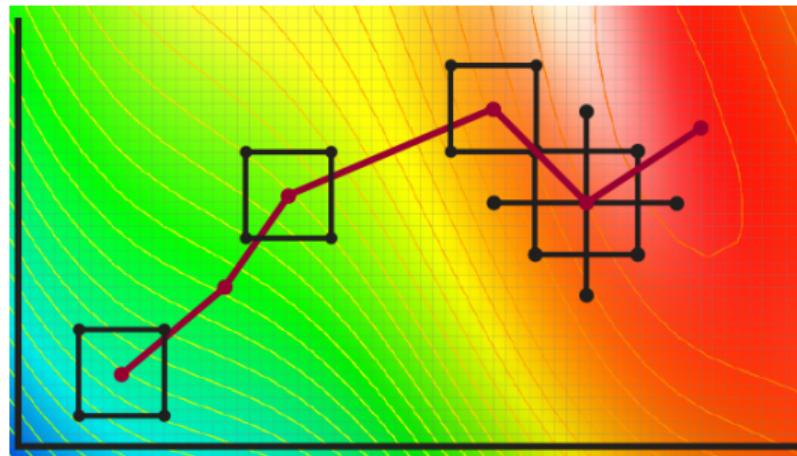


Experimentation for Improvement



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Design and Analysis of Experiments

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Outcome

Number of popped corn

Objective

Maximizing number of popped corn

This is equivalent to:

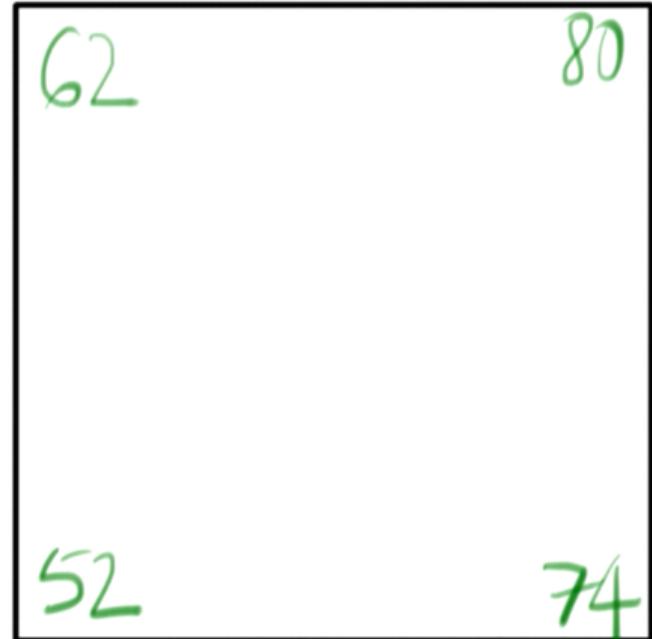
“minimize the number of unpopped corn”

+

Yellow
corn

B = Corn

— White
corn



A = Time

160s

200s

+

$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$



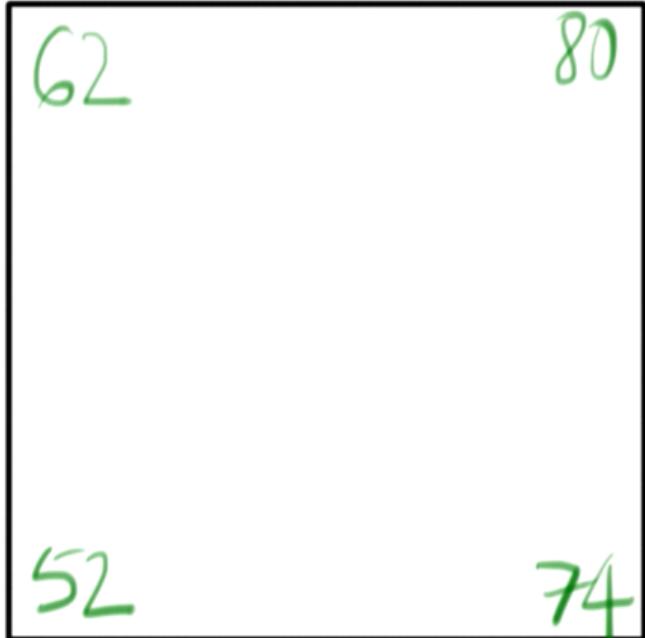
prediction

+

Yellow
corn

B = Corn

White
corn



160s

A = Time

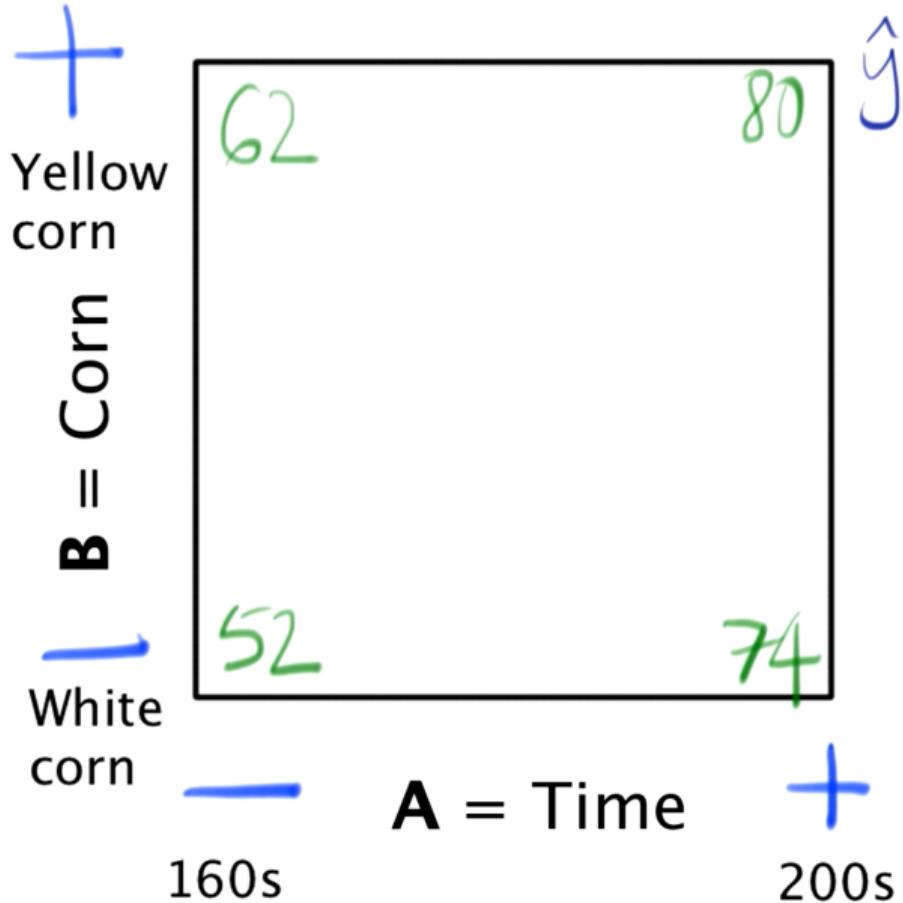
200s

+

$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

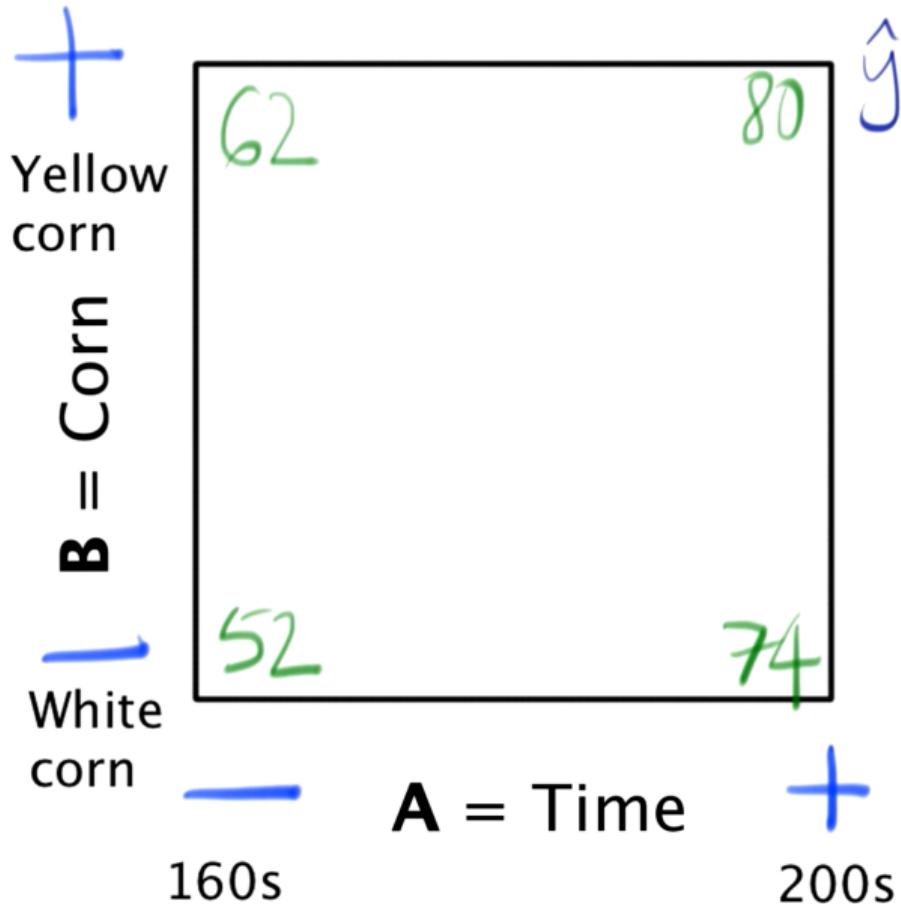


baseline or intercept



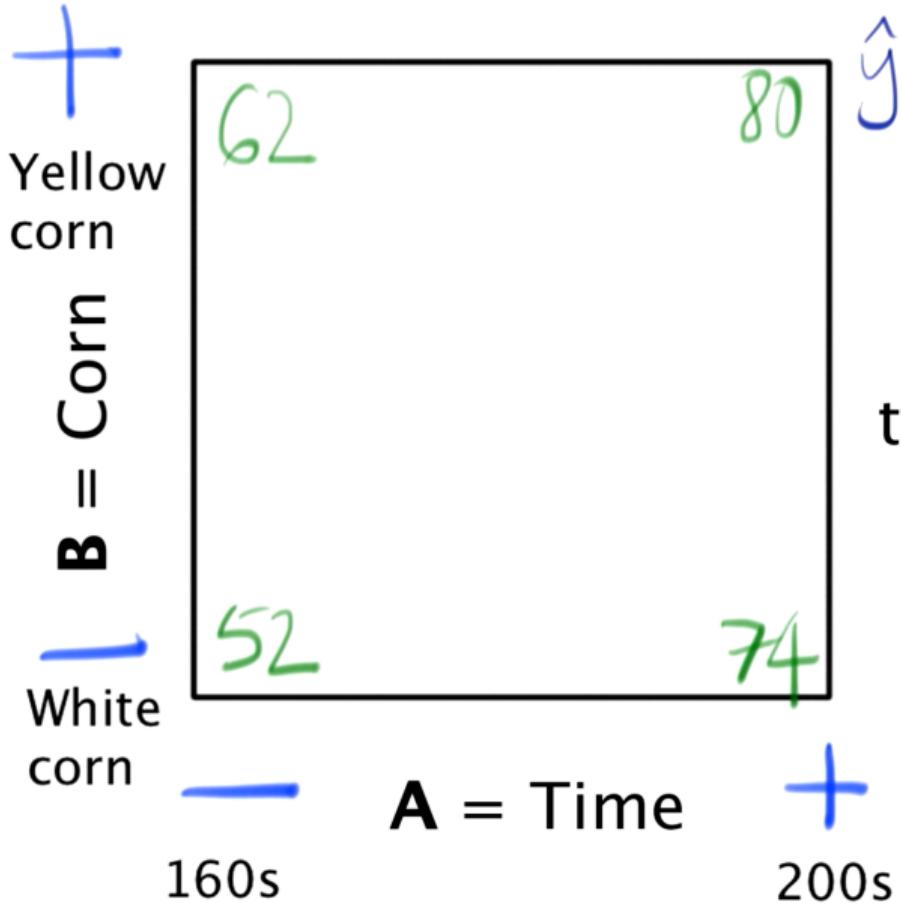
$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

Main effect of factor A



$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

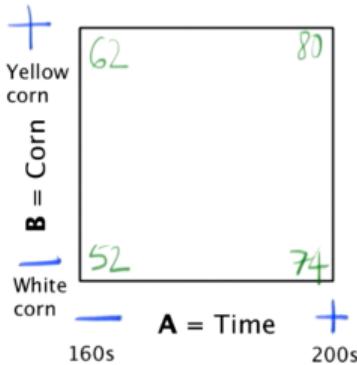
Main effect of factor **B**



$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

two factor interaction, AB

Exercise: make sure you can calculate the "-1" by hand.



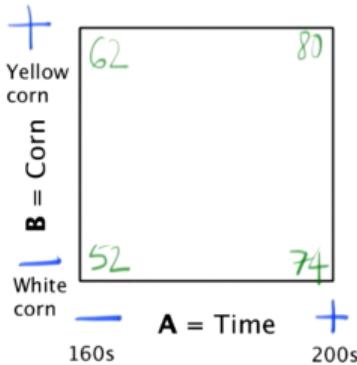
$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

$x_A \equiv$ coded cooking time value (factor A)

$$x_A = -1 \quad A = 160 \text{ seconds}$$

$$\underbrace{x_A = +1}_{\text{coded units}} \quad A = \underbrace{200 \text{ seconds}}_{\text{real-world units}}$$



$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

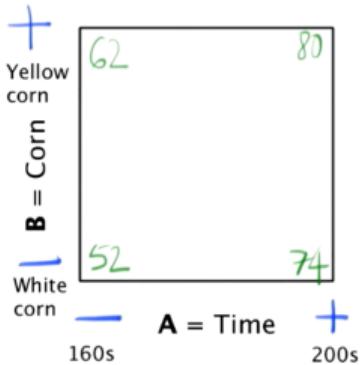
$x_B \equiv \text{coded corn type (factor B)}$

$$x_B = -1 \quad B = \text{white corn}$$

$$\underbrace{x_B = +1}_{\text{coded units}} \quad B = \underbrace{\text{yellow corn}}_{\text{real-world units}}$$



these can be switched around
(the sign will flip)

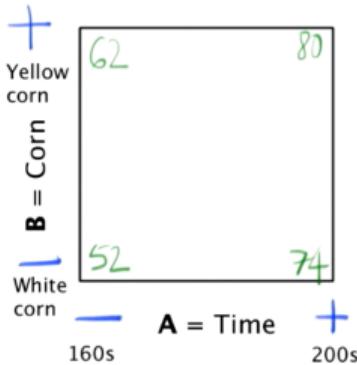


Experiment 1: $y_1 = b_0 + b_A x_{A-} + b_B x_{B-} + b_{AB} x_{A-} x_{B-}$

Experiment 2: $y_2 = b_0 + b_A x_{A+} + b_B x_{B-} + b_{AB} x_{A+} x_{B-}$

Experiment 3: $y_3 = b_0 + b_A x_{A-} + b_B x_{B+} + b_{AB} x_{A-} x_{B+}$

Experiment 4: $y_4 = b_0 + b_A x_{A+} + b_B x_{B+} + b_{AB} x_{A+} x_{B+}$



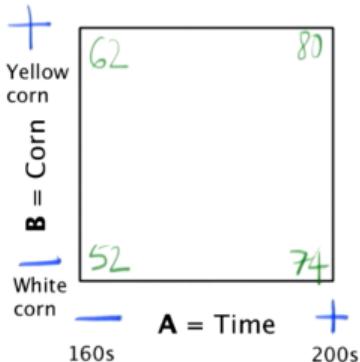
$$\hat{y} = 67 + 10x_A + 4x_B - 1x_A x_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

Experiment 1: $y_1 = b_0 + b_A x_{A-} + b_B x_{B-} + b_{AB} x_{A-} x_{B-}$

Some shorthand. Let ...

x_{A-}	represent	$x_A = -1$
x_{A+}	represent	$x_A = +1$
x_{B-}	represent	$x_B = -1$
x_{B+}	represent	$x_B = +1$

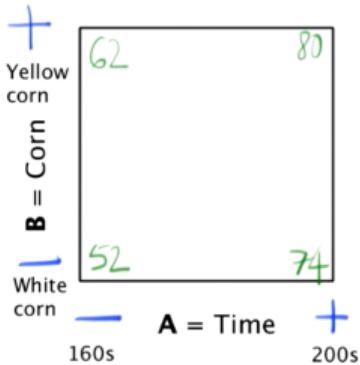


$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

- Experiment 1:** $y_1 = b_0 + b_A(-1) + b_B(-1) + b_{AB}(-1)(-1)$
- Experiment 2:** $y_2 = b_0 + b_A(+1) + b_B(-1) + b_{AB}(+1)(-1)$
- Experiment 3:** $y_3 = b_0 + b_A(-1) + b_B(+1) + b_{AB}(-1)(+1)$
- Experiment 4:** $y_4 = b_0 + b_A(+1) + b_B(+1) + b_{AB}(+1)(+1)$

Four unknowns above: b_0, b_A, b_B , and b_{AB}

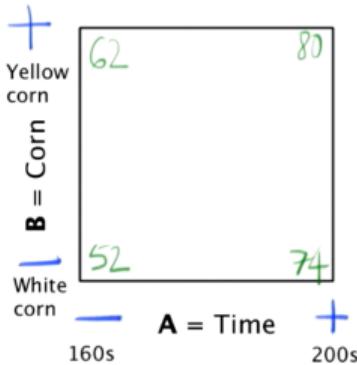


$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

For all experiments

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & (-1) & (-1) & (-1)(-1) \\ 1 & (+1) & (-1) & (+1)(-1) \\ 1 & (-1) & (+1) & (-1)(+1) \\ 1 & (+1) & (+1) & (+1)(+1) \end{pmatrix} \begin{pmatrix} b_0 \\ b_A \\ b_B \\ b_{AB} \end{pmatrix}$$



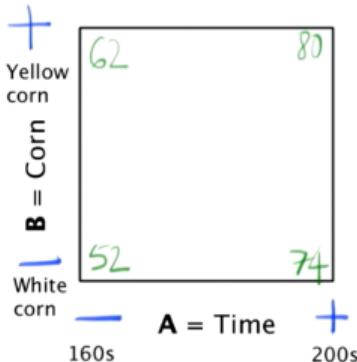
$$\hat{y} = 67 + 10x_A + 4x_B - 1x_Ax_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

For all experiments

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & +1 \\ 1 & +1 & -1 & -1 \\ 1 & -1 & +1 & -1 \\ 1 & +1 & +1 & +1 \end{pmatrix} \begin{pmatrix} b_0 \\ b_A \\ b_B \\ b_{AB} \end{pmatrix}$$

$\mathbf{y} = \mathbf{X} \mathbf{b}$



$$\hat{y} = 67 + 10x_A + 4x_B - 1x_A x_B$$

$$\hat{y} = b_0 + b_A x_A + b_B x_B + b_{AB} x_A x_B$$

The solution is: $\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & +1 \\ 1 & +1 & -1 & -1 \\ 1 & -1 & +1 & -1 \\ 1 & +1 & +1 & +1 \end{pmatrix} \mathbf{b}$$