ENPH 257 Lab Report

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hello world

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Comparison of Finite Differencing Methods

The following are methods of finite differencing, demonstrated using the simple example of the one-dimensional heat conduction equation

$$\frac{\partial}{\partial t}u(x,t) = \alpha \frac{\partial^2}{\partial x^2}u(x,t). \tag{A.1}$$

A.1 Explicit method

Finite difference equation

In the explicit method for finite differencing, the temperature for each spatial point can be independently determined from the previous step. That is,

$$u_i^{n+1} = u_i^n + \alpha \cdot \frac{\delta t}{\delta x^2} \cdot \left(u_{i-1}^n - 2u_i^n + u_{i+1}^n\right), \tag{A.2}$$

where the superscript $n \in [0, T]$ signifies the step in time t_n and the subscript $i \in [0, X]$ represents the position x_i. δt and δx are the time steps in time and position respectively.

If we define

$$\zeta = \alpha \cdot \frac{\delta t}{\delta x^2},$$

then this can be written as a linear relationship

$$\mathbf{u}^{n+1} = \mathbf{D}\mathbf{u}^n, \tag{A.3}$$

where $\mathbf{D}$ is the matrix defined by

$$\mathbf{D}_{ij} = \begin{cases} \text{BC}, & i = 0 \text{ or } i = X \\ \zeta, & 0 < i < X \text{ and } |i-j| = 1 \\ 1 - 2\zeta, & 0 < i < X \text{ and } i - j = 0 \\ 0, & \text{else} \end{cases} \tag{A.4}$$

The top and bottom rows ($i = 0$ and $i = X$) are determined from boundary conditions. Note that as ζ approaches 0, $\mathbf{D}$ approaches the identity matrix. Thus ζ should be as small as possible for the best approximtation. The condition for stability is

$$0 < \zeta < 1.$$

Error in time

The explicit method uses a forward difference in time. The expression for the derivative approximation is derived from the power series expansion in δt

$$u(x,t+\delta t) = u(x,t) + \frac{\partial}{\partial t}u(x,t)\cdot\delta t + \frac{\partial^2}{\partial t^2}u(x,t)\cdot\frac{\delta t^2}{2!} + \ldots \tag{A.5}$$

Thus we have an approximation for the time derivative

$$\frac{u(x,t+\delta t) - u(x,t)}{\delta t} = \frac{\partial}{\partial t}u(x,t) + O(\delta t). \tag{A.6}$$

The error in time due to this approximation is linear in δt.

Error in position

The explicit method uses a central second derivative in space. The expression for the second derivative approximation is derived from the power series expansion in δx.

$$u(x+\delta x,t) = u(x,t) + \frac{\partial}{\partial x}u(x,t)\cdot\delta x + \frac{\partial^2}{\partial x^2}u(x,t)\cdot\frac{\delta x^2}{2!} + \frac{\partial^3}{\partial x^3}u(x,t)\cdot\frac{\delta x^3}{3!} + \ldots \tag{A.7}$$

At $-\delta x$, this is

$$u(x-\delta x,t) = u(x,t) - \frac{\partial}{\partial x}u(x,t)\cdot\delta x + \frac{\partial^2}{\partial x^2}u(x,t)\cdot\frac{\delta x^2}{2!} - \frac{\partial^3}{\partial x^3}u(x,t)\cdot\frac{\delta x^3}{3!} + \ldots$$

Thus the approximation for the second spatial derivative is

$$\frac{u(x-\delta x,t) - 2u(x,t) + u(x+\delta x,t)}{\delta x^2} = \frac{\partial^2}{\partial x^2}u(x,t) + O(\delta x^2). \tag{A.8}$$

The error in position due to this approximation is quadratic in δx.

A.2 Implicit method

A.3 Crank-Nicolson method