ENPH 257 Lab Report

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Abstract

hello world

Chapter 1

Introduction

1.1 Conduction

Heat diffusion due to conduction is given by the heat equation

$$\left(\frac{\partial}{\partial t} u(\mathbf{r}, t)\right)_{\text{cond}} = \frac{k}{c\rho} \nabla^2 u(\mathbf{r}, t), \tag{1.1}$$

where $u(\mathbf{r}, t)$ is the temperature at position $\mathbf{r}$ and time t, k is the thermal conductivity, c is the specific heat of the material, and ρ is the mass density of the material.

1.2 Convection

Power gain due to convection for a differential surface element at position $\mathbf{r}$ may be approximated by

$$P_{\text{conv}}(\mathbf{r}, t) = -k_c \, \delta S(\mathbf{r}) \left(u(\mathbf{r}, t) - u_{\text{amb}}\right), \tag{1.2}$$

where k_c is the convection coefficient, $\delta S(\mathbf{r})$ is the differential surface element at $\mathbf{r}$, and u_{amb} is the ambient temperature of the surrounding fluid.

Now temperature change due to power for a differential mass element is given by

$$P = \frac{\partial}{\partial t} Q = c \, \rho \, \delta V \frac{\partial}{\partial t} u, \tag{1.3}$$

so the rate of change of temperature due to convection is given by

$$\left(\frac{\partial}{\partial t} u(\mathbf{r}, t)\right)_{\text{conv}} = -\frac{k_c \, \delta S(\mathbf{r})}{c \, \rho \, \delta V(\mathbf{r})} \left(u(\mathbf{r}, t) - u_{\text{amb}}\right). \tag{1.4}$$

1.3 Radiation

Power gain due to radiation for a differential surface element at position $\mathbf{r}$ is given by Planck's Law to be

$$P_{\text{rad}}(\mathbf{r}, t) = -\epsilon \, \sigma \, \delta S(\mathbf{r}) \left(u(\mathbf{r}, t)^4 - u_{\text{amb}}^4\right), \tag{1.5}$$

where ϵ is the emissivity and σ is the Stefan-Boltzmann constant.

Again from Equation (1.3), we can obtain the contribution to the rate of change of temperature due to radiation

$$\left(\frac{\partial}{\partial t} u(\mathbf{r}, t)\right)_{\text{rad}} = -\frac{\epsilon \, \sigma \, \delta S(\mathbf{r})}{c \, \rho \, \delta V(\mathbf{r})} \left(u(\mathbf{r}, t)^4 - u_{\text{amb}}^4\right). \tag{1.6}$$

1.4 Heat diffusion equation for a cylindrical rod

For a one-dimensional cylindrical rod of length L and radius R, the heat diffusion equation is

$$\frac{\partial}{\partial t}u(x,t) = \frac{1}{c\,\rho}\left(k\frac{\partial^2}{\partial x^2}u(x,t) - k_c\frac{\delta S(x)}{\delta V(x)}\left(u(x,t) - u_{\mathrm{amb}}\right) - \epsilon\,\sigma\frac{\delta S(x)}{\delta V(x)}\left(u(x,t)^4 - u_{\mathrm{amb}}^4\right) + \frac{P_{\mathrm{in}}(x,t)}{\delta V(x)}\right) \tag{1.7}$$

Chapter 2

Objective

Chapter 3

Method

3.1 Experimental setup

3.1.1 Rod dimensions

3.1.2 Thermocouple circuit

3.2 Calibration

3.3 Data aquisition

Chapter 4

Results

Chapter 5

Simulation and optimization

5.1 One-dimensional explicit-step simulation

A one-dimensional explicit-step finite difference program was constructed to simulate heat diffusion based on Equation (1.7). Based on the formalism in Appendix A, the explicit finite difference form of Equation (1.7) is

$$\mathbf{u}^{n+1} = \mathbf{u}^n + \delta t\left(\delta\mathbf{u}_{\text{cond}} + \delta\mathbf{u}_{\text{conv}} + \delta\mathbf{u}_{\text{rad}} + \delta\mathbf{u}_{\text{pow}}\right). \tag{5.1}$$

5.1.1 Finite difference equation

5.1.2 Implementation

5.2 Optimization of one-dimensional simulation

5.3 High-level simulation

Chapter 6

Analysis

Chapter 7

Conclusion and Summary

Appendix A

Comparison of Finite Differencing Methods

The following are methods of finite differencing, demonstrated using the simple example of the one-dimensional heat conduction equation

$$\frac{\partial}{\partial t}u(x,t) = \alpha \frac{\partial^2}{\partial x^2}u(x,t). \tag{A.1}$$

A.1 Explicit method

Finite difference equation

In the explicit method for finite differencing, the temperature for each spatial point can be independently determined from the previous step. That is,

$$u_i^{n+1} = u_i^n + \alpha \cdot \frac{\delta t}{\delta x^2} \cdot \left(u_{i-1}^n - 2u_i^n + u_{i+1}^n\right), \tag{A.2}$$

where the superscript $n \in [0,T]$ signifies the step in time t_n and the subscript $i \in [0,X]$ represents the position x_i. δt and δx are the time steps in time and position respectively.

If we define

$$\zeta = \alpha \cdot \frac{\delta t}{\delta x^2},$$

then this can be written as a linear relationship

$$\mathbf{u}^{n+1} = \mathbf{D}\mathbf{u}^n, \tag{A.3}$$

where $\mathbf{D}$ is the matrix defined by

$$\mathbf{D}_{ij} = \begin{cases} \text{BC}, & i = 0 \text{ or } i = X \\ \zeta, & 0 < i < X \text{ and } |i-j| = 1 \\ 1-2\zeta, & 0 < i < X \text{ and } i-j = 0 \\ 0, & \text{else} \end{cases} \tag{A.4}$$

The top and bottom rows ($i = 0$ and $i = X$) are determined from boundary conditions. Note that as ζ approaches 0, $\mathbf{D}$ approaches the identity matrix. Thus ζ should be as small as possible for the best approximtation. The condition for stability is

$$0 < \zeta < 1.$$

Error in time

The explicit method uses a forward difference in time. The expression for the derivative approximation is derived from the power series expansion in δt

$$u(x, t+\delta t) = u(x,t) + \frac{\partial}{\partial t}u(x,t)\cdot\delta t + \frac{\partial^2}{\partial t^2}u(x,t)\cdot\frac{\delta t^2}{2!} + \dots \tag{A.5}$$

Thus we have an approximation for the time derivative

$$\frac{u(x,t+\delta t) - u(x,t)}{\delta t} = \frac{\partial}{\partial t}u(x,t) + O(\delta t). \tag{A.6}$$

The error in time due to this approximation is linear in δt.

Error in position

The explicit method uses a central second derivative in space. The expression for the second derivative approximation is derived from the power series expansion in δx.

$$u(x+\delta x, t) = u(x,t) + \frac{\partial}{\partial x}u(x,t)\cdot\delta x + \frac{\partial^2}{\partial x^2}u(x,t)\cdot\frac{\delta x^2}{2!} + \frac{\partial^3}{\partial x^3}u(x,t)\cdot\frac{\delta x^3}{3!} + \dots \tag{A.7}$$

At $-\delta x$, this is

$$u(x-\delta x, t) = u(x,t) - \frac{\partial}{\partial x}u(x,t)\cdot\delta x + \frac{\partial^2}{\partial x^2}u(x,t)\cdot\frac{\delta x^2}{2!} - \frac{\partial^3}{\partial x^3}u(x,t)\cdot\frac{\delta x^3}{3!} + \dots$$

Thus the approximation for the second spatial derivative is

$$\frac{u(x-\delta x,t) - 2u(x,t) + u(x+\delta x,t)}{\delta x^2} = \frac{\partial^2}{\partial x^2}u(x,t) + O(\delta x^2). \tag{A.8}$$

The error in position due to this approximation is quadratic in δx.

A.2 Implicit method

A.3 Crank-Nicolson method