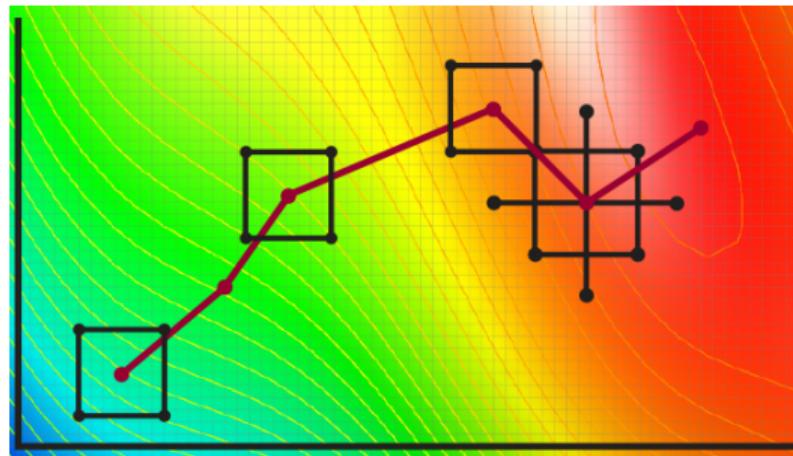


Experimentation for Improvement



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Design and Analysis of Experiments

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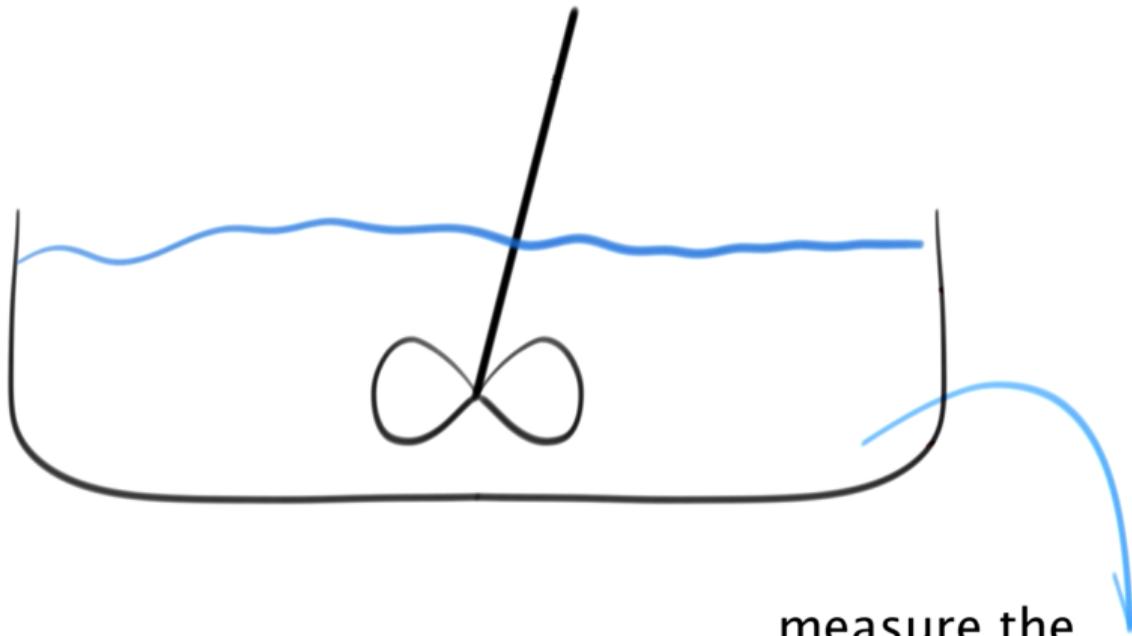
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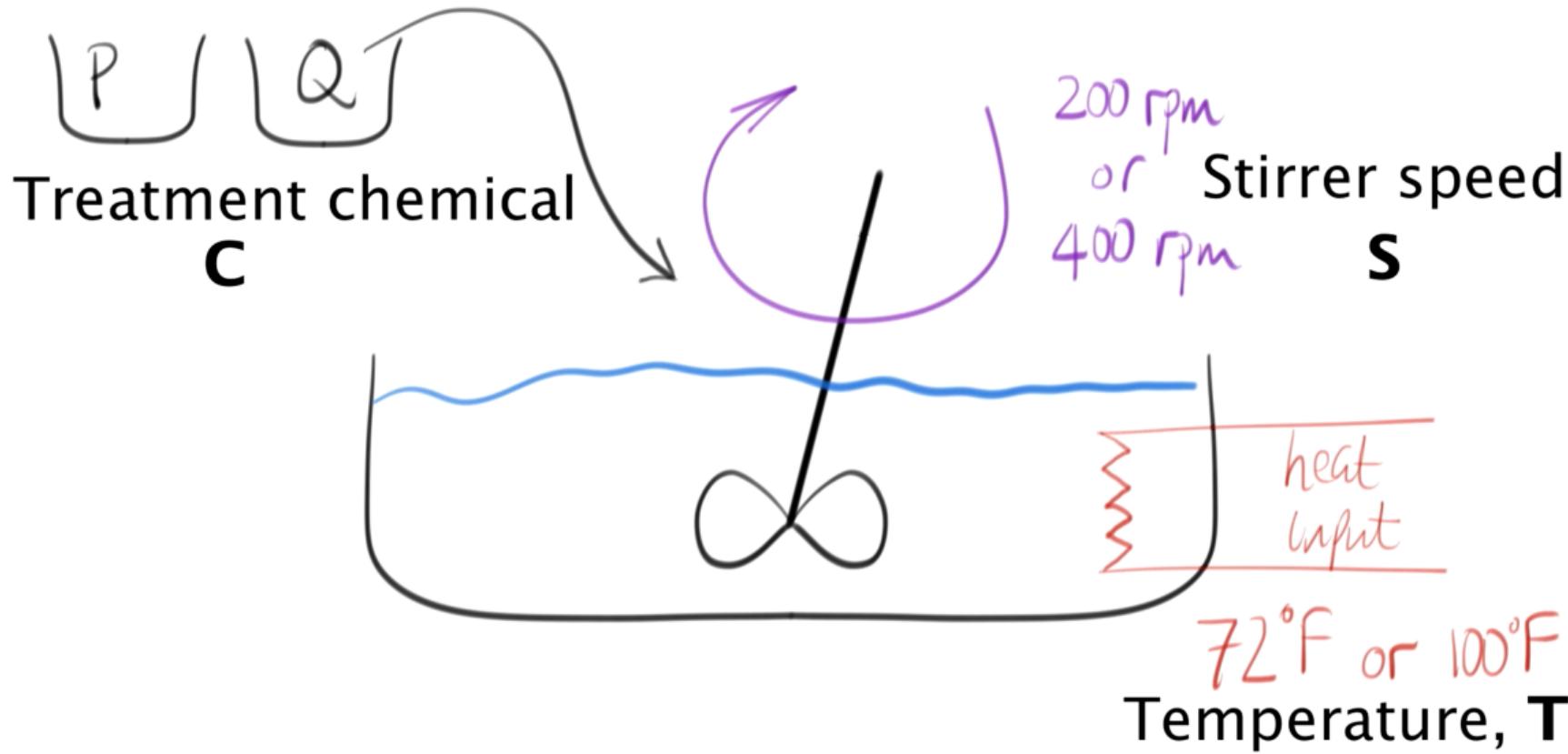
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Objective: minimize this amount

measure the
pollutant amount
afterwards $= y$



3 factors in total: **C, T, S**

Total experiments = 2^k

k = number of factors = 3

2 refers to the number of levels per factor

Waste water treatment example: analysis of the data by hand

Standard order	Actual order	<i>C</i>	<i>T</i>	<i>S</i>	Outcome [lbs]

Chemical Temp Stir speed

factors

Waste water treatment example: analysis of the data by hand

Standard order	Actual order	C	T	S	Outcome [lbs]
		-	-	-	
		+	-	-	
		-	+	-	
		+	+	-	
		-	-	+	
		+	-	+	
		-	+	+	
		+	+	+	


factors

Waste water treatment example: analysis of the data by hand

Standard order	Actual order	C	T	S	Outcome [lbs]
1	6	-	-	-	5
2	2	+	-	-	30
3	5	-	+	-	6
4	3	+	+	-	33
5	7	-	-	+	4
6	1	+	-	+	3
7	8	-	+	+	5
8	4	+	+	+	4

Waste water treatment example: analysis of the data by hand

Standard order	Actual order	C	T	S	Outcome [lbs]
1	6	-	-	-	5
2	2	+	-	-	30
3	5	-	+	-	6
4	3	+	+	-	33
5	7	-	-	+	4
6	1	+	-	+	3
7	8	-	+	+	5
8	4	+	+	+	4

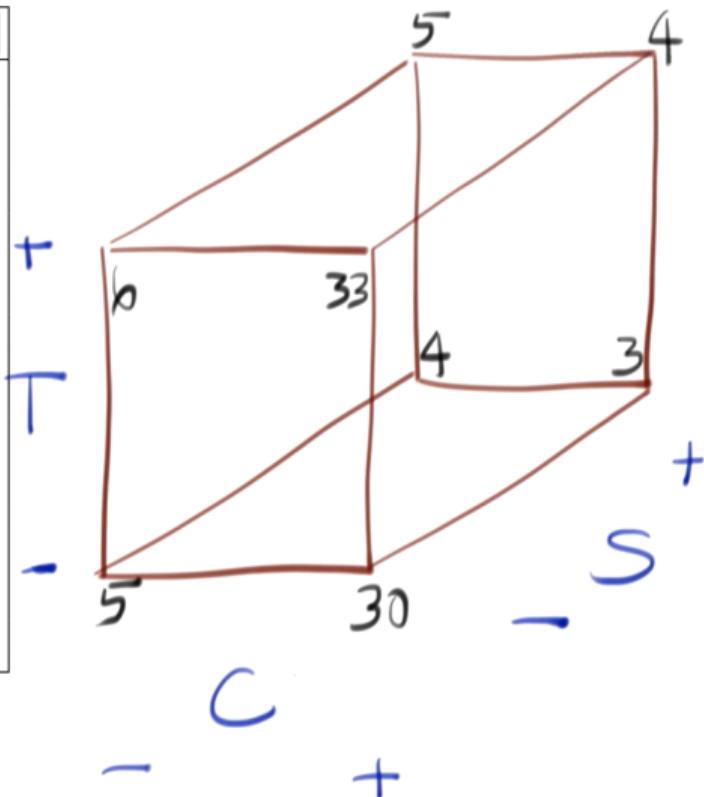
except for
these two

Waste water treatment example: analysis of the data by hand

Standard order	Actual order	C	T	S	Outcome [lbs]
1	6	-	-	-	5
2	2	+	-	-	30
3	5	-	+	-	6
4	3	+	+	-	33
5	7	-	-	+	4
6	1	+	-	+	3
7	8	-	+	+	5
8	4	+	+	+	4

Waste water treatment example: analysis of the data by hand

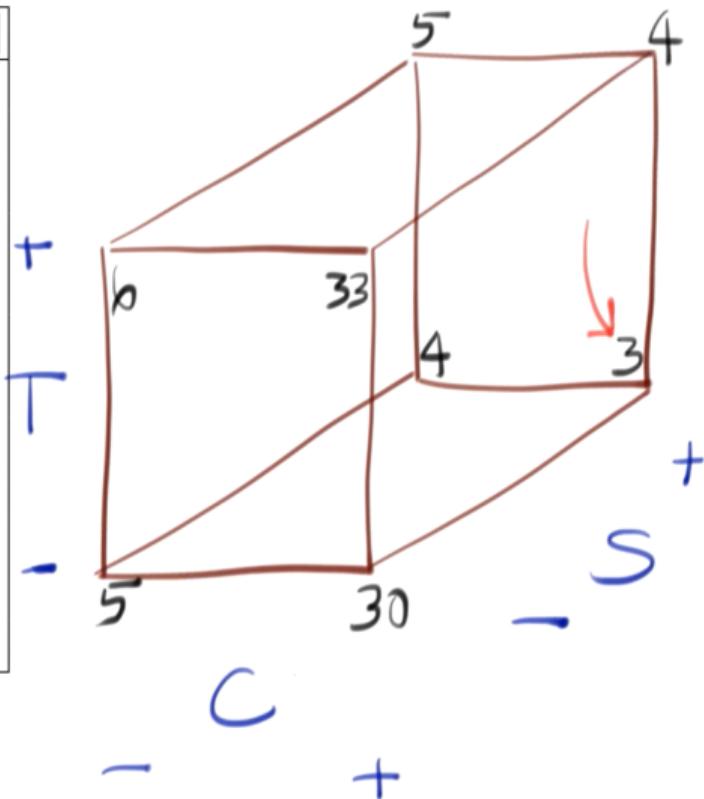
Standard order	C	T	S	Outcome [lbs]
1	-	-	-	5
2	+	-	-	30
3	-	+	-	6
4	+	+	-	33
5	-	-	+	4
6	+	-	+	3
7	-	+	+	5
8	+	+	+	4



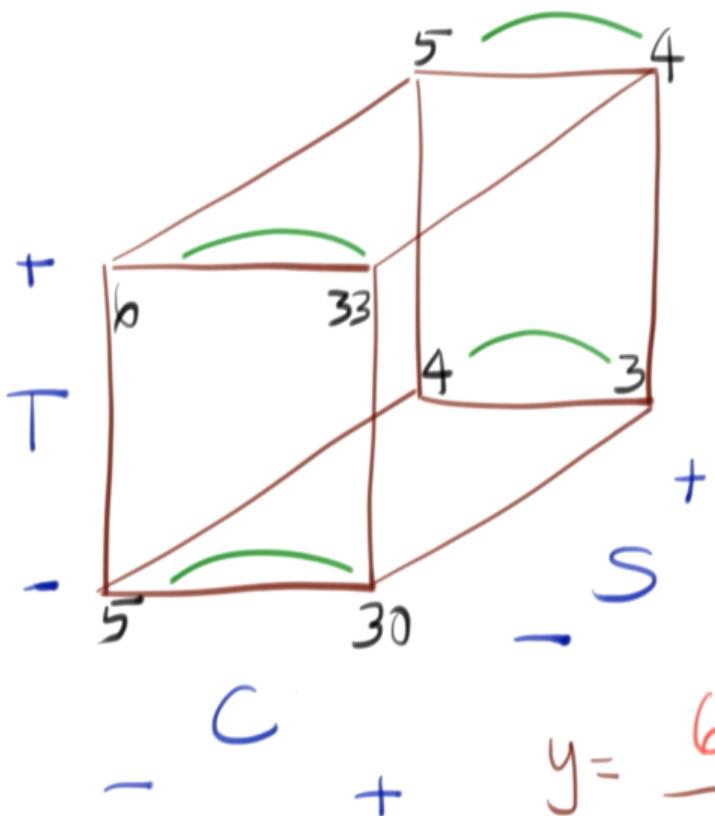
Waste water treatment example: analysis of the data by hand

Standard order	C	T	S	Outcome [lbs]
1	-	-	-	5
2	+	-	-	30
3	-	+	-	6
4	+	+	-	33
5	-	-	+	4
6	+	-	+	3
7	-	+	+	5
8	+	+	+	4

chemical Q low temperature fast speed



Waste water treatment example: analysis of the data by hand



Chemical C: $- \equiv P \quad + \equiv Q$

$+12.5 \text{ lbs pollution}$

T	S
+	+
+	-
-	+
-	-

$$4 - 5 = -1$$

$$33 - 6 = 27$$

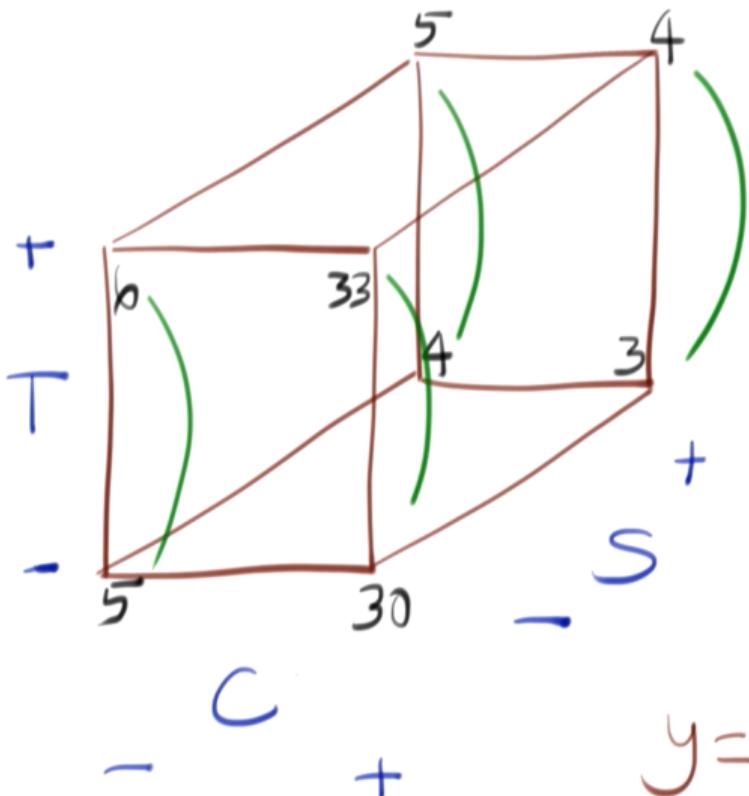
$$3 - 4 = -1$$

$$30 - 5 = 25$$

$$\text{Average} = \frac{50}{4}$$

Average effect of C = 12.5

Waste water treatment example: analysis of the data by hand



Temperature T: $\ominus \equiv 70^{\circ}\text{F}$ $\oplus \equiv 100^{\circ}\text{F}$

C	S
---	---

$$+ \quad + \quad 4 - 3 = 1$$

$$+ \quad - \quad 33 - 30 = 3$$

$$- \quad + \quad 5 - 4 = 1$$

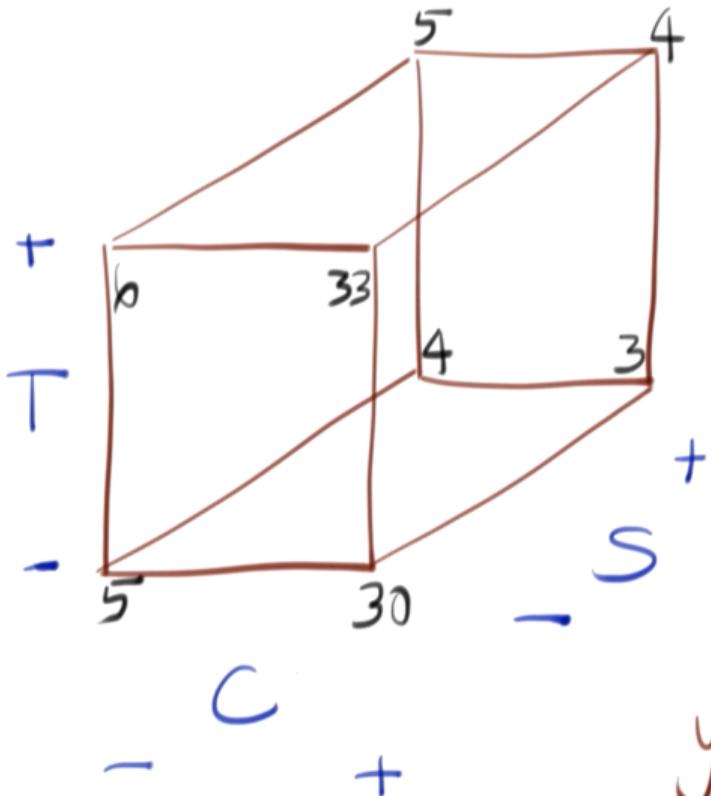
$$- \quad - \quad 6 - 5 = 1$$

$$\text{Average} = \frac{6}{4} = 1.5$$

$$y = \underline{0.75} x_T$$

1.5 lbs

Waste water treatment example: analysis of the data by hand



Stir speed S : $\ominus 200 \text{ rpm}$ $\oplus 400 \text{ rpm}$

$$\begin{array}{c} C \\ \hline T \end{array}$$

$$+ \quad + \quad 4 - 33 = -29$$

$$+ \quad - \quad 3 - 30 = -27$$

$$- \quad + \quad 5 - 6 = -1$$

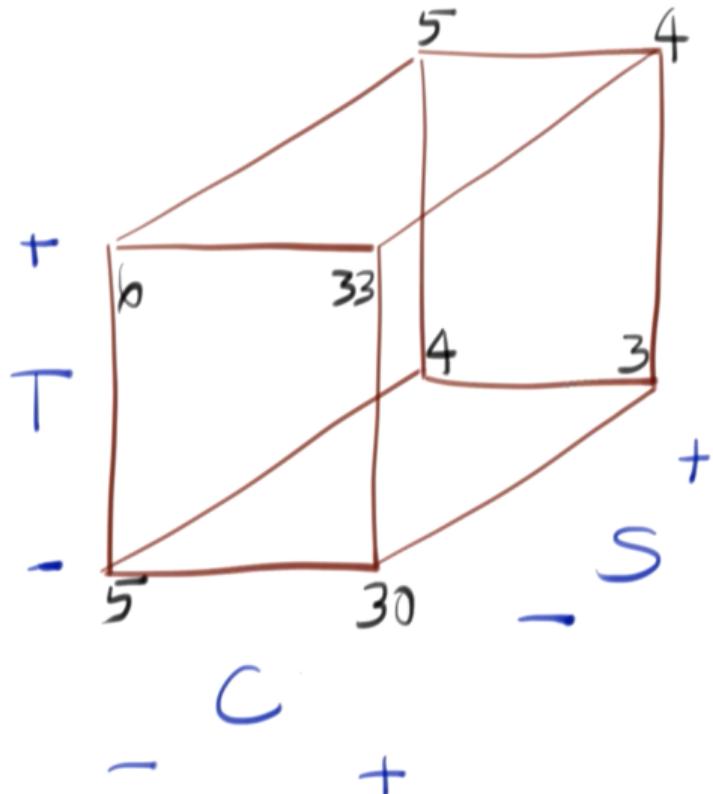
$$- \quad - \quad 4 - 5 = -1$$

$$\begin{array}{rcl} \text{Average} & = & -\frac{50}{4} \end{array}$$

$$= -14.5$$

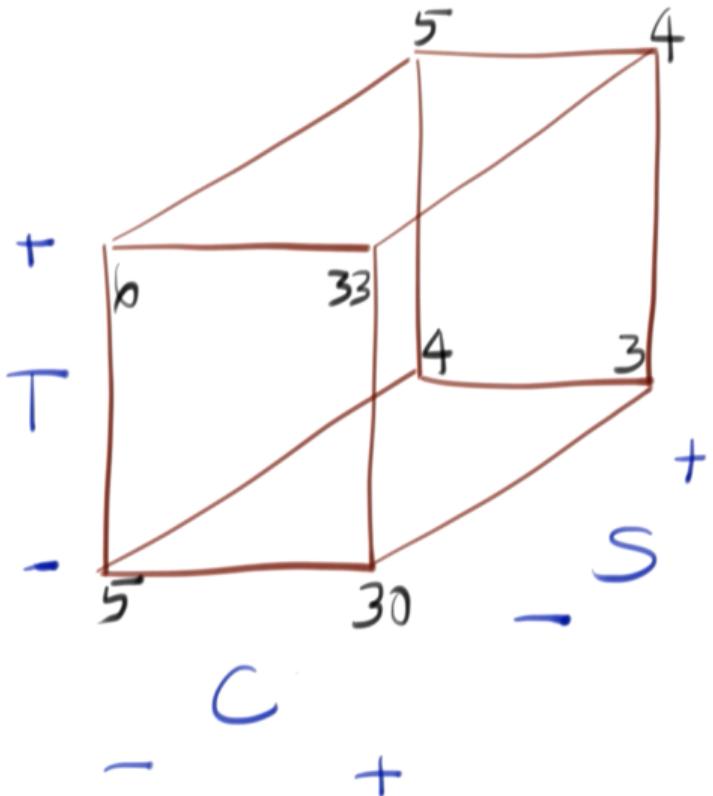
$$y = \underline{-7.25} x_S$$

Waste water treatment example: analysis of the data by hand



$$y = 11.25 + 6.25x_C + 0.75x_T - 7.25x_S$$

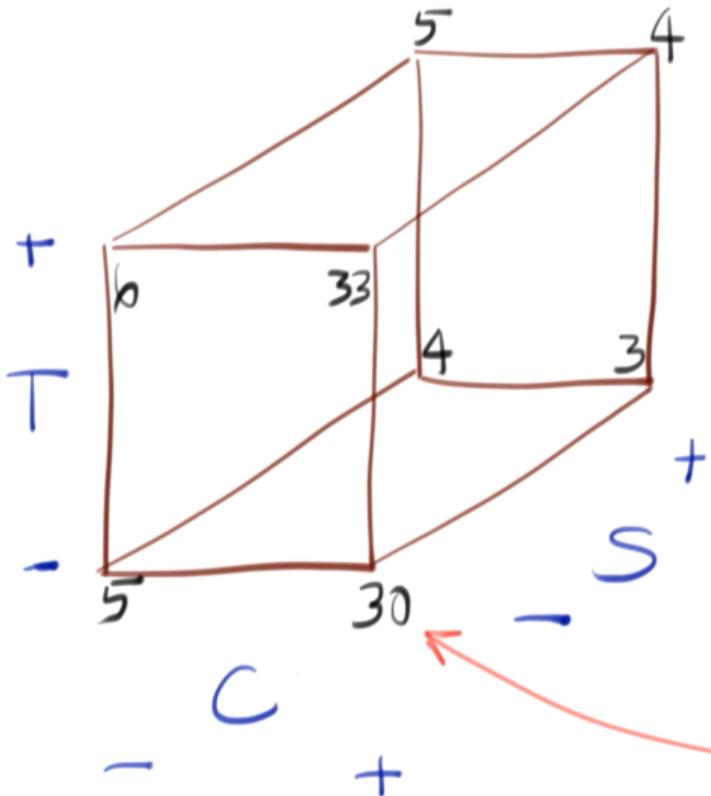
Waste water treatment example: analysis of the data by hand



$$y = 11.25$$

$$\frac{5+30+6+33+4+3+5+4}{8}$$

Waste water treatment example: analysis of the data by hand



$$y = 11.25 + 6.25x_C + 0.75x_T - 7.25x_S$$

Prediction
 $y = 11.25 + 6.25(+1) + 0.75(-1) - 7.25(-1)$

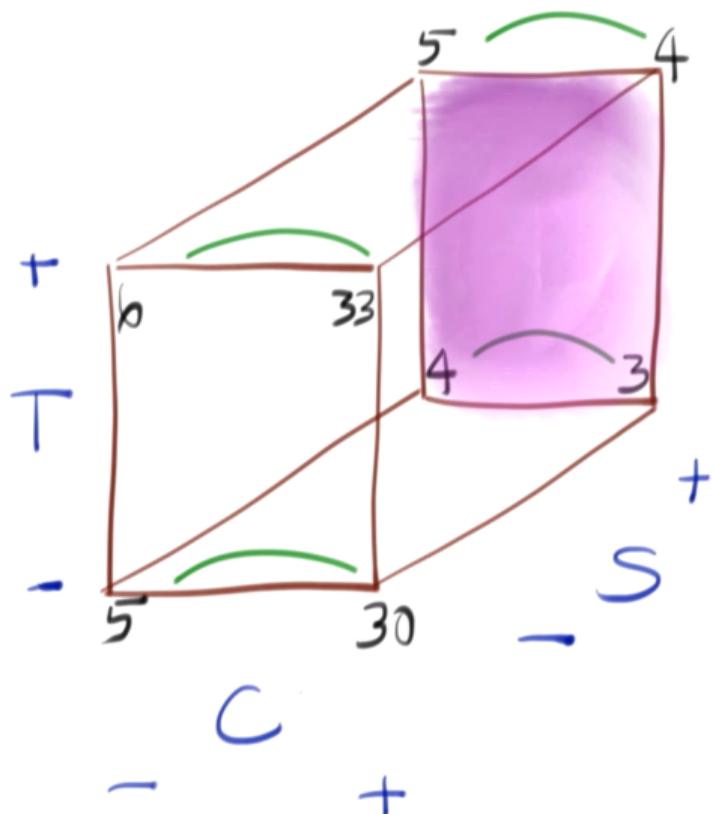
$$y = 24$$

Interaction is missing

The definition of “interaction”

The effect of one factor (for example, **A**) depends on the value, or the level, of another factor (**B**, for example).

Waste water treatment example: analysis of the data by hand



Chemical C: $- \equiv P$ $+ \equiv Q$

$+12.5 \text{ lbs pollution}$

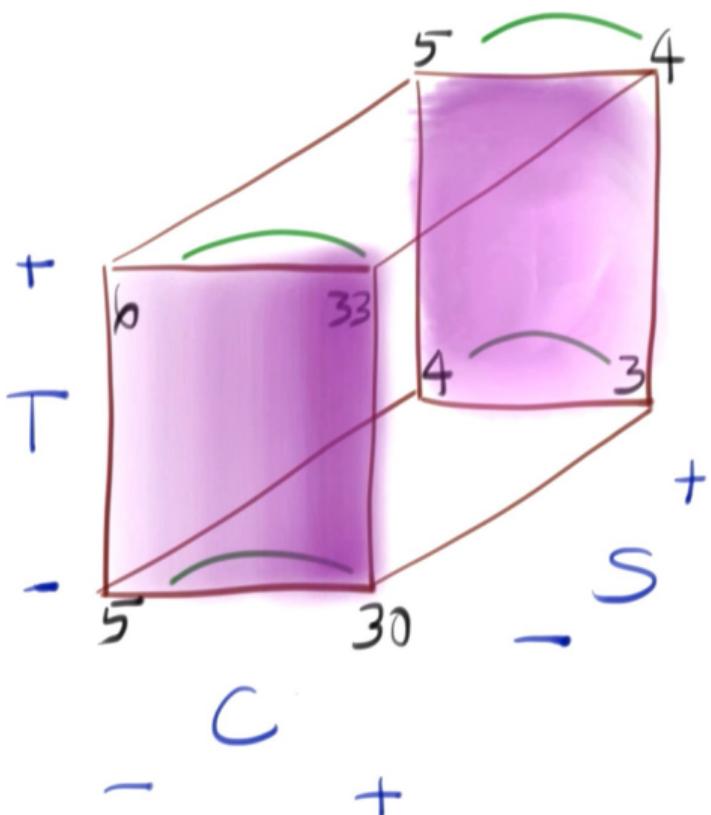
T	S
+	+
+	-
-	+
-	-

$$\begin{array}{r}
 4 - 5 = -1 \\
 33 - 6 = 27 \\
 3 - 4 = -1 \\
 30 - 5 = 25
 \end{array}$$

$$\text{Average} = 50/4$$

Average effect of C = 12.5

Waste water treatment example: analysis of the data by hand



Chemical C: $- \equiv P$ $+ \equiv Q$

$+12.5 \text{ lbs pollution}$

T	S
+	+
+	-
-	+
-	-

$$4 - 5 = -1$$

$$33 - 6 = 27$$

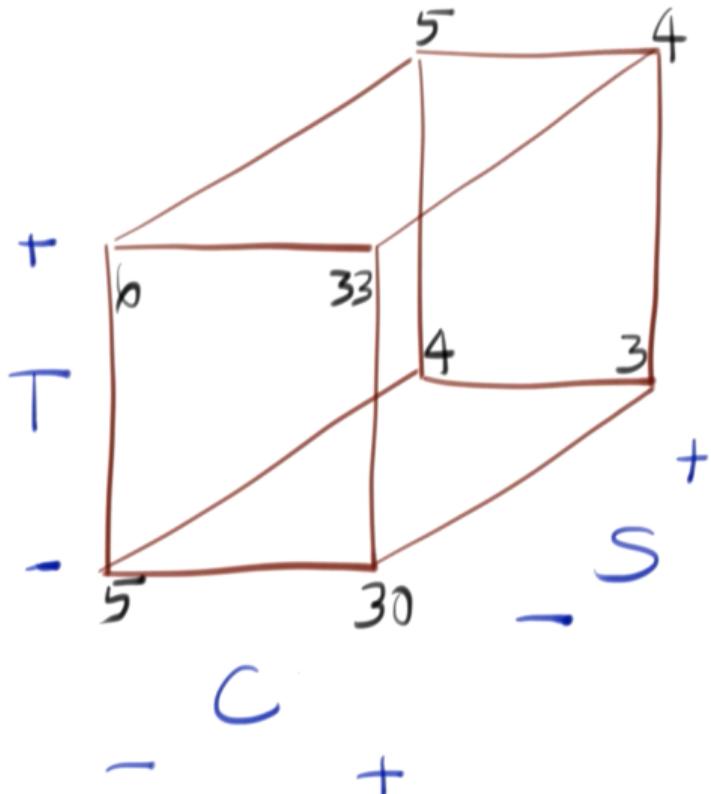
$$3 - 4 = -1$$

$$30 - 5 = 25$$

$$\text{Average} = 50/4$$

Average effect of C = 12.5

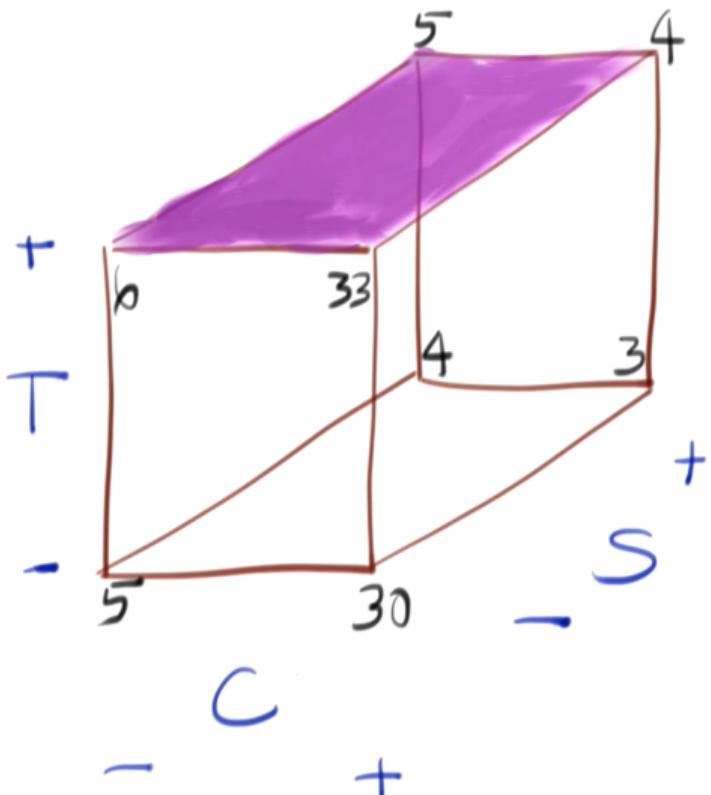
Waste water treatment example: analysis of the data by hand



$$\begin{aligned}y &= 11.25 & y &= b_{\text{intercept}} \\&+ 6.25x_C & + b_C x_C \\&+ 0.75x_T & + b_T x_T \\&- 7.25x_S & + b_S x_S \\&+ \boxed{}x_C x_S & + b_{CS} x_C x_S\end{aligned}$$

Waste water treatment example: analysis of the data by hand

Calculate the b_{CS} interaction
“How C interacts with S”



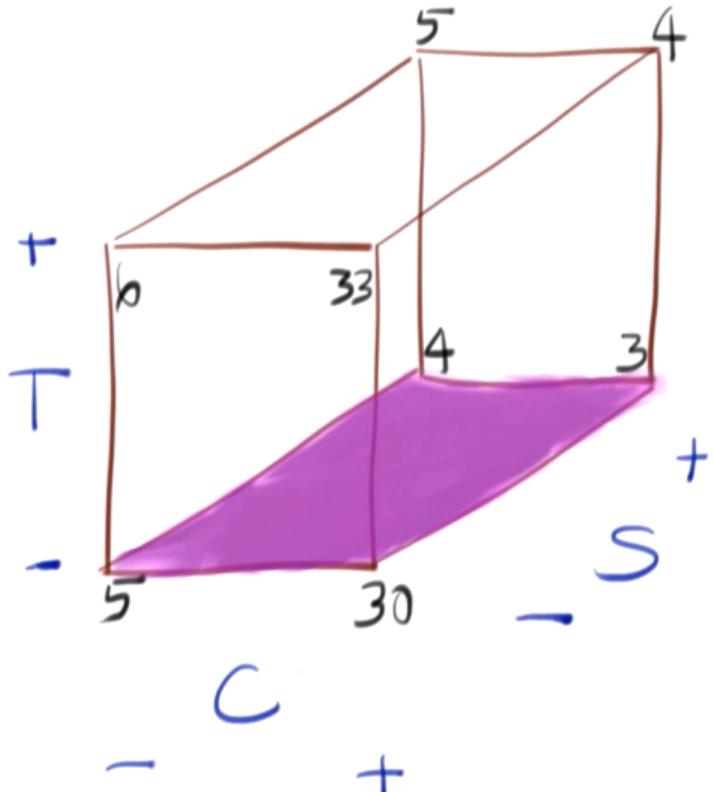
At high temperature, $T = +$

T	S	Effect of C
+	+	$4 - 5 = -1$
+	-	$33 - 6 = 27$

$$\frac{-1 - 27}{2} = -14$$

Waste water treatment example: analysis of the data by hand

Calculate the b_{CS} interaction
“How C interacts with S”

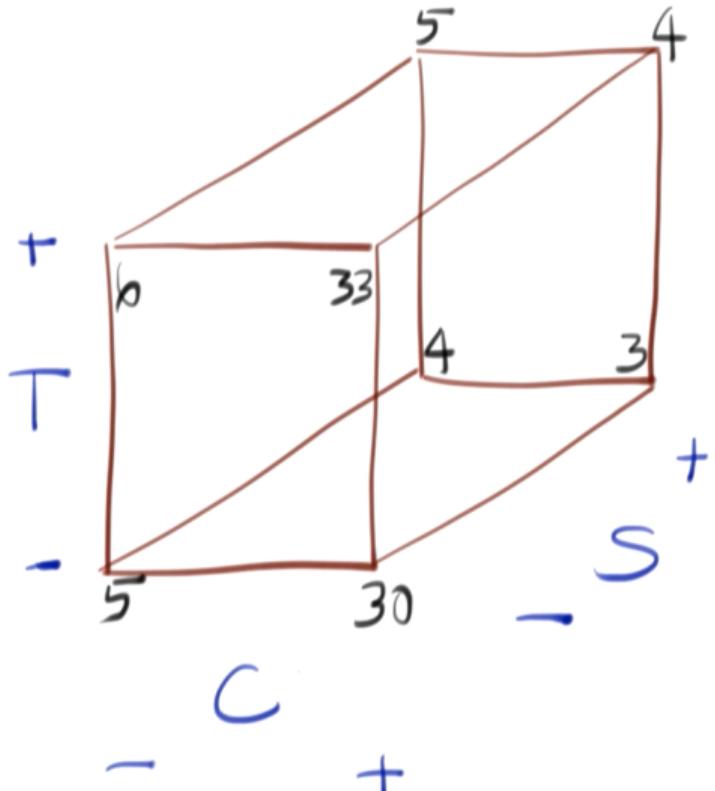


At low temperature, $T = -$

T	S	Effect of C
-	+	$3 - 4 = -1$
-	-	$30 - 5 = 25$

$$\frac{-1 - 25}{2} = -13$$

Waste water treatment example: analysis of the data by hand



Calculate the b_{CS} interaction
“How C interacts with S”

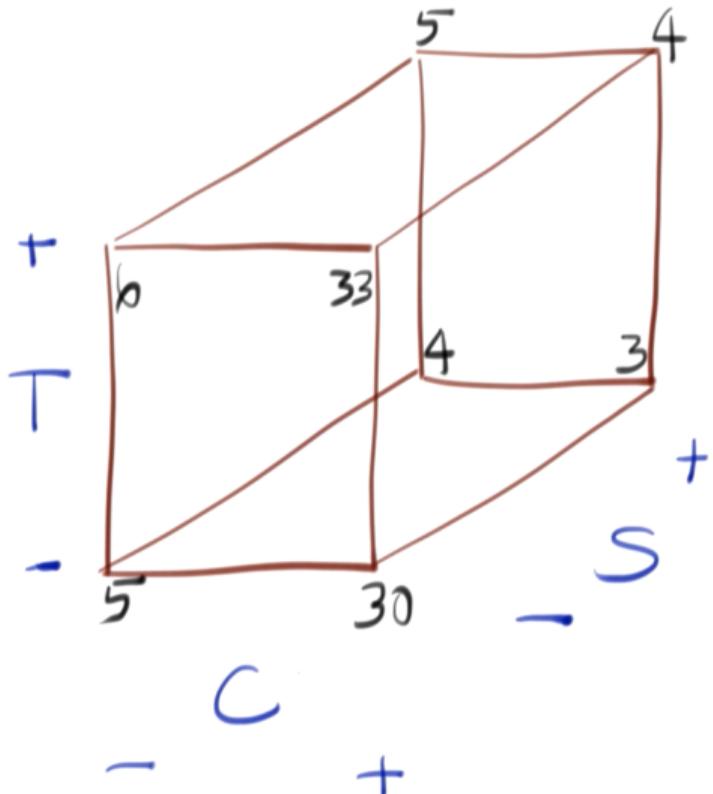
At high temperature, $T = +$
 CS interaction is -14

At low temperature, $T = -$
 CS interaction is -13

Average CS interaction is -13.5

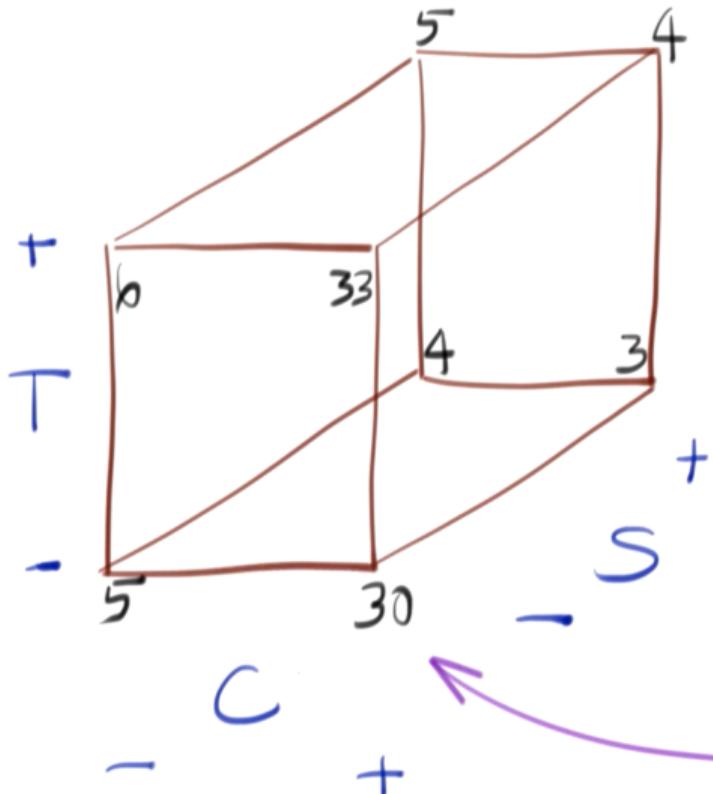
Reported as $b_{CS} = \frac{-13.5}{2} = -6.75$

Waste water treatment example: analysis of the data by hand



$$\begin{aligned}y &= 11.25 & y &= b_{\text{intercept}} \\&+ 6.25x_C & + b_C x_C \\&+ 0.75x_T & + b_T x_T \\&- 7.25x_S & + b_S x_S \\&- 6.75x_C x_S & + b_{CS} x_C x_S\end{aligned}$$

Waste water treatment example: analysis of the data by hand

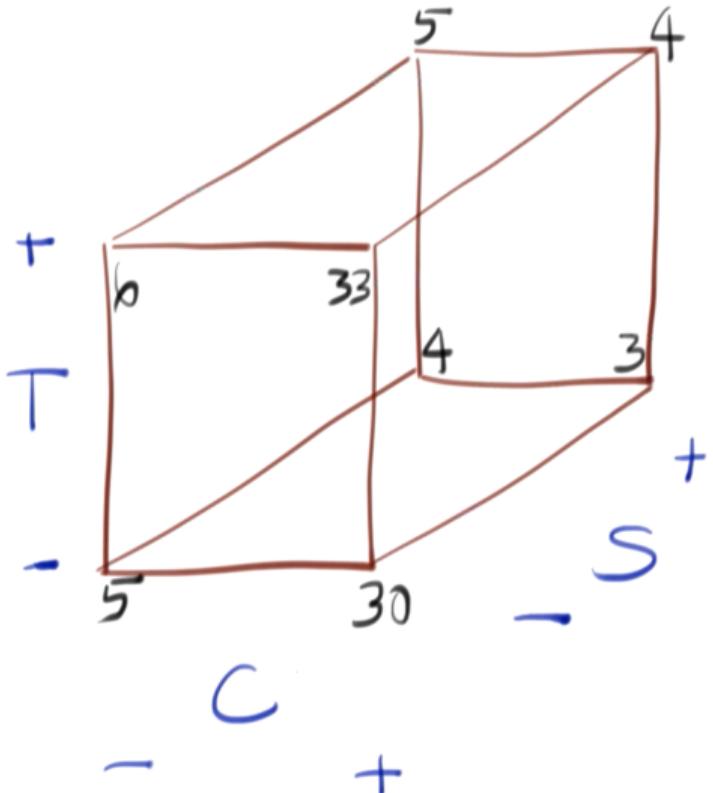


$$y = 11.25 + 6.25x_C + 0.75x_T - 7.25x_S - 6.75x_C x_S$$

Prediction

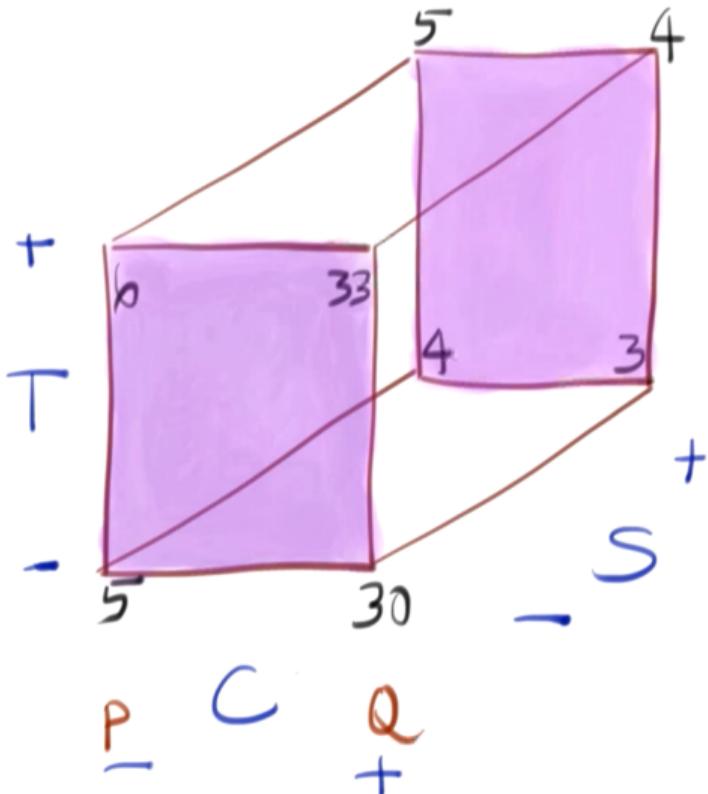
$$\hat{y} = 11.25 + 6.25(+1) + 0.75(-1) - 7.25(-1) - 6.75(+1)(-1)$$
$$\hat{y} = 30.75$$

Waste water treatment example: analysis of the data by hand



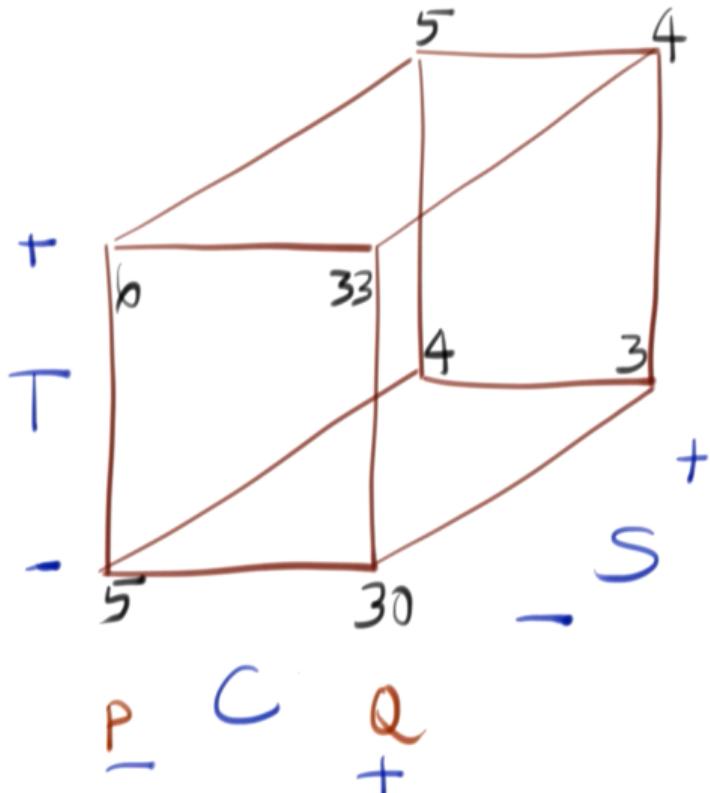
$$\begin{aligned}y &= 11.25 \\&+ 6.25x_C \\&+ 0.75x_T \\&- 7.25x_S \\&- 6.75x_C x_S \\&+ 0.25x_C x_T \\&- 0.25x_T x_S \\&- 0.25x_C x_T x_S\end{aligned}$$

Waste water treatment example: analysis of the data by hand



$$\begin{aligned}y &= 11.25 & y &= b_{\text{intercept}} \\&+ 6.25x_C && + b_C x_C \\&+ 0.75x_T && + b_T x_T \\&- 7.25x_S && + b_S x_S \\&\boxed{- 6.75x_C x_S} && + b_{CS} x_C x_S\end{aligned}$$

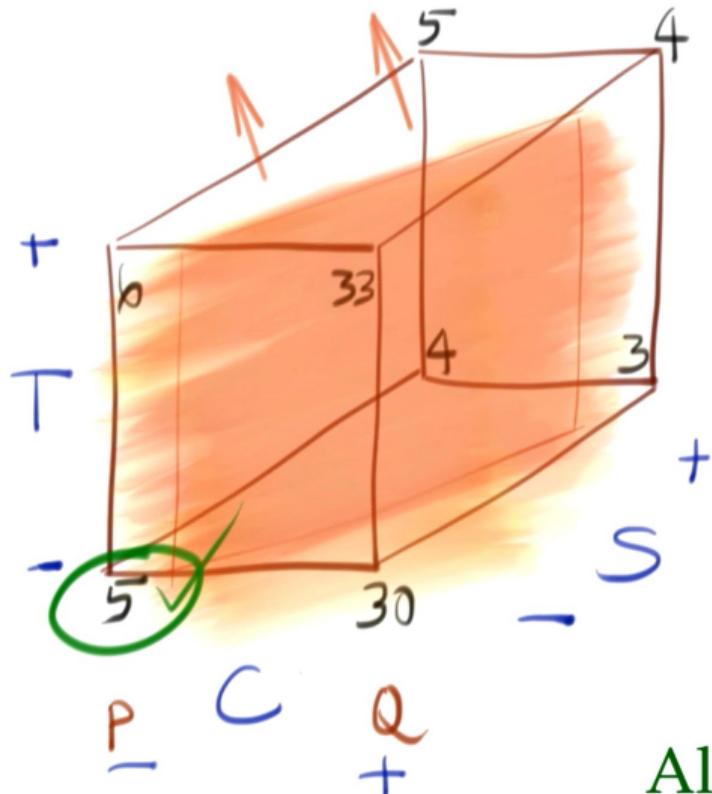
Waste water treatment example: analysis of the data by hand



$$\begin{aligned}y &= 11.25 & y &= b_{\text{intercept}} \\&+ 6.25x_C && + b_C x_C \\&+ 0.75x_T && + b_T x_T \\&- 7.25x_S && + b_S x_S \\&- 6.75x_C x_S && + b_{CS} x_C x_S\end{aligned}$$

Pollution < 10 lbs
Cost of Q is double cost of P

Waste water treatment example: analysis of the data by hand



$$\begin{aligned}y &= 11.25 & y &= b_{\text{intercept}} \\&+ 6.25x_C && + b_C x_C \\&+ 0.75x_T && + b_T x_T \\&- 7.25x_S && + b_S x_S \\&- 6.75x_C x_S && + b_{CS} x_C x_S\end{aligned}$$

Pollution < 10 lbs

Cost of Q is double cost of P

Always consider secondary outcomes