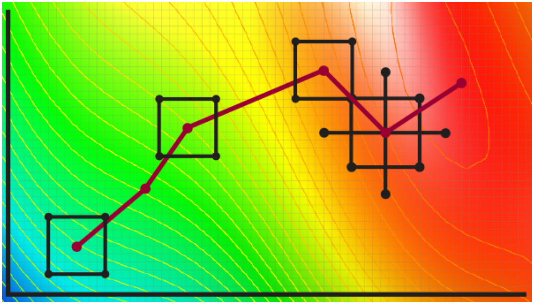
Experimentation for Improvement



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Design and Analysis of Experiments

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ADVICE

pick a design that meets the objective

- If you are just starting out, avoid eliminating factors to simply get a full factorial.
- Use the experimental evidence to eliminate factors.
- Remember: these are experimental building blocks. The experiments you run first can be extended on later.
- In the next example, we show how factors are eliminated, *based on evidence*.

[Flickr: rahego]

An example to demonstrate a saturated fractional factorial analyis

We have 7 factors: **A**, **B**, **C**, **D**, **E**, **F**, and **G**.

The fewest number of experiments is: 8 runs

Creating the standard order table for the fractional factorial

Experiment	A	B	C	D = AB	E = AC	F = BC	G = ABC
1	−	−	−	+	+	+	−
2	+	−	−	−	−	+	+
3	−	+	−	−	+	−	+
4	+	+	−	+	−	−	−
5	−	−	+	+	−	−	+
6	+	−	+	−	+	−	−
7	−	+	+	−	−	+	−
8	+	+	+	+	+	+	+

$$\boxed{2^{7\text{-}4}_{\mathrm{III}} \quad \begin{array}{l} \pm D=AB \\ \pm E=AC \\ \pm F=BC \\ \pm G=ABC \end{array}}$$

2^{k-p}

with $p = 4$

Creating the standard order table for the fractional factorial

Experiment	A	B	C	D = AB	E = AC	F = BC	G = ABC	y
1	$-$	$-$	$-$	$+$	$+$	$+$	$-$	
2	$+$	$-$	$-$	$-$	$-$	$+$	$+$	
3	$-$	$+$	$-$	$-$	$+$	$-$	$+$	
4	$+$	$+$	$-$	$+$	$-$	$-$	$-$	
5	$-$	$-$	$+$	$+$	$-$	$-$	$+$	
6	$+$	$-$	$+$	$-$	$+$	$-$	$-$	
7	$-$	$+$	$+$	$-$	$-$	$+$	$-$	
8	$+$	$+$	$+$	$+$	$+$	$+$	$+$	

$2^{7\text{-}4}_{\mathrm{III}}$
±D=AB
±E=AC
±F=BC
±G=ABC

2^{k-p}
with $p = 4$

Let's follow the recommended approach shown earlier in the video

1. Read the generators from the trade off table
   - **D = AB** and **E = AC** and **F = BC** and **G = ABC**
2. Rearrange the generators as **I = . . .**
   - **I = ABD** and **I = ACE** and **I = BCF** and **I = ABCG**
3. Form the **defining relationship** taking all combinations of the words: **I = . . .**

   I = ABD = ACE = BCF = ABCG = . . . ⟵ that's 5 so far
   BDCE = ACDF = CDG = ABEF = BEG = AFG = . . . ⟵ that's 11
   DEF = ADEG = CEFG = BDFG = . . . ⟵ we are up to 15
   ABCDEFG ⟵ we have all 16 here

   (ABD)(ACE) = BCDE (ABD)(BCF) = ACDF (ABD)(ACE)(BCF) = DEF
4. Ensure the defining relationship has 2^p words
   - $p = 4$, so we have 16 words.
5. Use the defining relationship to compute the aliasing pattern
6. Ensure the aliasing is acceptable

Check the alias patterns by using the defining relationship

$$
\begin{aligned}
\mathbf{A} &= \mathbf{BD} = \mathbf{CE} = \mathbf{FG} = \mathbf{BCG} = \mathbf{CDF} = \mathbf{BEF} = \mathbf{DEG} = \mathbf{ABCF} = \mathbf{ABEG} = \ldots \\
&\qquad = \mathbf{ACDG} = \mathbf{ADEF} = \mathbf{ABDCE} = \mathbf{ABDFG} = \mathbf{ACEFG} = \mathbf{BCDEFG} \\
\mathbf{B} &= \mathbf{AD} = \mathbf{CF} = \mathbf{EG} + \text{other higher order interactions} \\
\mathbf{C} &= \mathbf{AE} = \mathbf{BF} = \mathbf{DG} \\
\mathbf{D} &= \mathbf{AB} = \mathbf{CG} = \mathbf{EF} \\
\mathbf{E} &= \mathbf{AC} = \mathbf{BG} = \mathbf{DF} \\
\mathbf{F} &= \mathbf{BC} = \mathbf{AG} = \mathbf{DE} \\
\mathbf{G} &= \mathbf{CD} = \mathbf{BE} = \mathbf{AF}
\end{aligned}
$$

Finally! You get to do the experiments and record the outcome value

Experiment	**A**	**B**	**C**	**D = AB**	**E = AC**	**F = BC**	**G = ABC**	y
1	$-$	$-$	$-$	$+$	$+$	$+$	$-$	320
2	$+$	$-$	$-$	$-$	$-$	$+$	$+$	276
3	$-$	$+$	$-$	$-$	$+$	$-$	$+$	306
4	$+$	$+$	$-$	$+$	$-$	$-$	$-$	290
5	$-$	$-$	$+$	$+$	$-$	$-$	$+$	272
6	$+$	$-$	$+$	$-$	$+$	$-$	$-$	274
7	$-$	$+$	$+$	$-$	$-$	$+$	$-$	290
8	$+$	$+$	$+$	$+$	$+$	$+$	$+$	255

The reduced model: only has four main effect factors

Experiment	A	C	E	G	B	D	F	y
1	$-$	$-$	$+$	$-$	$-$	$+$	$+$	320
2	$+$	$-$	$-$	$+$	$-$	$-$	$+$	276
3	$-$	$-$	$+$	$+$	$+$	$-$	$-$	306
4	$+$	$-$	$-$	$-$	$+$	$+$	$-$	290
5	$-$	$+$	$-$	$+$	$-$	$+$	$-$	272
6	$+$	$+$	$+$	$-$	$-$	$-$	$-$	274
7	$-$	$+$	$-$	$-$	$+$	$-$	$+$	290
8	$+$	$+$	$+$	$+$	$+$	$+$	$+$	255

Those that are more math oriented: please verify that each column is uncorrelated with the others. So rebuilding the model implies the factor estimates are the same.

Other fractional factorial designs: Plackett-Burman designs

Plackett-Burman designs exists in multiples of 4:

- 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...
- Main effects are confounded with two-factor interactions, but in a complicated way.
- Such designs are most usefully created by software, unlike the fractional factorial designs shown in this section.
- e.g. a Placket-Burman design in 20 runs, can screen for 19 factors!

Number of factors, k

Number of runs	3	4	5	6	7	8
4	2^{3-1}_{III} ±C=AB					
8	2^{3} *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC	
16	2^{3} *twice*	2^{4} *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=BCD	2^{7-3}_{IV} ±E=ABC ±F=BCD ±G=ACD	2^{8-4}_{IV} ±E=BCD ±F=ACD ±G=ABC ±H=ABD
32	2^{3} *4 times*	2^{4} *twice*	2^{5} *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABCD ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=BCDE
64	2^{3} *8 times*	2^{4} *4 times*	2^{5} *twice*	2^{6} *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF

Definitive Screening Designs: a type of optimal design

Optimal designs

- There are several desirable mathematical criteria that can be optimal.
- We won't go into the details, but factorial designs often meet these optimal criteria.
- Interested in the details? Search for:
  - `D-optimal designs`
    it is the most common optimal design

Definitive screening designs

- Factors can be at 3 levels (not 2!)
- Small number of runs
- Main effects and 2-factor interactions **are not** aliased - a great advantage.

D-Optimal Fractions of Three-Level Factorial Designs

T. J. Mitchell and C. K. Bayne
Union Carbide Corporation
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D-optimal fractions of three-level factorial designs for p factors are c
effects models ($2 \le p \le 4$) and quadratic response surface models ($2 \le p$
generated using an exchange algorithm for maximizing $|X'X|$ and a
duces D-optimal balanced array designs. The design properties for the

[http://www.jstor.org/discover/10.2307/1267635]

A Class of Three-Level Designs for Definitive Screening in the Presence of Second-Order Effects

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[http://yint.org/dsdesign]

ADVICE

Practice, fail, start over, and persist

- There are case studies in the course textbook
- There are other textbooks, listed on the course website
- Create your own datasets
  - biscuits
  - coffee
  - growing plants, or
  - many of the experiments suggested in the course forums