$$\begin{bmatrix} v_i \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 + \dfrac{1}{sC_1} & -\dfrac{1}{sC_1} \\ -\dfrac{1}{sC_1} & R_2 + \dfrac{1}{sC_1} + \dfrac{1}{sC_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_i \\ 0 \end{bmatrix} = \begin{bmatrix} \dfrac{sR_1C_1 + 1}{sC_1} & -\dfrac{1}{sC_1} \\ -\dfrac{1}{sC_1} & \dfrac{sR_2C_1C_2 + C_1 + C_2}{sC_1C_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$v_o = i_2 \frac{1}{sC_2}$$

$$= \frac{\begin{vmatrix} \dfrac{sR_1C_1 + 1}{sC_1} & v_i \\ -\dfrac{1}{sC_1} & 0 \end{vmatrix}}{\begin{vmatrix} \dfrac{sR_1C_1 + 1}{sC_1} & -\dfrac{1}{sC_1} \\ -\dfrac{1}{sC_1} & \dfrac{sR_2C_1C_2 + C_1 + C_2}{sC_1C_2} \end{vmatrix}} \frac{1}{sC_2}$$

$$= v_i \frac{\dfrac{1}{s^2C_1C_2}}{\dfrac{sR_1C_1 + 1}{sC_1} \dfrac{sR_2C_1C_2 + C_1 + C_2}{sC_1C_2} - \dfrac{1}{s^2C_1^2}}$$

$$\frac{v_o}{v_i} = \frac{\dfrac{C_1}{s^2C_1^2C_2}}{\dfrac{sR_1C_1 + 1 \quad sR_2C_1C_2 + C_1 + C_2 \quad - C_2}{s^2C_1^2C_2}}$$

$$= \frac{C_1}{sR_1C_1 + 1 \quad sR_2C_1C_2 + C_1 + C_2 \quad - C_2}$$

$$= \frac{C_1}{s^2R_1R_2C_1^2C_2 + s \quad R_1C_1 \quad C_1 + C_2 \quad + R_2C_1C_2 \quad + C_1}$$

$$= \frac{1}{s^2R_1R_2C_1C_2 + s \quad R_1 \quad C_1 + C_2 \quad + R_2C_2 \quad + 1}$$

Denominator is of form

$$\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1$$

Hence,

$$\omega_n = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

Two coincident poles on the real axis corresponds to ζ equals 1.

[Proof:

A coincident double root can be described by

$$\left(\frac{s}{\omega_n} + 1\right)^2 = \frac{s^2}{\omega_n^2} + 2\frac{s}{\omega_n} + 1$$

Comparing this to the above standard form it is apparent that ζ must be 1 for this to be true.]

Coincident poles can then only happen if

$$\frac{2}{\omega_n} = 2\sqrt{R_1 R_2 C_1 C_2} = R_1 (C_1 + C_2) + R_2 C_2$$

Squaring both sides yields

$$\begin{aligned} 4R_1 R_2 C_1 C_2 &= R_1^2 (C_1 + C_2)^2 + 2R_1 R_2 C_2 (C_1 + C_2) + R_2^2 C_2^2 \\ &= R_1^2 (C_1 + C_2)^2 + 2R_1 R_2 C_1 C_2 + 2R_1 R_2 C_2^2 + R_2^2 C_2^2 \end{aligned}$$

Simplifying,

$$2R_1 R_2 C_1 C_2 = R_1^2 (C_1 + C_2)^2 + 2R_1 R_2 C_2^2 + R_2^2 C_2^2$$

Setting this up as a quadratic equation in R_1,

$$R_1^2 (C_1 + C_2)^2 + 2R_1 R_2 C_2 (C_2 - C_1) + R_2^2 C_2^2 = 0$$

The solution for this is given by

$$
\begin{aligned}
R_1 &= \frac{-2R_2 C_2 \left(C_2 - C_1\right) \pm \sqrt{4R_2^2 C_2^2 \left(C_1 - C_2\right)^2 - 4R_2^2 C_2^2 \left(C_1 + C_2\right)^2}}{2\left(C_1 + C_2\right)^2} \\
&= \frac{-R_2 C_2 \left(C_2 - C_1\right) \pm R_2 C_2 \sqrt{\left(C_1 - C_2\right)^2 - \left(C_1 + C_2\right)^2}}{\left(C_1 + C_2\right)^2} \\
&= -R_2 C_2 \frac{\left(C_2 - C_1\right) \mp \sqrt{C_1^2 - 2C_1 C_2 + C_2^2 - C_1^2 - 2C_1 C_2 - C_2^2}}{\left(C_1 + C_2\right)^2} \\
&= R_2 C_2 \frac{\left(C_2 - C_1\right) \mp 2i\sqrt{C_1 C_2}}{\left(C_1 + C_2\right)^2}
\end{aligned}
$$

But this requires R_1 to be complex which is not possible. Hence, coincident poles are not possible with physical components.