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ORIGINAL ARTICLE

A robust design for a closed-loop supply chain network under an uncertain environment

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Abstract This paper presents a robust design for a multi-product, multi-echelon, closed-loop logistic network model in an uncertain environment. The model includes a general network structure considering both forward and reverse processes that can be used in various industries, such as electronics, digital equipment, and vehicles. Because logistic network design is a time consuming and costly project as well as a strategic and sensitive decision (i.e. the change of such decision is difficult in the future), a robust optimisation approach is adopted to cope with the uncertainty of demand and the return rate described by a finite set of possible scenarios. Hence, to obtain robust solutions with better time, the scenario relaxation algorithm is employed for the proposed model. Numerical examples and a sensitivity analysis are presented to demonstrate the significance and applicability of the presented model. It is shown that solutions resulted from the suggested approach insure more situations, especially in worst case ones. The results show that although the profit values of the robust configuration are less than the deterministic configuration, the robust configuration is more reliable than the deterministic one because the deterministic configuration is infeasible under some demand and return rates (i.e. in the worst cases). Moreover, the results show the computing time superiority of the algorithm compared to the extensive form model as well as optimality of the resulted solutions.

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1 Introduction

Supply chain management (SCM) emerged in the early 1990s and has gained recent interest from researchers because the integrated management of an effective and efficient supply chain can reduce unexpected/negative occurrences and can definitively affect the profitability of an entire firm. One of crucial planning activities in SCM is to design the configuration of the supply chain network. In SCM, three planning levels (namely strategic, tactical and operational) are usually distinguished depending on the time horizon (namely long, medium, and short terms, respectively) [1]. The planning of the strategic level includes determining the number, location, capacity and technology type of facilities, and the tactical/operational level involves determining the quantities of purchasing, production, distribution and inventory holdings as well as shipments transferred between established facilities. The configuration of the supply chain is a key strategic decision that influences activities at a tactical/operational level and has a long-lasting effect on the network [2]. Therefore, the fact that a supply chain network design (SCND) problem invests a large amount of capital in new facilities over an extended time period makes the SCND problem an extremely important issue. Examples of models and solution procedures for SCND problems can be found in [3–6].

In the last 10 years, due to global warning in particular, growing attention has been paid to reverse logistics, which refers to activities that are dedicated to the collection, repair, recovery, recycling and/or disposal of the returned products within SCM. Many companies, such as Kodak and Xerox, have concentrated on remanufacturing and recycling activities and achieved important successes in this area [7]. The various aspects of these activities can account for reuse activities including the reduction of the use of primary resources, pollution prevention, waste management, environmental concern, social responsibility and government directives, e.g. the European Union WEEE Directive and customer pressures.

The literature dedicated to the reverse logistic networks can be divided into problems that fully focus on the backward network (i.e. recovery network) and those in which the backward network is integrated with the forward network (i.e. closed-loop network) [1]. Because designing the forward and reverse logistics separately results in sub-optimal designs with respect to objectives of the supply chain, the closed-loop supply chain (CLSC) network is critically important [8–10]. Closed-loop logistic management can guarantee the least waste of materials by following the conservation laws during the life cycle of the materials [11]. For more details on the importance of the closed-loop supply chain, readers may refer to [12, 13].

In the closed-loop supply chain, few studies have accounted for the capacity levels as decision variables [14, 15], whereas most studies have considered only a single capacity level for each facility (see Table 1). Nevertheless, the capacity level is an important decision variable in real-life applications because of its potent effect on logistic network efficiency.

On the other hand, as noted by Sabri and Beamon [16], uncertainty is one of the most challenging and significant problems in SCM. However, the literature integrating uncertainty with location decisions in the SCM and CLSC areas remains scare (see Table 1 and [1, 2]). Various sources of uncertainty can be found in the related literature, namely customer demands, production costs, transportation costs, return rates, supply lead times and exchange rates. Moreover, the majority of approaches to logistic network design problems in the literature have used stochastic programming by applying the expected effect and conducting partially the uncertainty of the parameters. Due to the difficulty of changing the network configuration in the future (i.e. strategic decisions), the long-lasting impact on the total chain and the large investment in the SCND project, the robust optimisation approach employing the worst-case effect is more reliable compared to the stochastic programming approach. Therefore, this paper adopts the robust optimisation approach to achieve the robust configuration of a supply chain network design.

Based on the aforementioned discussions, this paper presents a multi-product, closed-loop logistic network design model in which the demand and the return rate are uncertain and are described by a set of finite scenarios. The presented model attempts to determine the capacity level of a facility location and the amount of products to be shipped among the network entities. The structure of the closed-loop network includes three echelons in the forward direction (i.e. echelons constituted by the flow arcs among the entities of suppliers, plants, distribution centres and customers) and three echelons in the backward direction (i.e. echelons constituted by the flow arcs among the entities of customers, collection centres, repair centres and disposal centres). In spite of the case-based models, the proposed model consists of a general network structure considering both the forward and reverse processes; therefore, it can be used in various industries, such as electronics, digital equipment and vehicles. Because of the high impact and critical decisions required in a supply chain network design, a robust optimisation approach is applied to deal with the uncertainty. Therefore, a scenario relaxation algorithm is employed to determine a robust configuration with a faster computation time. In summary, the main contributions of this paper are as follows:

- Introducing a new model for multi-product closed-loop logistic network design that integrates strategic decisions (i.e. location of facilities) with tactical decisions (i.e. activities related to procurement, production/reproduction and distribution/redistribution) to extend the existing models found so far in the literature.
- Obtaining the applicability and efficiency of the scenario relaxation algorithm compared to the extensive form model to solve the SCND model that contemplates the uncertainty of the demand and the return rate as a finite number of possible scenarios.

The structure of this study is as follows. Section 2 provides a state-of-the-art of existing work in a closed-loop supply chain and then reviews the robust optimisation approach. Section 3 presents the model's assumptions and limitations. In Section 4, we present a mixed-integer linear programming (MILP) model of a robust closed-loop logistic network design. To obtain a robust configuration of the proposed model, the scenario relaxation algorithm is described in Section 5. Section 6 presents and analyses the computational results. Finally, we give the conclusions of this paper in Section 7.

2 Literature review

In this section, we inquire into the literature and categorise studies into two groups. The first group is the studies associated with the closed-loop supply chain, and the second contains the studies related to the robust optimisation.

Table 1 Review of the existing studies in closed-loop logistic network design

Reference	Logistic network echelons									Model features			
	Supply centres	Manufacturing centres	Distribution centres	Collection centres	Repair centres	Redistribution centres	Remanufacturing centres	Recycling centres	Disposal centres	Period		Product	
										Single	Multiple	Single	Multiple
Fleischmann et al. [7]		2	2	2			2			✓		✓	
Salema et al. [38]		2	2	2			2			✓		✓	
Lu and Bostel [39]		2	2	2			2		1	✓		✓	
Üster et al. [5]		1	1	2	2					✓			✓
Ko and Evans [18]		1	2		2		1				✓		✓
Salema et al. [17]		2	2	2			2			✓			✓
Min and Ko [19]		1	2		2		1						✓
Lee and Dong [8]		1	2	2			1			✓		✓	
Lee and Dong [20]		1	2	2		1	1				✓		✓
Wang and Hsu [10]	1	2	2	2			2	2		✓		✓	
El-Sayed et al. [2]	2	2	2	2		2	2		2		✓	✓	
Pishvaee et al. [21]		2	2	2			2		2	✓		✓	
Ramezani et al. [22]	1	2	2	2		2			2	✓			✓
Current paper	1	2	2	2	2	2	2	1	1	✓			✓

Reference	Model features						Variables					Goals		
	Parameter			Capacity expansion	Limitation of facilities	Limitation of capacity	Inventory	Transportation value	Demand satisfaction	Facility capacity	Location/ allocation	Responsiveness	Profit	Cost
	Certain	Uncertain												
		Stochastic	Robust											
Fleischmann et al. [7]	✓							✓	✓		✓			✓
Salema et al. [38]	✓					✓		✓	✓		✓			✓
Lu and Bostel [39]	✓							✓			✓			✓
Üster et al. [5]	✓							✓			✓			✓
Ko and Evans [18]	✓			✓		✓		✓			✓			✓
Salema et al. [17]		✓				✓		✓	✓		✓			✓
Min and Ko [19]	✓			✓		✓		✓			✓			✓
Lee and Dong [8]	✓				✓	✓		✓			✓			✓
Lee and Dong [20]		✓				✓		✓			✓			✓
Wang and Hsu [10]	✓					✓		✓			✓			✓
El-Sayed et al. [2]	✓				✓	✓		✓			✓		✓	
Pishvaee et al. [21]	✓					✓		✓		✓	✓			✓
Ramezani et al. [22]		✓				✓		✓			✓	✓	✓	

Table 1 (continued)

Reference	Model features						Variables					Goals		
	Parameter			Capacity expansion	Limitation of facilities	Limitation of capacity	Inventory	Transportation value	Demand satisfaction	Facility capacity	Location/ allocation	Responsiveness	Profit	Cost
	Certain	Uncertain												
		Stochastic	Robust											
Current paper			✓		✓	✓		✓		✓	✓		✓	

1 The facility layer is considered; *2* the location decision is made in the facility layer

2.1 Closed-loop supply chain

In recent years, the SCND problem has received a great deal of attention due to the ever-increasing development of the competitive environment. This competitiveness causes that firms be under a pressure to supply, produce, and distribute the products to survive in today's market and imposes the companies to assure their customers a high level of service and, at the same time, to control costs and maximise profits. The SCND problem has a key importance in the long-term strategic level required to be optimised for efficiency of the whole supply chain. The decisions in this problem aim in some way to increase shareholder value. A stream of the literature research in this field with focus on the closed-loop networks is reviewed in this section. The closed-loop supply chain network consists of two sections, namely, the forward and backward logistics. In the backward logistic, when the used products are shipped to the forward facilities, they should allow both forward and backward flows under the same capacity. As Table 1 indicates, the large numbers of models are deterministic in comparison with the uncertain problems. These models include the extensive scope of the formulations ranging from a simple-linear, uncapacitated, single-product facility location model to complex, non-linear, capacitated, multi-product models.

Fleischmann et al. [8] presented a supply chain design model that optimises the forward flow along with the return flow without considering the capacity limit. Salema et al. [17] extended Fleischmann's model [11] to a capacitated, multi-product, forward/reverse logistic network with uncertainty on demands and returns applied to an Iberian company. However, when suspending logistics between dismantlers and manufacturing facilities, both of the mentioned models did not account for the supplier side and did not include the relation between the forward and return flows.

Üster et al. [7] presented a multi-product, closed-loop, supply chain network model and solved it using the Bender decomposition technique. They considered manufacturing and remanufacturing separately and assumed a single sourcing for the customers. Ko and Evan [18] developed a dynamic, integrated, closed-loop network operated by the third party logistic (3PL) service provider. They applied a genetic algorithm (GA) to solve their model. Min and Ko [19] presented a dynamic design of a closed-loop logistic network and proposed a GA to solve their problem including the location and allocation for 3PLs. Lee and Dong [20] proposed a dynamic, closed-loop, logistic network under demand uncertainty. An approach integrating a recently proposed sampling method with a simulated annealing algorithm was developed to obtain solutions.

El-Sayed et al. [4] presented a multi-period, multi-echelon, closed-loop logistic network design model. They considered three echelons in the forward direction (i.e. suppliers, manufacturing plants and distribution centres) and two echelons in the return direction (i.e. disassembly and redistribution centres). The objective of their model was to maximise the profit of a supply chain. Wang and Hsu [11] developed a generalised closed-loop supply chain design model that determined the locations of manufactories, distribution, and dismantlers where the forward and backward flows use the distributions as hybrid processing facilities. A revised spanning-tree based GA employing determinant encoding representation was presented to solve the model. Pishvaee et al. [21] suggested a bi-objective, integrated, closed-loop supply chain design model in which the costs and the responsiveness of the logistic network are considered as objectives of model. They developed an efficient multi-objective, mimetic algorithm by applying three different local searches to determine the set of non-dominated solutions. Ramezani et al. [22] presented a multi-objective stochastic programming approach to design a forward/reverse supply chain with three performance measures: profit, customer responsiveness, and quality of suppliers (using Six Sigma concept). The Pareto optimal solutions along with the relevant financial risk are computed to illustrate tradeoff of objectives. The results give a proper insight for having a better decision making.

To review the existing studies on closed-loop supply chain network design and to specify the distinctiveness of this paper from others in the literature, we categorise the models in accordance with four characteristics, namely, logistic network echelons, model features, decision variables, and objectives. As shown in Table 1, the large part of models in the CLSC network design is deterministic and few researchers consider uncertainty. Moreover, the researches relevant to uncertainty have used stochastic programming and did not account for the robustness issue. Although the idea of robust optimisation has been considered in other areas, to the best of our knowledge, this is first time that the robustness aspect is considered in the closed-loop logistic network context under an uncertain environment. Furthermore, the model properties shown in Table 1 distinguish this paper from others in the literature. Therefore, we address the relevant literature of the robust optimisation approach in the following subsection.

2.2 Robust optimisation

The considered models are proposed in diverse contexts and are solved by the related solution approaches; however, the parameter fluctuations of these systems are basically unavoidable for the following reasons. First, the imprecision makes the real value of the parameters deviate from their theoretical value. Second, the parameters may gradually deviate because of the environmental factors when the system is in use. If these parameters are simply described as deterministic values, some impractical strategies may be created, which result in the waste of resources and the low efficiency of the entire system [23].

Uncertainty can be defined via the concept of a scenario that assigns plausible values to the model parameters. Determining the values of variables using the expected effect of a set of scenarios is not completely sufficient because the subject of how to protect against potentially high-impact events was only partly conducted. Therefore, the conventional approaches, such as deterministic and stochastic approaches, do not protect against high-impact problems, such as the creation of a costly SCND project. Moreover, decision makers encounter problems for which no adequate historical data are available for estimating the uncertain parameters, and the reliable estimation of scenario probabilities can be difficult. In other words, the robust optimisation approach is motivated as follows:

- Conducting the reliability uncertainty using the worst-case scenario to protect against potentially high-impact events, such as SCND problems.
- Coping with problems for which the information is not adequate to estimate the distribution function of the uncertain parameters or the scenario probabilities.

Therefore, the robust optimisation approach is appropriate in this context by searching the robust long-term decision setting that functions across all possible realisations without assigning an assumed probability to any imprecision parameter.

The robust optimisation comprises min–max and min–max regret approaches defined by Kouvelis and Yu [24]. Let S denote a finite set of scenarios and x denote the feasible solution of a given problem. For a minimisation problem, $\forall s \in S_s, Z_s(x)$ and Z_s^* denote the objective and the optimal objective of a problem under scenario s, respectively. The min–max version includes finding a solution with the best worst-case value across all possible scenarios, which can be formulated by:

$$\min_{x \in X} - \max_{s \in S} Z_s(x) \tag{1}$$

In the min–max regret version, the regret value of each scenario is defined by the difference between the objective value of a feasible solution (i.e. $Z_s(x)$) and the optimal objective value (i.e. Z_s^*). This difference can be denoted by the absolute regret or relative regret. The min–max regret and the min–max relative regret consist of finding a robust solution that minimises its maximum regret and its maximum relative regret, respectively, which can be stated by:

$$\min_{x\in X} - \max_{s\in S} Z_s(x) - Z_s^* \tag{2}$$

$$\min_{x\in X} - \max_{s\in S} \frac{Z_s(x) - Z_s^*}{Z_s^*} \tag{3}$$

Indeed, the corresponding max–min and min–max regret version can be denoted for maximisation problems.

Aissi et al. [25] reviewed the state-of-the-art in min–max regret and min–max relative regret robust optimisation and presented a comprehensive discussion of the incentive for these approaches and different aspects of employing robust optimisation in practice. Ben-Tal and Nemirovski [26, 27] and Ben-Tal et al. [28] engaged in robust optimisation by permitting the data to be ellipsoids and suggested efficient algorithms to solve convex optimisation problems under data ambiguity. Bertsimas and Sim [29, 30] presented an approach for discrete optimisation and network flow problems that provides the degree of conservatism of a solution. They demonstrated that the robust equivalent of an NP-hard, a-approximable, 0–1 discrete optimisation problem remains a-approximable. The application of semi-infinite programming in solving the min–max optimisation problems can be observed in a work by Zakovic and Rustem [31]. In addition, Rustem and Howe [32] presented algorithms for the worst-case design and some applications for risk management.

Some approaches have been designed to reduce the number of scenarios. Daskin et al. [33] proposed an a-reliable, min–max regret model and found a solution that minimises objective function with regard to a selected subset of scenarios whose occurrence probability is greater than the user-specified value a. Kalai et al. [34] presented another approach called lexicographic α-robustness that, instead of considering the worst case, considers all scenarios in lexicographic order from worst to best and also incorporates a tolerance threshold α to not allow a difference among solutions with similar values. Assavapokee et al. [35] proposed a scenario relaxation algorithm for min–max regret and min–max relative regret robust optimisation. The algorithm iteratively solves and updates a series of relaxation problems as an approximation to full problem formulation until both the feasibility and optimality conditions of the entire problem are satisfied. At first, the algorithm solves the problem for a subset of scenarios and then sequentially applies an enumerative search to examine all possible scenarios. Finally, the algorithm adds those scenarios disturbing optimality and/or feasibility conditions to the relaxation problem. The authors showed that the algorithm terminates at an optimal min–max regret robust solution (if one exists) in a finite number of iterations.

Aside from the min–max and min–max regret versions, the variability model is another formulation that is considered in literature to be a robust optimisation. It uses standard deviation, variance, and other criteria to handle the distribution of the objective value [36]. The purpose of this approach is to optimise the total average cost/profit and to minimise the variance of objective functions under various scenarios, which is often difficult to solve because of its non-linear features.

3 Model assumptions and limitations

From the above-discussed concept, we develop a closed-loop logistic network design model that is more complex and requires more effort to analyse than both forward and backward logistics simultaneously. The goal of this study is to design a robust, closed-loop logistic system that maximises total profit by determining the facility locations and flows between facilities along each capacity-constrained stage when the customer demands and return rates of products are uncertain. The proposed model considers the following assumptions and limitations.

1. The model is multi-product.
2. The supplier, customer and disposal locations are known and fixed.
3. The potential locations of manufacturing facilities, distribution centres, collection centres and repair centres are known.
4. The flow is only permitted to be transported between two consecutive stages. Moreover, there are no flows between facilities at the same stage.
5. The number of facilities that can be opened and their capacities are both restricted.
6. The quantity of demands and return rates are uncertain and are described by the set of scenarios. The return rate for each customer depends on the customer demand.
7. The cost values (i.e. fixed, material, manufacturing, operating, test and inspection, repair, remanufacturing and recycling costs) are known.

4 Model formulation

In this section, we first present a deterministic MILP model for the closed-loop logistic network design and then extend it to an uncertain situation in the following subsections.

4.1 Deterministic model

The model involves the following sets, parameters and decision variables.

Sets

V Set of fixed locations of suppliers, $\forall v \in V$.
I Set of potential locations of plants, $\forall i \in I$.
J Set of potential locations available for distribution centres, $\forall j \in J$.
C Set of fixed locations of customers, $\forall c \in C$.
K Set of potential locations available for collection centres, $\forall k \in K$.
L Set of potential locations available for repair centres, $\forall l \in L$.
D Set of fixed locations for disposal centres, $\forall d \in D$.
H Set of capacity level available for potential locations, $\forall h \in H$.
P Set of product types, $\forall p \in P$.

Parameters

D_{cp} Demand of customer c for product *p*
PR_{cp} Unit price product *p* at customer *c*
PC_{ip} Unit production cost of product *p* at plant *i*
OC_{jp} Unit operating cost of product *p* at distribution centre *j*
IC_{kp} Unit inspection and test cost of product *p* at collection centre *k*
RC_{lp} Unit repair cost of product *p* at repair centre *l*
RMC_{ip} Unit remanufacturing cost of product *p* at plant *i*
RSC_{vp} Unit recycling cost of product *p*
DC_{dp} Unit disposal cost of product *p*
FX_i^h Fixed cost for opening plant *i* with capacity level *h*
FY_j^h Fixed cost for opening distribution centre *j* with capacity level *h*
FZ_k^h Fixed cost for opening collection centre *k* with capacity level h
FW_l^h Fixed cost for repair centre *l* with capacity level *h*
CS_{vp} Capacity of supplier *v* for product *p*
CX_i^h Capacity of plant *i* with capacity level *h*
CY_j^h Capacity of distribution centre *j* with capacity level *h*
CZ_k^h Capacity of collection centre *k* with capacity level *h*
CW_l^h Capacity of repair centre *l* with capacity level *h*
TSP_{vip} Unit transportation and purchasing cost of product *p* shipped from supplier *v* to plant *i*
TPD_{ijp} Unit transportation cost of product *p* shipped from plant *i* to distribution centre *j*
TDC_{jcp} Unit transportation cost of product *p* shipped from distribution centre *j* to customer *c*
TCC_{ckp} Unit transportation cost of returned product *p* shipped from customer *c* to collection centre *k*
TCR_{klp} Unit transportation cost of repairable product *p* shipped from collection centre *k* to repair centre *l*
TRD_{ljp} Unit transportation cost of repaired product *p* shipped from repair centre *l* to distribution centre *j*
TCP_{kip} Unit transportation cost of product *p* shipped from collection centre *k* to plant *i* for remanufacturing
TCS_{kvp} Unit transportation cost of product *p* shipped from collection centre *k* to supplier *v* for recycling
m_p Capacity utilisation rate per unit of product *p*
RT Return ratio of used products at customers
RR Ratio of repair
RM Ratio of remanufacturing
RS Ratio of recycling
RD Ratio of disposal
ps_{vp} Percentage of the total capacity for recycling in supplier *v* for product *p*
px_i^h Percentage of the total capacity for remanufacturing in plant *i* with capacity level *h*

Decision variables

QSP_{vip} Quantity of product *p* produced at plant *i* using raw materials from supplier *v*
QPD_{ijp} Quantity of product *p* shipped from plant *i* to distribution centre *j*
QDC_{jcp} Quantity of product *p* shipped from distribution centre *j* to customer *c*
QCC_{ckp} Quantity of returned product *p* shipped from customer *c* to collection centre *k*
QCR_{klp} Quantity of repairable product *p* shipped from collection centre *k* to repair centre *l*
QRD_{ljp} Quantity of repaired product *p* shipped from repair centre *l* to distribution centre *j*
QCP_{kip} Quantity of product *p* shipped from collection centre *k* to plant *i* for remanufacturing
QCS_{kvp} Quantity of product *p* shipped from collection centre *k* to supplier *v* for recycling
QCD_{kdp} Quantity of product *p* shipped from collection centre *k* to disposal centre *d*

$$X_i^h = \begin{cases} 1 & \text{if plant } i \text{ with capacity level } h \text{ is opened,} \\ 0 & \text{otherwise;} \end{cases}$$

$$Y_j^h = \begin{cases} 1 & \text{if distribution center } j \text{ with capacity level } h \text{ is opened,} \\ 0 & \text{otherwise;} \end{cases}$$

$$Z_k^h = \begin{cases} 1 & \text{if collection center } k \text{ with capacity level } h \text{ is opened,} \\ 0 & \text{otherwise;} \end{cases}$$

$$W_l^h = \begin{cases} 1 & \text{if collection center } l \text{ with capacitylevel } h \text{ is opened,} \\ 0 & \text{otherwise;} \end{cases}$$

The general structure of the proposed closed-loop logistic network is illustrated in Fig. 1. In the forward direction, the suppliers are responsible for providing the raw material to manufacturing facilities. The new products are conveyed from plants to customers via distribution centres to meet the customer's demand. In the backward direction, the returned products from customers are transferred to collection centres for testing and inspection. After testing in collection centres, the recyclable, remanufactureable, repairable, and disposable products are shipped to suppliers, plants, repair centres, and disposal centres, respectively. Using this strategy, excessive transportation of returned products is reduced, and the returned products are directly transferred to the relevant facilities. In addition, the repaired products are shipped to distribution centres through repair centres.

Because the remanufacturing and redistribution process are fulfilled in forward facilities, and the recyclable and repaired products are also inserted in forward facilities, the proposed model is a closed-loop logistic network. This model is not a case-based logistic network due to its general nature; however, it can encompass various industries (e.g. electronic, digital equipment and vehicle industries). In terms of the aforementioned notations, the closed-loop logistic network design problem can be formulated as follows.

4.1.1 Objective function

The objective of the presented model is to maximise the total profit of the closed-loop network as follows.

$$\text{Total profit} = \text{total income} - \text{total cost} \tag{4}$$

The total income is calculated by multiplying the number of products shipped to customers by the unit price of each product as follows.

$$\text{Total income} = \sum_j \sum_c \sum_p \mathrm{QDC}_{jcp}\mathrm{PR}_{cp} \tag{5}$$

The total cost is the sum of the following costs.

$$\sum_i \sum_h \mathrm{FX}_i^h X_i^h + \sum_j \sum_h \mathrm{FY}_j^h Y_j^h + \sum_k \sum_h \mathrm{FZ}_k^h Z_k^h + \sum_l \sum_h \mathrm{FW}_l^h W_l^h \quad \text{Fixed cost} \tag{6}$$

$$\sum_i \sum_j \sum_p \mathrm{QPD}_{ijp} PC_{ip} \quad \text{Manufacturing cost} \tag{7}$$

$$\sum_j \sum_c \sum_p \mathrm{QDC}_{jcp}\mathrm{OC}_{jp} \quad \text{Operating cost} \tag{8}$$

$$\sum_c \sum_k \sum_p \mathrm{QCC}_{ckp}\mathrm{IC}_{kp} \quad \text{Inspection cost} \tag{9}$$

$$\sum_j \sum_l \sum_p \mathrm{QRD}_{ljp}\mathrm{RC}_{lp} \quad \text{Repairing cost} \tag{10}$$

$$\sum_i \sum_k \sum_p \mathrm{QCP}_{kip}\mathrm{RMC}_{ip} \quad \text{Remanufacturing cost} \tag{11}$$

$$\sum_v \sum_k \sum_p \mathrm{QCS}_{kvp}\mathrm{RSC}_{vp} \quad \text{Recycling cost} \tag{12}$$

$$\sum_k \sum_d \sum_p \mathrm{QCD}_{kdp}\mathrm{DC}_{dp} \quad \text{Disposal cost} \tag{13}$$

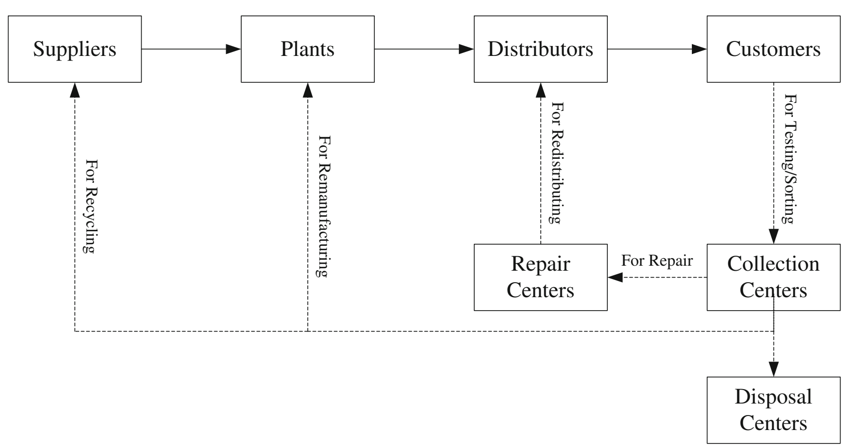

Fig. 1 Proposed closed-loop logistic network

$$\begin{aligned}&\sum_v\sum_i\sum_p \mathrm{QSP}_{vip}\mathrm{TIJ}_{ijr}+\sum_i\sum_j\sum_p \mathrm{QPD}_{ijp}\mathrm{TPD}_{ijp}+\sum_j\sum_c\sum_p \mathrm{QDC}_{jcp}\mathrm{TDC}_{jcp}\\&+\sum_c\sum_k\sum_p \mathrm{QCC}_{ckp}\mathrm{TCC}_{ckp}+\sum_k\sum_l\sum_p \mathrm{QCR}_{kjp}\mathrm{TCR}_{kjp}+\sum_l\sum_j\sum_p \mathrm{QRD}_{ljp}\mathrm{TRD}_{ijr}\quad \text{Transportation cost}\\&+\sum_{\mathrm{k}}\sum_{\mathrm{i}}\sum_{\mathrm{p}} \mathrm{QCP}_{\mathrm{kip}}\mathrm{TCP}_{\mathrm{kip}}+\sum_k\sum_v\sum_p \mathrm{QCS}_{kvp}\mathrm{TCS}_{kvp}\end{aligned}\tag{14}$$

4.1.2 Constraints

This section is a representation of the constraints of the presented model.

Balance constraints:

$$\sum_v \mathrm{QSP}_{vip}+\sum_k \mathrm{QCP}_{kip}=\sum_j \mathrm{QPD}_{ijp},\quad \forall i,p \tag{15}$$

$$\sum_i \mathrm{QPD}_{ijp}+\sum_l \mathrm{QRD}_{ljp}=\sum_c \mathrm{QDC}_{jcp},\quad \forall j,p \tag{16}$$

$$\sum_j \mathrm{QDC}_{jcp}=D_{cp},\quad \forall c,p \tag{17}$$

$$\sum_k \mathrm{QCC}_{ckp}=D_{cp}\mathrm{RT},\quad \forall c,p \tag{18}$$

$$\sum_l \mathrm{QCR}_{klp}=\sum_c \mathrm{QCC}_{ckp}\mathrm{RR},\quad \forall k,p \tag{19}$$

$$\sum_i \mathrm{QCP}_{kip}=\sum_c \mathrm{QCC}_{ckp}\mathrm{RM},\quad \forall k,p \tag{20}$$

$$\sum_v \mathrm{QCS}_{kvp}=\sum_c \mathrm{QCC}_{ckp}\mathrm{RS},\quad \forall k,p \tag{21}$$

$$\begin{aligned}\sum_c \mathrm{QCC}_{ckp}=&\sum_l \mathrm{QCR}_{klp}+\sum_i \mathrm{QCP}_{kip}\\&+\sum_v \mathrm{QCS}_{kvp}+\sum_d \mathrm{QCD}_{kdp},\end{aligned}\tag{22}$$

$\forall k,p$

$$\sum_k \mathrm{QCR}_{klp}=\sum_j \mathrm{QRD}_{ljp},\quad \forall l,p \tag{23}$$

Constraint (15) shows that, for each product, the sum of the flow entering each plant from all suppliers and collection centres is equal to the flow leaving the facility. Constraint (16) insures that, for each product, the sum of the flow entering each distribution centre from all plants and repair centres is equal to the flow leaving this distribution centre. Constraint (17) states that, for each product, the flow exiting the distribution centres must satisfy the demand of all customers. Constraint (18) describes the relationship of customer demands with the flow of the returned products that are transferred from customers to collection centres. Constraint (19) imposes that, for each product, the flow exiting the collection location to all repair centres to be repaired is equal to the flow entering each collection centre from all customers multiplied by the repair ratio. Constraint (20) shows that, for each product, the flow exiting from collection centre to all plants to be recovered is equal to the flow entering to each collection location from all customers multiplied by the remanufacturing ratio. Constraint (21) insures that, for each product, the flow exiting the collection location to all suppliers to be recycled is equal to the flow entering each collection centre from all customers multiplied by the recycling ratio. Constraint (22) shows that, for each product, the flow entering each collection location from all customers is equal to the sum of the flow going to each repair centre for repair, to each plant for remanufacturing, to each supplier for recycling, and to each disposal location for disposal. Constraint (23) ensures that, for each product, the flow entering each repair centre from all collection centres is equal to the flow going from this facility to distribution centres.

Capacity constraints:

$$\sum_i \mathrm{QSP}_{vip}\le \mathrm{CS}_{vp},\quad \forall v,p \tag{24}$$

$$\sum_j\sum_p m_p\mathrm{QPD}_{ijp}\le \sum_h \mathrm{CX}_i^h X_i^h,\quad \forall i \tag{25}$$

$$\sum_c\sum_p m_p\mathrm{QDC}_{jcp}\le \sum_h \mathrm{CY}_j^h Y_j^h,\quad \forall j \tag{26}$$

$$\begin{aligned}&\sum_l\sum_p m_p\mathrm{QCR}_{klp}+\sum_i\sum_p m_p\mathrm{QCP}_{kip}+\sum_v\sum_p m_p\mathrm{QCS}_{kvp}\\&+\sum_d\sum_p m_p\mathrm{QCD}_{kdp}\le \sum_h \mathrm{CZ}_k^h Z_k^h,\quad \forall k\end{aligned}\tag{27}$$

$$\sum_j \sum_p m_p \mathrm{QRD}_{ljp} \le \sum_h \mathrm{CW}_l^h W_l^h, \quad \forall l \tag{28}$$

$$\sum_k \sum_p m_p \mathrm{QCP}_{kip} \le \sum_h \mathrm{px}_i^h CX_i^h X_i^h, \quad \forall i \tag{29}$$

$$\sum_k \mathrm{QCS}_{kvp} \le \mathrm{ps}_{vp} \mathrm{CS}_{vp}, \quad \forall v, p \tag{30}$$

Constraint (24) ensures that, for each product, the sum of the flow exiting each supplier to all plants does not exceed the supplier's capacity. Constraint (25) states that the sum of the flow exiting each plant to all distribution centres does not exceed the plant's capacity. Constraint (26) shows that the sum of the flow exiting from each distribution centre to all customers does not exceed the distributor's capacity. Constraint (27) ensures that the sum of the flow exiting each collection centre to all suppliers, plants, repair centres, and disposal locations does not exceed the collector's capacity. Constraint (28) states that the sum of the flow exiting each repair centre to all redistributors does not exceed the repair centre's capacity. Constraint (29) ensures that the sum of the flow entering each plant from all collection centres does not exceed the remanufacturing capacity of the plant. Constraint (30) states that, for each product, the sum of the flow entering each supplier from all collection centres does not exceed the recycling capacity of the supplier.

Maximum number of the activated locations constraints:

$$\sum_i \sum_h X_i^h \le NX \tag{31}$$

$$\sum_j \sum_h Y_j^h \le NY \tag{32}$$

$$\sum_k \sum_h Z_k^h \le NZ \tag{33}$$

$$\sum_l \sum_h W_l^h \le NW \tag{34}$$

Constraints (31) to (34) restrict the facility number of plants, distributors, collection centres, and repair centres that can be opened, respectively.

Capacity level constraints and others:

$$\sum_h X_i^h \le 1, \quad \forall i \tag{35}$$

$$\sum_h Y_j^h \le 1, \quad \forall j \tag{36}$$

$$\sum_h Z_k^h \le 1, \quad \forall k \tag{37}$$

$$\sum_h W_l^h \le 1, \quad \forall l \tag{38}$$

$$X_i^h, Y_j^h, Z_k^h, W_l^h \in \{0,1\} \quad \forall i, j, k, l, h \tag{39}$$

$$\begin{aligned} &\mathrm{QSP}_{vip}, \mathrm{QPD}_{ijp}, \mathrm{QDC}_{jcp}, \mathrm{QCC}_{ckp}, \mathrm{QCR}_{klp}, \mathrm{QRD}_{ljp}, \mathrm{QCP}_{kip}, \\ &\quad \mathrm{QCS}_{kvp}, \mathrm{QCD}_{hdp} \ge 0 \quad \forall v, i, j, c, k, l, p \end{aligned} \tag{40}$$

Constraints (35) to (38) insure that each plant, distribution centre, collection centre, and repair centre can be assigned at most one capacity level, respectively. Constraints (39) and (40) impose binary and non-negativity restrictions on the corresponding decision variables, respectively.

4.2 Robust formulation

To extend the aforementioned deterministic model to an uncertain environment, we assume that the demand for products and the return rate in customer zones are uncertain and can assume values from a finite set of possible scenarios with an unknown joint probability distribution. The objective of the attempted closed-loop logistic network under uncertainty is to find a robust solution that minimises the maximum regret between the optimal objective function value and the resulting objective function for all possible scenarios. To obtain the regret of each scenario, the optimal objective functions are determined by solving the following model.

$$u_s = \begin{cases} \max f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right) \\ \text{s.t.} \\ \qquad \text{Eqs.}(15) - (40) \\ \qquad Q_s \ge 0 \quad \text{and} \quad X_i^h, Y_j^h, Z_k^h, W_l^h \in \{0,1\} \end{cases} \tag{41}$$

where f_s and Q_s are the resulting objective function and flow quantity between the nodes of the logistic network under scenario s ∈ S. When the scenario set S is a finite set, the min–max regret approach attempts to find binary choice decisions such that the maximum regret from the optimal objective function values is minimised. Model (41) consists of two types of decision variables. Decision variables specifying the network configuration, namely those binary variables that represent the location of plants, distribution centres, collection centres, and repair centres, are considered to be first-stage variables, which must be taken into account before the realisation of the uncertainty. The second-stage variables are continuous variables that are related to the amount of products to be shipped among the entities of network, which can be made after the uncertain parameters have been determined. The robust solution of the decision variables can be obtained by solving the following model.

$$\begin{aligned}&\min \delta\\&\text{s.t.}\\&\left.\begin{array}{r}\delta \geq u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)\\ \text{Equations}(15)-(40)\\ Q_s \geq 0 \quad \text{and} \quad X_i^h, Y_j^h, Z_k^h, W_l^h \in \{0,\ 1\}\end{array}\right\} \quad \forall s \in S\end{aligned} \tag{42}$$

This model is referred to as the extensive form model of the problem under deviation robust definition. Unfortunately, the size of the extensive form model presented in (42) can become unmanageably large when a large number of scenarios are considered. The implementation of this model requires a long computation time to obtain a robust solution with a large number of scenarios. For this reason, we use the scenario relaxation algorithm [35] to obtain a robust network configuration. The following is a description of this algorithm.

5 Scenario relaxation algorithm

The main idea of the scenario relaxation algorithm is that, in a problem with a large number of possible scenarios, only a small subset of scenarios is actually employed to search for an optimal robust solution. The flow diagram of the scenario relaxation algorithm is shown in Fig. 2. The algorithm classifies the scenario set into two types of scenarios. The first type is comprised of scenarios that are required to insure the feasibility of a robust solution for all possible scenarios. The second type includes scenarios that are required to satisfy the optimality of the robust solution. The scenario relaxation algorithm first considers the subset of all scenarios $\overline{S} \subset S$ and then solves the relaxed model (42) by only considering scenario set $\overline{S}$ instead of S. To incorporate the second subset, an alternative robust solution is applied to compute the regrets from best objective for all unconsidered scenarios. If optimal value of relaxed model is greater or equal to all regrets, the current solution is robust and the algorithm ends. Otherwise, a subset of unconsidered scenarios with regret value greater than optimal value of relaxed model will be added to $\overline{S}$ in next iteration. The algorithm guarantees termination at an optimal robust solution, if one exists.

The overall procedure of the scenario relaxation algorithm, proposed by Assavapokee et al. [35], can be summarised as follows.

Step 0: Select a subset $\overline{S} \subset S$, set LB=0 and UB=∞, determine a predetermined small nonnegative value ε , and then go to Step 1.

Step 1 Solve model (41) to obtain the optimal objective function $u_s \forall s \in S$. If this model is infeasible for any scenario, the algorithm is terminated (i.e. the problem is ill posed). Otherwise, proceed to step 2.

Step 2 Solve the relaxation model (42) by considering only the scenario set $\overline{S}$ instead of S. If the relaxed model (42) is infeasible, the algorithm end (i.e. no robust solution exists). Otherwise, set LB=δ^* (the optimal value from the relaxed model) and fix the binary variables in current optimal solution form the relaxed model, $X_i^h, Y_j^h, Z_k^h, W_l^h$.. If $\text{UB} - \text{LB} \leq \varepsilon$, the resulted robust solution is globally ε-optimal robust solution and algorithm is ended. Otherwise, go to Step 3.

Step 3 Solve the model (41) for all scenarios $S \backslash \overline{S}$.. Let $S_1 \subset S \backslash \overline{S}$ such that the model (41) is infeasible and let $S_2 \subset S \backslash \left(\overline{S}\right) \cup S_1$ such that $u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right) \geq \delta^*$.

Step 4 If $S_1 \neq \emptyset$ go to step 5. Otherwise update the upper bound as follows.

$$\text{UB} \leftarrow \min\left(\text{UB},\ \ \max_{s \in S}\left(u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)\right)\right)$$

If UB–LB≤ε, the resulted robust solution is globally ε-optimal robust solution and algorithm is ended. Otherwise go to step 6.

Step 5 Choose a subset $S_1{}' \subset S_1$, add to scenario set $\overline{S}$, and then proceed step 2. Here, we select set of scenarios with the highest objective function values from the model (42).

Step 6 Choose a subset $S_1{}' \subset S_1$, add to scenario set $\overline{S}$, and then proceed step 2. Here, we select set of scenarios with the highest value of $u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)$.

The algorithm involves two formulations (41) and (42). The formulation (41) is a model to obtain optimal solution under each scenario, which the resulted objective is used in formulation (42), and the relaxed model (42) is a status of the extensive form model. Notice that the feasible region of

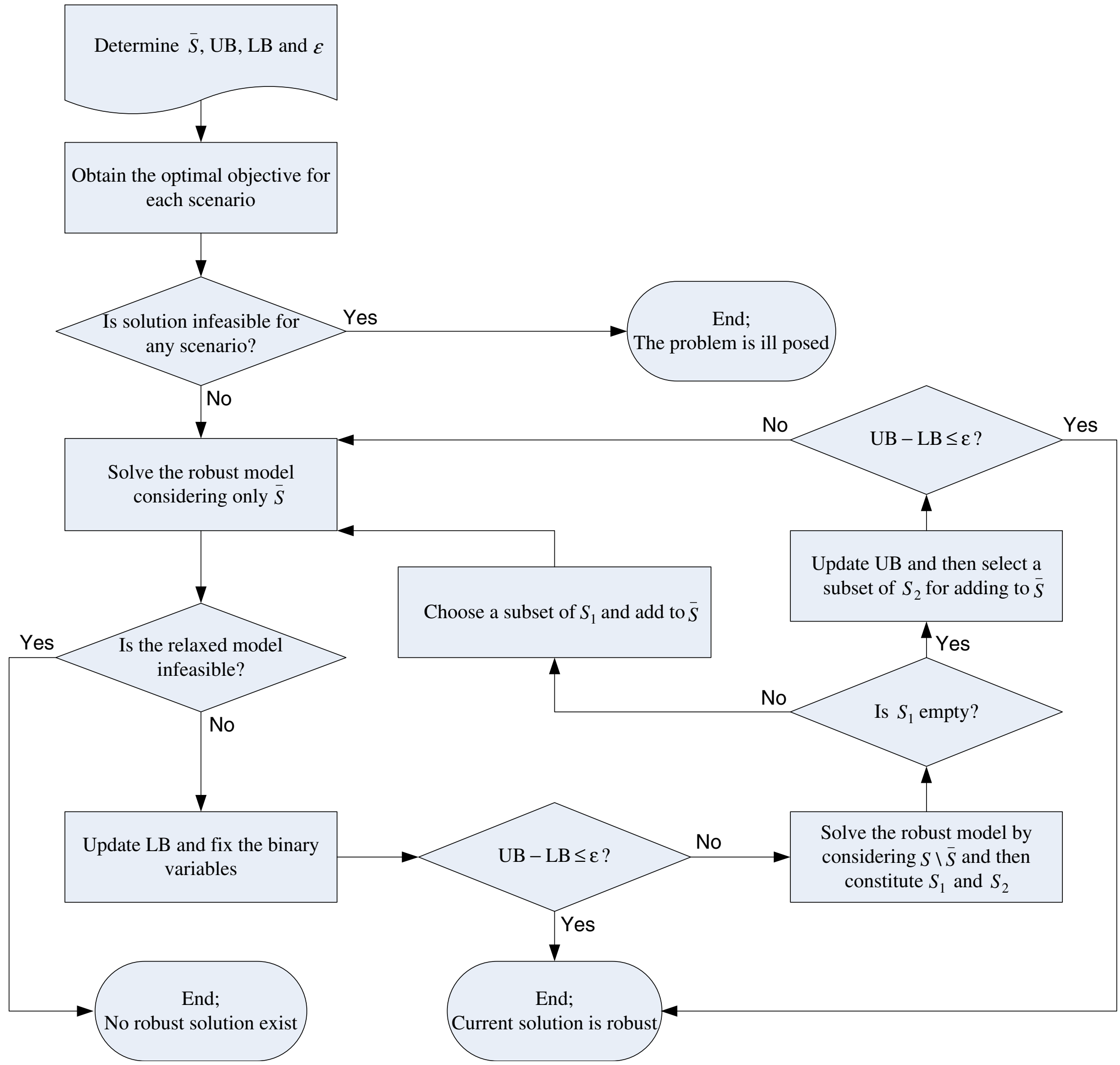

Fig. 2 Procedure of the scenario relaxation algorithm

formulation (41) includes the feasible region of the extensive form model. Hence, four situations to be implemented:

1. If model (41) is infeasible even under one of scenarios, the extensive form model (original problem) is also infeasible;
2. If the relaxed model (42) is infeasible, the extensive form model is also infeasible;
3. $\mathrm{LB} \leq \min\left(\max_{s\in S}\left(u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)\right)\right)$ under robust definition for all iterations; and
4. The optimal solution to the extensive form model is a feasible solution to the relaxed model (42).

Based on the aforementioned points, it is obvious that if either model (41) or (42) is infeasible, the algorithm terminates and thus the extensive form model is infeasible. Now it is assumed that the algorithm terminates in steps 2 or 4 with UB=LB. Notice that the algorithm can terminate in these steps only if the current robust solution is feasible to the extensive form model, or $S_1=\emptyset$, and $\mathrm{UB} = \max_{s\in S}\left(u_s - f_s\left(Q_s, X_i^{h*}, Y_j^{h*}, Z_k^{h*}, W_l^{h*}\right)\right) \geq \min \left(\max_{s\in S}\left(u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)\right)\right)$ Therefore if UB = LB, then

$$\max_{s\in S}\left(u_s - f_s\left(Q_s, X_i^{h*}, Y_j^{h*}, Z_k^{h*}, W_l^{h*}\right)\right)$$
$$= \min\left(\max_{s\in S}\left(u_s - f_s\left(Q_s, X_i^h, Y_j^h, Z_k^h, W_l^h\right)\right)\right)$$

As can be resulted, the current robust solution is an optimal solution to the extensive form model and since a finite number of possible scenarios exist, the algorithm terminates in finite number of iterations. Therefore, the algorithm is terminated with an optimal robust solution, if one exists, or detects infeasibility to problem.

6 Computational results

To illustrate the suitability and consistency of the proposed approach, a numerical example is first presented and analysed for the robust closed-loop logistic network design under an uncertain environment. The scale of the numerical experiment is as follows: the number of suppliers is three; the number of potential locations for establishing the plants is five, and the maximum number of plants that can be opened is two; the number of potential locations for establishing the distribution centres is six, and the maximum number of distributors that can be opened is four; the number of customer zones is 10; the number of potential locations for establishing collection and repair centres is four, and the maximum number that can be opened is two; and there is one disposal centre. The parameters of the given problem are given in Table 2. The transportation costs are defined as corresponding to the distance between nodes on each layer of the supply chain network. Moreover, the fixed cost and capacity of the same facilities are discriminated according to the relevant levels. The behaviour of the proposed model has been studied in the case when the demand of customers and the return rate of products from customers are uncertain. It is assumed that each uncertain parameter can have a value from 80 to 110 % of the values given in Table 2. Each uncertain parameter is defined by a finite number of possible scenarios, and the other parameters have constant values shown in Table 2.

Table 2 Values of parameters used in the numerical experiments

Parameter	Value
D_{cp}	Uniform(150, 280)
RT	0.5
RR	0.45
RM	0.25
RS	0.15
RD	0.15
$\mathrm{ps}_{\mathrm{vp}}$	0.25
px_i^h	0.25
m_p	Uniform (2, 4)
$\mathrm{PR}_{\mathrm{ip}}$	Uniform (120, 140)
$\mathrm{PC}_{\mathrm{ip}}$	Uniform (30, 35)
OC_{j}	Uniform (9, 12)
$\mathrm{RMC}_{\mathrm{ip}}$	Uniform (12, 14)
IC_{k}	Uniform (2, 4)
RC_{ι}	Uniform (4, 6)
$\mathrm{RSC}_{\mathrm{p}}$	Uniform (5, 7)
DC_{p}	Uniform (1, 3)
FX_i^h	Uniform (50,000, 80,000)
FY_j^h	Uniform (10,000, 15,000)
FZ_k^h	Uniform (4,000, 7,000)
FW_l^h	Uniform (8,000, 12,000)
$\mathrm{CS}_{\mathrm{vp}}$	Uniform (750, 1,000)
CX_i^h	Uniform (6,500, 7,500)
CY_j^h	Uniform (4,300, 6,000)
CZ_k^h	Uniform (4,700, 5,600)
CW_l^h.	Uniform (1,900, 2,600)

In our problem assumptions, all uncertain parameters are described as a finite set of possible scenarios. Because there is a lack of complete knowledge about the probability distribution of uncertain parameters in this problem, decision makers are not able to search for the first-stage decision values (long-term decisions) that produce the best long-run average performance. In this situation, criteria for the first-stage decisions can be to minimise the maximum regret between the optimal objective function value under perfect information and the resulting objective function value under the robust decisions over all possible realisations of the uncertain parameters (scenarios) in the model (provides a reasonable objective function value). The resulting first-stage decision setting is referred to as the robust configuration.

Traditionally, min–max regret robust solution can be obtained by solving a scenario-based extensive form model of the problem. The size of this extensive form model grows substantially with the number of scenarios used to represent uncertainty. Hence, the scenario relaxation algorithm is efficiently used to determine the robust first-stage decision setting when the only information available to decision makers at the time of making the first-stage decisions is a finite set of possible scenarios with unknown probability distribution. The key insight of the scenario relaxation algorithm is that in a problem with a large number of possible scenarios only a small subset of scenarios actually are employed to obtain a robust solution, that it lead to a less computation time compared with the extensive form model.

First, we solve the deterministic model of the first experiment. The closed-loop logistic network that is relevant to this deterministic model is shown in Fig. 3. To better describe the uncertainty and obtain a confident network configuration, the number of scenarios is increased. Because the size of the problem becomes unmanageably large when increasing the number of scenarios, the scenario relaxation algorithm is applied to obtain a robust configuration of the network. The number of possible scenarios varies from 10 to 3,000 scenarios. The problem with different scenarios is solved by both the extensive form model and the scenario relaxation algorithm with $\varepsilon^{\circ}=0$ on a computer with an Intel core2 Duo 2.00 GHz processor and 2.00 GB of RAM using the GAMS and CPLEX solver. Table 3 indicates the performance of the extensive form model and the scenario relaxation algorithm for the various numbers of scenarios. If the algorithm cannot obtain a solution within 10 h, a computation time of "–" is shown. The scenario relaxation algorithm

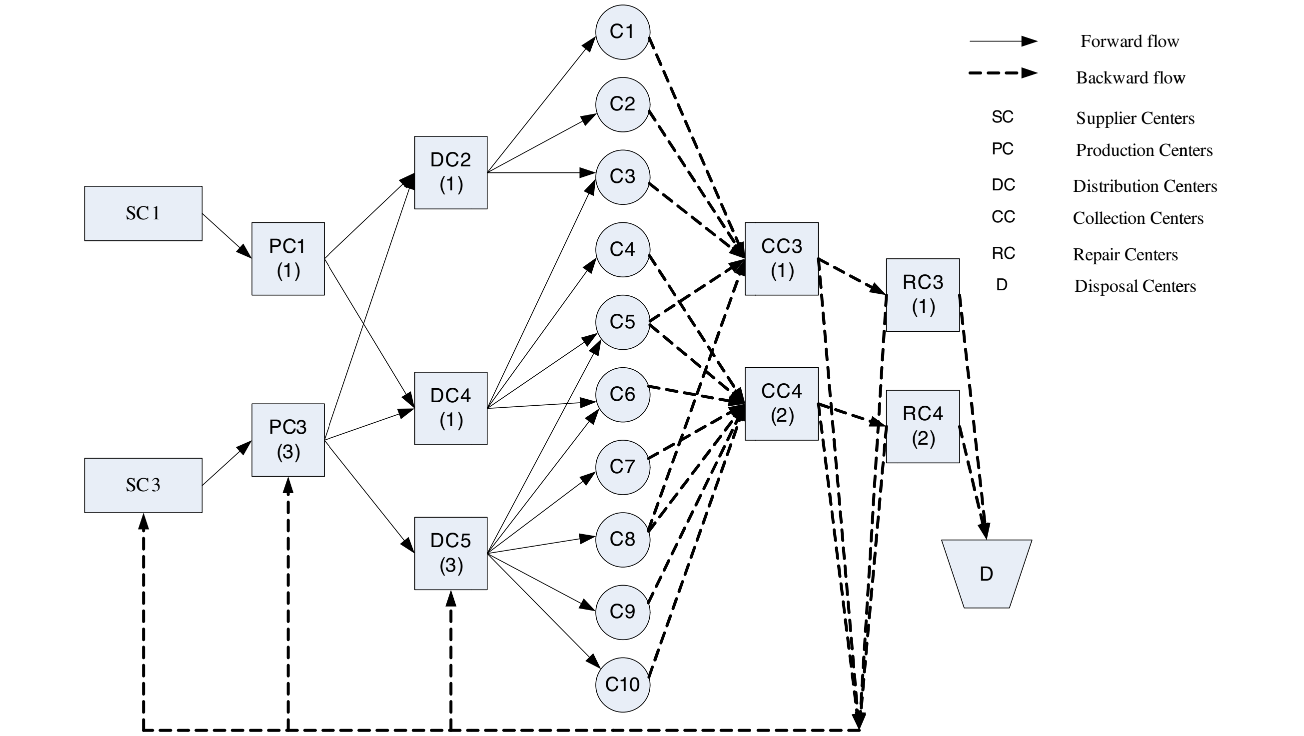

Fig. 3 Logistic network of the deterministic model for the numerical example

decreases the computation time compared to the extensive form model, especially when the number of scenarios is increased.

In addition, Table 3 shows the network configuration when the number of scenarios is increased. The highlighted cells indicate the capacity levels of the facilities

Table 3 Results of the numerical example

No.	Number of scenarios	Total scenarios generated by the algorithm	Plant locations				Distribution locations						Collection centers				Repair locations				Time of the scenario relaxation algorithm (s)	Time of the extensive form model (s)
			a	b	c	d	a	b	c	d	e	f	a	b	c	d	a	b	c	d		
1	10	3	1		3		2			1	3			1	3				1	3	39	35
2	20	3	1		3		1			2	3		1		2				1	3	17	52
3	30	3	1		3		2			1	3				3	1			1	3	75	223
4	40	3	1		3		1			2	3			1	3				1	3	27	351
5	50	4	1		3		2			1	3				2	2			1	3	36	248
6	100	6	1		3		3			1	3			1	3				1	3	84	1392
7	200	10	1		3		3			1	3		1		2				2	2	356	> 2 hour
8	300	6	1		3			1		2	3				1	3			1	3	166	> 2 hour
9	400	6	1		3			1		2	3				3	1			1	3	259	> 2 hour
10	500	8	1		3			1		2	3				3	1			1	3	404	> 2 hour
11	600	8	1		3			1		2	3				3	1			2	2	576	> 2 hour
12	700	9	1		3			1		2	3			1	3				1	3	717	> 2 hour
13	800	9	1		3		3			1	3				3	1			1	3	778	> 2 hour
14	900	10	1		3			1		2	3				3	1			2	2	991	> 2 hour
15	1000	11	1		3			1		2	3				3	1			1	3	1183	> 2 hour
16	1200	13	1		3			1		2	3			1	3				1	3	1708	> 4 hour
17	1400	13	1		3			1		2	3			1	3				1	3	2291	> 4 hour
18	1600	15	1		3			1		2	3			1	3				1	3	2852	> 4 hour
19	1800	16	1		3			1		2	3				3	1			1	3	3679	-
20	2000	17	1		3			1		2	3				3	1			1	3	4607	-
21	2500	21	1		3			1		2	3			1	3				1	3	9361	-
22	3000	30	1		3			1		2	3			1	3				1	3	17541	-

a–f signify the potential location of facilities

1, 2, and 3 illustrate the capacity levels for the potential locations

The highlighted cells show that the capacity level is not changed for the related facility location

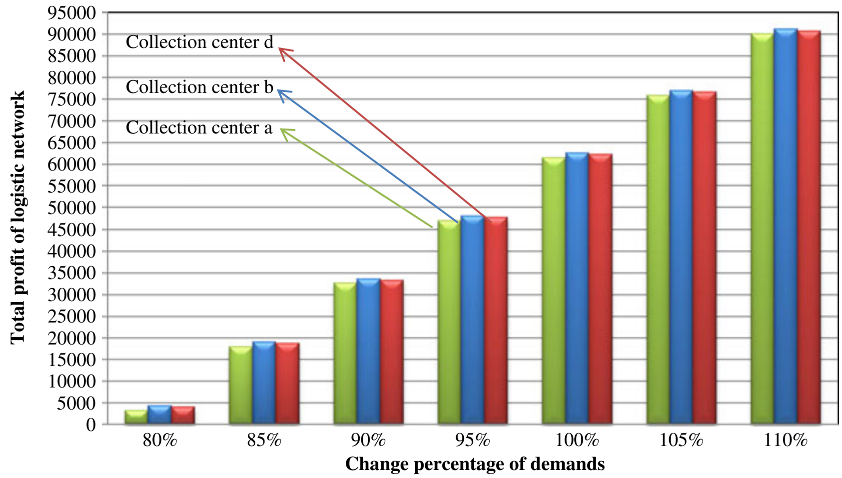

Fig. 4 Total profit of the network in various collectors

that do not change by increasing the number of scenarios. As shown in Table 3, the capacity level and location of the second collection centre is unknown (the location of Collection Centre *c* with a capacity level of 3 does not change, but the other collection locations, i.e. Collection Centres a, b and d, do change). Therefore, to select one of these potential locations, we fix the highlighted configuration in Table 3 along only one out of Collection Centres a, b, and d with a capacity level of 1 to solve the given problem. As observed in Fig. 4, the objective function of the problem including Collection Centre b is better than other problems under different demands. Thus, the highlighted settings in Table 3 and Collection Centre 2 with a capacity level of 1 are considered as the most robust configuration of the closed-loop logistic network; this result differs from the configuration of the deterministic model shown in Fig. 3.

To illustrate the efficiency of the resulting robust configuration, a sensitivity analysis of uncertain parameters is conducted. Table 4 shows the resulting profit values of the robust and deterministic configuration with respect to changes in the demand and the return rates. The deterministic configuration is shown in Fig. 3 and the robust configuration is illustrated in Table 3. The profit values of the robust configuration are less than the deterministic configuration, whereas the robust configuration is more reliable than the deterministic one because the deterministic configuration is infeasible under some demand and return rates (i.e. in the worst cases). It mean that when the demands and return rates are increased the capacity of the deterministic configuration cannot be responsible for the increased demands and return rates, and thus the model is infeasible in the relevant conditions. In contrary, the robust configuration is feasible and responsible for all conditions. Hence, these results justify the robust optimization approach in this problem and show it as a reliable tool for making strategic and sensitive decisions, where satisfying demands of customers and responsibility is very important for such problem in the current competitive environment. The resulting robust configuration is obtained using the max–min criterion, aiming at constructing a solution minimising the maximum deviation between the value of the solution and the optimal value of corresponding scenario over all possible scenarios. As is also pointed out by Aissi et al. [25], the study of this criterion is motivated by application where anticipation of the worst case is crucial, e.g. the sensor placement problem, SCND problem, and so on. Hence, the

Table 4 Sensitivity analysis of demand and return ratio on the deterministic and robust configuration

Return ratio	Change percentage of demands							
	80 %		90 %		100 %		110 %	
	DC	RC	DC	RC	DC	RC	DC	RC
0.4	14,328.11	4,762.195	43,835.71	34,157.01	72,626.06	62,864.74	92,080.6	91,483.38
0.45	14,176.83	4,531.783	43,666.59	33,924.51	72,567.61	62,712.38	91,935.37	91,315.34
0.5	14,025.56	4,301.371	43,487.03	33,664.07	72,379.14	62,514.78	Infeasible	91,140
0.55	13,874.28	4,070.96	43,232.35	33,403.62	Infeasible	62,246.75	Infeasible	90,928.5

DC deterministic configuration, *RC* robust configuration

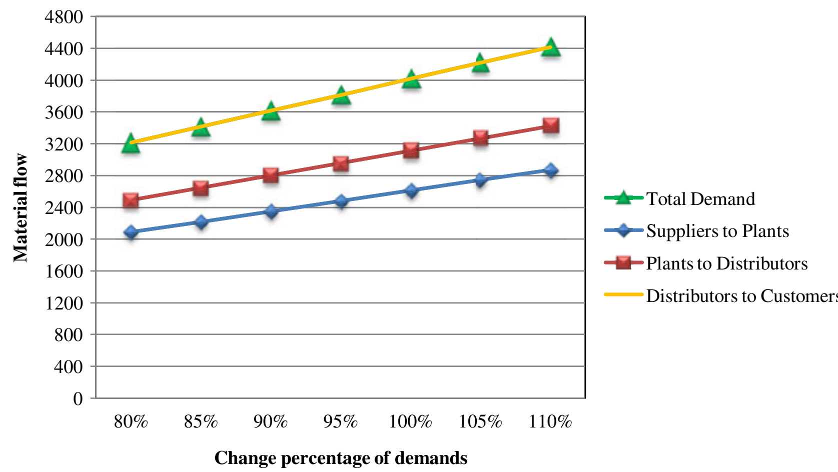

Fig. 5 Material flow between forward facilities in various demands

standard approaches will fail to protect against such high-impact events [25]. Therefore, these discussion and result illustrates the importance of such approach where demand and/or other parameters can easily change in today's market. Moreover, Table 4 shows consistent results for changes of the demand and the return rate in both the robust and deterministic configuration. When the demand increases or decreases, the profit increases or decreases in the same way. Conversely, if the return rate increases, the profit decreases as expected. Another conclusion reached from Table 4 is that the parameter of the return rate is less sensitive than the parameter of the demand in determining the profit of the logistic network.

Figures 5 and 6 indicate the consistency of outputs among the forward and backward locations of the resulting robust configuration under different demands, respectively. It is evident from Fig. 5 that the demands of all customers are satisfied (two lines representing the total demand and the output from distributors to customers are coincident with each other). The flow shipped from plants to distributors is less than total demand, from Fig. 5, that the difference of these flows is justified by the flow shipped from repair centres to redistributors, from Fig. 6. In a similar way, the difference of output from plants to distributors and output from suppliers to plants, from Fig. 5, is justified by output from collectors to plants, from Fig. 6. Moreover, Fig. 6 shows that the flow shipped from customers to collection centres is equal to the sum of the flow shipped from collectors to repair centres, plants, suppliers, and disposal centres. It is also evident from Fig. 6 that flow balance exists in repair centres (the two lines representing the input to repair centres from collection centres and output from repair centres to redistribution centres are coincide with each other). In addition, Fig. 6 shows that the flow shipped from collection centres to supplier and disposal centres are identical because of the same assumed return ratio to two destinations (0.15 of the return quantity). The results in Figs. 5 and 6 verify the solutions of the proposed model.

To show the optimality of the scenario relaxation algorithm, the values of objective function of the algorithm are computed. Figure 7 shows the objective function value of both extensive form model and the scenario relaxation algorithm. The solutions of the extensive form model are optimal because these solutions are obtained by CPLEX

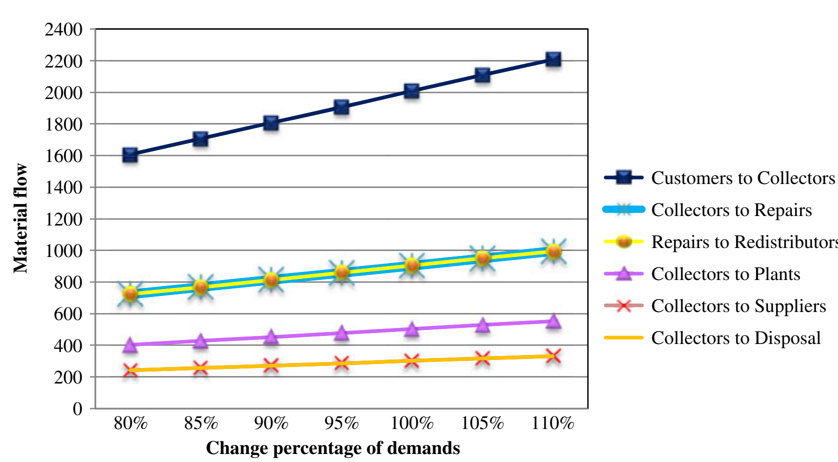

Fig. 6 Material flow between backward facilities versus various demands

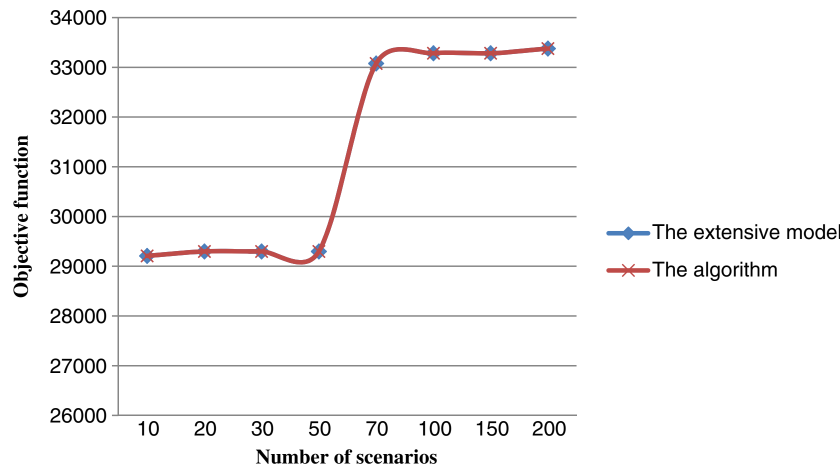

Fig. 7 Objective of the extensive model and the algorithm versus number of scenarios

solver. As it can be seen from Fig. 7, the two lines representing objective function of the extensive form model and the scenario relaxation algorithm are coincident with each other. This issue confirms that the solutions of the scenario relaxation algorithm also are optimal.

Furthermore, to show the validity of the proposed model and the applicability of the scenario relaxation algorithm, four test problems are presented and their results are reported. Both the extensive form model and the scenario relaxation algorithm are tested on four test problems of different sizes (see Table 5) and parameters, in such a way reported in the recent literature (e.g. [4, 37]). The number of possible scenarios for the test problems varies from 50 to 150. Euclidean distances are used to define the transportation costs between nodes on each layer of network. The demand for each type of product is based on $U[150, 280]$. The price for each type of product is generated by $U[100, 140]$. The unit cost of production, operating, inspection, repair, remanufacturing, recycling, and disposal are generated corresponding to 25, 8, 2, 4, 10, 5, and 1 % of price for each product. Moreover, after calculating the total capacity of plants as $1.5\sum_c \sum_p m_p D_{cp}$, the capacity of plants are defined randomly by sharing the total capacity into plants. In a similar way, the capacity of suppliers, distribution centres, collection centres and repair centres are defined. Other parameters are generated by uniform distribution with corresponding ranges of Table 2. Table 6 show the corresponding results of the scenario relaxation algorithm compared with the extensive form model in test problems 1–4 with various numbers of scenarios. If the extensive from model does not reach a solution within 2 h while the algorithm achieves a solution, the computation time of "–" is reported for the extensive from model in this table because the computing time superiority of the scenario relaxation algorithm is obvious in this case.

Table 6 shows that the scenario relaxation algorithm decreases the computational time compared to the extensive form model. The number of scenarios that are actually employed by the scenario relaxation algorithm to find an optimal robust solution is also shown in Table 6. Increases and decreases in a number of these scenarios proportionally affect the computation time of the algorithm. On the other hand, the random choice of subsets $\overline{S}$,, $Snewapos;_1$, and $Snewapos;_2$ in the algorithm can have an effect on the computational time of the algorithm. In other words, the total number of the generated scenarios, the number of scenarios employed by the algorithm, and the random choice of subsets all affect the computational time of the scenario relaxation algorithm. The associated results illustrate the computing time superiority of the scenario relaxation algorithm compared to the extensive form model, especially in the presence of a large number of scenarios.

Table 5 Size of test problems

Test problem	Suppliers	Plants	Distribution centres	Collection centres	Repair centres	Disposal centres	Product types	Capacity level	Customers
1	3	5	5	5	5	2	2	3	20
2	4	6	6	6	6	2	2	3	30
3	5	7	7	7	7	3	2	3	40
4	6	8	8	8	8	3	2	3	50

Table 6 The results of test problems with different scenarios

Test problem	Category	Total number of scenarios	Number of scenarios employed by the algorithm	Time of the algorithm (s)	Time of the extensive form model (s)
1	1	50	4	182	5,271
	2	100	5	149	3,396
	3	150	5	182	6,650
2	1	50	4	1,491	–
	2	100	5	1,757	–
	3	150	6	2,009	–
3	1	50	6	1,844	–
	2	100	8	4,358	–
	3	150	9	3,577	–
4	1	50	3	732	–
	2	100	3	793	–
	3	150	4	1,186	–

Consequently, the results shown in the tables and figures confirm that the model gives consistent results. Therefore, it can be considered as a reliable tool for making strategic and sensitive decisions. These results also demonstrate the applicability and effectiveness of the scenario relaxation algorithm for solving max–min robust optimisation SCND problems.

7 Conclusion

We proposed a new approach for a supply chain network design in which the demand and the return rate were uncertain. The proposed model was successful in designing a closed-loop, logistic network while including four layers (i.e. suppliers, plants, distributors and customers) in the forward direction and four layers (i.e. customers, collection centres, repair centres and disposal centres) in the backward direction. The paper used the robust optimisation approach with the min–max regret criterion motivated by high-impact problems such as SCND one in which the situation of worst case is critical. To find a robust solution with a faster time, the scenario relaxation algorithm was employed. The results showed that the robust configuration is more reliable than the deterministic one because the deterministic configuration is infeasible under some demand and return rates (i.e. in the worst cases) while the robust configuration is feasible and responsible for all conditions, although the profit values of the robust configuration are less than the deterministic configuration. Moreover, the results demonstrated the computing time superiority of the scenario relaxation algorithm compared to the extensive form model, especially in the presence of a large number of scenarios. These results verified the applicability and effectiveness of the scenario relaxation algorithm in the proposed model.

For future research in this area, we suggest the development of some heuristic and meta-heuristic methods for large-sized problems as well as exact decomposition techniques. Furthermore, the model can be improved with several extensions, such as addressing risk, budget constraints and financial considerations or adding some characteristics to make it closer to a realistic scenario.

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