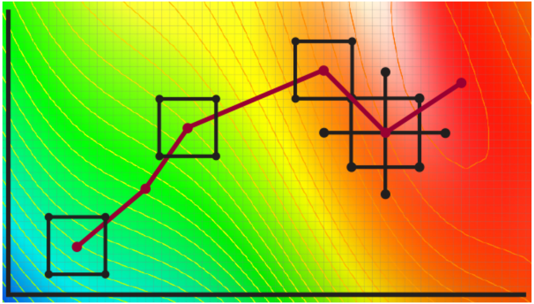
Experimentation for Improvement



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Design and Analysis of Experiments

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Number of factors, *k*

Number of runs	3	4	5	6	7	8	9
4	2^{3-1}_{III} ±C=AB						
8	2^{3} *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC		
16	2^{3} *twice*	2^{4} *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=ABD	2^{7-3}_{IV} ±E=ABC ±F=ABD ±G=ACD	2^{8-4}_{IV} ±E=ABC ±F=ABD ±G=ACD ±H=BCD	2^{9-5}_{III}
32	2^{3} *4 times*	2^{4} *twice*	2^{5} *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABC ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=ACDE	2^{9-4}_{IV}
64	2^{3} *8 times*	2^{4} *4 times*	2^{5} *twice*	2^{6} *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF	2^{9-3}_{IV}

increasing cost

Number of factors, k

Number of runs	3	4	5	6	7	8	9
4	2^{3-1}_{III} ±C=AB						
8	2^{3} *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC		
16	2^{3} *twice*	2^{4} *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=ABD	2^{7-3}_{IV} ±E=ABC ±F=ABD ±G=ACD	2^{8-4}_{IV} ±E=ABC ±F=ABD ±G=ACD ±H=BCD	2^{9-5}_{III}
32	2^{3} *4 times*	2^{4} *twice*	2^{5} *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABC ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=ACDE	2^{9-4}_{IV}
64	2^{3} *8 times*	2^{4} *4 times*	2^{5} *twice*	2^{6} *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF	2^{9-3}_{IV}

increasing information about additional factors

lower resolution greater aliasing

Number of factors, *k*

Number of runs	3	4	5	6	7	8	9
4	2^{3-1}_{III} ±C=AB						
8	2^{3} *full*	2^{4-1}_{IV} ±D=ABC	2^{5-2}_{III} ±D=AB ±E=AC	2^{6-3}_{III} ±D=AB ±E=AC ±F=BC	2^{7-4}_{III} ±D=AB ±E=AC ±F=BC ±G=ABC		
16	2^{3} *twice*	2^{4} *full*	2^{5-1}_{V} ±E=ABCD	2^{6-2}_{IV} ±E=ABC ±F=ABD	2^{7-3}_{IV} ±E=ABC ±F=ABD ±G=ACD	2^{8-4}_{IV} ±E=ABC ±F=ABD ±G=ACD ±H=BCD	2^{9-5}_{III}
32	2^{3} *4 times*	2^{4} *twice*	2^{5} *full*	2^{6-1}_{VI} ±F=ABCDE	2^{7-2}_{IV} ±F=ABC ±G=ABDE	2^{8-3}_{IV} ±F=ABC ±G=ABD ±H=ACDE	2^{9-4}_{IV}
64	2^{3} *8 times*	2^{4} *4 times*	2^{5} *twice*	2^{6} *full*	2^{7-1}_{VII} ±G=ABCDEF	2^{8-2}_{V} ±G=ABCD ±H=ABEF	2^{9-3}_{IV}

For your own case: be prepared to remap your letters ... temporarily

Original factor names

- **C**: chemical variable
- **T**: temperature variable
- **S**: stirring speed variable

Factor names in the table

- **A**
- **B**
- **C**

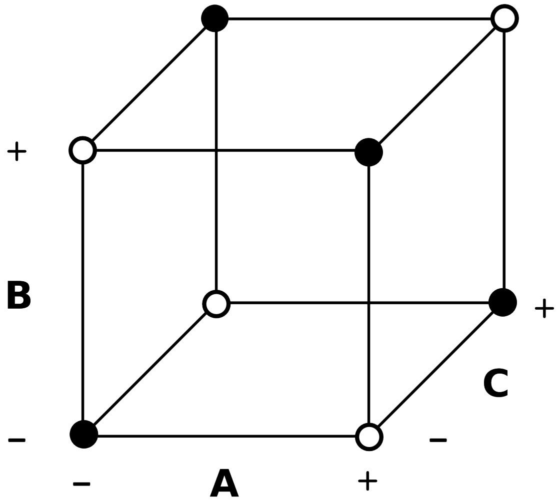
$+\mathbf{S} = \mathbf{CT}$

$+\mathbf{C} = \mathbf{AB}$

$-\mathbf{S} = \mathbf{CT}$

$-\mathbf{C} = \mathbf{AB}$

Setting up the half-fraction in 3 factors: using the complementary half



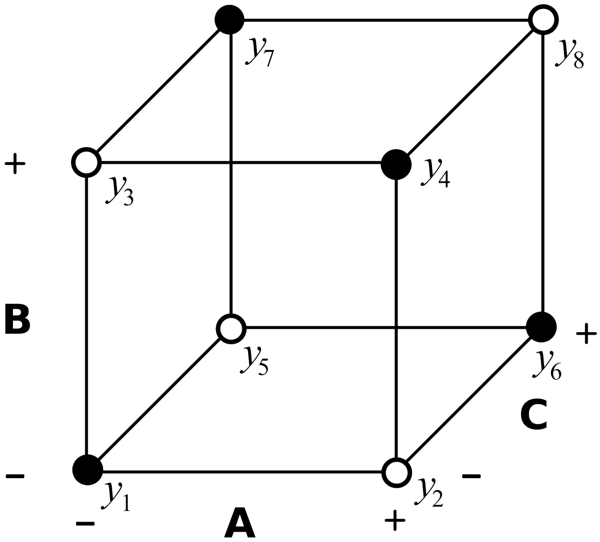
Experiment	**A**	**B**	**C** $= -$**AB**
1	$-$	$-$	$-(-)(-) = -$
2	$+$	$-$	$-(+)(-) = +$
3	$-$	$+$	$-(-)(+) = +$
4	$+$	$+$	$-(+)(+) = -$

$-$**C** $=$ **AB** can be rewritten as **C** $= -$**AB**

Notice how the 4 runs generated with **C** $= -$**AB** correspond to the closed circles.

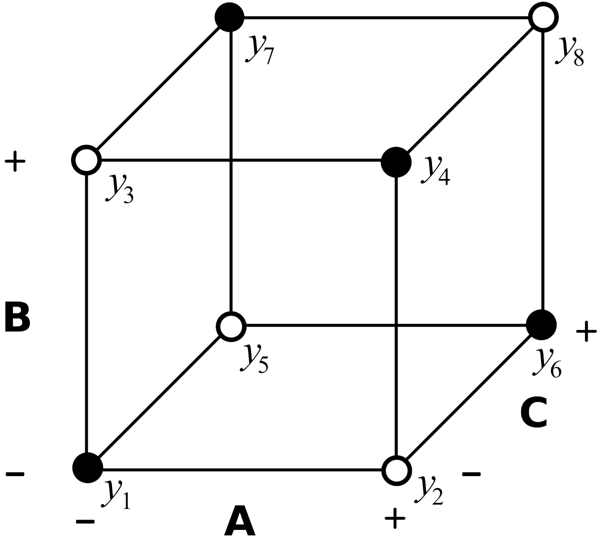
The 4 runs generated with **C** $= +$**AB** correspond to the open circles.

Creating and understanding fractional factorials from a half-fraction example



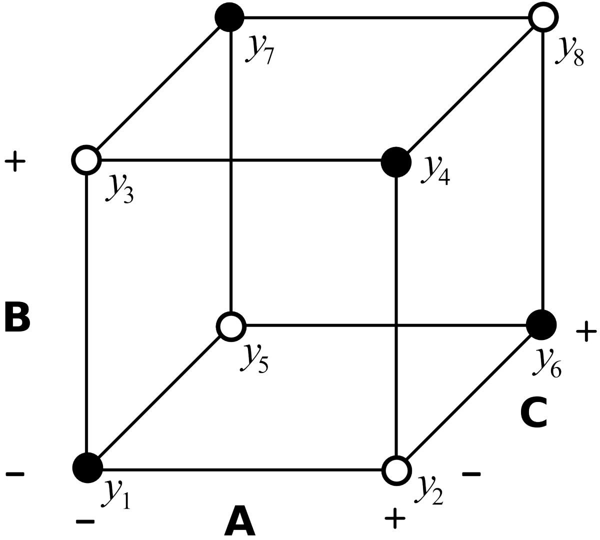
Experiment	**A**	**B**	**C**
1	−	−	−
2	+	−	−
3	−	+	−
4	+	+	−
5	−	−	+
6	+	−	+
7	−	+	+
8	+	+	+

Creating and understanding fractional factorials from a half-fraction example



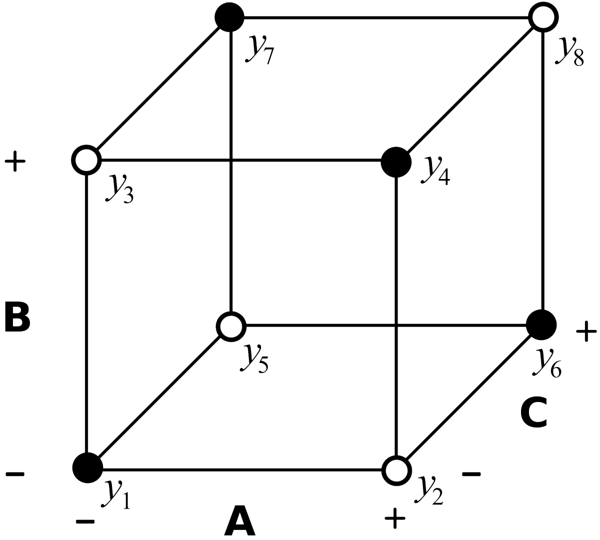
Experiment	A	B	C
2	+	−	−
3	−	+	−
5	−	−	+
8	+	+	+

Creating and understanding fractional factorials from a half-fraction example



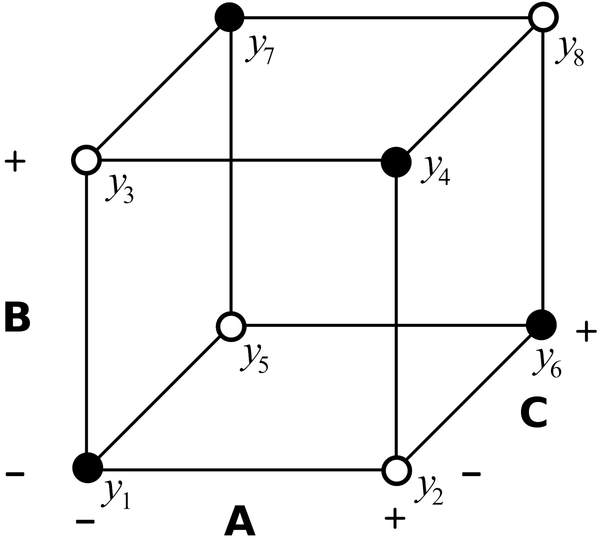
Experiment	**A**	**B**	**C**
5	$-$	$-$	$+$
2	$+$	$-$	$-$
3	$-$	$+$	$-$
8	$+$	$+$	$+$

Creating and understanding fractional factorials from a half-fraction example



Experiment	A	B	C
5	−	−	+
2	+	−	−
3	−	+	−
8	+	+	+

Creating and understanding fractional factorials from a half-fraction example



Experiment	**A**	**B**	**C = AB**
5	−	−	+
2	+	−	−
3	−	+	−
8	+	+	+

Our predictive model $\hat{y} = b_0 + b_{\mathsf{C}} x_{\mathsf{C}} + b_{\mathsf{T}} x_{\mathsf{T}} + b_{\mathsf{S}} x_{\mathsf{S}} + b_{\mathsf{CT}} x_{\mathsf{C}} x_{\mathsf{T}} + b_{\mathsf{CS}} x_{\mathsf{C}} x_{\mathsf{S}} + b_{\mathsf{TS}} x_{\mathsf{T}} x_{\mathsf{S}} + b_{\mathsf{CTS}} x_{\mathsf{C}} x_{\mathsf{T}} x_{\mathsf{S}}$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{array}{cccccccc} b_0 & b_{\mathsf{C}} & b_{\mathsf{T}} & b_{\mathsf{S}} & b_{\mathsf{CT}} & b_{\mathsf{CS}} & b_{\mathsf{TS}} & b_{\mathsf{CTS}} \end{array} \\ \begin{pmatrix} +1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 \\ +1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 \\ +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \end{pmatrix} \end{array} \begin{pmatrix} b_0 \\ b_{\mathsf{C}} \\ b_{\mathsf{T}} \\ b_{\mathsf{S}} \\ b_{\mathsf{CT}} \\ b_{\mathsf{CS}} \\ b_{\mathsf{TS}} \\ b_{\mathsf{CTS}} \end{pmatrix}$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_A x_A + b_B x_B + b_C x_C + b_{AB} x_A x_B + b_{AC} x_A x_C + b_{BC} x_B x_C + b_{ABC} x_A x_B x_C$

$$
\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix}
=
\begin{array}{c}
\begin{array}{cccccccc} b_0 & b_A & b_B & b_C & b_{AB} & b_{AC} & b_{BC} & b_{ABC} \end{array} \\
\begin{pmatrix}
+1 & -1 & -1 & -1 & +1 & +1 & +1 & -1 \\
+1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\
+1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\
+1 & +1 & +1 & -1 & +1 & -1 & -1 & -1 \\
+1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\
+1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 \\
+1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 \\
+1 & +1 & +1 & +1 & +1 & +1 & +1 & +1
\end{pmatrix}
\end{array}
\begin{pmatrix} b_0 \\ b_A \\ b_B \\ b_C \\ b_{AB} \\ b_{AC} \\ b_{BC} \\ b_{ABC} \end{pmatrix}
$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_{\mathsf{A}}x_{\mathsf{A}} + b_{\mathsf{B}}x_{\mathsf{B}} + b_{\mathsf{C}}x_{\mathsf{C}} + b_{\mathsf{AB}}x_{\mathsf{A}}x_{\mathsf{B}} + b_{\mathsf{AC}}x_{\mathsf{A}}x_{\mathsf{C}} + b_{\mathsf{BC}}x_{\mathsf{B}}x_{\mathsf{C}} + b_{\mathsf{ABC}}x_{\mathsf{A}}x_{\mathsf{B}}x_{\mathsf{C}}$

$$\begin{array}{cccc} & & \begin{array}{cccccccc} b_0 & b_{\mathsf{A}} & b_{\mathsf{B}} & b_{\mathsf{C}} & b_{\mathsf{AB}} & b_{\mathsf{AC}} & b_{\mathsf{BC}} & b_{\mathsf{ABC}} \end{array} & \\ \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \end{pmatrix} & = & \begin{pmatrix} + & - & - & - & + & + & + & - \\ + & + & - & - & - & - & + & + \\ + & - & + & - & - & + & - & + \\ + & + & + & - & + & - & - & - \\ + & - & - & + & + & - & - & + \\ + & + & - & + & - & + & - & - \\ + & - & + & + & - & - & + & - \\ + & + & + & + & + & + & + & + \end{pmatrix} & \begin{pmatrix} b_0 \\ b_{\mathsf{A}} \\ b_{\mathsf{B}} \\ b_{\mathsf{C}} \\ b_{\mathsf{AB}} \\ b_{\mathsf{AC}} \\ b_{\mathsf{BC}} \\ b_{\mathsf{ABC}} \end{pmatrix} \end{array}$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_{\mathsf{A}} x_{\mathsf{A}} + b_{\mathsf{B}} x_{\mathsf{B}} + b_{\mathsf{C}} x_{\mathsf{C}} + b_{\mathsf{AB}} x_{\mathsf{A}} x_{\mathsf{B}} + b_{\mathsf{AC}} x_{\mathsf{A}} x_{\mathsf{C}} + b_{\mathsf{BC}} x_{\mathsf{B}} x_{\mathsf{C}} + b_{\mathsf{ABC}} x_{\mathsf{A}} x_{\mathsf{B}} x_{\mathsf{C}}$

$$\begin{pmatrix} \\ y_2 \\ y_3 \\ \\ y_5 \\ \\ \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{matrix} b_0 & b_{\mathsf{A}} & b_{\mathsf{B}} & b_{\mathsf{C}} & b_{\mathsf{AB}} & b_{\mathsf{AC}} & b_{\mathsf{BC}} & b_{\mathsf{ABC}} \end{matrix} \\ \begin{pmatrix} & & & & & & & \\ + & + & - & - & - & - & + & + \\ + & - & + & - & - & + & - & + \\ & & & & & & & \\ + & - & - & + & + & - & - & + \\ & & & & & & & \\ & & & & & & & \\ + & + & + & + & + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 \\ b_{\mathsf{A}} \\ b_{\mathsf{B}} \\ b_{\mathsf{C}} \\ b_{\mathsf{AB}} \\ b_{\mathsf{AC}} \\ b_{\mathsf{BC}} \\ b_{\mathsf{ABC}} \end{pmatrix}$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_{\mathsf{A}}x_{\mathsf{A}} + b_{\mathsf{B}}x_{\mathsf{B}} + b_{\mathsf{C}}x_{\mathsf{C}} + b_{\mathsf{AB}}x_{\mathsf{A}}x_{\mathsf{B}} + b_{\mathsf{AC}}x_{\mathsf{A}}x_{\mathsf{C}} + b_{\mathsf{BC}}x_{\mathsf{B}}x_{\mathsf{C}} + b_{\mathsf{ABC}}x_{\mathsf{A}}x_{\mathsf{B}}x_{\mathsf{C}}$

$$\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ \\ \\ \\ \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{array}{cccccccc} b_0 & b_{\mathsf{A}} & b_{\mathsf{B}} & b_{\mathsf{C}} & b_{\mathsf{AB}} & b_{\mathsf{AC}} & b_{\mathsf{BC}} & b_{\mathsf{ABC}} \end{array} \\ \begin{pmatrix} + & - & - & + & + & - & - & + \\ + & + & - & - & - & - & + & + \\ + & - & + & - & - & + & - & + \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ + & + & + & + & + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 \\ b_{\mathsf{A}} \\ b_{\mathsf{B}} \\ b_{\mathsf{C}} \\ b_{\mathsf{AB}} \\ b_{\mathsf{AC}} \\ b_{\mathsf{BC}} \\ b_{\mathsf{ABC}} \end{pmatrix}$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_A x_A + b_B x_B + b_C x_C + b_{AB} x_A x_B + b_{AC} x_A x_C + b_{BC} x_B x_C + b_{ABC} x_A x_B x_C$

$$
\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \\ \\ \\ \\ \\ \end{pmatrix}
=
\begin{array}{c}
\begin{array}{cccccccc} b_0 & b_A & b_B & b_C & b_{AB} & b_{AC} & b_{BC} & b_{ABC} \end{array} \\
\begin{pmatrix}
+ & - & - & + & + & - & - & + \\
+ & + & - & - & - & - & + & + \\
+ & - & + & - & - & + & - & + \\
+ & + & + & + & + & + & + & + \\
 & & & & & & & \\
 & & & & & & & \\
 & & & & & & & \\
 & & & & & & &
\end{pmatrix}
\end{array}
\begin{pmatrix} b_0 \\ b_A \\ b_B \\ b_C \\ b_{AB} \\ b_{AC} \\ b_{BC} \\ b_{ABC} \end{pmatrix}
$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

Our predictive model $\hat{y} = b_0 + b_A x_A + b_B x_B + b_C x_C + b_{AB} x_A x_B + b_{AC} x_A x_C + b_{BC} x_B x_C + b_{ABC} x_A x_B x_C$

$$
\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} =
\begin{array}{c}
\begin{array}{cccccccc} b_0 & b_A & b_B & b_C & b_{AB} & b_{AC} & b_{BC} & b_{ABC} \end{array} \\
\begin{pmatrix}
+ & - & - & + & + & - & - & + \\
+ & + & - & - & - & - & + & + \\
+ & - & + & - & - & + & - & + \\
+ & + & + & + & + & + & + & +
\end{pmatrix}
\end{array}
\begin{pmatrix} b_0 \\ b_A \\ b_B \\ b_C \\ b_{AB} \\ b_{AC} \\ b_{BC} \\ b_{ABC} \end{pmatrix}
$$

$$\mathbf{y} = \mathbf{X}\mathbf{b}$$

We have “aliasing” taking place here

Who is *Jorge Mario Bergoglio*?

Wikipedia

We have “aliasing” taking place here

Who is *Kevin George Dunn*?

- My email address
- A username for a website

Aliasing: when we have more than one name for the same thing

What is aliased in this experimental design (i.e. which columns are the same)?

- **A=BC**
- **B=AC**
- **C=AB** (this was intentional: read from the "tradeoff" table)
- **ABC = Intercept** (the intercept is indicated as b_0)

Remove the aliases by collapsing identical columns

$$\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{array}{cccccccc} b_0 & b_{\mathsf{A}} & b_{\mathsf{B}} & b_{\mathsf{C}} & b_{\mathsf{AB}} & b_{\mathsf{AC}} & b_{\mathsf{BC}} & b_{\mathsf{ABC}} \end{array} \\ \begin{pmatrix} + & - & - & + & + & - & - & + \\ + & + & - & - & - & - & + & + \\ + & - & + & - & - & + & - & + \\ + & + & + & + & + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 \\ b_{\mathsf{A}} \\ b_{\mathsf{B}} \\ b_{\mathsf{C}} \\ b_{\mathsf{AB}} \\ b_{\mathsf{AC}} \\ b_{\mathsf{BC}} \\ b_{\mathsf{ABC}} \end{pmatrix}$$

- 4 equations (rows) in 8 unknowns (the entries shown in the last matrix)
- a
- s
- d

Remove the aliases by collapsing identical columns

$$\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{array}{cccc} b_0 + b_{\mathsf{ABC}} & b_{\mathsf{A}} + b_{\mathsf{BC}} & b_{\mathsf{B}} + b_{\mathsf{AC}} & b_{\mathsf{C}} + b_{\mathsf{AB}} \end{array} \\ \begin{pmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & - \\ + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 + b_{\mathsf{ABC}} \\ b_{\mathsf{A}} + b_{\mathsf{BC}} \\ b_{\mathsf{B}} + b_{\mathsf{AC}} \\ b_{\mathsf{C}} + b_{\mathsf{AB}} \end{pmatrix}$$

- 4 equations (rows) in 4 unknowns (the entries in the last matrix)
- a
- s
- d

Remove the aliases by collapsing identical columns

$$\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{array}{cccc} b_0 + b_{\mathsf{ABC}} & b_{\mathsf{A}} + b_{\mathsf{BC}} & b_{\mathsf{B}} + b_{\mathsf{AC}} & b_{\mathsf{C}} + b_{\mathsf{AB}} \end{array} \\ \begin{pmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & - \\ + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 + b_{\mathsf{ABC}} \\ \hat{b}_{\mathsf{A}} \\ b_{\mathsf{B}} + b_{\mathsf{AC}} \\ b_{\mathsf{C}} + b_{\mathsf{AB}} \end{pmatrix}$$

- Let's call the merged coefficient $\hat{b}_{\mathsf{A}} = b_{\mathsf{A}} + b_{\mathsf{BC}}$
- a
- s
- d

Remove the aliases by collapsing identical columns

$$\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} = \begin{array}{c} \begin{matrix} b_0 + b_{ABC} & b_A + b_{BC} & b_B + b_{AC} & b_C + b_{AB} \end{matrix} \\ \begin{pmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & - \\ + & + & + & + \end{pmatrix} \end{array} \begin{pmatrix} b_0 + b_{ABC} \\ \hat{b}_A \\ \hat{b}_B \\ b_C + b_{AB} \end{pmatrix}$$

- Let's call the merged coefficient $\hat{b}_A = b_A + b_{BC}$
- $\hat{b}_B = b_B + b_{AC}$
- s
- d

Remove the aliases by collapsing identical columns

$$
\begin{pmatrix} y_5 \\ y_2 \\ y_3 \\ y_8 \end{pmatrix} =
\begin{array}{c}
\begin{array}{cccc} b_0 + b_{\mathsf{ABC}} & b_{\mathsf{A}} + b_{\mathsf{BC}} & b_{\mathsf{B}} + b_{\mathsf{AC}} & b_{\mathsf{C}} + b_{\mathsf{AB}} \end{array} \\
\begin{pmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & - \\ + & + & + & + \end{pmatrix}
\end{array}
\begin{pmatrix} \hat{b}_0 \\ \hat{b}_{\mathsf{A}} \\ \hat{b}_{\mathsf{B}} \\ \hat{b}_{\mathsf{C}} \end{pmatrix}
$$

- $\hat{b}_{\mathsf{A}} = b_{\mathsf{A}} + b_{\mathsf{BC}}$
- $\hat{b}_{\mathsf{B}} = b_{\mathsf{B}} + b_{\mathsf{AC}}$
- $\hat{b}_{\mathsf{C}} = b_{\mathsf{C}} + b_{\mathsf{AB}}$
- $\hat{b}_0 = b_0 + b_{\mathsf{ABC}}$

Now that we understand aliasing; how can we work with it in our system?

We'd like to take advantage of doing half the work, but still get the most benefit. But, recall we showed that this will lead to a loss of some accuracy:

Full factorial model

$$
\begin{aligned}
\hat{y} = {} & 11.25 \\
& + 6.25\, x_{\mathsf{A}} \\
& + 0.75\, x_{\mathsf{B}} \\
& - 7.25\, x_{\mathsf{C}} \\
& + 0.25\, x_{\mathsf{A}} x_{\mathsf{B}} \\
& - 6.75\, x_{\mathsf{A}} x_{\mathsf{C}} \\
& - 0.25\, x_{\mathsf{B}} x_{\mathsf{C}} \\
& - 0.25\, x_{\mathsf{A}} x_{\mathsf{B}} x_{\mathsf{C}}
\end{aligned}
$$

Fractional factorial model

$$
\begin{aligned}
\hat{y} = {} & 11.0 \\
& + 6.0\, x_{\mathsf{A}} \\
& - 6.0\, x_{\mathsf{B}} \\
& - 7.0\, x_{\mathsf{C}} \\
& + \cancel{b_{\mathsf{AB}}\, x_{\mathsf{A}} x_{\mathsf{B}}} \\
& + \cancel{b_{\mathsf{AC}}\, x_{\mathsf{A}} x_{\mathsf{C}}} \\
& + \cancel{b_{\mathsf{BC}}\, x_{\mathsf{B}} x_{\mathsf{C}}} \\
& + \cancel{b_{\mathsf{ABC}}\, x_{\mathsf{A}} x_{\mathsf{B}} x_{\mathsf{C}}}
\end{aligned}
$$