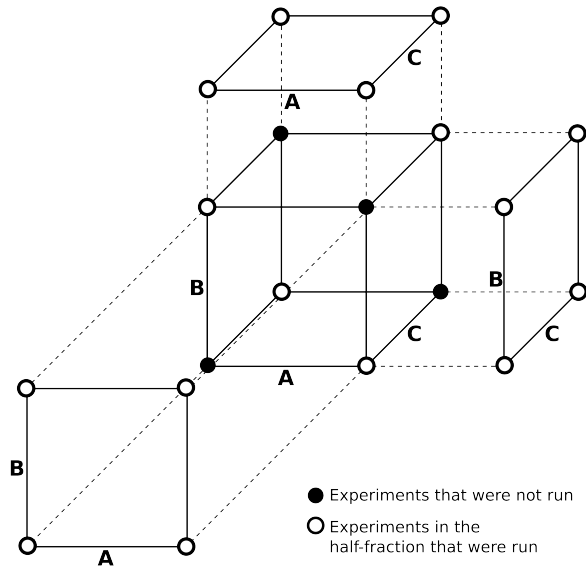
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

If you'd like to do half the work, which 4 experiments would you pick?



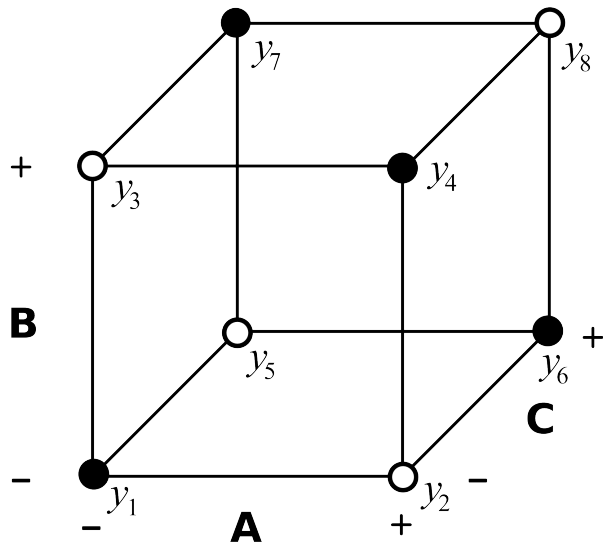
Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

There are embedded full factorials inside the fractional factorial



If we choose our runs in a smart way, then fractional factorials will collapse to full factorials if an effect is insignificant.

If you'd like to do half the work, which 4 experiments would you pick?



Experiment	A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

Only consider the 4 open circles.

Model comparison between the full and fractional factorials

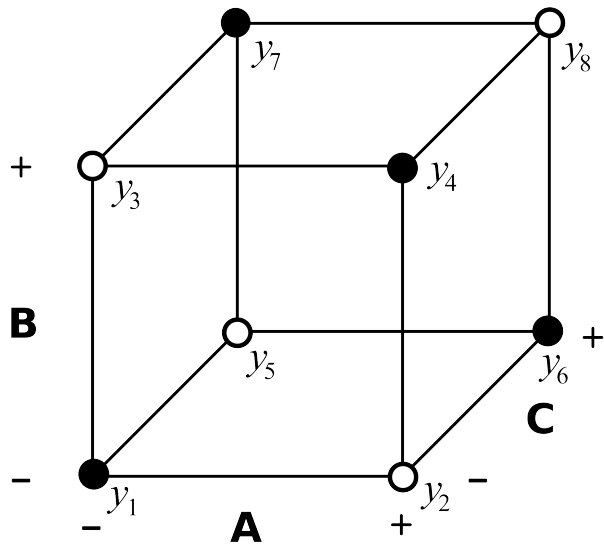
Full factorial model

$$\begin{aligned}\hat{y} = & 11.25 \\ & + 6.25 x_A \\ & + 0.75 x_B \\ & - 7.25 x_C \\ & + 0.25 x_A x_B \\ & - 6.75 x_A x_C \\ & - 0.25 x_B x_C \\ & - 0.25 x_A x_B x_C\end{aligned}$$

Fractional factorial model

$$\begin{aligned}\hat{y} = & 11.0 \\ & + 6.0 x_A \\ & - 6.0 x_B \\ & - 7.0 x_C \\ & + \cancel{b_{AB} x_A x_B} \\ & + \cancel{b_{AC} x_A x_C} \\ & + \cancel{b_{BC} x_B x_C} \\ & + \cancel{b_{ABC} x_A x_B x_C}\end{aligned}$$

The mathematics behind a fractional factorial

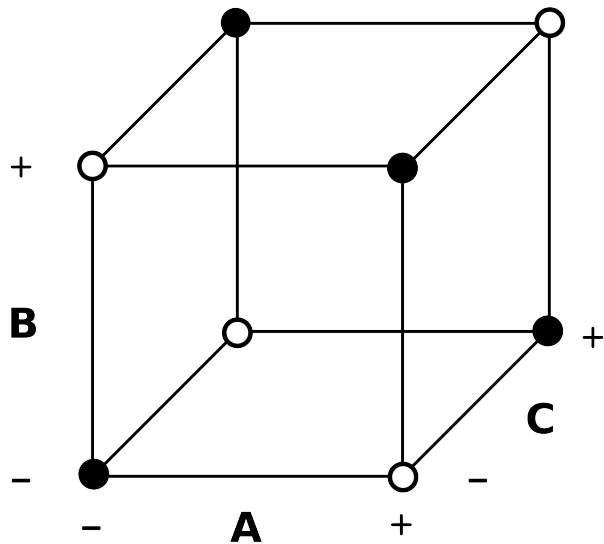


$$2^3 = 8$$

$$\frac{2^3}{2} = 4$$

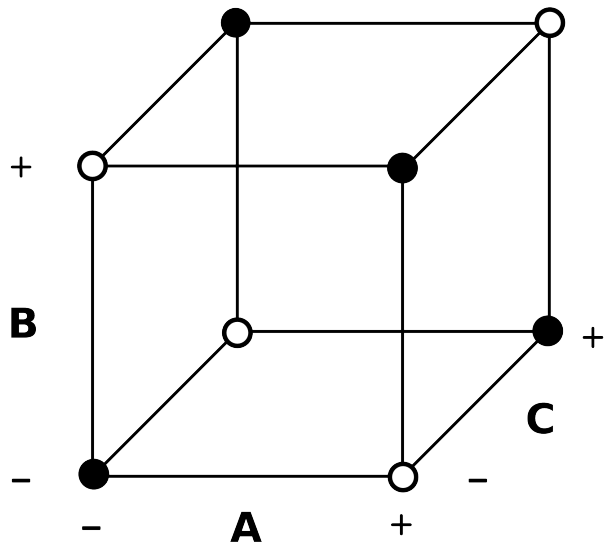
$$2^{3-1} = 2^2 = 4$$

Setting up the half-fraction in 3 factors



Experiment	A	B
1	-	-
2	+	-
3	-	+
4	+	+

Setting up the half-fraction in 3 factors



Experiment	A	B	C = AB
1	-	-	$(-)(-) = +$
2	+	-	$(+)(-) = -$
3	-	+	$(-)(+) = -$
4	+	+	$(+)(+) = +$

on when fractional-factorials are suitable

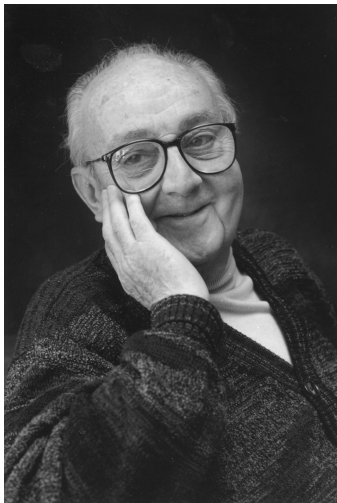
Screening is when you evaluate a new system

- ▶ lab-scale exploration
- ▶ making a new product
- ▶ troubleshooting a problem to isolate major causes

Optimization: where you need that prediction accuracy

- ▶ avoid optimizing prematurely
- ▶ a less-fractionated design is used for optimization (more on this later)

Quote from George Box



"In an ongoing investigation, a rough rule is that only a portion (say 25%) of the experimental effort and budget should be invested in the first design."

In the next class ...

We learn how to create half-fractions for any general system.

For example, how did we get $\mathbf{C} = \mathbf{AB}$?