Normalized LMS Algorithm – Interference Canceling

Mohit Mewara
Department of Computer and Information Science and Engineering
University of Florida
Gainesville, USA
Email: mohitmewara@ufl.edu

***Abstract*— This paper describes noise cancellation from a corrupted signal using normalized least mean square algorithm. The method uses 2 signals – "primary signal" and "reference signal". The primary signal is the corrupted signal containing the original signal and the noise. The reference signal is the noise which is correlated with the primary signal in some form. The reference signal is filtered adaptively and subtracted from the primary signal to give error, which is the original signal. The paper includes the experiments performed over different filter orders and step size. Learning curve and ERLE improvement are used as parameters to compute optimum weight to obtain original signal.**

***Index Terms*— NLMS, Adaptive Filter**

I. Introduction

Noise free image is required in many real world applications for effective communication. The noise from surrounding environment corrupts the original signal and the resulting signal may be unbearable to hear. The primary signal has low speech quality and different filtering techniques can be used to compute the original signal back. NLMS is one of those filtering techniques which can reduce noise from the primary signal and enhance the quality of signal.

The adaptive filter technique is useful when the noise and input speech signal is random is nature. Over several decades, a significant amount of research attention has been focused on the signal processing technique that are able to extract a desired speech signal and reduce the effects of unwanted noise [2]. Depending on the number of sensors used in the system, these approaches can be classified into three basic categories, namely temporal filtering techniques using only a single microphone, adaptive noise cancellation utilizing a primary sensor to pick up the noisy signal and reference sensor to measure the noise field and beamforming techniques exploiting an array of sensors [3]. This paper presents an adaptive noise cancellation technique using normalized least mean square (NLMS) adaptive filter. The objective of this paper is to improve the performance of speech signal operating in noisy environments.

II. Normalized least mean square (NLMS)

NLMS filter is similar to LMS filter. Both the filters differ in the way the step size used in both is defined. The step size controls the adjustment of filter's weights. The Least Mean Square does not scale well with the increasing input. As a result, an optimum step size is very hard to find which guarantees the stability of algorithm. Therefore, we use Normalized Lean Mean Square which solves this problem by normalizing with the power of the input. In normalized LMS, the step size is adapted according to the signal amplitude and power [1]. This makes Normalized LMS more favorable. The block diagram of figure 1 describes an adaptive filter for noise cancellation using reference input [2]. The primary input vector $s(n) + v_1(n)$ is a corrupted signal containing both original signal and noise. The reference input $v_2(n)$ is the noisy signal which is correlated to the signal $v_1(n)$. The reference input is passed into the adaptive filter to obtain $y(n)$, which is the actual output signal. The output signal $y(n)$ is subtracted from primary input to obtain error signal $e(n)$. The $e(n)$ is used by learning algorithm as a feedback for weight adjustment of the filter. This sequence is repeated for many iterations until steady state of filter is reached. The error signal is actually the original signal which we want to retrieve. Over calculation of optimal weight, the optimal weight is used to calculate the error signal or the actual signal.

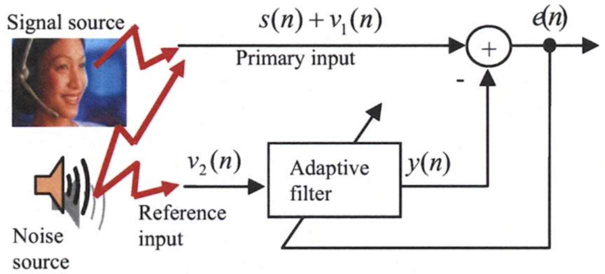

Figure 1. Block Diagram of Adaptive noise cancellation using reference input (noise)

The output vector of an NLMS filter is given by:

$$y(n) = v_2(n)\,w(n)H \qquad (1)$$

where, $w(n)$ is the current weight, $v_2(n)$ is the input signal and $y(n)$ is the output signal.
Error signal is calculated by:

$$e(n) = d(n) - y(n) \qquad (2)$$

where $e(n)$ is the error signal and $d(n)$ is the desired signal.
The weight in NLMS algorithm is given by:

$$w(n+1) = w(n) + \frac{\mu}{\varepsilon + ||v_2(n)||^2} v_2(n)e(n) \qquad (3)$$

where, w(n+1) is the weight at (n+1) time, ε and μ are constants. ε is a very small non-zero number to avoid division by 0 in denominator.

III. SNR AND ERLE

Signal-to-noise ratio compares the level of a desired signal to the level of background noise. It is defined as the ratio of the wanted signal power to the unwanted noise power, often expressed in decibels. A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise [5].

$$SNR = \frac{P_{signal}}{P_{noise}} \tag{4}$$

Where, P is average power. Both signal and noise power must be measured at the same or equivalent points in a system, and within the same system bandwidth.

Signals are often expressed using the logarithmic decibel scale. SNR may be expressed in decibels as

$$SNR_{db} = 10\log_{10}\frac{P_{signal}}{P_{noise}} \tag{5}$$

Echo return loss enhancement (ERLE) is the ratio of send-in power and the power of a residual error signal (e) immediately after the cancellation. ERLE measures the amount of loss introduced by the adaptive filter alone. ERLE is measured in dB.

$$ERLE = 10\log_{10}\frac{E[d]^2}{E[e]^2} \tag{6}$$

Where, d is the primary (Speech + Noise) signal and e is the error or output signal from filter. ERLE can be used to judge performance of the filter.

IV. EXPERIMENTS

The primary signal is used as the input signal which is the combination of Speech signal and Noise. The reference signal is the noise which is captured by another microphone present near the microphone of primary signal. Initially, when the signal is drawn on plot, it can be seen that a lot of noise is present in the signal. The noise is so intense that nothing can be heard from the primary signal.

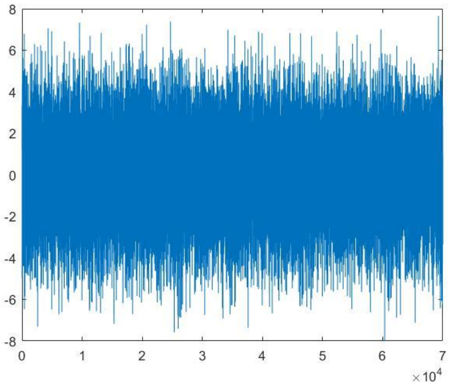

Figure 2. Primary Signal

A. NLMS Algorithm

The initial analysis begins for filter order 2. The MATLAB program is written for NLMS algorithm. The NLMS algorithm is summarized as below,

(i) *Inputs:*

x = input signal
d_n = desired signal
M = Filter Length
μ = Step-size factor
E = epsilon

(ii) Outputs:

y = output of filter
e = error signal

(iii) Pseudo-code

Initialize:
Filter coefficient w(1) = 0
Loop1:
For every i, do
Calculate $e(i) = d(i) - w(i)^T x(i)$
Update $w(i) = \frac{\mu\, e(i)\, x(i)}{(E + ||x(i)||^2)}$
End

B. Performance Surface Contours

MSE is calculated for each weight and performance surface contours are generated with the weight tracks for the two weights filter case. The contour plot shows the error surface steepness. If the error surface is parabolic then there exists a single point where the error surface attains the minimum value. This is very important because if the given Mean Square Error is not converging to a single point or a hyperplane then it does not have an optimal weight. The performance surface and contour plot for filter order 2 is shown in Figure 2 and 3 and it suggests that the MSE is converging for the primary signal to a hyperplane of Weight 1 and Weight 2. Further analysis can be performed using this information.

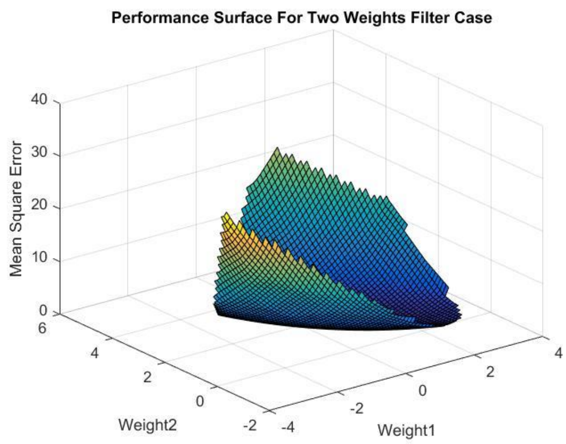

Figure 3. Two-Dimensional Quadratic Error Surface (Performance Surface)

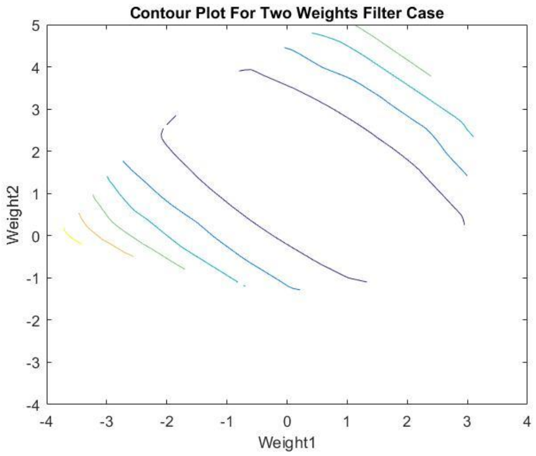

Figure 4. Contour Plot of the Mean Square Error

The contour plot is constructed to represent the performance surface by plotting constant MSE slices (contours) on a 2-dimensional area of weight1 and weight2.

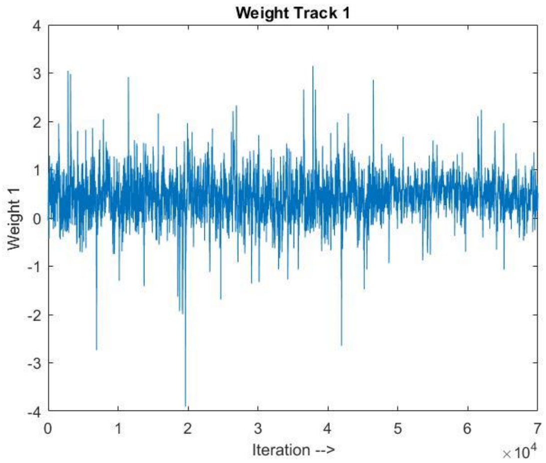

Figure 5. Weight Track 1 for Filter order 2

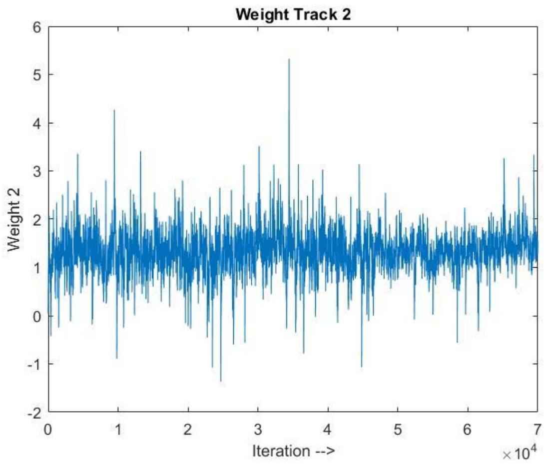

Figure 6. Weight Track 2 for Filter order 2

Figure 5 and 6 are the Weight Tracks. The Weight Track 1 and Weight Track 2 are the plot of weights over iteration. The above figure shows that they are not converging to an optimal weight.

C. Learning Curve

A learning curve is used to understand the performance of an adaptive filter, including convergence speed, steady state error, and stability [4]. A learning curve is a plot of the mean square error (MSE) of the adaptive filter versus time or iteration. The learning curve for filter order 2 and $\mu = [0.1, 0.5, 1]$ is computed. It is observed that as the iteration increases, the adaptive filter MSE remains almost same and is not converging for higher μ. So, a lesser value of μ (0.01) is used and the learning curve started converging. The learning curve took 70K iterations to converge and the MSE is minimum at the optimal weight. Figure 7 shows that the MSE reaches minimum (close to 0) at the end of iteration. However, the output signal generated using the optimum weight is still having a lot of noise and it is not audible. This means that apart from MSE, SNR should be considered for better results.

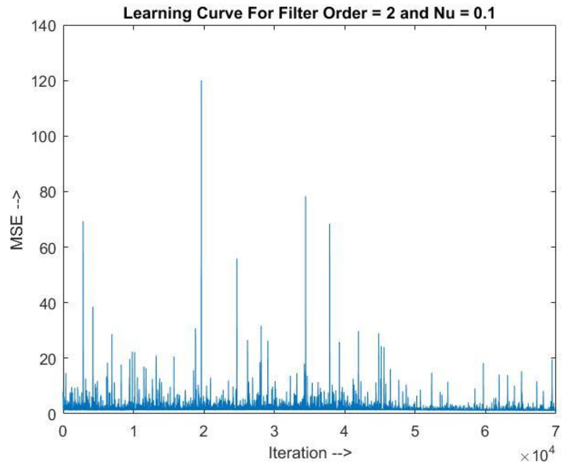

Figure 7. Learning Curve for Filter Order = 2 and $\mu = 0.1$ is not converging

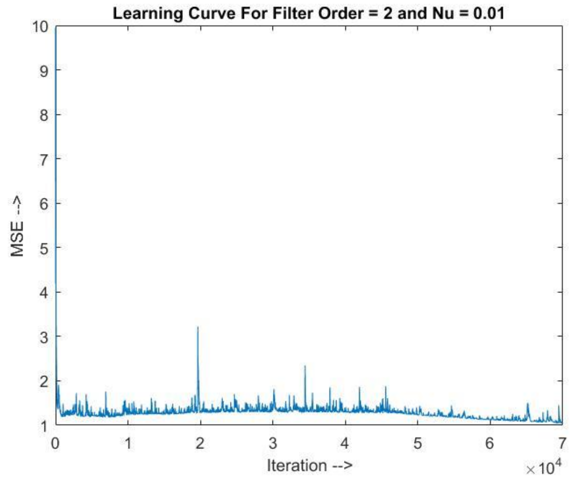

Figure 8. Learning Curve for Filter Order = 2 and $\mu = 0.01$ is converging

The learning curve is converging for step size 0.01 at 46K iterations.

The error signal contains a lot of noise and due to this the speech is not clearly heard. However, it can be seen from the error signal plot that the noise is reduced after passing the primary signal from the filter.

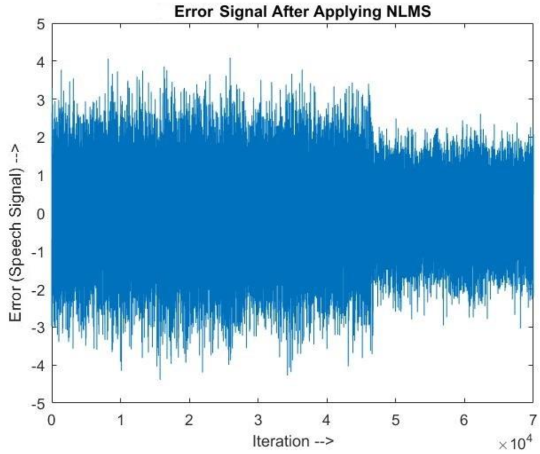

Figure 9. Error signal for filter order 2

D. ERLE Calculation

The $ERLE_{order2}$ which is the ratio of the primary signal with filter output signal (error or speech signal) is computed for all step sizes. The ERLE has increased but the voice is still not audible for the speech signal which means that higher orders should be used for better ERLE and thus the performance.

E. Cross Validation

Cross Validation is a model validation technique for assessing how the results of a statistical analysis will generalize to an independent data set. It is used in prediction problem to accurately predict the performance of a model.

The goal of cross validation is to define a dataset to "test" the model in the training phase (i.e., the validation dataset), in order to limit problems like overfitting, give an insight on how the model will generalize to an independent dataset [8].

There are three types of non-exhaustive cross-validation:

1. k-fold cross-validation
   In this validation original data is subsampled in k equal size. One subsample is used to test the model and the others are used as training data. The cross validation is repeated k times with each subsample used once in validation data. The k results from the folds can then be averaged to produce a single estimation.
2. 2-fold cross-validation
   In this technique the data is divided into two datasets. One dataset is used for training and the other is used for testing.
3. Repeated random sub-sampling validation
   In this technique, the data is randomly splitted into training and validation data. For each such split, the model is fit to the training data, and predictive accuracy is assessed using the validation data. The results are then averaged over the splits.

The cross-validation can be used based on the ERLE in this filter. ERLE is a performance parameter and any performance parameter can be used for cross validation.

F. Calculation Using Higher Filter Orders

The process used to calculate learning curve and ERLE is repeated on higher filter orders to improve the error. The task is performed on multiple step sizes for orders – 10, 20, 40 and 50. It can be noted that as the filter order is increasing, the ERLE value is also increasing. The best results are shown for the filter order 50, which means that the order 50 will give best results.

The error signal for all the filters does not showed any impressive results for the speech between 1 and 46.5k iterations but the speech becomes more audible after 46.5k with increase in filter order. Looking at the figure 10 and 11, it is visible that the error is improved in late iterations.

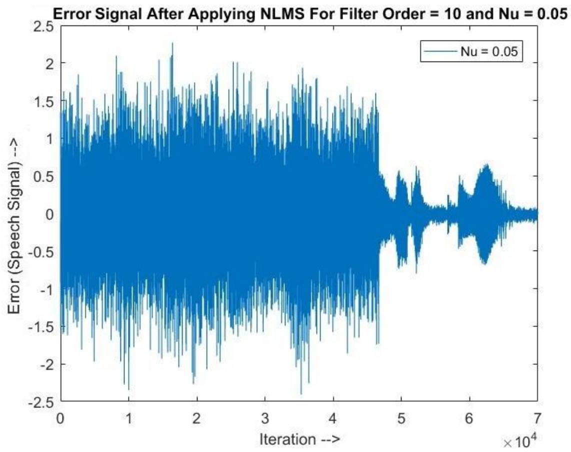

Figure 10. Error signal for filter order 10

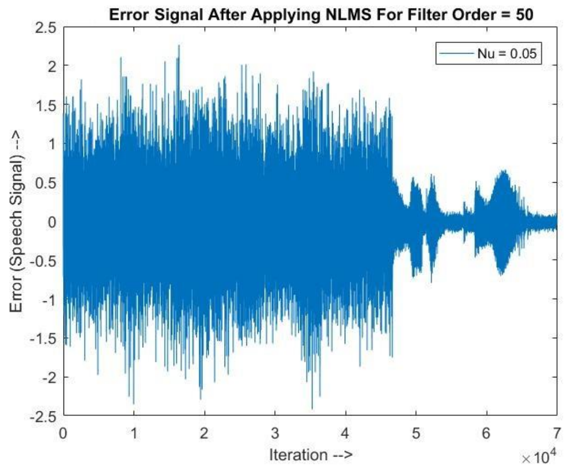

Figure 11. Error Signal for Filter Order 50

It can be seen in figure 12 and 13 that the learning curve is not behaving as expected. The learning curve shows that the convergence is achieved quickly in initial iterations and it remained stable till 46.5k but after that the curve diverges and again start convergence. The learning curves for all orders are also following the same pattern – converging till 46.5k iteration, then make a spike and then again converges till 70K. This pattern suggests that the signal has multiple trends and the intensity of noise changes a lot after 46.5k.

The best filter order among all would be 50 as it has maximum ERLE but only the later part of speech signal is clearly audible. There are multiple trends in the primary signal and so two segment filter order should be used to determine best filter.

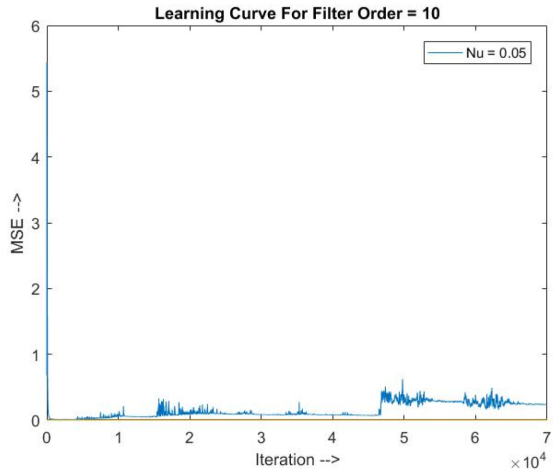

Figure 12. Learning Curve for Filter Order 10

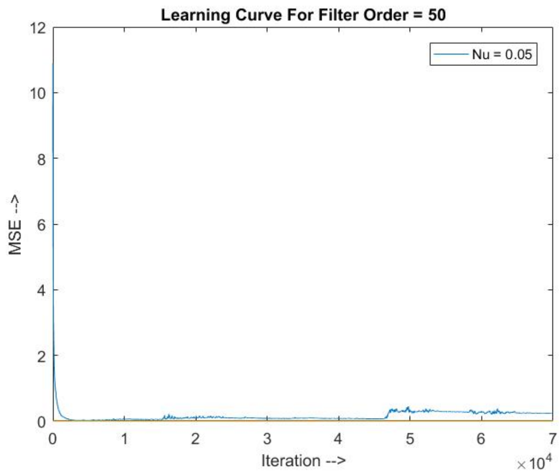

Figure 13. Learning Curve for Filter Order 50

G. Two Segment Model Adaptive Filter

From the previous analysis, it is clear that a single segment model will not fit well for this signal. The changes in signal shows that a 2 segment model filter should be used. The 2 segment model filter would be a filter which will compute 2 optimal weights for primary signal. These weights are computed by dividing the data into 2 parts. The optimal weight W^*_{model1} would be calculated by taking the data between 1 and 46.5k iteration and W^*_{model2} by taking the data after 46.5k till the end of iteration.

The SNR improvement in dB by the ERLE is impressive for the two segment model adaptive filter. There are two error signals for this model as the primary signal is divided into two segments. Both the error signals are concatenated into a single final error signal. The ERLE is computed using the final error signal. The results show that the performance is increased a lot for each filter order.

There are 2 learning curves for each filter order and best step size. Both the learning curves are converging for all filter orders. Figure 14, 15, 16 and 17 shows the learning curves of each segment for filter order 10 and 50 respectively.

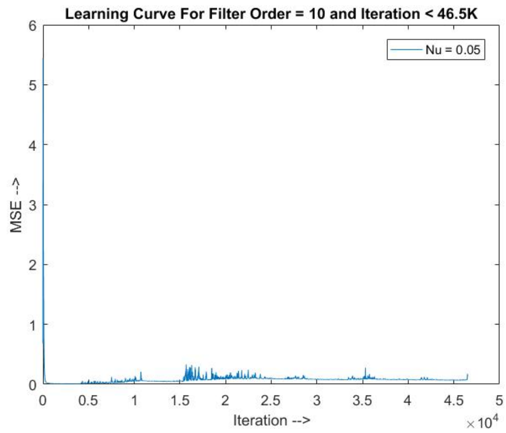

Figure 14. Learning Curve for First Segment for Filter Order 10

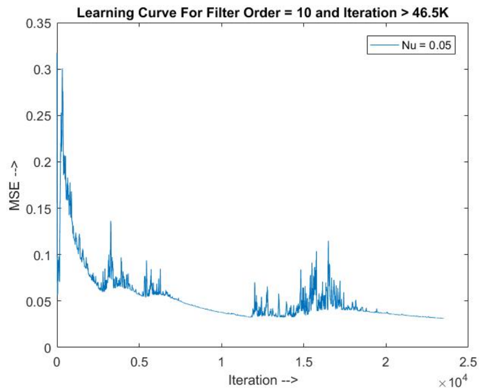

Figure 15. Learning Curve for Second Segment for Filter Order 10

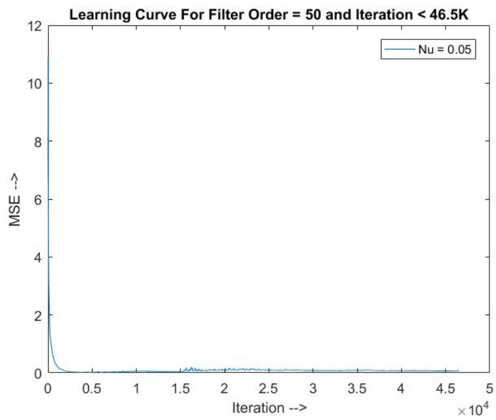

Figure 16. Learning Curve for First Segment for Filter Order 50

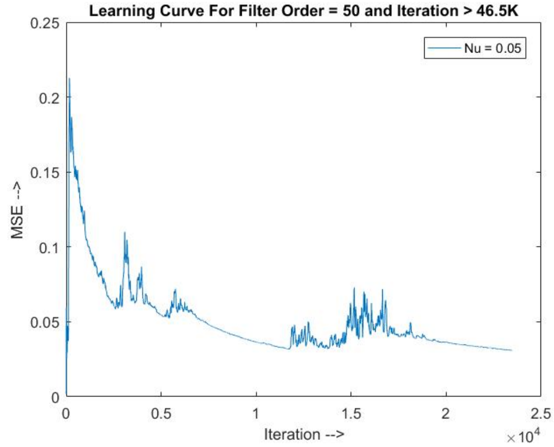

Figure 17. Learning Curve for Second Segment for Filter Order 50

The best filter order is 50 in terms of ERLE. The error signal (speech signal) for order 50 is very good and the sound is clearly audible.

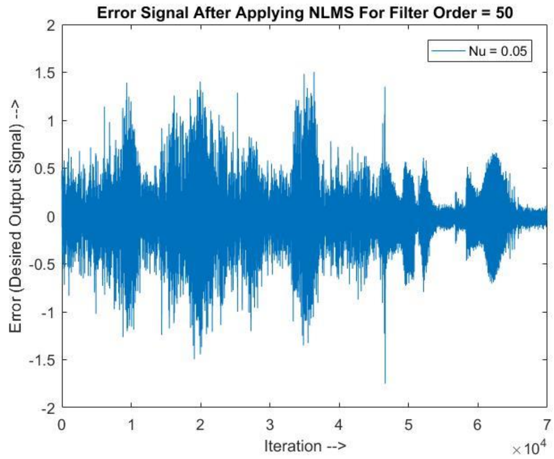

Figure 18. Error signal for Two Segment Model Adaptive Filter

V. Results

ERLE is calculated for multiple step size for filter order 2.

TABLE I. ERLE results for non-segmented adaptive filter for filter order 2

Step Size	0.001	0.01	0.05	0.1	0.5
ERLE	5.9504	5.9799	5.9814	5.9955	0.8071

The maximum ERLE for Filter Order 2 is found at step size 0.01. The optimal weight of this step size was used to calculate error signal but the output was not satisfactory as clear audio was not audible.

Two approaches are used to obtain noise free signal – Non Segmentation and Two Segment Adaptive Filter of the primary signal. The filter with segmentation performed well in comparison with the other filter. The below table shows the comparison between 2 filters in terms of ERLE. The step size for Two Segment Filter and Non Segment Filter for comparison is kept 0.05.

TABLE II. ERLE results for non-segmented adaptive filter for filter order 2

Order	10	20	40	50
ERLE for Non Segment	12.540513	12.533159	12.538914	12.548963
ERLE for Two Segment	16.134295	15.792093	16.476541	16.882364

The ERLE for highest filter order 50 is best. The ERLE for primary input is 3.739603 and for Two Segment is 16.882364. The Two Segment Filter has better ERLE and the speech signal is clearly audible using this filter. The statement in the speech signal is “I will not condone a course of action that will lead us to war”.

VI. Conclusion

NLMS algorithm is used to design an adaptive filter which cancels the noise from primary signal to give speech signal as the output. NLMS is used for this process because the step size is adapted according to the signal amplitude and power. Two types of filters are proposed to obtain the best error signal. The Two Segment Model Adaptive Filter gave the best result and the voice of signal can be clearly heard using it.

References

[1] Least Mean Squares filter – “https://en.wikipedia.org/wiki/Least_mean_squares_filter”

[2] Dr.R.D.Kharadkar , “Acoustic Noise Cancellation using Adaptive Filters: A Survey”, International Conference on Electronic Systems, Signal Processing and Computing Technologies, 2014

[3] Pramod R. Bokde, Nitin K. Choudhari , “Implementation of Adaptive filtering algorithms for removal of Noise from ECG signal”, Choudhari Pramod R Bokde et al, Int.J.Computer Technology & Applications,Vol 6 (1),51-56

[4] Learning Curve, “http://www.ni.com/example/31220/en/”

[5] Mohd Zaizu Ilyas Ali O. Abid Noor Khairul Anuar Ishak Aini Hussain Salina Abdul Samad"Normalized Least Mean Square Adaptive Noise Cancellation Filtering for Speaker Verification in Noisy Environments", International Conference on Electronic Design, 2008 H. Yiteng, B. Jacob and C. Jingdong. Acoustic MIMO signal processin: Signal and Communication Technology. Springer.2006.

[6] Learning curve http://www.ni.com/example/31220/en/.

[7] SNR, “https://en.wikipedia.org/wiki/Signal-to-noise_ratio”

[8] Cross Validation, “https://en.wikipedia.org/wiki/Cross-validation_(statistics)”