Contents Circle and Polygon intersection $\ \ldots \ \ldots \ \ldots \ \ldots$ 11.6 11.7 Basic Rotating Calipers 11.9 1.1 1.2 1.3 12 Ad hoc 12.1 1.4 Joseph Problem Segment Max Segment Sum Stone Merge Manhattan Spanning Tree K Cover Tree M Segments' Maximum Sum Minimum Enclosing Cycle Rotating Sweep Line 12.2 Pragma 17 124 12.5 1.8 12.6 23 12.8 2.1 Hilbert Curve 12.9 2.2 2.3 1 Basic 2.4 1.1.vimrc Theorem and Formula syntax on Data Structure Cext/pb_ds> Unordered Map Hash Rope se ru nu ai se ts=4 sts=4 sw=4 st=4 smarttab laststatus=2 expandtab inoremap {<ENTER> {}<LEFT><ENTER>;<LEFT><ENTER><UP><TAB> "se mouse=a expandtab 4.4 4.5 4.6 1.2 Check 4.7 Dancing Link for i in \$(seq 1 10000); ./gen > input Flow ./ac < input > out_ac 5.1 ./wa < input > out_wa 5.2 diff out_ac out_wa || break 5.3 5.4 1.3 Factor Count List 6 Tree 6.1 Zhu Liu Algo Centroid Decomposition 6.2 ζi. 6.3 factor number of i) 6.4 50400 10080 72, 168, 108, 110880 144 6.5 192, 221760 332640 498960 200 6.6 554400 216, 665280 720720 240, 256, 1081080 2162160 320, 3603600 360, Graph 4324320 384, 6486480 400, 7207200 432, 448, 10810800 576, 720. 8648640 480, 21621600 7.132432400 672, 600, 43243200 61261200 7.2768, 110270160 800, 1152, 551350800 1200, 1344, 1102701600 1440, 73513440 245044800 1008 7.3 367567200 1200, 698377680 1280, 7.4 735134400 1396755360 1536 1.4 Default Math Extended Euclidean Big Integer Gaussian Elimination Linear Basis Build Prime Miller Rabin Pollard Rho Build Phi and Mu Primitive Root Cipolla's Algorithm Discrete Log Integer Partition Meissel-Lehmer Algorithm De Bruijn Simplex Algorithm Middle Speed Linear Recursion 8.1 8.2 // Compile with "g++ -std=c++11 -Wall -Wextra -Wconversion -Wshadow -fsanitize=undefined -Dlawfung" 8.4 #ifdef lawfung 8.5 13 8.6 13 8.7 13 8.8 _DO(__VA_ARGS__);\ 8.10 }while(0) 8.11 14 template<typename I> void _DO(I&&x) {cerr << x << '\n';}</pre> 8.12 14 template<typename I, typename ...T> void _DO(I&&x,T&&...tail) { cerr << x << ", "; _DO(tail...);}</pre> 8.13 8.14 15 #define IOS 8.15 #else #define debug(...) 8.17 #define IOS ios_base::sync_with_stdio(0);cin.tie(0) #endif 16 Convolution 9.1 16 9.2 16 1.5 Pragma 9.3 9.4 17 #pragma GCC optimize("Ofast", "unroll-loops") #pragma GCC optimize("no-stack-protector") 9.5 17 #pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.2,popcnt,abm, 10 String mmx, avx, tune=native") 10.1#pragma GCC diagnostic ignored "-W" 10.2 17 10.3 17 10.4 1.6 Random Int 10.5 10.6 #include <random> Lexicographically Smallest Rotation mt19937 rng(chrono::steady_clock::now().time_since_epoch(). count()); 11 Geometry int rand_int(int lb, int ub) Circle 11.1 Circle Half Plane Intersection Convex Hull 3D Dynamic convexhull Polar Angle Sort { return uniform_int_distribution<int>(lb, ub)(rng); } double rand_double(double lb, double ub) { return uniform_real_distribution<double>(lb, ub)(rng); }

11.5

1.7 Increase Stack Size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bak = (char*)rsp;
|_asm__("movq %0, %%rsp\n"::"r"(p));
// main
|_asm__("movq %0, %%rsp\n"::"r"(bak));</pre>
```

1.8 FasterIO

```
| static inline char getRawChar() {
| static char buf[1 << 16], *p = buf, *end = buf;
| if (p == end) {
| if ((end = buf + fread_unlocked(buf, 1, 1 << 16, stdin)) ==
| buf) return '\0';
| p = buf;
| }
| return *p++;
| }
| while (c = getRawChar() && (unsigned)(c - '0') > 10U) n = n *
| 10 + (c - '0');
```

2 Bitwise Trick

2.1 Builtin Function

```
// count left 0s
|int __builtin_clz (unsigned int x) // 31 - __builtin_clz is lg
|int __builtin_clzll (unsigned long long x) // 63 - clz
|// count number of 1's
|int __builtin_popcount (unsigned int x)
|int __builtin_popcountll (unsigned long long x)
```

2.2 Subset Enumeration

```
int subset_enumeration(int s) {
    for (int now = s; now > 0; now = (now - 1) & s) {
        cout << now << ' ';
    }
    cout << "0\n";
}</pre>
```

2.3 Next Permutation on Binary

```
|ll next_perm(ll v) {
| ll t = v | (v - 1);
| return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1))
| ;
|}
```

2.4 SOS DP

```
// 0 is 0, 1 can be 1 or 0
| for (int i = 0; i < n; ++i)
| for (int j = 0; j < (1 << n); ++j)
| if ( j & (1 << i) )
| a[j] += a[ j ^ (1 << i) ];</pre>
```

3 Theorem and Formula

- Pick's theorem $A = i + \frac{b}{2} 1$
- Laplacian matrix L = D A
- Derangement $D_n = (n-1)(D_{n-1} + D_{n-2})$
- Möbius function $\sum_{i|n} \mu(i) = [n=1]$
- Euler's totient function $\sum\limits_{i\,|\,n}\phi(i)=n$
- Inversion formula

$$\begin{split} f(n) &= \sum_{i=0}^{n} \binom{n}{i} g(i), \ g(n) = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} f(i) \\ f(n) &= \sum_{d \mid n} g(d), \ g(n) = \sum_{d \mid n} \mu(\frac{n}{d}) f(d) \end{split}$$

• Sum of powers

$$\sum_{k=1}^{n} k^{m} = \frac{1}{m+1} \sum_{k=0}^{m} {m+1 \choose k} B_{k}^{+} n^{m+1-k}$$

$$\sum_{j=0}^{m} {m+1 \choose j} B_{j}^{-} = 0$$

$$\text{note}: B_{1}^{+} = -B_{1}^{-} B_{i}^{+} = B_{i}^{-}$$

• Cipolla's algorithm

$$\left(\frac{u}{p}\right) = u^{\frac{p-1}{2}}$$

$$1. \left(\frac{a^2 - n}{p}\right) = -1$$

2.
$$x = (a + \sqrt{a^2 - n})^{\frac{p+1}{2}}$$

• High order residue

```
[d^{\frac{p-1}{(n,p-1)}} \equiv 1] (p is odd prime and p \( d)
```

· Packing and Covering

 $|{\rm Maximum~Independent~Set}| \, + \, |{\rm Minimum~Vertex~Cover}| = |{\rm V}|$

Kőnig's theorem

|Maximum matching|(easy) = |Minimum vertex cover|

• Dilworth's theorem

width = |smallest chain decomposition| (vertex split and matching) = |largest antichain| = |maximim clique in Complement| (easy)

· Mirsky's theorem

 $\begin{array}{l} \mbox{height} = |\mbox{longest chain}|(\mbox{easy DP}) = |\mbox{smallest antichain decomposition}| \\ = |\mbox{minimum anticlique partition}| \ (\mbox{subset DP}) \end{array}$

• Triangle center

```
-G: (1,1,1)
-O: (a^{2}(b^{2}+c^{2}-a^{2}), \cdots) = (\sin 2A, \sin 2B, \sin 2C)
-I: (a,b,c) = (\sin A, \sin B, \sin C)
-E: (-a,b,c) = (-\sin A, \sin B, \sin C)
-H: (\frac{1}{b^{2}+c^{2}-a^{2}}, \cdots) = (\tan A, \tan B, \tan C)
```

• $\lfloor \frac{n}{i} \rfloor$ enumeration $T_0=1, T_i=\lfloor \frac{n}{\lfloor \frac{n}{T_{i-1}+1} \rfloor} \rfloor$

4 Data Structure

$4.1 < ext/pb_ds >$

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/priority_queue.hpp>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
using namespace std;
__gnu_pbds::priority_queue<int> pq;
__gnu_pbds::priority_queue<int>::point_iterator idx[10];
__gnu_pbds::priority_queue<int, less<int>, pairing_heap_tag>
pairing_heap_tag, thin_heap_tag, binomial_heap_tag
rc_binomial_heap_tag, binary_heap_tag
*/
idx[0] = pq.push(1);
pq.modify(idx[0], 2); // change the iterator's value to 2
pq1.join(pq2);
typedef tree<int,null_type,less<int>,rb_tree_tag,
     tree_order_statistics_node_update> TREE;
TREE name;
*name.find_by_order(0);
name.order_of_key(1);
name.insert(2);
name.delete(3);
name.split(v, b); /// value < v of a split to b</pre>
name.join(another TREE);
```

4.2 Unordered Map Hash

```
| struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
    };
    typedef unordered_map<Key, int, KeyHasher> map_t;
```

4.3Rope

```
#include <ext/rope>
using namespace __gnu_cxx;
int main() {
  rope<int> v;
               // can be cout directly if it's char
  rope<int> v1(v);
 rope<int> v2(arr, arr + 10); //int arr[100];
 v.find(3); // return the first positoin of 3
 v.push_back(4); v.pop_back();
  //append not for iterator
  v.insert(pos, s); // pos can be iterator, integer. s can be
 rope, int, array
v.replace(pos, len, s); // (pos, len) can be (it1, it2). s is
        same as insert.
  v.erase(pos, len); // or v.erase(it1, it2)
  v2 = v.substr(pos, len); // same as erase
 v.copy(pos, len, arr); // int arr[100]; (pos, len) can be
       omitted
 v[0], v[1]
  auto it1 = v.mutable_begin(), it2 = v.mutable_end();
```

4.4 Disjoint Set

```
struct DJS{
  int p[N], rk[N];
  vector<pair<int*,int>> memo;
  vector<size_t> stk;
  void save(){
    stk.push_back(memo.size());
  }
  void undo(){
    while(memo.size() > stk.back()){
      *memo.back().first = memo.back().second;
      memo.pop_back();
    stk.pop_back();
  void assign(int *x, int v){
    memo.push_back({x, *x});
     *x=v;
  //assign(&a, b); //a = b
|} djs;
```

4.5 Persistent Treap

```
#include <bits/stdc++.h>
using namespace std;
struct Treap {
 static Treap mem[P];
Treap *lc,*rc;
  char c; int sz;
  Treap(){}
  Treap(char _c) : lc(NULL),rc(NULL),sz(1),c(_c){}
 Treap::mem[P], *ptr=Treap::mem ;
int Sz(Treap* t) {
 return t?t->sz:0;
void pull(Treap* t) {
 if (!t) return
  t->sz = Sz(t->lc) + Sz(t->rc) + 1;
Treap* merge(Treap* a,Treap* b) {
  if (!a || !b) return a?a:b;
  Treap* ret;
 if (myRnd() \% (Sz(a) + Sz(b)) < Sz(a)) {
    ret = new (ptr++) Treap(*a);
    ret->rc = merge(a->rc,b);
  else {
    ret = new(ptr++) Treap(*b);
    ret->lc=merge(a,b->lc);
  pull(ret);
  return ret;
void split(Treap* t,int k,Treap* &a,Treap* &b) {
  if (!t) a=b=NULL;
  else if (Sz(t\rightarrow lc) + 1 \ll k) {
    a = new(ptr++) Treap(*t);
    split(t->rc,k-Sz(t->lc)-1,a->rc,b);
    pull(a);
  else {
```

```
3
     b=new(ptr++) Treap(*t);
     split(t->lc,k,a,b->lc);
     pull(b);
   }
 }
 int d;
 char buf[M];
 Treap* ver[N];
 ptr = Treap::mem;
 v_cnt++
 ver[v_cnt] = ver[v_cnt-1];
 split(ver[v_cnt],p,tl,tr);
tl = merge(tl,new(ptr++)Treap(buf[j]));
 4.6 Link Cut Tree
 struct SplayNode {
     static SplayNode HOLE;
     SplayNode *ch[2], *par;
     bool rev:
     SplayNode(): par(&HOLE), rev(false) { ch[0] = ch[1] = &HOLE}
     bool isRoot() {
         return (par->ch[0] != this && par->ch[1] != this);
     void push() {
         if (rev) {
              if (ch[0]) ch[0]->rev ^= 1;
              if (ch[1]) ch[1]->rev ^= 1;
              swap(ch[0], ch[1]);
              rev ^= 1;
     void pushFromRoot() {
         if (!isRoot()) par->pushFromRoot();
         push();
     void pull() {
          if (ch[0]) ch[0]->d = d + ch[0]->parLen;
         if (ch[1]) ch[1]->d = d + ch[1]->parLen;
     void rotate() {
    SplayNode *p = par, *gp = p->par;
         bool dir = (p->ch[1] == this);
          if (!p->isRoot()) gp->ch[gp->ch[1] == p] = this;
         p \rightarrow ch[dir] = ch[dir \land 1];
         p->ch[dir]->par = p;
         p->par = this;
ch[dir ^ 1] = p
         p->pull(), pull();
     void splay() {
         pushFromRoot();
         while (!isRoot()) {
              if (!par->isRoot()) {
                  SplayNode *gp = par->par;
                   if ((gp->ch[0] == par) == (par->ch[0] == this))
                         rotate();
                  else par->rotate();
              rotate();
         }
 } SplayNode::HOLE;
 namespace LCT {
     SplayNode *access(SplayNode *x) {
    SplayNode *last = &SplayNode::HOLE;
          while (x != &SplayNode::HOLE) {
              x->splay();
              x \rightarrow ch[1] = last;
              x->pull();
              last = x:
              x = x - par;
         return last;
     void makeRoot(SplayNode *x) {
         access(x);
         x->splay()
          x->rev ^= 1;
     void link(SplayNode *x, SplayNode *y) {
         makeRoot(x);
         x -> par = y;
```

void cut(SplayNode *x, SplayNode *y) {

```
makeRoot(x);
                                                                           #define MAX 1050
                                                                           #define INF 0x3f3f3f3f
         access(y)
         y->splay();
                                                                           struct DLX{
         y->ch[0] = &SplayNode::HOLE;
         x->par = &SplayNode::HOLE;
    void cutParent(SplayNode *x) {
                                                                                     100];
        access(x);
                                                                                int ans;
         x->splay();
                                                                                void init(int n){
         x - ch[0] - par = \&SplayNode::HOLE;
                                                                                    this \rightarrow n = n;
         x - ch[0] = &SplayNode::HOLE;
                                                                                    ans = INF;
                                                                                    REP(i , 0 , n + 1){
    u[i] = d[i] = i;
    l[i] = i - 1;
    SplayNode *findRoot(SplayNode *x) {
         x = access(x)
         while (x->ch[0] != \&SplayNode::HOLE) x = x->ch[0];
                                                                                         r[i] = i + 1;
         x->splay();
                                                                                    r[n] = 0, l[0] = n; sz = n + 1;
                                                                                    MEM(s , 0);
    SplayNode *query(SplayNode *x, SplayNode *y) {
        makeRoot(x);
         return access(y);
                                                                                    int tmp = sz;
    SplayNode *queryLca(SplayNode *x, SplayNode *y) {
                                                                                    for(auto to : sol){
                                                                                         l[sz] = sz - 1;
        access(x);
         auto lca = access(y);
                                                                                         r[sz] = sz + 1;
         x->splay();
                                                                                         d[sz] = to;
         return lca \rightarrow data + lca \rightarrow ch[1] \rightarrow sum + (x == lca ? 0 : x
                                                                                         u[sz] = u[to];
                                                                                         d[u[to]] = sz, u[to] = sz;
    void modify(SplayNode *x, int data) {
                                                                                         s[to] ++ , sz ++;
        x->splay();
x->data = data;
         x->pull();
                                                                                way[i])
                                                                                void remove(int c){
4.7 Li Chao Tree
                                                                                    l[r[c]] = l[c];
                                                                                    r[l[c]] = r[c];
                                                                                    FOR(i , d , c) FOR(j , r , i){
    u[d[j]] = u[j];
    d[u[j]] = d[j];
struct line {
    ll a, b;
    line(): a(0), b(0) {}
    line(ll a, ll b): a(a), b(b) {}
                                                                                         --s[col[j]];
    11 operator()(ll x) const { return a * x + b; }
                                                                                int restore(int c){
   FOR(i , u , c) FOR(j , l , i){
struct lichao {
    line st[NN];
                                                                                         ++s[col[j]];
    int sz, lc[NN], rc[NN];
                                                                                         u[d[j]] = j;
    int gnode() {
                                                                                         d[u[j]] = j;
         st[sz] = line(0, -1e18); //min: st[sz] = line(0, 1e18); lc[sz] = -1, rc[sz] = -1;
                                                                                    l[r[c]] = c;
         return sz++;
                                                                                    r[l[c]] = c;
    void init() {
                                                                                void DFS(int floor){
         sz = 0; gnode();
                                                                                    if(r[0] == 0){
                                                                                         ans = min(ans , floor);
    void add(int l, int r, line tl, int o) {
                                                                                         return;
         bool lcp = st[o](l) < tl(l); //min: change < to >
                                                                                    if(floor >= ans) return;
```

bool mcp = st[o]((1 + r) / 2) < tl((1 + r) / 2); //min:change < to > if (mcp) swap(st[o], tl); if (r - l == 1) return; if (lcp != mcp) { if (lc[o] == -1) lc[o] = gnode(); add(1, (1 + r) / 2, tl, lc[o]);if (rc[o] == -1) rc[o] = gnode(); add((l + r) / 2, r, tl, rc[o]); il query(int l, int r, int x, int o) { if (r - l == 1) return st[o](x); if (x < (l + r) / 2) { if (lc[o] == -1) return st[o](x); return max(st[o](x), query(l, (l + r) / 2, x, lc[o])); } else { if (rc[o] == -1) return st[o](x); return max(st[o](x), query((l + r) / 2, r, x, rc[o } } solver;

4.8 Dancing Link

```
int n , sz , s[MAX];
int row[MAX * 100] , col[MAX * 100];
    int l[MAX * 100] , r[MAX * 100] , u[MAX * 100] , d[MAX * 100]
    void AddRow(int rr , vector<int> sol){
             row[sz] = rr , col[sz] = to;
         r[sz - 1] = tmp , l[tmp] = sz - 1;
#define FOR(i , way , to) for(int i = way[to] ; i != to ; i =
         int c = r[0];
         FOR(i, r, 0) if(s[i] < s[c]) c = i;
         remove(c);
         FOR(i \ , \ d \ , \ c)\{
             FOR(j , r , i) remove(col[j]);
DFS(floor + 1);
             FOR(j , l , i) restore(col[j]);
         restore(c);
} solver;
int n , m;
int32_t main(){
    ĪOS;
    while(cin >> n >> m){
         solver.init(m);
         REP(i , 0 , n){
              int nn , in;
              cin >> nn;
              vector<int> sol;
              REP(j, 0, nn) cin >> in, sol.pb(in);
              solver.AddRow(i , sol);
         solver.DFS(0);
         if(solver.ans == INF) cout << "No" << endl;
else cout << solver.ans << endl;</pre>
    return 0;
```

|}

4.9 Range Modify and Query BIT

```
|int n, m, k;
int bit[4][MAX][MAX];
void update(int c[MAX][MAX], int a, int b, int val) {
  for(int i = a + 10; i < MAX; i += i \& -i)
     for(int j = b + 10; j < MAX; j += j \& -j)
       c[i][i] += val;
int update(int x, int y, int val) {
  update(bit[0], x, y, val);
  update(bit[1], x, y, -val * x);
update(bit[2], x, y, -val * y);
  update(bit[3], x, y, val * x * y);
void update(int a, int b, int x, int y, int val){
  update(a, b, val);
  update(a, y + 1, -val);
  update(x + 1, b, -val);
  update(x + 1, y + 1, val);
int query(int c[MAX][MAX], int a, int b){
  int cnt = 0;
  for(int i = a + 10; i > 0; i -= i \& -i)
    for(int j = b + 10 ; j > 0 ; j -= j & -j)
  cnt += c[i][j];
  return cnt;
int query(int x, int y){
  int cnt = 0;
  cnt += query(bit[0], x, y) * (x + 1) * (y + 1);
  cnt += query(bit[1], x, y) * (y + 1);
cnt += query(bit[2], x, y) * (x + 1);
  cnt += query(bit[3], x, y);
  return cnt:
int query(int a, int b, int x, int y){
  int cnt = 0;
  cnt += query(a - 1, b - 1);
cnt -= query(a - 1, y);
  cnt -= query(x, b - 1);
  cnt += query(x, y);
  return cnt;
}
/* usage:
void update(x1, y1, x2, y2, val);
int query(x1, y1, x2, y2); */
```

5 Flow

5.1 ISAP with bound

```
Maximum density subgraph ( \sum W_e + \sum W_v  ) / |V|
Binary search on answer:
For a fixed D, construct a Max flow model as follow:
Let S be Sum of all weight( or inf)
1. from source to each node with cap = S
2. For each (u,v,w) in E, (u->v,cap=w), (v->u,cap=w)
3. For each node v, from v to sink with cap = S + 2 * D - deg[v]
    ] - 2 * (W of v)
where deg[v] = \sum weight of edge associated with v
If maxflow < S * IVI, D is an answer.
Requiring subgraph: all vertex can be reached from source with
edge whose cap > 0.
//Be careful that it's zero base !!!!!!!!
#include <bits/stdc++.h>
using namespace std;
typedef long long ll;
#define SZ(x) ((int)(x).size())
#define eb emplace_back
const ll INF = 0x3f3f3f3f3f3f3f3f3f;
const 11 N = 5e2 + 5;
struct isap{
 struct edge{
    int t, r;
    11 c;
    edge(int _t, int _r, ll _c) : t(_t), r(_r), c(_c) {}
 int n, S, T;
  vector<edge> adj[N];
```

```
int dis[N], gap[N], ok;
   isap(int _n, int _s, int _t) : n(_n), S(_s), T(_t) {
  for(int i = 0; i < n + 2; ++ i) adj[i].clear();</pre>
   void add(int u, int v, ll c){
  adj[u].eb( v, adj[v].size(), c );
  adj[v].eb( u, adj[u].size() - 1, 0 );
   ll dfs(int now, ll f){
      if(now == T) return f;
      int mi = n;
      for(edge &e : adj[now]){
        if(e.c){
           11 x;
           if( dis[now] == dis[e.t] + 1 && (x = dfs(e.t, min(f, e.
                c))) ){
             adj[e.t][e.r].c += x;
           mi = min(mi, dis[e.t]);
        }
      if( --gap[dis[now]] == 0) ok = 0;
      dis[now] = mi + 1;
      gap[ dis[now] ]++;
      return 0;
   il flow(){
      memset(dis, 0, n * 4);
      memset(gap, 0, n * 4);
      gap[0] = n;
     ok = 1;
ll r = 0;
      while(dis[S] < n && ok) r += dfs(S, INF);
   // below for bounded only
   11 D[N];
   void bounded_init() {
     memset(D, 0, n * 8);
   void add2(int u, int v, ll b, ll c) {
      add(u, v, c - b);
      D[u] -= b;
      D[v] += b;
   11 bounded_flow() {
      int SS = n, TT = n + 1;
      ll base = 0;
      for(int i = 0; i < n; ++ i) {</pre>
        if (D[i] > 0) base += D[i];
if (D[i] > 0) add(SS, i, D[i]);
if (D[i] < 0) add(i ,TT, -D[i]);</pre>
      add(T, S, INF);
      int tmps = S, tmpt = T;
n += 2; S = SS, T = TT;
     ll f = flow();
n -= 2; S = tmps; T = tmpt;
      return f == base ? flow() : -1LL;
};
        Min Cost Max Flow
```

```
int dis[N], par[N], par_id[N];
     pair<int, int> flow() {
   int flow = 0, cost = 0;
          while (true) {
              for (int i = 0; i \le n; ++i) {
                   dis[i] = INF, in_que[i] = false;
              queue<int> que; que.push(s);
              while (!que.empty()) {
                   int t = que.front(); que.pop();
int ptr = 0;
                    in_que[t] = false
                    for (Edge e: G[t]) {
                        if (e.cap > 0) {
                             if (dis[e.to] > dis[t] + e.cost) {
                                 dis[e.to] = dis[t] + e.cost;
                                  par[e.to] = t, par_id[e.to] = ptr;
                                  if (!in_que[e.to]) {
                                      que.push(e.to);
                                      in_que[e.to] = true;
                                 }
                             }
                        ++ptr:
                   }
               if (dis[t] == INF) break;
              int mn_flow = INF;
               for (int i = t; i != s; i = par[i]) {
                   mn_flow = min(mn_flow, G[ par[i] ][ par_id[i]
                         ].cap);
              flow += mn_flow;
cost += mn_flow * dis[t];
for (int i = t; i != s; i = par[i]) {
    G[ par[i] ][ par_id[i] ].cap -= mn_flow;
                   G[ i ][ G[ par[i] ][ par_id[i] ].rev ].cap +=
                         mn_flow;
          return make_pair(flow, cost);
     }
} flow;
```

5.3 S-W Global Min Cut

```
struct SW {
  //find global min cut in O(V^3)
  //points are ZERO-BASE!!!
  static const int N = 506;
  int adj[N][N], wei[N], n;
  bool vis[N], del[N];
  void init(int _n) {
    memset(adj, 0, sizeof(adj));
    memset(del, 0, sizeof(del));
  void add_edge(int x, int y, int w) {
    adj[x][y] += w;
    adj[y][x] += w;
  void search(int & s, int & t) {
    memset(wei, 0, sizeof(wei));
    memset(vis, 0, sizeof(vis));
    s = t = -1;
    while (true) {
      int mx = -1, mx_id = 0;
for (int i = 0; i < n; ++i) {
  if (!del[i] && !vis[i] && mx < wei[i]) {</pre>
          mx_id = i
          mx = wei[i];
      if (mx == -1) break;
      vis[mx_id] = true;
      t = mx_id;
      for (int i = 0; i < n; ++i)
         if (!vis[i] && !del[i])
          wei[i] += adj[mx_id][i];
    }
  int solve() {
    int ret = INF;
    for (int i = 0; i < n - 1; ++i) {
      int x, y;
```

```
search(x, y);
ret = min(ret, wei[y]);
       del[y] = true;
       for (int j = 0; j < n; ++j) {
         adj[x][j] += adj[y][j];
         adj[j][x] += adj[y][j];
     return ret;
   }
} SW;
```

Gomory Hu Tree

```
def cut(G,s,t) :
   return minimum s-t cut in G
 def gomory_hu(G):
   T = \{\}
   p = [1] * |V(G)|
   for s in [2,n] :
     t = p[s]
     w(C) = cut(G, s, t)
     add(s, t, w(C)) to T
for i in [s + 1, n] :
        if p[i] == t and s-i path exists in G\setminus C:
          p[i] = s
   return T;
```

Tree

Minimum Steiner Tree 6.1

```
// Minimum Steiner Tree
 // 0(V 3^T + V^2 2^T)
 struct SteinerTree{
 #define V 33
 #define T 8
 #define INF 1023456789
       int n , dst[V][V] , dp[1 << T][V] , tdst[V];</pre>
       void init( int _n ){
            n = _n;
             for( int i = 0 ; i < n ; i ++ ){</pre>
                  for( int j = 0 ; j < n ; j ++ )
   dst[ i ][ j ] = INF;</pre>
                  dst[ i ][ i ] = 0;
            }
       void add_edge( int ui , int vi , int wi ){
   dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
   dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
       void shortest_path(){
            for( int k = 0 ; k < n ; k ++ )
    for( int i = 0 ; i < n ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            dst[i][j] = min( dst[i][j],
                                          dst[ i ][ k ] + dst[ k ][ j ] );
       int solve( const vector<int>& ter ){
   int t = (int)ter.size();
            for( int i = 0 ; i < ( 1 << t ) ; i ++ )
    for( int j = 0 ; j < n ; j ++ )
        dp[ i ][ j ] = INF;
for( int i = 0 ; i < n ; i ++ )
        dp[ 0 ][ i ] = 0;</pre>
             for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){</pre>
                   if( msk == ( msk & (-msk) ) ){
                        int who = __lg( msk );
for( int i = 0 ; i < n ; i ++ )
   dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];</pre>
                        continue:
                  dp[ msk ^ submsk ][ i ] );
                  for( int i = 0 ; i < n ; i ++ ){</pre>
                        tdst[ i ] = INF;
                        for( int i = 0 ; i < n ; i ++ )</pre>
```

```
Zhu Liu Algo
6.2
  //1 base edge and vertex
  static const int N=556,M=2660, MM = M * 10,inf=1e9; //MM = M * log N
  struct bian{
     int u,v,w,use,id;
  }b[M],a[MM];
  int n,m=0,ans,pre[N],id[N],vis[N],root,In[N],h[N],len,way[M];
  void init(int _n,int _root){
    for (int i = 0; i < MM; ++i) {
       a[i] = \{0, 0, 0, 0, 0\};
    n=_n; m=0; b[0].w=1e9; root=_root;
  }
  void add(int u,int v,int w){
    b[++m]=(bian)\{u,v,w,0,m\};
    a[m]=b[m];
  int work(){
    len=m;
     for (;;){
       for (int i=1;i<=n;i++){pre[i]=0; In[i]=inf; id[i]=0; vis[</pre>
             i]=0; h[i]=0;}
       for (int i=1;i<=m;i++)</pre>
         if (b[i].u!=b[i].v&&b[i].w<In[b[i].v]){</pre>
            \label{eq:prebasis} \begin{split} \mathsf{pre}[b[\mathsf{i}].\mathsf{v}] = & b[\mathsf{i}].\mathsf{u}; \ \mathsf{In}[b[\mathsf{i}].\mathsf{v}] = & b[\mathsf{i}].\mathsf{w}; \ \mathsf{h}[b[\mathsf{i}].\mathsf{v}] = & b[\mathsf{i}]. \end{split}
       for (int i=1;i<=n;i++) if (pre[i]==0&&i!=root) return 0;</pre>
       int cnt=0; In[root]=0;
       for (int i=1;i<=n;i++){</pre>
          if (i!=root) a[h[i]].use++;
          int now=i; ans+=In[i];
         while (vis[now]==0&&now!=root){
            vis[now]=i; now=pre[now];
          if (now!=root&&vis[now]==i){
            cnt++; int kk=now;
            while (1){
              id[now]=cnt; now=pre[now];
              if (now==kk) break;
       if (cnt==0) return 1;
       for (int i=1;i<=n;i++) if (id[i]==0) id[i]=++cnt;</pre>
       for (int i=1;i<=m;i++){</pre>
          int k1=In[b[i].v]; int k2=b[i].v;
         b[i].u=id[b[i].u]; b[i].v=id[b[i].v];
          if (b[i].u!=b[i].v){
            b[i].w-=k1; a[++len].u=b[i].id; a[len].v=h[k2];
            b[i].id=len;
         }
       n=cnt;
       root=id[root];
    return 1;
  }
  int getway(){
     for (int i=1;i<=m;i++) way[i]=0;</pre>
     for (int i=len;i>m;i--){
       a[a[i].u].use+=a[i].use; a[a[i].v].use-=a[i].use;
     for (int i=1;i<=m;i++) way[i]=a[i].use;</pre>
     int ret = 0;
     for (int i = 1; i <= m; ++i){</pre>
       if (way[i] == 1) {
         ret += a[i].w;
       }
    return ret;
} zl;
//if zl.work() == 0, then it is not connected
//otherwise, use zl.getway() to check bian is selected or not
```

6.3 Centroid Decomposition

```
const int Mlg = __lg(MAX) + 2;
struct edge {
  int to, weight;
   edge(int _to,int _w):to(_to),weight(_w){}
};
vector<edge> edg[MAX];
struct Cen {
   ll val;
   int p, sz, dep;
   Cen(){}
   Cen(int _p,int _d):val(0),p(_p),sz(0),dep(_d){}
} cen[MAX];
ll dis[Mlg][MAX];
bool visit[MAX];
vector<int> v
int sz[MAX], mx[MAX];
void dfs_sz(int id) {
  visit[id]=1;
   v.push_back(id);
   sz[id]=1;
   mx[id]=0;
   for (edge i:edg[id]) {
     if (!visit[i.to]) {
       dfs_sz(i.to);
       mx[id] = max(mx[id], sz[i.to]);
       sz[id] += sz[i.to];
     }
  }
void dfs_dis(int id,int cen_dep,ll weight) {
   dis[cen_dep][id] = weight;
   visit[id]=1;
   for (edge i:edg[id])
     if (!visit[i.to])
       dfs_dis(i.to,cen_dep,weight+i.weight);
void build(int id,int cen_dep,int p) {
  dfs sz(id):
   int nn=v.size();
   int ccen=-1;
   for (int i:v) {
     if (max(nn-sz[i],mx[i])*2 <= nn)</pre>
       ccen=i
     visit[i]=0;
   dfs_dis(ccen,cen_dep,0);
   for (int i:v)
                   visit[i]=0;
   v.clear();
   visit[ccen]=1;
   cen[ccen] = Cen(p,cen_dep);
   for (edge i:edg[ccen])
     if (!visit[i.to])
       build(i.to,cen_dep+1,ccen);
}
void add(int id, int d) {
   for(int p=id;p!=-1;p=cen[p].p){
     cen[p].val += dis[cen[p].dep][id]*d;
cen[p].val -= dis[cen[p].dep-1][id]*d;
     cen[p].sz += d;
  }
ll query(int id) {
    ll ret=0;
   int pre_sz=0;
   for(int p=id;p!=-1;p=cen[p].p){
     ret += cen[p].val;
     ret += (cen[p].sz - pre_sz)*dis[cen[p].dep][id];
     pre_sz = cen[p].sz;
  }
   return ret;
}
// edg[u].push_back(edge(v,w));
// edg[v].push_back(edge(u,w));
// memset(visit,0,sizeof(visit));
// build(1,1,-1);
// add(u, d)
// query(u)
```

6.4 Dynamic MST

```
/* Dynamic MST 0( Q lg^2 Q )
                                                                         |int siz[MAX] , son[MAX] , dep[MAX] , ffa[MAX];
|int top[MAX] , idx[MAX] , idpo = 0;
 (qx[i], qy[i])->chg weight of edge No.qx[i] to qy[i]
 delete an edge: (i, \infty)
                                                                         int n , m;
 add an edge: change from \infty to specific value
                                                                         int e[MAX][3];
                                                                         vector<int> v[MAX];
                                                                         struct node{ int big , sml; } st[MAX * 4];
const int SZ=M+3*MXQ;
int a[N],*tz;
                                                                         void init(){
int find(int xx){
                                                                              REP(i , 0 , MAX) v[i].clear();
                                                                             MEM(siz , 0) , MEM(son , 0) , MEM(dep , 0) , MEM(ffa , 0);
MEM(top , 0) , MEM(idx , 0) , idpo = 0;
  int root=xx; while(a[root]) root=a[root];
  int next; while((next=a[xx])){a[xx]=root; xx=next; }
  return root;
                                                                         void DFS1(int now , int fa , int deep){
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }</pre>
                                                                              siz[now] = 1;
int kx[N],ky[N],kt, vd[N],id[M], app[M];
                                                                              dep[now] = deep;
bool extra[M];
                                                                              ffa[now] = fa;
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,int *z,int
                                                                              int big = 0;
REP(i , 0 , v[now].size()){
      m1,long long ans){
  if(Q==1){
                                                                                  int to = v[now][i];
if(to != fa){
    for(int i=1;i<=n;i++) a[i]=0;</pre>
    z[qx[0]]=qy[0]; tz = z
                                                                                      DFS1(to , now , deep + 1);
    for(int i=0;i<m1;i++) id[i]=i;</pre>
                                                                                       siz[now] += siz[to];
    sort(id,id+m1,cmp); int ri,rj;
                                                                                       if(siz[to] > big) big = siz[to] , son[now] = to;
    for(int i=0;i<m1;i++){</pre>
                                                                                  }
      ri=find(x[id[i]]); rj=find(y[id[i]]);
      if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
                                                                         void DFS2(int now , int fa , int root){
    printf("%lld\n",ans);
                                                                              top[now] = root;
    return;
                                                                              idx[now] = ++idpo;
                                                                              if(son[now] != 0) DFS2(son[now] , now , root);
  int ri, rj;
                                                                              REP(i , 0 , v[now].size()){
  //contract
                                                                                  int to = v[now][i];
  kt=0;
                                                                                  if(to != fa && to != son[now]) DFS2(to , now , to);
  for(int i=1;i<=n;i++) a[i]=0;</pre>
                                                                              }
  for(int i=0;i<Q;i++){</pre>
    ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
                                                                         void solveinit(){
                                                                              DFS1(1 , 0 , 0);
DFS2(1 , 0 , 1);
  int tm=0;
  for(int i=0;i<m1;i++) extra[i]=true;</pre>
                                                                              REP(i , 2 , n + 1){
int a = e[i][0] , b = e[i][1] , c = e[i][2];
  for(int i=0;i<Q;i++) extra[ qx[i] ]=false;</pre>
  for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;</pre>
                                                                                  if(dep[a] < dep[b]) swap(a , b);</pre>
  tz=z; sort(id,id+tm,cmp);
                                                                                  update(1 , 1 , n , idx[a] , c);
  for(int i=0;i<tm;i++){</pre>
                                                                             }
    ri=find(x[id[i]]); rj=find(y[id[i]]);
    if(ri!=rj){
                                                                         void query(int a , int b){
      a[ri]=rj; ans += z[id[i]];
                                                                              node ans;
      kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
                                                                              ans.big = -INF, ans.sml = INF;
int t1 = top[a], t2 = top[b];
                                                                              while(t1 != t2){
  for(int i=1;i<=n;i++) a[i]=0;</pre>
                                                                                  if(dep[t1] < dep[t2]) swap(t1 , t2) , swap(a , b);</pre>
  for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);</pre>
                                                                                  ans = pull(ans , query(1 , 1 , n , idx[t1] , idx[a]));
  int n2=0;
                                                                                  a = ffa[t1], t1 = top[a];
  for(int i=1;i<=n;i++) if(a[i]==0)</pre>
  vd[i]=++n2;
                                                                              if(dep[a] > dep[b]) swap(a , b);
  for(int i=1;i<=n;i++) if(a[i])</pre>
                                                                              if(a != b) ans = pull(ans , query(1 , 1 , n , idx[son[a]] ,
  vd[i]=vd[find(i)];
                                                                                    idx[b]));
  int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
                                                                              return cout << ans.sml << " " << ans.big << endl , void();</pre>
  for(int i=0;i<m1;i++) app[i]=-1;</pre>
  for(int i=0;i<Q;i++) if(app[qx[i]]==-1){</pre>
                                                                         init();
    Nx[m2]=vd[\ x[\ qx[i]\ ]\ ];\ Ny[m2]=vd[\ y[\ qx[i]\ ]\ ];\ Nz[m2]=z[
                                                                         REP(i \ , \ 2 \ , \ n \ + \ 1)\{
          qx[i] ];
                                                                              int a , b , c; cin >> a >> b >> c;
    app[qx[i]]=m2; m2++;
                                                                              e[i][0] = a, e[i][1] = b, e[i][2] = c;
                                                                              v[a].pb(b); v[b].pb(a);
  for(int i=0;i<0;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }</pre>
  for(int i=1;i<=n2;i++) a[i]=0;</pre>
                                                                         solveinit();
  for(int i=0;i<tm;i++){</pre>
                                                                         query(a, b);
    ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
    if(ri!=rj){
                                                                         6.6 Block tree
      a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
      Ny[m2]=vd[y[id[i]]]; Nz[m2]=z[id[i]]; m2++;
                                                                         #include <bits/stdc++.h>
    }
                                                                         using namespace std;
 }
  int mid=0/2;
                                                                         const int N = 30006;
  solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
                                                                         const int K = 177; // 2 2 2 2 2 2 2 2 K 2 2
  solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
                                                                         int w[N], sum[N], mx[N];
                                                                         int root[N], sz[N], fa[N], dep[N];
vector<int> G[N], T[N]; // T @ @ @ @ @ @
// root[i] @ @ @ i @ @ @ @ @ @ @
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
  scanf("%d%d",&n,&m);
  for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);</pre>
                                                                         scanf("%d",&Q);
                                                                         void dfs1(int now, int par, int depth) {
   fa[now] = par; dep[now] = depth;
  for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]--; }</pre>
                                                                              if (!root[now]) { // @@@@@@@@@@@@@@
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }
                                                                                  root[now] = now;
                                                                                  sz[now] = 1;
```

for (int i = 0; i < (int)G[now].size(); ++i) {</pre>

6.5 Heavy-Light Decomposition

```
int to = G[now][i];
        if (to == par) continue;
        T[now].push_back(to);
            root[to] = root[now];
            ++sz[ root[now] ];
        dfs1(to, now, depth + 1);
    }
}
void dfs2(int now, int pre_sum, int pre_mx) {
   // ② ② ② ② ② ② ② mx, sum
    sum[now] = pre_sum, mx[now] = pre_mx;
    for (int i = 0; i < (int)T[now].size(); ++i) {</pre>
        int to = T[now][i];
        dfs2(to, pre_sum + w[to], max(pre_mx, w[to]));
}
void change(int pos, int val) {
    w[pos] = val;
    dfs2(root[pos], w[ root[pos]], w[ root[pos]]);
// @ @ @ @ @ @ @ @ @ @ @ @ @ @ @ pos @ @ @ @
}
void qmax(int u, int v) {
    // @ @ @ @ @ @ qsum @ @ @ @ @ @ @ @ @ @ @
void qsum(int u, int v) {
    int ans = 0;
    while (u != v) {
        if (dep[u] < dep[v]) swap(u, v);</pre>
            ans += w[u];
            u = fa[u];
               if (dep[ root[u] ] < dep[ root[v] ]) swap(u, v);</pre>
            ans += sum[u];
            u = fa[ root[u] ];
    ans += w[u]; // 2 2 2 2 2 LCA 2 2 2
    printf("%d\n", ans);
|}
```

7 Graph

7.1 Biconnected Component

```
int low[N], dfn[N];
bool vis[N];
int cnt[N], e[N], x[N], y[N]; // e[i] = x[i] ^ y[i]
int stamp, bcc_no = 0;
vector<int> G[N], bcc[N];
stack<int> sta;
void dfs(int now,int par) {
    vis[now] = true;
     dfn[now] = low[now] = (++stamp);
     for (int i : G[now]) {
         int to = ( e[i] ^ now );
         if (to == par) continue;
         if (!vis[to]) {
             sta.push(i); dfs(to,now);
             low[now] = min(low[now], low[to]);
             if (low[to] >= dfn[now]) {
    ++bcc_no; int p; // p is edge index
                  do {
                      p = sta.top(); sta.pop();
                      bcc[bcc_no].push_back(p);
                 } while (p != i);
             }
         else if (dfn[to] < dfn[now]) {</pre>
             sta.push(i);
             low[now] = min(low[now], dfn[to]);
    }
į }
```

7.2 General Graph Macthing

```
const int N = 100006, E = (2e5) * 2;
 struct Graph{
     //1-index
     int to[E],bro[E],head[N],e;
     int lnk[N],vis[N],stp,n;
     int per[N];
     void init( int _n ){
   //remember to set every array to 0
          stp = 0; e = 1; n = _n;
for( int i = 1 ; i <= n ; i ++ )
              head[i] = lnk[i] = vis[i] = 0, per[i] = i;
          //random_shuffle(per+1, per+n+1);
     void add_edge(int u,int v){
          u=per[u], v=per[v];
          to[e]=v,bro[e]=head[u],head[u]=e++;
          to[e]=u,bro[e]=head[v],head[v]=e++;
     bool dfs(int x){
          vis[x]=stp;
          for(int i=head[x];i;i=bro[i]){
               int v=to[i];
               if(!lnk[v]){
                   lnk[x]=v,lnk[v]=x;
return true;
              }else if(vis[lnk[v]]<stp){</pre>
                   int w=lnk[v];
                   lnk[x]=v, lnk[v]=x, lnk[w]=0;
                   if(dfs(w)){
    return true;
                   lnk[w]=v, lnk[v]=w, lnk[x]=0;
              }
          return false;
     int solve(){
          int ans = 0;
for(int i=1;i<=n;i++)
               if(!lnk[i]){
                   stp++; ans += dfs(i);
          return ans;
} graph;
 7.3 KM
```

```
const int INF = 0x3f3f3f3f;
const int maxn = 610;
int n, w[maxn][maxn], lx[maxn], ly[maxn], slk[maxn];
int s[maxn], t[maxn], good[maxn];
int match(int now) {
    s[now] = 1;
    for (int to = 1; to <= n; to ++) {
         if(t[to]) continue;
         if(lx[now] + ly[to] == w[now][to]) {
             t[to] = 1;
             if(good[to] == 0 || match(good[to]))
                 return good[to] = now, 1;
         else slk[to] = min(slk[to], lx[now] + ly[to] - w[now][
              to]);
    return 0;
void update() {
    int val = INF;
    for (int i = 1; i <= n; i ++)
         if(t[i] == 0) val = min(val, slk[i]);
    for (int i = 1; i <= n; i ++) {
        if(s[i]) lx[i] -= val;
if(t[i]) ly[i] += val;
void run_km() {
    for (int i = 1; i <= n; i ++) {
    lx[i] = w[i][1];</pre>
         for (int j = 1; j \le n; j ++)
             lx[i] = max(lx[i], w[i][j]);
    for (int i = 1; i <= n; i ++)
         ly[i] = 0, good[i] = 0;
```

```
for (int i = 1; i <= n; i ++) {
    for (int j = 1; j <= n; j ++) slk[j] = INF;
    while(1) {
        for (int j = 1; j <= n; j ++)
            s[j] = t[j] = 0;
        if(match(i)) break;
        else update();
    }
}

/* how_to_use:
1. put edge in w[i][j]
2. run_km
3. match: (good[i], i)
*/</pre>
```

7.4 Maximum Weighted Matching(General Graph)

```
struct WeightGraph {
   static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){}
        edge(int ui,int vi,int wi)
            :u(ui),v(vi),w(wi){}
    int n,n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2],slack[N*2],st[N*2],pa[N*2];
    int flo_from[N*2][N+1],S[N*2],vis[N*2];
   vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    void update_slack(int u,int x){
        if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x]))</pre>
             slack[x]=u;
   void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;++u)</pre>
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    void q_push(int x){
        if(x<=n)q.push(x);</pre>
        else for(size_t i=0;i<flo[x].size();i++)</pre>
            q_push(flo[x][i]);
    void set_st(int x,int b){
        st[x]=b;
        if(x>n)for(size_t i=0;i<flo[x].size();++i)</pre>
            set_st(flo[x][i],b);
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].
             begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
   void set_match(int u,int v){
        match[u]=g[u][v].v;
        if(u<=n) return;</pre>
        edge e=g[u][v];
        int xr=flo_from[u][e.u],pr=get_pr(u,xr);
        for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i^1]);</pre>
        set_match(xr,v);
        rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end());
   void augment(int u,int v){
        for(;;){
            int xnv=st[match[u]];
            set_match(u,v);
            if(!xnv)return;
            set_match(xnv,st[pa[xnv]]);
            u=st[pa[xnv]],v=xnv;
        }
    int get_lca(int u,int v){
        static int t=0:
        for(++t;ullv;swap(u,v)){
```

```
if(u==0)continue;
        if(vis[u]==t)return u;
        vis[u]=t;
        u=st[match[u]];
        if(u)u=st[pa[u]];
void add_blossom(int u,int lca,int v){
    int b=n+1;
    while(b<=n_x&&st[b])++b;</pre>
    if(b>n_x)++n_x
    lab[b]=0,S[b]=0;
    match[b]=match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for(int x=u,y;x!=lca;x=st[pa[y]])
        flo[b].push\_back(x), flo[b].push\_back(y=st[match[x
             ]]),q_push(y);
    reverse(flo[b].begin()+1,flo[b].end());
    for(int x=v,y;x!=lca;x=st[pa[y]])
        flo[b].push_back(x),flo[b].push_back(y=st[match[x
             ]]),q_push(y);
    set_st(b,b);
    for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;</pre>
    for(int x=1;x<=n;++x)flo_from[b][x]=0;</pre>
    for(size_t i=0;i<flo[b].size();++i){</pre>
        int xs=flo[b][i];
        for(int x=1;x<=n_x;++x)</pre>
            if(g[b][x].w==0|le_delta(g[xs][x])<e_delta(g[b]
                 ][x]))
                g[b][x]=g[xs][x],g[x][b]=g[x][xs];
        for(int x=1;x<=n;++x)</pre>
            if(flo_from[xs][x])flo_from[b][x]=xs;
    set_slack(b);
void expand_blossom(int b){
    for(size_t i=0;i<flo[b].size();++i)</pre>
        set_st(flo[b][i],flo[b][i]);
    int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
    for(int i=0;i<pr;i+=2){</pre>
        int xs=flo[b][i],xns=flo[b][i+1];
        pa[xs]=g[xns][xs].u;
        S[xs]=1,S[xns]=0;
        slack[xs]=0,set_slack(xns);
        q_push(xns);
    S[xr]=1,pa[xr]=pa[b];
    for(size_t i=pr+1;i<flo[b].size();++i){</pre>
        int xs=flo[b][i];
        S[xs]=-1, set\_slack(xs);
    st[b]=0;
bool on_found_edge(const edge &e){
    int u=st[e.u],v=st[e.v];
    if(S[v]==-1){
        pa[v]=e.u,S[v]=1;
        int nu=st[match[v]];
        slack[v]=slack[nu]=0;
        S[nu]=0,q_push(nu);
    }else if(S[v]==0){
        int lca=get_lca(u,v);
        if(!lca)return augment(u,v),augment(v,u),true;
        else add_blossom(u,lca,v);
    return false;
bool matching(){
    memset(S+1,-1,sizeof(int)*n_x);
    memset(slack+1,0,sizeof(int)*n_x);
    q=queue<int>();
    for(int x=1;x<=n_x;++x)</pre>
        if(st[x]==x\&\&!match[x])pa[x]=0,S[x]=0,q_push(x);
    if(q.empty())return false;
    for(;;){
        while(q.size()){
            int u=q.front();q.pop();
             if(S[st[u]]==1)continue;
            for(int v=1;v<=n;++v)</pre>
                 if(g[u][v].w>0&&st[u]!=st[v]){
                     if(e_delta(g[u][v])==0){
                         if(on_found_edge(g[u][v]))return
                              true;
                     }else update_slack(u,st[v]);
```

```
int d=INF;
             for(int b=n+1;b<=n_x;++b)</pre>
                 if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
             for(int x=1;x<=n_x;++x)</pre>
                 if(st[x]==x\&slack[x]){
                      if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x])
                      else if(S[x]==0)d=min(d,e_delta(g[slack[x
                           ]][x])/2);
             for(int u=1;u<=n;++u){</pre>
                 if(S[st[u]]==0){
                      if(lab[u]<=d)return 0;</pre>
                 lab[u]-=d;
}else if(S[st[u]]==1)lab[u]+=d;
             for(int b=n+1;b<=n_x;++b)</pre>
                 if(st[b]==b){
                      if(S[st[b]]==0)lab[b]+=d*2;
                      else if(S[st[b]]==1)lab[b]-=d*2;
             q=queue<int>();
             for(int x=1;x<=n_x;++x)</pre>
                 if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta
                      (g[slack[x]][x])==0)
                      if(on_found_edge(g[slack[x]][x]))return
                           true;
             for(int b=n+1;b<=n_x;++b)</pre>
                 if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(
                      b);
         }
         return false;
    pair<long long,int> solve(){
        memset(match+1,0,sizeof(int)*n);
        n_x=n;
         int n_matches=0;
        long long tot_weight=0;
         for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();</pre>
         int w_max=0;
         for(int u=1;u<=n;++u)</pre>
             for(int v=1; v<=n; ++v){</pre>
                 flo_from[u][v]=(u==v?u:0);
                 w_{max}=max(w_{max},g[u][v].w);
         for(int u=1;u<=n;++u)lab[u]=w_max;</pre>
        while(matching())++n_matches;
         for(int u=1;u<=n;++u)</pre>
             if(match[u]&&match[u]<u)</pre>
                 tot_weight+=g[u][match[u]].w;
         return make_pair(tot_weight,n_matches);
    void add_edge( int ui , int vi , int wi ){
        g[ui][vi].w = g[vi][ui].w = wi;
    void init( int _n ){
        n = _n;
         for(int u=1;u<=n;++u)</pre>
             for(int v=1;v<=n;++v)</pre>
                 g[u][v]=edge(u,v,0);
} graph;
7.5 Minimum Mean Cycle
/* minimum mean cycle O(VE) */
struct MMC{
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
```

```
double d[V][V];
void init( int _n )
{ n = _n; m = 0; }
// WARNING: TYPE matters
void addEdge( int vi , int ui , double ci )
\{ e[m ++ \bar{]} = \{ vi, ui, ci \}; \}
void bellman_ford() {
     for(int i=0; i<n; i++) d[0][i]=0;
for(int i=0; i<n; i++) {</pre>
          fill(d[i+1], d[i+1]+n, inf);
for(int j=0; j<m; j++) {
               int v = e[j].v, u = e[j].u;
               if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {
                    d[i+1][u] = d[i][v]+e[j].c;
```

```
prv[i+1][u] = v;
                       prve[i+1][u] = j;
                   }
              }
          }
     double solve(){
          // returns inf if no cycle, mmc otherwise
          double mmc=inf;
          int st = -1;
          bellman_ford();
          for(int i=0; i<n; i++) {</pre>
               double avg=-inf;
               for(int k=0; k<n; k++) {</pre>
                   if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i</pre>
                        ])/(n-k));
                   else avg=max(avg,inf);
               if (avg < mmc) tie(mmc, st) = tie(avg, i);</pre>
          FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
for (int i=n; !vst[st]; st=prv[i--][st]) {
              vst[st]++
               edgeID.PB(prve[i][st]);
               rho.PB(st);
          while (vst[st] != 2) {
              int v = rho.back(); rho.pop_back();
              cycle.PB(v);
              vst[v]++;
          reverse(ALL(edgeID));
          edgeID.resize(SZ(cycle));
          return mmc;
} mmc;
```

7.6 Maximum Clique

```
struct BKB{
    static const int MAX_N = 50;
    typedef bitset<MAX_N> bst;
    bst N[MAX_N];
    int n;
    ll wei[MAX_N], ans, cc;
BKB(int _n = 0): n(_n), ans(0), cc(0){
         for(int i = 0; i < _n; ++ i)
             N[i].reset();
    void add_edge(int a, int b) {
         N[a][b] = N[b][a] = 1;
    void set_wei(int a, ll w) {
         wei[a] = w;
    11 CNT(bst P) {
         //if vertices have no weight: return P.count();
         11 rt = 0;
         for(int i = P._Find_first(); i < n; i = P._Find_next(i)</pre>
             rt += wei[i];
         return rt;
    void pro(bst P, ll cnt = 0) {
         if (!P.any()){
             if(cnt == ans)
                  ++ cc;
             else if(cnt > ans) {
                  ans = cnt;
                  cc = 1;
             return:
         ^{\prime\prime} // "<" can be change to "<=" if we don't need to count
         if ( CNT(P) + cnt < ans)
             return:
         int u = P._Find_first();
bst now = P & ~N[u];
         for (int i = now._Find_first(); i < n; i = now.</pre>
              _Find_next(i) ) {
             pro(P & N[i], cnt + wei[i]);
             P[i] = 0;
         return;
    pll solve() {
         bst tmp;
```

8 Math

8.1 Extended Euclidean

```
|// ax + by = gcd(a, b)
|ll exgcd(ll a, ll b, ll &x, ll &y){
| if(a == 0) return x = 0, y = 1, b;
| ll g = exgcd(b % a, a, y, x);
| x -= b / a * y;
| return g;
|}
```

```
8.2 Big Integer
struct Bigint{
    static const int LEN = 60;
    static const int BIGMOD = 10000;
    int vl, v[LEN];
     // vector<int> v;
    Bigint() : s(1) \{ vl = 0; \}
    Bigint(long long a) {
    s = 1; vl = 0;
         if (a < 0) { s = -1; a = -a; }
         while (a) {
              push_back(a % BIGMOD);
a /= BIGMOD;
         }
    Bigint(string str) {
    s = 1; vl = 0;
         int stPos = 0, num = 0;
         if (!str.empty() && str[0] == '-') {
              stPos = 1;
              s = -1;
         for (int i=SZ(str)-1, q=1; i>=stPos; i--) {
  num += (str[i] - '0') * q;
  if ((q *= 10) >= BIGMOD) {
                   push_back(num);
                   num = 0; q = 1;
              }
         if (num) push_back(num);
         n();
    int len() const {
         return vl;//return SZ(v);
    bool empty() const { return len() == 0; }
    void push_back(int x) {
         v[v]++] = x; //v.PB(x);
    void pop_back() {
         vl--; //v.pop_back();
    int back() const {
         return v[vl-1]; //return v.back();
    void n() {
         while (!empty() && !back()) pop_back();
    void resize(int nl) {
         vl = nl; //v.resize(nl);
         fill(v, v+vl, 0); //fill(ALL(v), 0);
    void print() const {
         if (empty()) { putchar('0'); return; }
if (s == -1) putchar('-');
printf("%d", back());
         for (int i=len()-2; i>=0; i--) printf("%.4d",v[i]);
    friend std::ostream& operator << (std::ostream& out, const</pre>
          Bigint &a) {
         if (a.empty()) { out << "0"; return out; }
if (a.s == -1) out << "-";</pre>
         out << a.back();
         for (int i=a.len()-2; i>=0; i--) {
              char str[10];
```

```
snprintf(str, 5, "%.4d", a.v[i]);
          out << str;
    return out;
int cp3(const Bigint &b)const {
    if (s != b.s) return s - b.s;
if (s == -1) return -(-*this).cp3(-b);
     if (len() != b.len()) return len()-b.len();//int
    for (int i=len()-1; i>=0; i--)
   if (v[i]!=b.v[i]) return v[i]-b.v[i];
     return 0;
bool operator<(const Bigint &b)const
{ return cp3(b)<0; }
bool operator <= (const Bigint &b) const
{ return cp3(b)<=0; }
bool operator == (const Bigint &b)const
{ return cp3(b)==0; }
bool operator!=(const Bigint &b)const
{ return cp3(b)!=0; }
bool operator>(const Bigint &b)const
{ return cp3(b)>0; }
bool operator>=(const Bigint &b)const
{ return cp3(b)>=0; }
Bigint operator - () const {
    Bigint r = (*this);
r.s = -r.s;
     return r;
Bigint operator + (const Bigint &b) const {
  if (s == -1) return -(-(*this)+(-b));
  if (b.s == -1) return (*this)-(-b);
     Bigint r;
    int nl = max(len(), b.len());
    r.resize(nl + 1);
for (int i=0; i<nl; i++) {</pre>
          if (i < len()) r.v[i] += v[i];</pre>
          if (i < b.len()) r.v[i] += b.v[i];</pre>
          if(r.v[i] >= BIGMOD) {
              r.v[i+1] += r.v[i] / BIGMOD;
              r.v[i] %= BIGMOD;
    }
    r.n();
    return r;
Bigint operator - (const Bigint &b) const {
    if (s == -1) return -(-(*this)-(-b));
if (b.s == -1) return (*this)+(-b);
     if ((*this) < b) return -(b-(*this));</pre>
    Bigint r;
     r.resize(len());
     for (int i=0; i<len(); i++) {</pre>
         r.v[i] += v[i];
          if (i < b.len()) r.v[i] -= b.v[i];</pre>
          if (r.v[i] < 0) {
              r.v[i] += BIGMOD;
              r.v[i+1]--;
         }
    }
    r.n();
return r;
Bigint operator * (const Bigint &b) {
    Bigint r;
    r.resize(len() + b.len() + 1);
r.s = s * b.s;
    for (int i=0; i<len(); i++) {
    for (int j=0; j<b.len(); j++) {
        r.v[i+j] += v[i] * b.v[j];
    }</pre>
               if(r.v[i+j] >= BIGMOD) {
                   r.v[i+j+1] += r.v[i+j] / BIGMOD;
                   r.v[i+j] %= BIGMOD;
              }
         }
    r.n();
    return r;
Bigint operator / (const Bigint &b) {
    Bigint r:
     r.resize(max(1, len()-b.len()+1));
     int oriS = s;
    Bigint b2 = b; // b2 = abs(b)
     s = b2.s = r.s = 1;
     for (int i=r.len()-1; i>=0; i--) {
```

```
int d=0, u=BIGMOD-1;
while(d<u) {
    int m = (d+u+1)>>1;
    r.v[i] = m;
    if((r*b2) > (*this)) u = m-1;
    else d = m;
}
r.v[i] = d;
}
s = oriS;
r.s = s * b.s;
r.n();
return r;
}
Bigint operator % (const Bigint &b) {
    return (*this)-(*this)/b*b;
}
};
```

8.3 Gaussian Elimination

```
const int GAUSS_MOD = 100000007LL;
struct GAUSS{
     int n;
     vector<vector<int>> v;
     int ppow(int a , int k){
         if(k == 0) return 1;
         if(k % 2 == 0) return ppow(a * a % GAUSS_MOD , k >> 1);
         if(k % 2 == 1) return ppow(a * a % GAUSS_MOD , k >> 1)
* a % GAUSS_MOD;
     vector<int> solve(){
         vector<int> ans(n);
         REP(now , 0 , n){ 
 REP(i , now , n) if(v[now][now] == 0 && v[i][now]
                   != 0)
              swap(v[i] , v[now]); // det = -det;
if(v[now][now] == 0) return ans;
              int inv = ppow(v[now] [now] , GAUSS_MOD - 2);
              REP(i , 0 , n) if(i != now){
                  int tmp = v[i][now] * inv % GAUSS_MOD;
                  REP(j, now, n + 1) (v[i][j] += GAUSS_MOD -
                       tmp * v[now][j] % GAUSS_MOD) %= GAUSS_MOD;
              }
         REP(i, 0, n) \ ans[i] = v[i][n + 1] * ppow(v[i][i],
              GAUSS_MOD - 2) % GAUSS_MOD;
         return ans;
     // gs.v.clear() , gs.v.resize(n , vector<int>(n + 1 , 0));
|} as:
```

8.4 Linear Basis

```
const int MAX_M = 500; //maximum number of variable
typedef bitset<MAX_M+1> bst;
struct linear_basis{
 int m;
 bst mat[MAX_M];
 linear_basis(int _m):m(_m){
    for(int i = 0; i < _m; ++ i) mat[i].reset();</pre>
  // True means "No solution"
 int add_constraint(bst now) {
    for(int j = 0; j < m; ++ j) {
      if(now[j]){
        if(mat[j][j]) now ^= mat[j];
        else{
          mat[j] = now;
          for(int k = j + 1; k < m; ++ k)
            if(mat[j][k])
              mat[j] ^= mat[k];
          for(int k = 0; k < j; ++ k)
            if(mat[k][j])
              mat[k] ^= mat[j];
          return 0;
       }
     }
   }
   return now[m];
 }
  // get one possible solution
 bst get_ans() {
   bst rt; rt.reset();
    for(int i = 0; i < m; ++ i)
      if(mat[i][i] && mat[i][m])
        rt[i] = 1;
```

```
return rt;
}
};
/* usage :
1. Init it with # of variables
2. Adding constraint with format x1,x2...,xm,C
3. get_ans return one possible solution
*/
```

8.5 Build Prime

```
// MAX, eb
void build_prime(int min_fc[], vector<int> &P){
    for(int i = 2; i < MAX; ++ i){
        if(min_fc[i] == 0) min_fc[i] = i , P.eb(i);
        for(auto j : P){
            if(i * j >= MAX) break;
            min_fc[i * j] = j;
            if(i % j == 0) break;
        }
    }
}
```

8.6 Miller Rabin

```
11 mul(l1 a,l1 b,l1 mod) {
  //calculate a*b % mod
  11 r=0; a%=mod; b%=mod;
  while (b) {
     if (b&1) r=(a+r>=mod?a+r-mod:a+r);
     a=(a+a>=mod?a+a-mod:a+a);
     b>>=1;
  return r;
il power(ll a,ll n,ll mod) {
   if (n==0) return 1ll;
  else if (n==1) return a%mod;
  return mul( power(mul(a,a,mod),n/2,mod),n%2?a:1,mod );
const bool PRIME = 1, COMPOSITE = 0;
bool miller_robin(ll n,ll a) {
  if (__gcd(a,n) == n) return PRIME;
if (__gcd(a,n) != 1) return COMPOSITE;
ll d=n-1,r=0,ret;
  while (d%2==0) {
    r++; d/=2;
  ret = power(a,d,n);
   if (ret==1 ||ret==n-1) return PRIME;
  while (r--) {
     ret = mul(ret,ret,n);
     if (ret==n-1) return PRIME;
  return COMPOSITE;
bool isPrime(ll n) {
  //for int: 2,7,61
  ll as[7] = {2,325,9375,28178,450775,9780504,1795265022};
for (int i=0;7>i;i++) {
     if (miller_robin(n,as[i]) == COMPOSITE) return COMPOSITE;
  return PRIME;
}
```

8.7 Pollard Rho

```
|// isPrime (miller rabin)
map<ll, int> cnt
void PollardRho(ll n) {
   if (n == 1) return;
   if (isPrime(n)) return ++cnt[n], void();
   if (n % 2 == 0) return PollardRho(n / 2), ++cnt[2], void();
ll x = 2, y = 2, d = 1, p = 1;
   auto f = [\&](auto x, auto n, int p) { return (mul(x, x, n) +
        p) % n; }
   while (true) {
     if (d != n && d != 1) {
       PollardRho(n / d);
       PollardRho(d);
       return;
     if (d == n) ++p;
     x = f(x, n, p); y = f(f(y, n, p), n, p);
     d = \__gcd(abs(x - y), n);
   }
}
```

8.8 Build Phi and Mu

```
void build_phi(int ax[], int n){
  for(int i = 1; i <= n; ++i)
    ax[i] = i;
  for(int i = 1; i <= n; ++i)
    for(int j = i + i; j <= n; j += i)
    ax[j] -= ax[i];
}
void build_mu(int ax[], int n){
  for(int i = 1; i <= n; ++i)
    ax[i] = 0;
  ax[1] = 1;
  for(int i = 1; i <= n; ++i)
    for(int i = 1; i <= n; ++i)
    ax[j] -= ax[i];
}</pre>
```

8.9 Primitive Root

```
|// build_phi, power, eb
 // M has primitive root when M = 2, 4, p^n, 2p^n
ll Primitive_root(ll n) {
   if(n == 2) return 1;
vector<ll> sol;
   ll val = phi[n];
   for(ll i = 2; i * i <= val; ++ i){
    if(val % i == 0){
         sol.eb(i);
         while(val % i == 0) val /= i;
     }
   if(val != 1) sol.eb(val);
for(ll i = 2; i < n; ++ i){
   if(__gcd(i, n) != 1) continue;</pre>
     if(__gcd(i
ll ok = 1;
      for(auto to : sol){
         if(power(i , phi[n] / to , n) == 1){
           ok = 0:
           break;
        }
      if(ok)
        return i;
    return -1;
| }
```

8.10 Cipolla's Algorithm

```
struct Cipolla {
     ll p, n, a, w;
     Cipolla(ll _p, ll _n) : p(_p), n(_n){
          a = -1;
     il power(ll a, ll x) {
   if(x == 0) return 1;
          return power(a * a % p, x >> 1) * (x & 1 ? a : 1) % p;
     inline int lgd(ll x) {
          return power(x, (p - 1) / 2);
     ll rnd() {
          return ( ((11)rand() << 28) + rand() );</pre>
     pll mul(pll a, pll b) {
    return pll( (a.F * b.F + a.S * b.S % p * w) % p,
                       (a.F * b.S + a.S * b.F) \% p);
     pll power(pll ii, ll x) {
          if(x == 0) return pll(1, 0);
          return mul(power(mul(ii, ii), x >> 1), (x & 1 ? ii :
               pll(1, 0)));
     11 solve() {
          if(p == 2)
return n & 1;
          if(lgd(n) == p - 1)
                                    return -1:
          if(n == 0) return 0;
          while(a = rnd() % p, lgd((a * a - n + p)% p) == 1);
w = (a * a - n + p) % p;
          pll ii = power(pll(a, 1), (p + 1) / 2);
          assert(ii.S == 0);
          return ii.F;
};
```

8.11 Discrete Log

```
// power
 int DiscreteLog_with_s(int s, int x, int y, int m) {
      int kStep = max((int)sqrt(m), 10);
      unordered_map<int, int> p;
      int b = 1:
      for (int i = 0; i < kStep; ++i) {</pre>
          p[y] = i;
y = 1LL * y * x % m;
b = 1LL * b * x % m;
      for (int i = 0; i < m + 10; i += kStep) {
    s = 1LL * s * b % m;
          if (p.find(s) != p.end()) return i + kStep - p[s];
      return -1;
 int DiscreteLog(int x, int y, int m) {
   // x ^ ? === y % m
      if (m == 1) return 0;
   // y %= m;
      int s = 1;
      for (int i = 0; i < 70; ++i) {
          if (s == y) return i;
s = 1LL * s * x % m;
      if (s == y) return 70;
      int p = 70 + DiscreteLog_with_s(s, x, y, m);
      if (power(x, p, m) != y) return -1;
      return p:
1 }
```

8.12 Integer Partition

```
void build_partition(int _dp[], int n, int mod){
      dp[0] = 1;
      for(int i = 1; i \le n; ++ i){
          for(int j = 1; j <= n; ++ j){
int tmp = j * (j * 3 - 1) / 2;
               if(tmp > i) break;
               else if(j % 2 == 1) _{dp[i]} = (_{dp[i]} + _{dp[i} - _{tmp})
                     ]) % mod;
               else if(j % 2 == 0) _{dp[i]} = (_{dp[i]} - _{dp[i - tmp]}
                      + mod) % mod;
          for(int j = 1; j <= n; ++ j){
  int tmp = j * (j * 3 + 1) / 2;</pre>
               if(tmp > i) break;
               else if(j % 2 == 1) _{dp[i]} = (_{dp[i]} + _{dp[i - tmp]}
                     7) % mod;
               else if(j % 2 == 0) _{dp[i]} = (_{dp[i]} - _{dp[i - tmp]}
                      + mod) % mod;
          }
      return;
1}
```

8.13 Meissel-Lehmer Algorithm

```
// count number of prime that is <= n</pre>
int64_t PrimeCount(int64_t n) {
  if (n <= 1) return 0;</pre>
  const int v = sqrt(n);
  vector<int> smalls(v + 1);
   for (int i = 2; i \le v; ++i) smalls[i] = (i + 1) / 2;
  int s = (v + 1) / 2;
  vector<int> roughs(s);
  for (int i = 0; i < s; ++i) roughs[i] = 2 * i + 1;
vector<int64_t> larges(s);
  for (int i = 0; i < s; ++i) larges[i] = (n / (2 * i + 1) + 1) / 2;
  vector<bool> skip(v + 1);
   int pc = 0;
   for (int p = 3; p <= v; ++p) {
     if (smalls[p] > smalls[p - 1]) {
  int q = p * p; pc++;
  if (1LL * q * q > n) break;
       skip[p] = true;
       for (int i = q; i <= v; i += 2 * p) skip[i] = true;</pre>
       int ns = 0;
       for (int k = 0; k < s; ++k) {
          int i = roughs[k];
          if (skip[i]) continue;
          int64_t d = 1LL * i *
          int64_t d = 1LL * i * p;
larges[ns] = larges[k] - (d <= v ? larges[smalls[d] -</pre>
               pc] : smalls[n / d]) + pc;
```

x = i;

mi = INF;

int y = 0;

} }

if(abs(mi) < eps) return 0; // sigma <= 0</pre>

if(arr[i][x] > eps && arr[i][n + m + 1] / arr[i][x] <

mi = arr[i][n + m + 1] / arr[i][x];

// theta

for(int i = 1; i <= m; i ++){

y = i;

```
roughs[ns++] = i:
                                                                               assert(v):
                                                                               double weed = arr[y][x];
                                                                               for(int i = 1; i <= n + m + 1; ++ i)
    arr[y][i] /= weed;</pre>
       s = ns;
       for (int j = v / p; j >= p; --j) {
         int c = smalls[j] - pc;
                                                                               // now arr[y][n + m + 1] == theta
                                                                               for(int i = 0; i <= m; i ++){
   if(i == y) continue;
   double f = arr[i][x];</pre>
         for (int i = j * p, e = min(i + p, v + 1); i < e; ++i)
              smalls[i] -= c;
       }
    }
                                                                                   for(int j = 1; j <= m + n + 1; j ++)

arr[i][j] -= f * arr[y][j];
  for (int k = 1; k < s; ++k) {
     const int64_t m = n / roughs[k];
                                                                               return 1;
     int64_t s = larges[k] - (pc + k - 1);
for (int l = 1; l < k; ++l) {
                                                                          int main(){
       int p = roughs[l];
if (1LL * p * p > m) break;
                                                                               cin >> n;
                                                                               cin >> m;
       s = smalls[m / p] - (pc + l - 1);
                                                                               memset(arr, 0, sizeof arr);
                                                                               // input C
     larges[0] -= s;
                                                                               for(int i = 1 ; i <= n; i++ ){</pre>
  }
                                                                                   cin >> arr[0][i];
  return larges[0];
                                                                                   arr[0][i] = - arr[0][i];
                                                                               for(int i = 1; i <= m; i++){</pre>
                                                                                    // input A
8.14 De Bruijn
                                                                                    for(int j = 1; j <= n; j++)</pre>
// sz_lim, MAX, MAX_len
                                                                                        cin >> arr[i][j];
                                                                                    arr[i][n + i] = 1;
int res[MAX], aux[MAX_len];
void db(int t, int p, int len, int k, int &sz) {
                                                                                   // input b
     if (sz >= sz_lim) return;
                                                                                   cin >> arr[i][n + m + 1];
     if (t > len) {
         if (len % p == 0) {
                                                                               while(pro());
              for (int i = 1; i <= p && sz < sz_lim; ++i) res[sz
                                                                               \texttt{cout} << \texttt{arr[0][n + m + 1]} << "\n";
                                                                               return 0;
                   ++] = aux[i];
     } else {
                                                                                   Middle Speed Linear Recursion
                                                                          8.16
         aux[t] = aux[t - p];
         db(t + 1, p, len, k, sz);
                                                                          #define MAX 100000
         for (int i = aux[t - p] + 1; i < k; ++i) {
                                                                           #define INF 0x3f3f3f3f
              aux[t] = i;
                                                                          #define mod 10000
              db(t + 1, t, len, k, sz);
                                                                          int n , k , x[MAX] , c[MAX];
         }
                                                                           vector<int> mul(vector<int> a , vector<int> b){
    }
                                                                               vector < int > ans(n + n + 1);
                                                                               REP(i , 1 , n + 1) REP(j , 1 , n + 1) 
 ans[i + j] = (ans[i + j] + (a[i] * b[j])) \% mod;
// return cyclic string such that every string of length len
     using k character appears as a substring.
                                                                               RREP(i, n+n, n+1){
int de_bruijn(int k, int len) {
                                                                                   REP(j , 1 , n + 1) ans[i - j] = (ans[i - j] + ans[i] *
     if (k == 1) {
                                                                                         c[j]) % mod;
         res[0] = 0;
                                                                                   ans[i] = 0;
         return 1;
                                                                               return ans;
     for (int i = 0; i < k * len; i++) aux[i] = 0;
                                                                          }
     int sz = 0;
                                                                          vector<int> ppow(vector<int> a , int k){
     db(1, 1, len, k, sz);
return sz; // k^n
                                                                               if(k == 1) return a;
                                                                               if(k % 2 == 0) return
                                                                                                            ppow(mul(a , a) , k >> 1);
į }
                                                                               if(k % 2 == 1) return mul(ppow(mul(a , a) , k >> 1) , a);
8.15 Simplex Algorithm
                                                                          int main(){
                                                                               IOS;
                                                                               while(cin >> n && n){
maximize Cx under
                                                                                   REP(i , 1 , n + 1) cin >> x[i];
REP(i , 1 , n + 1) cin >> c[i];
Ax <=b
x >= 0
                                                                                   vector < int > v(n + n + 1);
                                                                                   v[1] = 1;
n variables
                                                                                   cin >> k , k
m constraints
                                                                                    v = ppow(v, k);
A is m by n */
                                                                                   int ans = 0;
                                                                                   REP(i , 1 , n + 1) ans = (ans + x[i] * v[i]) % mod; cout << ans << endl;
const int MAX = 45;
int n, m;
double arr[MAX][MAX];
                                                                               return 0;
bool pro(){
                                                                          }
     double mi = 0;
     int x = 1;
                                                                           8.17 Chinese Remainder Theorem
     for(int i = 1; i <= n + m; i ++)</pre>
                                            if(arr[0][i] < mi){</pre>
         mi = arr[0][i];
```

```
const int INF = 0x3f3f3f3f
void extgcd(ll a , ll b , ll &d , ll &x , ll &y){
    if(b == 0) d = a, x = 1, y = 0;
    else extgcd(b , a % b , d , y , x) , y -= (a / b) * x;
ĺl n;
vectór<ll> v , m;
int main(){
    while(cin >> n){
        v.clear() , m.clear();
ll ans , mod , d , x , y;
        REP(i , \emptyset , n) cin >> mod >> ans , m.pb(mod) , v.pb(ans
```

```
mod = m[0] , ans = v[0];
REP(i , 1 , n){
    ll res = ((v[i] - ans) % m[i] + m[i]) % m[i];
    extgcd(mod , m[i] , d , x , y);
    if(res % d != 0){ ans = -1; break; }

    res = (res / d * x % m[i] + m[i]) % m[i];
    ans = ans + res * mod;
    mod = mod * m[i] / d;
}
if(ans == -1) cout << ans << endl;
else cout << ans % mod << endl;
}
return 0;
}</pre>
```

9 Convolution

9.1 FFT

```
#include <bits/stdc++.h>
using namespace std;
const int MAXN = 2*262144;
typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0,1);
cplx omega[MAXN+1];
void pre_fft() {
 for (int i=0;i<=MAXN;i++) {</pre>
    omega[i] = exp(i*2*PI/MAXN*I);
 }
void fft(int n,cplx a[],bool inv=false) {
  int basic=MAXN/n;
  int theta=basic;
  for (int m=n;m>=2;m>>=1) {
    int mh=m>>1;
    for (int i=0;i<mh;i++) {</pre>
      cplx w=omega[inv?MAXN-(i*theta%MAXN):i*theta%MAXN];
      for (int j=i;j<n;j+=m) {</pre>
        int k=j+mh;
        cplx x=a[j]-a[k];
        a[j] += a[k];
        a[k] = w*x;
    theta = (theta*2)%MAXN;
 }
  int i=0;
  for (int j=1;j<n-1;j++) {</pre>
    for (int k=n>>1;k>(i^=k);k>>=1);
    if (j<i) swap(a[i],a[j]);</pre>
  if (inv) {
    for (int i=0;i<n;i++) a[i]/=n;</pre>
cplx a[MAXN],b[MAXN],c[MAXN];
//how to use
pre_fft();
fft(n,a);
fft(n,b);
for (int i=0;n>i;i++) {
 c[i] = a[i]*b[i];
fft(n,c,1);
*/
```

9.2 NTT

```
// Remember coefficient are mod P
/*
|(mod, root)
|(65537, 3)
|(23068673, 3)
|(998244353, 3)
|(1107296257, 10)
|(2013265921, 31)
|(2885681153, 3)
*/
|typedef long long ll;
|const int maxn = 65536;
||struct NTT{
```

```
11 \mod = 2013265921, root = 31;
      ll\ omega[maxn + 1];
      void prentt() {
          ll x=fpow(root, (mod - 1) / maxn);
          omega[0] = 1;
          for (int i=1;i<=maxn;++i) {</pre>
               omega[i] = omega[i - 1] * \times % mod;
      void real_init(ll _mod, ll _root) {
          mod = _mod;
root = _root;
          prentt();
      ll fpow(ll a, ll n) {
          (n += mod-1) \%= mod - 1;
          ll r = 1;
for (; n; n>>=1) {
    if (n&1) (r*=a)%=mod;
               (a*=a)%=mod;
          return r;
      void bitrev(vector<ll> &v, int n) {
          int z = __builtin_ctz(n)-1;
          for (int i=0;i<n;++i) {</pre>
               int x=0;
               for (int j=0; j<=z;++j) x ^= ((i>>j&1) << (z-j));
               if (x>i) swap(v[x], v[i]);
      void ntt(vector<ll> &v, int n) {
          bitrev(v, n);
          for (int s = 2; s <= n; s <<= 1) {
               int z = s \gg 1;
               for (int i = 0; i < n; i += s) {
                    for (int k = 0; k < z; ++k) {
                        ll x = v[i + k + z] * omega[maxn / s * k] %
                              mod;
                        v[i + k + z] = (v[i + k] + mod - x) mod;
                        (v[i+k] += x) \% = mod;
                   }
              }
          }
      void intt(vector<ll> &v, int n) {
          ntt(v, n);
          reverse(v.begin() + 1, v.end());
          ll inv = fpow(n, mod - 2);
for (int i = 0; i < n; ++i) {</pre>
               (v[i] *= inv) %= mod;
      vector<ll> conv(vector<ll> a, vector<ll> b) {
          int sz=1;
          while (sz < a.size() + b.size() - 1) sz <<= 1;</pre>
          vector<ll> c(sz);
          while (a.size() < sz) a.push_back(0);</pre>
          while (b.size() < sz) b.push_back(0);</pre>
          ntt(a, sz), ntt(b, sz);
          for (int i=0;i<sz;++i) c[i] = (a[i] * b[i]) % mod;</pre>
          intt(c, sz);
          while (c.size() && c.back() == 0) c.pop_back();
};
ll chinese(ll b1, ll m1, ll b2, ll m2) {
     ll a1 = bigpow(m2, m1 - 2, m1) * b1 % m1;
ll a2 = bigpow(m1, m2 - 2, m2) * b2 % m2;
ll ret= (a1 * m2 + a2 * m1) % (m1 * m2);
      assert(ret % m1 == b1 && ret % m2 == b2);
      return ret;
}
```

9.3 FWT

```
National Taiwan University LYB
                   a[j + d] = y - x; //IFWT OR
| }
 9.4 Subset Convolution
 for(int i = 0; i <= n; ++ i) {</pre>
   // f[\_builtin\_popcount(s)][s] = s, otherwise = 0. So is g[i]
   FWT(f[i], n) // OR
   FWT(g[i], n) // OR
   for(int s = 0; s < (1 << n); ++ s)
for(int j = 0; j <= i; ++ j)
h[i][s] += f[j][s] * g[i - j][s]
   IFWT(h[i], n) // OR
   for(int s = 0; i < (1 << n); ++ s)
     h[__builtin_popcount(s)][s] // is the real answer
 9.5 Ternary Xor
 pii operator*(const pii &p1, const pii &p2) {
     return {subb(mull(p1.F, p2.F) - mull(p1.S, p2.S)),
              subb(addd(mull(p1.F, p2.S) + mull(p1.S, p2.F)) -
                    mull(p1.S, p2.S))};
 pii cal1(pii p) {
     return {subb(-p.S), subb(p.F - p.S)};
 pii cal2(pii p) {
     return {subb(p.S - p.F), subb(-p.F)};
 //C is the size of a
 void DFT(vector<pii> &a) {
     for (int mid = 1; mid < C; mid *= 3) {</pre>
          for (int j = 0; j < C; j += mid * 3) {
              for (int k = 0; k < mid; ++k) {
   pii x = a[j + k], y = a[j + k + mid], z = a[j +
                          k + (mid \ll 1);
                   a[j + k] = x + y + z;

a[j + k + mid] = x + cal1(y) + cal2(z);
                   a[j + k + (mid << 1)] = x + cal2(y) + cal1(z);
              }
          }
     }
 const int invn = ppow(C, mod - 2);
 void IDFT(vector<pii> &a) {
      for (int mid = 1; mid < C; mid *= 3) {</pre>
          for (int j = 0; j < C; j += mid * 3) {
    for (int k = 0; k < mid; ++k) {
        pii x = a[j + k], y = a[j + k + mid],
                        z = a[j + k + (mid << 1)];
                   a[j + k] = x + y + z;

a[j + k + mid] = x + cal2(y) + cal1(z);
                   a[j + k + (mid << 1)] = x + cal1(y) + cal2(z);
              }
          }
     for (int i = 0; i < C; ++i) {
          a[i].F = mull(a[i].F, invn);
 void ff(vector<pii> &a, vector<pii> b) {
     DFT(a); DFT(b);
for (int i = 0; i < C; ++i) {</pre>
          a[i] = a[i] * b[i];
     IDFT(a);
 10
         String
 10.1 KMP
 const KMP_SIZE = ;
 struct KMP{
     string s;
      int f[KMP_SIZE] , pos;
     void solve(){
          f[0] = pos = -1;
          REP(i , 1 , s.size()){
```

while(pos != -1 && s[pos + 1] != s[i]) pos = f[pos

if(s[pos + 1] == s[i]) pos ++;

f[i] = pos;

}

```
};
 10.2 Z value
 const int ZVALUE_SIZE = ;
struct Z_VALUE{
     string s;
     int l = 0,
                 r = 0, z[ZVALUE\_SIZE];
     void solve(){
         REP(i , 0 , s.size()){
   z[i] = max(min(z[i - l] , r - i) , OLL);
              while(i + z[i] < s.size() && s[z[i]] == s[i + z[i]
                  l = i , r = i + z[i];
                  z[i] ++;
             }
         }
};
          Longest Palindrome
 10.3
const int PALINDROME_MAX = 2 * ???;
 struct Palindrome{
     string s , ss; // ss = input
     int z[PALINDROME_MAX];
     void solve(){
         s.resize(ss.size() + ss.size() + 1 , '.')
         REP(i , 0 , ss.size()) s[i + i + 1] = ss[i];
int l = 0 , r = 0;
         REP(i , 0 , s.size()){}
             z[i] = max(min(z[l + l - i] , r - i) , 1);
while(i - z[i] >= 0 && i + z[i] < s.size() && s[i -
                    z[i]] == s[i + z[i]]){
                  l = i , r = i + z[i];
                  z[i] ++;
             }
         }
 10.4 Aho-Corasick Algorithm
struct AC_Automata {
     static const int N = 2e4 + 6;
     static const int SIGMA = 26;
     int ch[N][SIGMA], val[N], sz;
     int last[N],fail(N);
     int que[N],qs,qe, cnt[N];
     void init() {
         sz = 1;
         memset(ch[0],0,sizeof(ch[0]));
         qs = qe = 0;
         memset(cnt,0,sizeof(cnt)); memset(val,0,sizeof(val));
              memset(last,0,sizeof(last));
     int idx(char c) {
    return c-'a';
     int insert(string s,int v) {
         int now=0;
         int n=s.size();
         for (int i = 0; i < n; ++i) {
              int c=idx(s[i]);
              if (!ch[now][c]) {
                  memset(ch[sz],0,sizeof(ch[sz]));
                  val[sz] = 0; ch[now][c] = sz++;
             now = ch[now][c];
         val[now] = v;
         return now;
     void print(int j) {
         if (j) {
              //now we match string v[j]
              print(last[j]); //may match multiple strings
     void getFail() {
    qs=0,qe=0; fail[0]=0;
         for (int c = 0; c < SIGMA; c++) {
              int now=ch[0][c];
              if (now) {
```

fail[now] = 0;

que[qe++] = now;

```
last[now] = 0;
                                                                                        m = p + 1;
              }
                                                                                   }
                                                                              }
                                                                              void getlcp(){
         while (qs != qe) {
                                                                                    int tmp = 0 , n = s.size();
              int t=que[qs++];
                                                                                   REP(i , 0 , n) rk[sa[i]] = i;
              for (int c = 0; c < SIGMA; c++) {
   int now=ch[t][c];</pre>
                                                                                   REP(i, 0, n){
                                                                                        if(rk[i] == 0) lcp[0] = 0;
                   if (!now) continue;
                   que[qe++] = now;
                   int v=fail[t];
                                                                                             if(tmp) tmp --;
                   while (v && !ch[v][c]) v=fail[v];
                                                                                             int po = sa[rk[i] - 1];
                                                                                             while(tmp + po < n && tmp + i < n && s[tmp + i] ==
                   fail[now] = ch[v][c];
last[now] = val[ fail[now] ]? fail[now]:last[
                                                                                                   s[tmp + po]) tmp ++;
                         fail[now] ];
                                                                                             lcp[rk[i]] = tmp;
              }
                                                                                        }
         }
                                                                                   }
    }
    void AC_evolution() {
                                                                               void getsp(){
         for (qs=0;qs!=qe;) {
                                                                                    int n = s.size();
                                                                                   REP(i , 0 , n) sp[rk[i]][0] = s.size() - i;
REP(i , 1 , n) sp[i - 1][1] = lcp[i];
              int now=que[qs++];
              for (int i=0; SIGMA>i; i++) {
                   if (ch[now][i] == 0) ch[now][i] = ch[fail[now][i]
                                                                                    REP(i
                                                                                            2 , logn){
                                                                                        }
    }
    void build() {
                                                                                        }
         getFail();
                                                                                   }
         AC_evolution();
                                                                               int Query(int L , int R){
    void Find(string s) {
                                                                                    int tmp = (L == R) ? 0 : 32 - __builtin_clz(R - L);
                                                                                    if(tmp == 0) return sp[L][0];
         int n=s.size(), now=0;
         for (int i=0;n>i;i++) {
                                                                                    else return min(sp[L][tmp], sp[R - (1 << (tmp - 1))][tmp])
              int c=idx(s[i]);
              while (now && !ch[now][c]) now = fail[now];
              now = ch[now][c];
                                                                               int Find(string ss){
                                                                                   int L = 0, R = s.size(), now;
              cnt[now]++;
                                                                                   while(R - L > 1){
                                                                                        now = (L + R) / 2;
         for (int i=qe-1;i>=0;i--) {
                                                                                        if(s[sa[now]] == ss[0]) break;
else if(s[sa[now]] > ss[0]) R = now;
else if(s[sa[now]] < ss[0]) L = now;</pre>
              cnt[ fail[que[i]] ] += cnt[ que[i] ];
    }
} ac;
                                                                                    if(s[sa[now]] != ss[0]) return 0;
const int N = 156;
                                                                                    REP(i , 1 , ss.size()){
                                                                                        int pre = now , ty = 0;
if(sa[now] + i >= s.size()) L = now , ty = 0;
string s[N];
int ed[N];
                                                                                        else if(s[sa[now] + i] == ss[i]) continue;
else if(s[sa[now] + i] > ss[i]) R = now , ty = 1;
ac.init();
ac.insert(s[i],i); // insert small strings
                                                                                        else if(s[sa[now] + i] < ss[i]) L = now , ty = 0;
ac.build();
ac.Find(large_string);
                                                                                        while(R - L > 1){
ac.cnt[ ac.insert(s[i],i) ]; // number of small string
                                                                                             now = (L + R) / 2;
                                                                                             if(sa[now] + i >= s.size()){
                                                                                                  if(ty == 0) R = now;
10.5 Suffix Array
                                                                                                  if(ty == 1) L = now;
const int SA_SIZE = ;
const int logn = 1 + ;
                                                                                             else if(ty == 0 \& Query(pre , now) < i) R = now;
                                                                                             else if(ty == 1 && Query(now , pre) < i) L = now;</pre>
string s;
                                                                                             else if(s[sa[now] + i] == ss[i]) break;
else if(s[sa[now] + i] > ss[i]) R = now;
int sa[SA_SIZE] , rk[SA_SIZE] , lcp[SA_SIZE];
int tma[2][SA_SIZE] , c[SA_SIZE] , sp[SA_SIZE][logn];
                                                                                             else if(s[sa[now] + i] < ss[i]) L = now;
int getsa(){
     -> update m = ? // how many char
int *x = tma[0] , *y = tma[1] , n = s.size() , m = 200;
                                                                                        if(sa[now] + i >= s.size()) return 0;
    REP(i , 0 , m) c[i] = 0;

REP(i , 0 , n) c[x[i] = s[i]] ++;

REP(i , 1 , m) c[i] += c[i - 1];
                                                                                        if(s[sa[now] + i] != ss[i]) return 0;
                                                                                   L = now , R = now;
RREP(i , 19 , 0){
   if(R + (1 << i) >= s.size()) continue;
    RREP(i , n - 1 , 0) sa[--c[x[i]]] = i;
for(int k = 1 ; k <= n ; k <<= 1){
                                                                                        else if(Query(L , R + (1 \ll i)) >= ss.size()) R += (1
         REP(i , 0 , m) c[i] = 0;

REP(i , 0 , n) c[x[i]] ++;

REP(i , 1 , m) c[i] += c[i - 1];
                                                                                   RREP(i , 19 , 0){
    if(L - (1 << i) < 0) continue;
         int p = 0;
         REP(i, n-k, n) y[p++] = i;
                                                                                        else if(Query(L - (1 \ll i) , R) >= ss.size()) L -= (1
         REP(i , 0 , n) if(sa[i] >= k) y[p ++] = sa[i] - k;

RREP(i , n - 1 , 0) sa[--c[x[y[i]]]] = y[i];

y[sa[0]] = p = 0;
                                                                                              << i):
                                                                                    return R - L + 1;
         REP(i , 1 , n) {
              if( x[sa[i]] == x[sa[i - 1]] && sa[i] + k < n && sa
                   [i - 1] + k < n \&\&
                                                                              how to use :
                                                                              1. cin >> s;
                   x[sa[i] + k] == x[sa[i - 1] + k]);
                                                                              2. getsa() , getlcp() , getsp();
              else \bar{p} ++;
              y[sa[i]] = p;
                                                                              3. strina ss:
                                                                              4. cin >> ss;
                                                                              5. cout << Find(ss) << endl;
*/</pre>
         swap(x , y);
         if(p + 1 == n) break;
```

10.6 Palindromic Tree

```
//MAXN
 const int N = 26;
 struct Palindromic_Tree {
   int next[MAXN][N];//trie tree edge
   int len[MAXN];//tree edge depth*2 (-1)
   int fail[MAXN];//fail link
   int num[MAXN];//fail link depth
   int cnt[MAXN];//# of this Palindrom
   int S[MAXN];//string
   int p;//# of different Palindrom + 2
   int n;//string len
   int last;
   int newnode(int 1) {
     memset(next[p], 0, N * 4);
     cnt[p] = num[p] = 0;
len[p] = 1;
     return p ++;
   void init() {
     p = n = 0;
     last = 1;
     newnode (0);
     newnode (-1);
     S[n] = -1;
     fail[0] = 1;
   int get_fail(int x){
     while (S[n - len[x] - 1] != S[n]) x = fail[x];
     return x;
  void add(int c) {
    c -= 'a';
     S[++ n] = c;
     int cur = get_fail ( last );
     if ( !next[cur][c] ){
       int now = newnode ( len[cur] + 2 );
       fail[now] = next[get_fail ( fail[cur] )][c];
       next[cur][c] = now;
       num[now] = num[fail[now]] + 1;
     last = next[cur][c];
     cnt[last] ++;
   void count () {
     for (int i = p - 1; i >= 0; -- i) cnt[fail[i]] += cnt[i];
|};
```

10.7 Lexicographically Smallest Rotation

```
string s;
 const int N = 4000006;
 int f[N];
 void solve() {
     s = s + s;
int n = (int)s.size();
     for (int i=0;i<n;++i) f[i] = -1;</pre>
     int k=0;
     for (int j=1;j<n;++j) {</pre>
          char sj = s[j];
int i = f[j-k-1];
while (i != -1 && sj != s[k+i+1]) {
               if (sj < s[k+i+1]) {
                    k = j-i-1;
               i = f[i];
          if (sj != s[k+i+1]) {
               if (sj < s[k]) {
                    k = j;
               f[j-k] = -1;
          else f[j-k] = i+1;
     n>>=1;
     if (k >= n) k-= n;
     for (int i=k;i<k+n;++i) {</pre>
          cout << s[i];</pre>
      cout << endl;
| }
```

```
11
        Geometry
         Circle
11.1
//Note that this code will crash if circle A and B are the same
 typedef pair<double, double> pdd;
pdd rtcw(pdd p){return pdd(p.Y, -p.X); }
vector<pdd> circlesintersect(pdd A, pdd B, double r1, double r2
     vector<pdd> ret;
     double d = dis(A, B);
     if(d > r1 + r2 | | d + min(r1, r2) < max(r1, r2))
     return ret;
double x = (d * d + r1 * r1 - r2 * r2) / (2 * d);
     double y = sqrt(r1 * r1 - x * x);
    pdd v = (B - A) / d;
     ret.eb(A + v * x + rtcw(v) * y);
     if(y > 0)
         ret.eb(A + v * x - rtcw(v) * y);
     return ret;
| }
         Half Plane Intersection
Pt interPnt( Line 11, Line 12, bool &res ){
    tie(p1, p2) = l1; tie(q1, q2) = l2;

double f1 = (p2 - p1) ^ (q1 - p1);

double f2 = (p2 - p1) ^ (p1 - q2);
     double f = (f1 + f2);
     if( fabs(f) < eps){ res=0; return {0, 0}; }</pre>
     res = true
     return q1 * (f2 / f) + q2 * (f1 / f);
bool isin( Line 10, Line 11, Line 12 ){
     // Check inter(l1, l2) in l0
    bool res; Pt p = interPnt(l1, l2, res);
     return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}
/* If no solution, check: 1. ret.size() < 3</pre>
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^{\wedge} (p - l.F) > 0
 */
/* --^-- Line.FI --^-- Line.SE --^-- */
vector<Line> halfPlaneInter( vector<Line> lines ){
     int sz = lines.size();
     vector<double> ata(sz), ord(sz);
     for( int i=0; i<sz; i++) {</pre>
         ord[i] = i;
         Pt d = lines[i].SE - lines[i].FI;
         ata[i] = atan2(d.Y, d.X);
     sort( ord.begin(), ord.end(), [&](int i, int j) {
             return ata[i] < ata[j];</pre>
             });
     vector<Line> fin;
     for (int i=0; i<sz; i++)
    if (!i or fabs(ata[ord[i]]] - ata[ord[i-1]]) > eps)
             fin.PB(lines[ord[i]]);
     deque<Line> dq;
     for (int i=0; i<(int)(fin.size()); i++) {</pre>
         while((int)(dq.size()) >= 2 and
                 not isin(fin[i], dq[(int)(dq.size())-2],
                      dq[(int)(dq.size())-1]))
             dq.pop_back();
         while((int)(dq.size()) >= 2 and
                 not isin(fin[i], dq[0], dq[1]))
             dq.pop_front()
```

11.3 Convex Hull 3D

dq.push_back(fin[i])

while((int)(dq.size()) >= 3 and

while((int)(dq.size()) >= 3 and

vector<Line> res(dq.begin(),dq.end());

not isin(dq[0], dq[(int)(dq.size())-2],

not isin(dq[(int)(dq.size())-1], dq[0], dq[1]))

dq[(int)(dq.size())-1]))

```
| #define SIZE(X) (int(X.size()))
```

dq.pop_back();

dq.pop_front()

return res:

}

#define PI 3.14159265358979323846264338327950288 struct Pt{ Pt cross(const Pt &p) const { return Pt(y * p.z - z * p.y, z * p.x - x * p.z, x * p.y $y * p.x); }$ } info[N]; int mark[N][N],n, cnt;; double mix(const Pt &a, const Pt &b, const Pt &c) { return a * (b ^ c); } double area(int a, int b, int c) { return norm((info[b] - info[a]) ^ (info[c] - info[a])); } double volume(int a, int b, int c, int d) { return nix(info[b] - info[a], info[c] - info[a], info[d] info[a]); } struct Face{ int a, b, c; Face(){} Face(int a, int b, int c): a(a), b(b), c(c) {} int &operator [](int k) { if (k == 0) return a; if (k == 1) return b; return c; } vector<Face> face; void insert(int a, int b, int c) { face.push_back(Face(a, b, c)); } void add(int v) { vector <Face> tmp; int a, b, c; cnt++; for (int i = 0; i < SIZE(face); i++) { a = face[i][0]; b = face[i][1]; c = face[i][2]; }</pre> if(Sign(volume(v, a, b, c)) < 0) mark[a][b] = mark[b][a] = mark[b][c] = mark[c][b] =</pre> mark[c][a] = mark[a][c] = cnt; else tmp.push_back(face[i]); } face = tmp; for (int i = 0; i < SIZE(tmp); i++) { a = face[i][0]; b = face[i][1]; c = face[i][2]; if (mark[a][b] == cnt) insert(b, a, v); if (mark[b][c] == cnt) insert(c, b, v); if (mark[c][a] == cnt) insert(a, c, v); int Find(){ for (int i = 2; i < n; i++) { Pt ndir = (info[0] - info[i]) ^ (info[1] - info[i]); if (ndir == Pt()) continue; swap(info[i], info[2]); for (int j = i + 1; j < n; j++) if (Sign(volume(0, 1, 2, j)) != 0) { swap(info[j], info[3]); insert(0, 1, 2); insert(0, 2, 1); return 1; } } return 0; } int main() { for (; scanf("%d", &n) == 1;) { for (int i = 0; i < n; i++) info[i].Input();</pre> sort(info, info + n); n = unique(info, info + n) - infoface.clear(); random_shuffle(info, info + n); if (Find()) { memset(mark, 0, sizeof(mark)); cnt = 0; for (int i = 3; i < n; i++) add(i); vector<Pt> Ndir for (int i = 0; i < SIZE(face); ++i) { Pt p = (info[face[i][0]] - info[face[i][1]]) ^</pre> (info[face[i][2]] - info[face[i][1]]); p = p / norm(p); Ndir.push_back(p); } sort(Ndir.begin(), Ndir.end()); int ans = unique(Ndir.begin(), Ndir.end()) - Ndir. begin(); printf("%d\n", ans); } else printf("1\n"); } } double calcDist(const Pt &p, int a, int b, int c) { return fabs(mix(info[a] - p, info[b] - p, info[c] - p) / area (a, b, c)); } //compute the minimal distance of center of any faces double findDist() { //compute center of mass double totalWeight = 0; Pt center(.0, .0, .0); Pt first = info[face[0][0]]; for (int i = 0; i < SIZE(face); ++i) { Pt p = (info[face[i][0]]+info[face[i][1]]+info[face[i][2]]+first)*.25; double weight = mix(info[face[i][0]] - first, info[face [i][1]] - first, info[face[i][2]] - first); totalWeight += weight; center = center + p * weight; } center = center / totalWeight; double res = 1e100; //compute distance for (int i = 0; i < SIZE(face); ++i) res = min(res, calcDist(center, face[i][0], face[i][1],</pre> face[i][2])); return res; }

11.4 Dynamic convexhull

```
/* Given a convexhull, answer querys in O(\lg N)
     CH should not contain identical points, the area should
     be > 0, min pair(x, y) should be listed first */
 double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
  struct Conv{
       int n:
       vector<Pt> a;
       vector<Pt> upper, lower;
       Conv(vector < Pt > \_a) : a(\_a){}
           n = a.size();
            int ptr = 0;
            for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;</pre>
           for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
for(int i=ptr; i<n; ++i) upper.push_back(a[i]);</pre>
           upper.push_back(a[0]);
       int sign( LL x ){ // fixed when changed to double
  return x < 0 ? -1 : x > 0; }
       pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
            int l = 0, r = (int)conv.size() - 2;
            for(; l + 1 < r; ){
                int mid = (1 + r) / 2;
                 if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
                 else 1 = mid:
            return max(make_pair(det(vec, conv[r]), r),
                     make_pair(det(vec, conv[0]), 0));
      void upd_tang(const Pt &p, int id, int &i0, int &i1){
   if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
   if(det(a[i1] - p, a[id] - p) < 0) i1 = id;</pre>
       void bi_search(int l, int r, Pt p, int &i0, int &i1){
            if(l == r) return;
           upd_tang(p, l % n, i0, i1);
int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
           for(; l + 1 < r; ) {
  int mid = (l + r) / 2;
                 int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
                 if (smid == sl) l = mid;
                 else r = mid;
           upd_tang(p, r % n, i0, i1);
       int bi_search(Pt u, Pt v, int l, int r) {
   int sl = sign(det(v - u, a[l % n] - u));
           for(; l + 1 < r; ) {
int mid = (l + r) / 2;
                 int smid = sign(det(v - u, a[mid % n] - u));
                 if (smid == sl) l = mid;
                 else r = mid;
            return 1 % n;
       // 1. whether a given point is inside the CH
      bool contain(Pt p) {
            if (p.X < lower[0].X || p.X > lower.back().X) return 0;
            int id = lower_bound(lower.begin(), lower.end(), Pt(p.X
                  , -INF)) - lower.begin();
           if (lower[id].X == p.X) {
   if (lower[id].Y > p.Y) return 0;

            }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;</pre>
            id = lower_bound(upper.begin(), upper.end(), Pt(p.X,
                 INF), greater<Pt>()) - upper.begin();
           if (upper[id].X == p.X) {
   if (upper[id].Y < p.Y) return 0;</pre>
           }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;</pre>
            return 1;
       ^{\prime}// 2. Find 2 tang pts on CH of a given outside point
      // return true with i0, i1 as index of tangent points
       // return false if inside CH
      bool get_tang(Pt p, int &i0, int &i1) {
   if (contain(p)) return false;
            i0 = i1 = 0;
            int id = lower_bound(lower.begin(), lower.end(), p) -
                 lower.begin();
           bi_search(0, id, p, i0, i1);
bi_search(id, (int)lower.size(), p, i0, i1);
            id = lower_bound(upper.begin(), upper.end(), p, greater
           <Pt>()) - upper.begin();
bi_search((int)lower.size() - 1, (int)lower.size() - 1
                  + id, p, i0, i1);
```

```
bi_search((int)lower.size() - 1 + id, (int)lower.size()
                 - 1 + (int)upper.size(), p, i0, i1);
          return true:
     // 3. Find tangent points of a given vector
      // ret the idx of vertex has max cross value with vec
     int get_tang(Pt vec){
          pair<LL, int> ret = get_tang(upper, vec);
ret.second = (ret.second+(int)lower.size()-1)%n;
          ret = max(ret, get_tang(lower, vec));
          return ret.second;
     // 4. Find intersection point of a given line
     // return 1 and intersection is on edge (i, next(i))
     // return 0 if no strictly intersection
     bool get_intersection(Pt u, Pt v, int &i0, int &i1){
          int p0 = get_tang(u - v), p1 = get_tang(v - u);
if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){</pre>
               if (p0 > p1) swap(p0, p1);
              i0 = bi_search(u, v, p0, p1);
              i1 = bi_search(u, v, p1, p0 + n);
              return 1;
          return 0:
     }
|};
```

11.5 Polar Angle Sort

```
| #define is_neg(_k) (_k.Y < 0 || (_k.Y == 0 && _k.X < 0) )
| bool cmp(pll a,pll b){
| int A = is_neg(a), B = is_neg(b);
| return (A == B ? (a ^ b) > 0 : A < B);
| }</pre>
```

11.6 Circle and Polygon intersection

```
struct Circle_and_Segment_Intersection {
   const ld eps = 1e-9;
     vector<pdd> solve(pdd p1, pdd p2, pdd cen, ld r) {
           //please notice that p1 != p2
           //condiser p = p2 + (p1 - p2) * t, 0 <= t <= 1
           vector<pdd> ret;
           vector<paa> ret;

p1 = p1 - cen; p2 = p2 - cen;

ld a = (p1 - p2) * (p1 - p2);

ld b = 2 * (p2 * (p1 - p2));

ld c = p2 * p2 - r * r;

ld bb4ac = b * b - 4 * a * c;
           if (bb4ac < -eps) return ret; //no intersection</pre>
           vector<ld> ts;
           if ( (bb4ac) <= eps) {
                 ts.push_back(-b / 2 / a);
           }
           else {
                ts.push_back( (-b + sqrt(bb4ac)) / (a * 2) );
ts.push_back( (-b - sqrt(bb4ac)) / (a * 2) );
           sort(ts.begin(), ts.end());
for (ld t: ts) {
                 if (-eps <= t && t <= 1 + eps) {</pre>
                       t = max(t, 0.0);
                      t = min(t, 1.0);
                      pdd pt = p2 + t * (p1 - p2);
                      pt = pt + cen;
                       ret.push_back(pt);
           return ret;
     }
} solver;
double f(ld a, ld b) {
   ld ret = b - a;
     while (ret <= -pi - eps) ret += 2 * pi;
     while (ret >= pi + eps) ret -= 2 * pi;
      return ret:
ld solve_small(pdd cen, ld r, pdd p1, pdd p2) {
   p1 = p1 - cen, p2 = p2 - cen;
   cen = {0, 0};
      vector<pdd> inter = solver.solve(p1, p2, cen, r);
     ld ret = 0.0;
      if ((int)inter.size() == 0) {
           if (in_cir(cen, r, p1)) {
    ret = (p1 ^ p2) / 2;
           }
           else {
```

```
ret = (r * r * f(atan2(p1.Y, p1.X), atan2(p2.Y, p2.
                    X))) / 2;
          }
     else if ( (int)inter.size() == 1) {
          if (!in_cir(cen, r, p1) && !in_cir(cen, r, p2)) {
              //outside cut
               ret = (r * r * f(atan2(p1.Y, p1.X), atan2(p2.Y, p2.X))
                    X))) / 2;
          else if (!in_cir(cen, r, p1)) {
              pdd _p1 = inter[0];
              ret += ((_p1 ^ p2) / 2);
ret += (r * r * f(atan2(p1.Y, p1.X), atan2(_p1.Y,
                    _p1.X))) / 2;
          else if (!in_cir(cen, r, p2)) {
              pdd _p2 = inter[0];
              ret += ((p1 ^ _p2) / 2);
ret += (r * r * f(atan2(_p2.Y, _p2.X), atan2(p2.Y,
                     p2.X))) / 2;
          }
     else if ( (int)inter.size() == 2) {
          pdd _p2 = inter[0], _p1 = inter[1];

ret += ((_p1 ^ _p2) / 2);

ret += (r * r * f(atan2(_p2.Y, _p2.X), atan2(p2.Y, p2.X))
               X))) / 2;
          ret += (r * r * f(atan2(p1.Y, p1.X), atan2(_p1.Y, _p1.
               X))) / 2;
     return ret;
}
ld solve(pdd cen, ld r, vector<pdd> pts) {
     ld ret = 0;
     for (int i = 0; i < (int)pts.size(); ++i) {</pre>
          ret += solve_small(cen, r, pts[i], pts[(i + 1) % int(
               pts.size())]);
     ret = max(ret, -ret);
     return ret;
i }
```

11.7 Segment Intersection

```
int intersect(PII a , PII b , PII c , PII d){
    if(max(a.F , b.F) < min(c.F , d.F)) return 0;
    if(max(c.F , d.F) < min(a.F , b.F)) return 0;
    if(max(a.S , b.S) < min(c.S , d.S)) return 0;
    if(max(c.S , d.S) < min(a.S , b.S)) return 0;
    if(cross(b - a , c - a) * cross(b - a , d - a) == 1) return 0;
    if(cross(d - c , a - c) * cross(d - c , b - c) == 1) return 0;
    return 1;
}</pre>
```

11.8 Line Intersection Point

11.9 Rotating Calipers

```
#define NXT(x) ((x + 1) \% m)
int main () {
    vector<pii> v; // v is the input points
    sort(v.begin(), v.end());
    vector<pii> up, down;
    for (pii p: v) {
        while (SZ(down) >= 2 \&\& sgn((p - down[SZ(down) - 2]) ^
             (p - down.back())) >= 0) {
            down.pop_back();
        down.push_back(p);
    reverse(v.begin(), v.end());
    for (pii p: v) {
        while (SZ(up) >= 2 \&\& sgn((p - up[SZ(up) - 2]) \land (p -
             up.back())) >= 0) {
            up.pop_back();
        up.push_back(p);
```

```
vector<pii> all;
    for (pii p: down) { all.push_back(p); } all.pop_back();
    for (pii p: up) { all.push_back(p); }
    all.pop_back();
    int m = all.size();
    int ptr = (int)down.size() - 1;
    for (int i = 0; i < m; ++i) {
        while (((all[NXT(ptr)] - all[ptr]) ^ (all[NXT(i)] - all
             [i])) > 0) {
            ptr = NXT(ptr);
    }
] }
```

Ad hoc 12

12.1Joseph Problem

```
// O(m + log N)
// n people, k-th dead. Find out the last alive person
int main() {
 long long n, k, i, x = 0, y;
scanf( "%I64d%I64d", &n, &k );
  for( i = 2; i \le k \&\& i \le n; ++i ) x = (x + k) \% i;
  for( ; i <= n; ++i ) {</pre>
      y = (i - x - 1)/k;
      if(i + y > n) y = n - i;
      i += y;
      x = (x + (y + 1) \% i * k) \% i;
 printf( "%I64d\n", x + 1 );
  return 0;
```

12.2 Segment Max Segment Sum

```
int n , m , x[MAX];
class N{
public: int tag , sml , sum , none;
} b[MAX * 4];
void Pull(int now , int l , int r){
    if(l == r){
        if(b[now].tag){
             b[now].sum = b[now].tag;
             b[now].none = 0;
             b[now].sml = b[now].tag;
         else{
             b[now].sum = 0;
             b[now].none = 1
             b[now].sml = INF;
    else {
        b[now].sml = min(b[ls].sml , b[rs].sml);
         if(b[now].tag) b[now].sml = min(b[now].sml , b[now].tag
        b[now].sum = b[ls].sum + b[rs].sum;
        b[now].none = b[ls].none + b[rs].none;
         if(b[now].tag) b[now].sum += b[now].tag * b[now].none ,
              b[now].none = 0;
void take_tag(int now , int l , int r , int val){
   if(b[now].tag && b[now].tag < val) b[now].tag = 0;</pre>
    if(l != r && b[ls].sml < val) take_tag(ls , l , mid , val);</pre>
    if(l != r \&\& b[rs].sml < val) take_tag(rs , mid + 1 , r ,
         val);
    Pull(now , l , r);
void Build(int now , int l , int r){
    b[now].none = 0;
    if(l == r) b[now].tag = b[now].sml = b[now].sum = x[l];
    else {
        Build(ls , l , mid) , Build(rs , mid + 1 , r);
Pull(now , l , r);
void update(int now , int l , int r , int ql , int qr , int val
    if(b[now].tag >= val) return ;
    if(ql \ll l \& r \ll qr)
        take_tag(now , l , r , val);
        b[now].tag = val;
```

```
Pull(now , l , r);
     else{
          if(qr <= mid) update(ls , l , mid , ql , qr , val);</pre>
         else if(mid + 1 \leftarrow ql) update(rs , mid + 1 , r , ql ,
               qr , val);
         Pull(now , l , r);
PII query(int now , int l , int r , int ql , int qr){
     if(ql <= l && r <= qr) return mp(b[now].sum , b[now].none);</pre>
         PII ans = mp(0, 0);
         if(qr <= mid) ans = query(ls , l , mid , ql , qr);</pre>
         else if(mid + 1 \leftarrow ql) ans = query(rs , mid + 1 , r ,
               ql , qr);
              PII a = query(ls , l , mid , ql , qr);
PII b = query(rs , mid + 1 , r , ql , qr);
ans = mp(a.A + b.A , a.B + b.B);
         if(b[now].tag != 0) ans.A += ans.B * b[now].tag , ans.B
         return ans;
REP(i , 1 , n + 1) cin >> x[i];
Build(1 , 1 , n);
update(1 , 1 , n , l , r , v);
cout << query(1 , 1 , n , l , r).A << endl;
```

12.3 Stone Merge

```
| int n, x[MAX], ans = 0;
 vector<int> v;
 int DFS(int now) {
      int val = v[now] + v[now + 1];
      ans += val;
     v.erase(v.begin() + now);
      v.erase(v.begin() + now);
      int id = 0;
     for (int i = now - 1; i >= 0; -- i)
    if(v[i] >= val) { id = i + 1; break; }
v.insert(v.begin() + id, val);
     while(id >= 2 \& v[id - 2] <= v[id]) {
          int dis = v.size() - id;
          DFS(id - 2);
          id = v.size() - dis;
     }
 int32_t main() {
     IOS;
      cin >> n;
      for (int i = 0; i < n; ++ i) cin >> x[i];
      for (int i = 0; i < n; ++ i) {
          v.emplace_back(x[i]);
          while(v.size() >= 3 && v[v.size() - 3] <= v[v.size() -</pre>
               1])
               DFS(v.size() - 3);
     while(v.size() > 1) DFS(v.size() - 2);
      cout << ans << endl;
      return 0;
|}
```

12.4 Manhattan Spanning Tree

```
#define edge pair<int, PII>
int n, sol[maxn];
PII x[maxn];
vector<edge> v;
class djs{
public:
     int x[maxn];
     void init(){ for (int i = 0; i < maxn; ++ i) x[i] = i; }
     int Find(int now){ return x[now] == now ? now : x[now] =
     Find(x[now]); }
void Union(int a, int b){ x[Find(a)] = Find(b); }
     int operator[](int now){ return Find(now); }
PII bit[maxn];
void update(int from, int val, int id){
     for(int i = from; i < maxn; i += i & -i)
         bit[i] = maxn(bit[i], mp(val, id));
}
```

```
int query(int from){
    PII res = bit[from];
    for(int i = from; i > 0; i = i & -i)
        res = maxn(res, bit[i]);
    return res.B;
int cmp(int a, int b){
    return x[a] < x[b];
int DIS(int q, int w){
    return abs(x[q].A - x[w].A) + abs(x[q].B - x[w].B);
void BuildEdge(){
    vector<int> uni;
    for (int i = 0; i < maxn; ++ i)</pre>
        bit[i] = mp(-INF, -1);
    for (int i = 0; i < n; ++ i) sol[i] = i;
    for (int i = 0; i < n; ++ i) uni.pb(x[i].B - x[i].A);
    sort(ALL(uni));
    uni.resize(unique(ALL(uni)) - uni.begin());
    sort(sol, sol + n, cmp);
    for (int i = 0; i < n; ++ i){
        int now = sol[i];
        int tmp = x[sol[i]].B - x[sol[i]].A;
        int po = lower_bound(ALL(uni), tmp) - uni.begin() + 1;
        int id = query(po);
        if(id >= 0) v.pb(mp(DIS(id, now), mp(id, now)));
        update(po, x[now].A + x[now].B, now);
void Build(){
    BuildEdge();
    for (int i = 0; i < n; ++ i) swap(x[i].A, x[i].B);
    BuildEdge();
    for (int i = 0; i < n; ++ i) x[i].A *= -1;
    BuildEdge();
    for (int i = 0; i < n; ++ i) swap(x[i].A, x[i].B);
    BuildEdge();
int solveKruskal(){
    ds.init();
    sort(ALL(v)):
    int res = 0;
    for (int i = 0; i < v.size(); ++ i) {</pre>
        int dis = v[i].A;
        PII tmp = v[i].B;
        if(ds[tmp.A] != ds[tmp.B]){
            ds.Union(tmp.A, tmp.B);
            res += dis;
        }
    return res;
int32_t main(){
    IOS;
    cin >> n:
    for (int i = 0; i < n; ++ i) cin >> x[i].A >> x[i].B;
    Build();
    int ans = solveKruskal();
    cout << ans << endl;
    return 0;
12.5 K Cover Tree
int n , k , dp[MAX] , ans;
```

```
vector<int> v[MAX];
void DFS(int now , int fa){
    if(v[now].size() == 1 && v[now][0] == fa)
        return dp[now] = -1 , void();
    int sml = INF , big = -INF;
    for(auto to : v[now]) if(to != fa){
        DFS(to , now);
        sml = min(sml , dp[to]);
big = max(big , dp[to]);
    if(sml == -k) dp[now] = k , ans ++;
    else if(big - 1 \ge abs(sml)) dp[now] = big - 1;
    else dp[now] = sml - 1;
int32_t main(){
    IOS;
    cin >> n >> k;
    REP(i , 2 , n + 1){
int a , b; cin >> a >> b;
        v[a].pb(b); v[b].pb(a);
```

```
if(k == 0) cout << n << endl;
else {
    DFS(0 , 0) , ans += dp[0] < 0;
    cout << ans << endl;
}
return 0;
}</pre>
```

12.6 M Segments' Maximum Sum

```
-----Greedy------
int n, m, fr[MAX], ba[MAX];
int v[MAX], idx = 1;
set<PII> cc:
void erase(int id){
    if(id == 0) return;
    int f = fr[id], b = ba[id];
    ba[fr[id]] = b, fr[ba[id]] = f;
    cc.erase(mp(abs(v[id]), id));
int32_t main(){
    cin >> n >> m;
    int sum = 0, pos = 0, ans = 0;
    REP(i, 0, n){
        int tmp; cin >> tmp;
        if(tmp == 0) continue;
        if((tmp >= 0 \&\& sum >= 0) || (tmp <= 0 \&\& sum <= 0)){}
            sum += tmp;
        else {
            if(sum > 0) ans += sum, pos ++;
            v[idx ++] = sum, sum = tmp;
    if(sum) v[idx ++] = sum;
    if(sum > 0) ans += sum, pos ++;
    REP(i, 0, idx){
        fr[i + 1] = i;
        ba[i] = i + 1;
        if(i) cc.insert(mp(abs(v[i]), i));
    ba[idx - 1] = 0;
    while(pos > m){
        auto tmp = cc.begin();
        int val = (*tmp).A, id = (*tmp).B;
        cc.erase(tmp);
        if(v[id] < 0 \&\& (fr[id] == 0 || ba[id] == 0)) continue;
        if(v[id] == 0) continue;
        ans -= val, pos --;
        v[id] = v[fr[id]] + v[id] + v[ba[id]];
        cc.insert(mp(abs(v[id]), id));
        erase(fr[id]), erase(ba[id]);
    cout << ans << endl;
    return 0;
}
            -----Aliens-----
int n, k, x[MAX];
PII dp[MAX], rd[MAX]; // max value, times, can be buy, times
int judge(int now){
    dp[i] = max(dp[i - 1], mp(rd[i - 1].A + x[i] - now, rd[
            i - 1].B + 1));
        rd[i] = max(rd[i - 1], mp(dp[i - 1].A - x[i])
                                                          , dp[
            i - 1].B));
    return dp[n].B;
int32_t main(){
    ĪŌS;
    cin >> n >> k;
    for (int i = 2; i \le n + 1; ++ i)
        cin >> x[i];
    for (int i = 1; i <= n; ++ i)
        x[i] += x[i - 1];
    if(judge(0) <= k) cout << dp[n].A << endl;</pre>
    else {
        int l = 0, r = 10000000000000L;
while(r - l > 1){
            int mid = l + ((r - l) \gg 1), res = judge(mid);
            if(res == k) return cout << dp[n].A + dp[n].B * mid</pre>
                  << endl, 0;
            else if(res < k) r = mid;</pre>
            else if(res > k) l = mid;
        judge(1);
```

```
National Taiwan University LYB

| cout << dp[n].A + k * l << endl;
| }
| return 0;
|}

12.7 Minimum Enclosing Cycle
|#define pdd pair<double, double>
|#define F first
|#define S second
| int n;
| pdd a[maxn]:
```

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
      count());
double dis(pdd p1, pdd p2) {
  return hypot(p1.F - p2.F, p1.S - p2.S);
inline double sq(double x) {
  return x * x;
pdd external(pdd p1, pdd p2, pdd p3) {
  double a1 = p1.F - p2.F, a2 = p1.F - p3.F;
  double b1 = p1.S - p2.S, b2 = p1.S - p3.S;
  double c1 = (sq(p1.F) - sq(p2.F)
                 + sq(p1.S) - sq(p2.S)) / 2;
  double c2 = (sq(p1.F) - sq(p3.F)
  + sq(p1.5) - sq(p3.5)) / 2;

double dd = a1 * b2 - a2 * b1;

return make_pair((c1 * b2 - c2 * b1) / dd

, (a1 * c2 - a2 * c1) / dd);
}
int main() {
  cin >> n;
  for (int i = 0; i < n; ++ i)
     cin >> a[i].F >> a[i].S;
  shuffle(a, a + n, rnq);
  pdd center = a[0];
   double r = 0;
  for (int i = 0; i < n; ++ i) {
     if (dis(center, a[i]) <= r) continue;</pre>
     center = a[i], r = 0;
      for (int j = 0; j < i; ++ j) {
        if (dis(center, a[j]) <= r) continue;</pre>
        center.F = (a[i].F + a[j].F) / 2;
        center.S = (a[i].S + a[j].S) / 2;
        r = dis(center, a[i]);
for (int k = 0; k < j; ++ k) {
  if (dis(center, a[k]) <= r) continue;</pre>
           center = external(a[i], a[j], a[k]);
           r = dis(center, a[i]);
        }
     }
  cout << fixed << setprecision(10) << r << endl; cout << center.F << " " << center.S << " \n";
   return 0;
```

12.8 Rotating Sweep Line

```
PII p[maxn];
int n, idx[maxn], pos[maxn];
vector<PII> v;
inline PII operator + (PII x, PII y) {
  return make_pair(x.F + y.F, x.S + y.S); }
inline PII operator - (PII x, PII y) {
  return make_pair(x.F - y.F, x.S - y.S); }
inline long long cross(PII x, PII y) {
  return 1ll * x.F * y.S - 1ll * x.S * y.F; }
inline int cmp(PII x, PII y) {
 x = p[x.S] - p[x.F];

y = p[y.S] - p[y.F];
  return cross(x, y) > 0;
int32_t main() {
  cin.tie(0), cout.sync_with_stdio(0);
  cin >> n >> wnt, wnt += wnt;
  for (int i = 1; i <= n; ++ i)
  cin >> p[i].F >> p[i].S;
  sort(p + 1, p + 1 + n);
  for (int i = 1; i \le n; ++ i)
    idx[i] = i, pos[i] = i;
```

```
for (int i = 1; i <= n; ++ i)
for (int j = i + 1; j <= n; ++ j)
        v.emplace_back(i, j);
    sort(v.begin(), v.end(), cmp);
   for(auto line : v) {
      int fr = pos[line.F], ba = pos[line.S], now;
      if(fr > ba) swap(fr, ba);
      // [TODO] points:
      // p[idx[
                      1]] more farther
      // p[idx[
                      2]] farther
      // p[idx[
                   fr]] ... p[idx[ba]]
      // p[idx[n - 1]] farther
// p[idx[n - 0]] more farther
      swap(idx[fr], idx[ba]);
      swap(pos[line.F], pos[line.S]);
      return 0;
1}
           Hilbert Curve
 12.9
// soring Mo's with hilbert(nn, L, R) can be faster !! // needed: nn >= n, no need to change n, nn = 2^k // usage: sort (ql_i, qr_i) by hilbert(nn, ql_i, qr_i)
 11 hilbert(int nn, int x, int y) {
   ll res = 0;
    for (int s = nn / 2; s; s >>= 1) {
      int rx = (x \& s) > 0;
      int ry = (y \& s) > 0;
      res += s * 1ll * s * ((3 * rx) ^ ry);
      if (ry == 0) {
        if (rx == 1) {
 x = s - 1 - x;
          y = s - 1 - y;
        swap(x, y);
     }
   return res;
}
```