Datum Transformations of NAV420 Reference Frames

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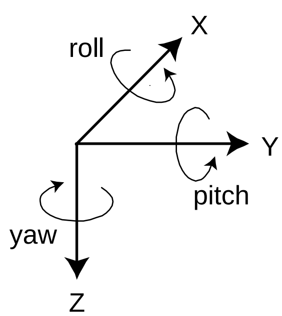
This application note explains how to convert various reference frames for Crossbow's Inertial Systems in general and NAV420CA in particular. The Crossbow's Navigation System or NAV420CA uses a 3-axis accelerometer and a 3-axis rate sensor to make a complete measurement of the dynamics of the system. The addition of a 3-axis magnetometer inside the Crossbow AHRS allows it to make a true measurement of magnetic heading without an external flux valve. When GPS receiver is added to the system, the combined system becomes a low-cost INS that can output location, velocity and acceleration.

Inertial Coordinate Frames

The NAV420CA has a label on one face illustrating the NAV420CA coordinate system as shown in Figure 1. With the connector facing you, and the mounting plate down, the axes are defined as:



X-axis – from face with connector through the NAV420CA
Y-axis – along the face with connector from left to right
Z-axis – along the face with the connector from top to bottom

Figure 1: NAV420CA Coordinate System

The axes form an orthogonal right-handed coordinate system. Its origin is nominally located at the vehicle CG. In Figure 2, the body frame of the system is shown relative to the tangent frame or local level horizontal frame.

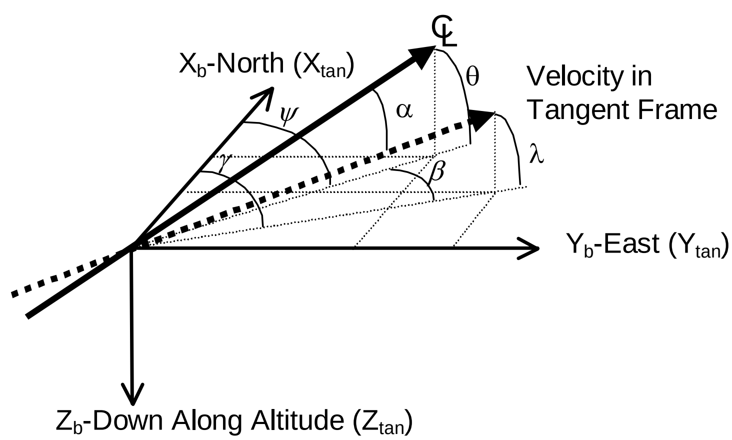


Figure 2: Coordinate Frame Angle Definitions

In this formulation, the body axis has been chosen to point towards north along its x-axis when there is no yaw angle. Therefore when the vehicle attitude is zero, or when the Euler angles roll, pitch, and yaw are zero, the transformation from body to tangent frame is simply:

$$CB2T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here the matrix $CB2T$ represents the "**C**osine rotation matrix, which takes you from **B**ody (**2**) to **T**angent frame." This serves as the starting point for further rotation of the body in the tangent frame due to vehicle attitude change. Several angles defined in the Figure 2 are of importance for the vehicle. The bold line represents the vehicle's centerline, and the dashed line represents the vehicle's velocity vector in the tangent frame. The angles, which the velocity vector makes with respect to the vehicle centerline, are the typical aerodynamic control angles, angle of attack α, and angle of sideslip β. The angles that the velocity vector makes with respect to the tangential plane are the typical air velocity angles, flight path angle λ and heading angle γ. The Euler body angles, which the centerline of the vehicle body makes with respect to the tangential frame, are the pitch θ and yaw ψ angles. The vehicle body roll angle ϕ is rotated along its centerline. The Euler angles describe the vehicle attitude and form a 3-2-1 rotation of the body in the tangent frame. In explicit terms the rotation matrix is:

$$CB2T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$CB2T = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\theta + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

From this rotation matrix, which will transform a vector from the body frame into the tangent frame, the attitude Euler angles can be derived as follows:

$$\phi_{(roll)} = \mathrm{atan}\left(\frac{CB2T(3,2)}{CB2T(3,3)}\right)$$

$$\theta_{(pitch)} = -\mathrm{asin}\left(CB2T(3,1)\right)$$

$$\psi_{(yaw)} = \mathrm{atan}\left(\frac{CB2T(2,1)}{CB2T(1,1)}\right)$$

GPS Coordinate Frames

Coordinates representing positions on the earth can be given in two formats, Spherical or Cartesian. Spherical or Geodetic coordinates are three dimensional with the components of latitude (φ), longitude (λ) and height above ellipsoid (h). With two of the components being non-linear with angular units, computations are more complex for coordinate geometry problems.
Alternatively, Cartesian coordinates are entirely linear and provide for an excellent platform for mathematics. The origin and orientation of the coordinate frame are dependant on the user's application and many well defined systems already exist. For global applications the system known as earth centered - earth fixed (ECEF) is preferred. Figure 3 illustrates the relationship between Spherical coordinates (φ, λ, h) and Cartesian ECEF coordinates (X, Y, Z) with respect to a reference ellipsoid.

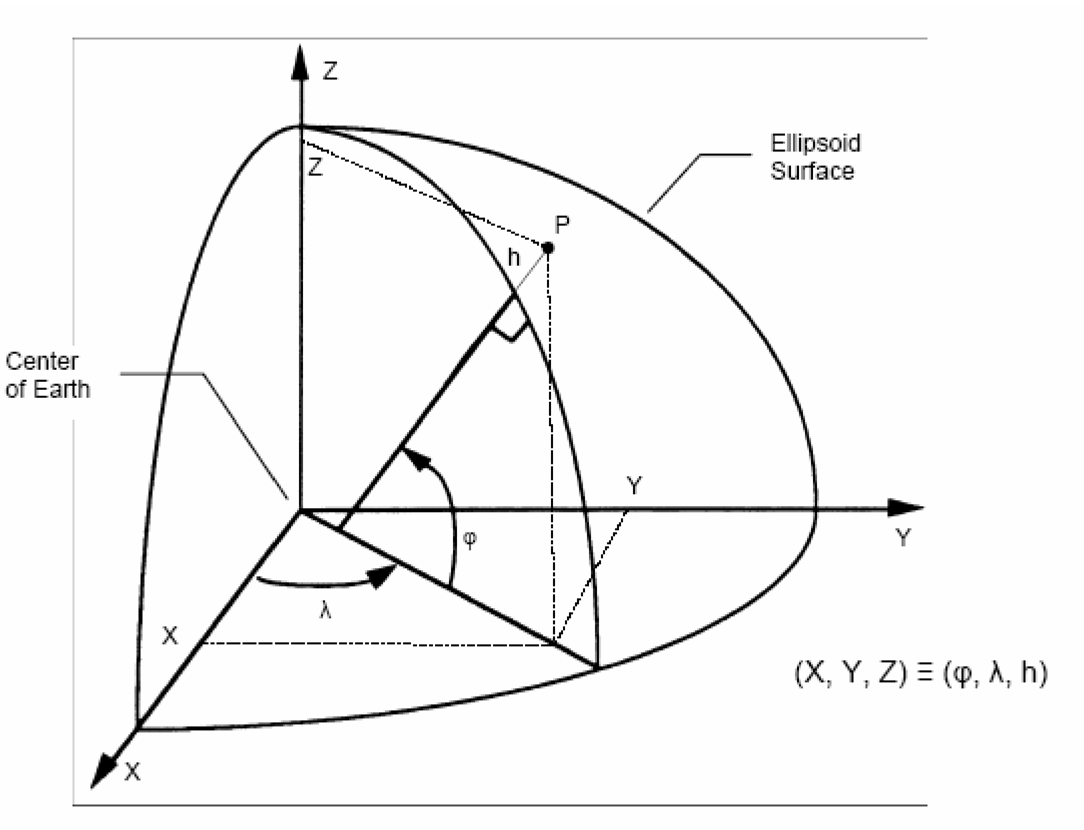


Figure 3: Relationship between Cartesian and Spherical Coordinate Systems

LLA Coordinate System

The most commonly used coordinate system today is the latitude, longitude, and altitude (LLA) system. The origin of the system is at the mass center of the earth. The Prime Meridian and the Equator are the reference planes used to define latitude and longitude. The geodetic latitude (there are many other defined latitudes) of a point is the angle from the equatorial plane to the vertical direction of a line normal to the reference ellipsoid.

The geodetic longitude of a point is the angle between a reference plane and a plane passing through the point, both planes being perpendicular to the equatorial plane.

The geodetic altitude at a point is the distance from the reference ellipsoid to the point in a direction normal to the ellipsoid.

ECEF Coordinate System

Earth Centered, Earth Fixed (ECEF) Cartesian coordinates are also used to define three-dimensional positions. Earth centered, earth-fixed, X, Y, and Z, Cartesian coordinates (XYZ) define three-dimensional positions with respect to the center of mass of the reference ellipsoid and follow rotations of the earth. The origin of the system is at the mass center of the earth. The Z-axis is along the axis of rotation and points toward the North Pole. The X-axis is defined by the intersection of the plane define by the prime meridian and the equatorial plane. The Y-axis completes a right-handed orthogonal system by a plane 90° east of the X-axis and its intersection with the equator.

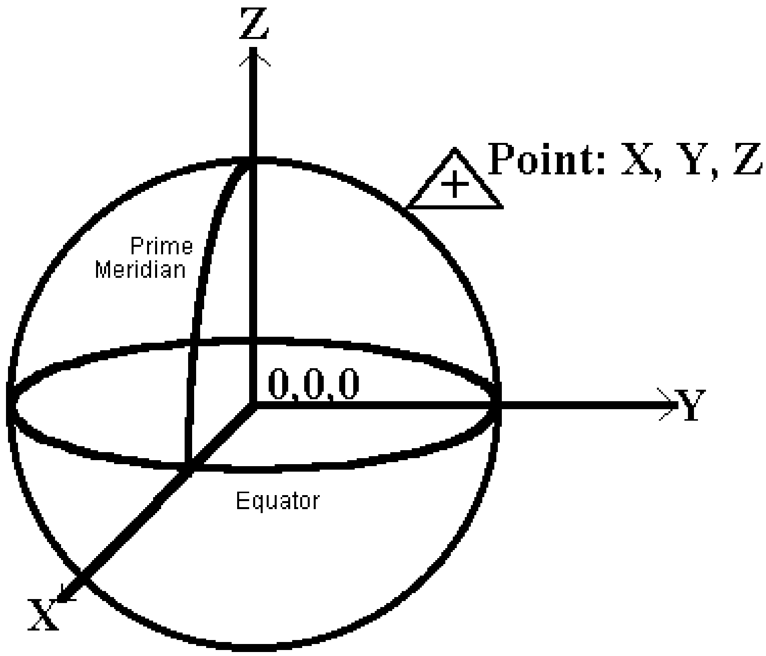


Figure 4: ECEF Coordinate Reference Frame

The Global Positioning System (GPS) is based on the World Geodetic System of 1984 (WGS84) datum. WGS84 is a geocentric system, which provides an excellent mathematical representation in relation to the orbiting satellite constellation. Upon the introduction of satellite navigation, several national geodetic organizations immediately grasped the technology to update their datums with modernized geocentric ellipsoids and to reduce existing distortions.

A reference ellipsoid can be described by a series of parameters that define its shape and which include a semi-major axis (*a*), a semi-minor axis (*b*) and its first eccentricity (*e*) and its second eccentricity (*e'*) as shown in Figure 5. Depending on the formulation used, ellipsoid flattening (*f*) may be required. This ellipsoid has its origin coincident with the ECEF origin. The X-axis pierces the Greenwich meridian (where longitude = 0 degrees) and the XY-plane make up the equatorial plane (latitude = 0 degrees). Altitude is described as the perpendicular distance above the ellipsoid.

WGS84 Parameters:

$$a = 6378137$$

$$\mathrm{b} = a(1 - f) = 6356752.31424518$$

$$\mathrm{f} = \frac{1}{298.257223563}$$

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$e' = \sqrt{\frac{a^2 - b^2}{b^2}}$$

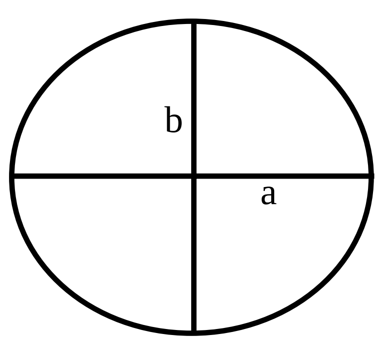


Figure 5: Reference Ellipsoid Parameters

Conversion between ECEF and Local Tangential Plane

GPS coordinate frame conversions are accomplished by various methods. Complete datum conversion is based on seven parameter transformations that include three translation parameters, three rotation parameters and a scale parameter. Simple three parameter conversion between latitude, longitude, and height in different datums can be accomplished by conversion through ECEF X, Y, Z Cartesian coordinates in one reference datum and three origin offsets that approximate differences in rotation, translation and scale.

LLA to ECEF

The conversion from LLA to ECEF is shown below.

$$X = (N + h)\cos\varphi\cos\lambda$$
$$Y = (N + h)\cos\varphi\sin\lambda$$
$$Z = \left(\frac{b^2}{a^2}N + h\right)\sin\varphi$$

where,

$$\varphi = latitude$$
$$\lambda = longitude$$
$$h = \text{height above ellipsoid}$$
$$\text{N = Radius of Curvature, defined as :}$$
$$= \frac{a}{\sqrt{1 - e^2\sin^2\varphi}}$$

Vertical Datums

Knowledge of the geoidal undulation of a particular position, allows the orthometric height to be derived from GPS measured ellipsoidal heights. Vertical datums can refer to either a surface such as a geoid or the surface of a reference ellipsoid. The height determined by GPS measurements relates to the perpendicular distance above the reference ellipsoid and should not be confused with the more well-known height datum Mean Sea Level (MSL). The datum that defines the MSL (also called the geoid) is a complex surface that requires dense and accurate gravity data to define its shape. The WGS84 ellipsoid approximates the geoid on a worldwide basis with deviations between the two datums never exceeding 100 meters. The relationship between the geoid and ellipsoid is shown in Figure 6 and the algebraic difference between the two is known as the geoidal undulation (N).

The conversion between the two reference datums is shown by:

$$h = H + N$$

where,

h = ellipsoidal height (Geodetic),
H = orthometric height (MSL),
N = geoid separation (undulation)

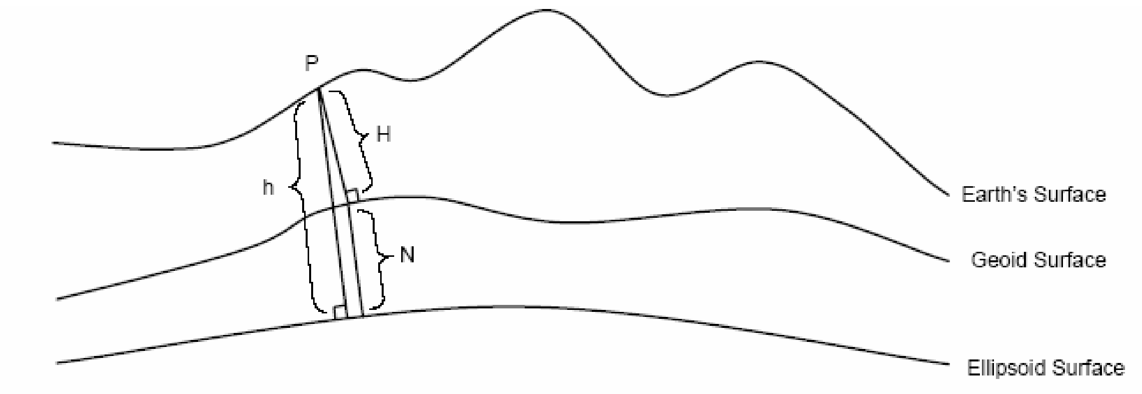


Figure 6: Ellipsoid Height and Geoid Height Relationship

Representing geoidal undulations for a relatively large area with a mathematical model becomes difficult due to high frequency spherical harmonics. To overcome the problem of irregular geoidal undulations, an identical approach to the horizontal datum transformations can be done with the use of two dimensional data grids. Each node on the grid has a geoidal undulation (N) value and intermediate locations are interpolated.

Converting ECEF Velocities to Local Tangent Plane Velocities

GPS also resolves the speed and direction of travel in the ECEF XYZ reference frame. To convert these values to a local tangent plane (LTP), the velocity vector must be rotated to reflect directions in terms more usable to the user. The LTP uses the orientation of North, East, and Down, (NED) which is consistent with the geodetic coordinates LLA. To transform the velocity vector, you use the following direction cosine matrix (North, East, Down) and solving for each component results in the following matrix transformation:

$$\begin{bmatrix} V_{north} \\ V_{east} \\ V_{down} \end{bmatrix} = \begin{bmatrix} -\sin(\varphi)\cos(\lambda) & \sin(\varphi)\sin(\lambda) & \cos(\varphi) \\ -\sin(\lambda) & \cos(\lambda) & 0 \\ -\cos(\varphi)\cos(\lambda) & -\cos(\varphi)\sin(\lambda) & -\sin(\varphi) \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

The speed and heading data can be derived from the velocity information using the following relationship.

$$Speed = \sqrt{V_{north}^2 + V_{east}^2}$$

$$Heading = \arctan \frac{V_{east}}{V_{north}}$$

References

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